ESSAYS ON DIGITAL PLATFORMS: NETWORK EFFECTS, MARKET STRUCTURE, AND PLATFORM STRATEGY

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Declaration

This thesis is the result of the author's original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.

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Abstract

This thesis investigates the economic functioning of digital platform markets, highlighting how their unique characteristics challenge industrial organisation (IO) theory applied to traditional markets. Using a theoretical IO framework, it develops models to analyse platform strategies, competition, and user surplus, providing insights relevant for both economic research and policy design.

Chapter 1: The Economics of Digital Markets The first chapter aims to develop a unified theoretical framework that brings together diverse strands of research on two-sided markets and advertising. By doing so, it organises the existing literature in a systematic and rigorous way, providing a clearer understanding of the distinctive features of digital markets, such as multi-sided interactions, network effects, and monetisation strategies, and their implications for economic policy. The key contribution of this chapter lies in integrating fragmented theoretical approaches into a unified and flexible framework that can be applied across different types of digital platforms and market structures, enabling analysis of competitive interactions, user switching behaviour, and mergers. Thus, it provides a consistent analytical framework for comparing price-based and advertisement (ad)-supported platforms, analysing platform strategies, policy interventions, and welfare outcomes, and capturing emerging trends such as multi-homing, platform consolidation, and evolving monetisation models.

Chapter 2: Competition and Mergers among Digital Platforms Building on the framework developed by Tan and Zhou (2021) in *The Effects of Competition and Entry in Multi-sided Markets* published in the REStud paper, this chapter develops a theoretical model to examine how competition and market consolidation of digital platforms affect user surplus. The model accounts for both symmetric and asymmetric platforms and examines transitions from triopoly to duopoly and from duopoly to monopoly. Through analytical reasoning, illustrated with MATLAB-based numerical simulations, the chapter identifies conditions under which reducing the number of platforms from three to two can enhance user surplus while showing that consolida-

tion to a monopoly generally reduces user surplus due to higher prices. This chapter contributes by examining how network effects and platform asymmetry drive changes in user surplus and, in turn, shape our understanding of the welfare impact of platform mergers. It identifies the conditions under which platform mergers can improve or reduce welfare, offering guidance for regulators on assessing consolidation, and emphasising the importance of maintaining competitive market structures.

Chapter 3: Advertising Spending and User Welfare in Digital Platform Markets This chapter examines advertising spending on ad-based platforms and its impact on user surplus across different scenarios, including multi-homing, single-homing, duopoly, and post-merger monopoly settings. A model of an ad-based monopoly platform illustrates how advertising expenditure is determined, highlighting trade-offs between platform revenue and user surplus. The analysis shows that multi-homing does not always enhance welfare, particularly when platforms are highly substitutable, and ad exposure is high. In contrast to price-based markets, mergers between highly substitutable ad platforms can increase user surplus by reducing duplicate content and cumulative ad nuisance. These findings provide a structural understanding of digital advertising markets and offer detailed policy guidance on platform regulation, advertising standards, and merger assessment.

Overall, this thesis advances the understanding of digital platform markets by developing a unified theoretical framework applicable across different monetisation models, analysing the interplay between network effects, competition, and mergers, and clarifying the welfare implications of multi-homing and advertising. Its contributions inform both economic theory and regulatory policy, offering tools to assess platform strategies, market structure, and user welfare in evolving digital markets.

Acknowledgements

It does not matter where you come from. What truly matters is where you are determined to go.

I grew up in a small town, where I studied in a vernacular medium and spoke little English until my 20s. After six years in the finance industry, I took a leap of faith and pursued my master's degree in Germany in 2019. Two years later, I found myself moving to the UK to begin a PhD, something I never imagined possible when I first started.

Coming from a non-economics background, I initially struggled to grasp the core concepts. But my master's studies and PhD journey changed that. They gave me not only a solid foundation in economics but also a new way of thinking, one that moved from a profit-driven business outlook to a broader understanding of welfare and social impact.

During my PhD, I also had the opportunity to teach undergraduate students. Standing in front of a classroom and teaching in English, a language I once struggled with, was both humbling and empowering. It reminded me of how far I have come and filled me with gratitude for the opportunity to learn while helping others grow.

I owe deep thanks to everyone who has shaped my path. My parents for their unwavering belief in me. My siblings and their families, for their encouragement and patience. And especially my supervisor, Prof. Alex Dickson, who saw potential in my ideas, guided me with clarity and kindness, and helped me grow as a researcher and a person. His mentorship has been a constant source of inspiration.

For anyone striving toward a goal, the background does not define the future. What truly defines success is persistence, the courage to keep moving forward, and the belief in creating something meaningful. I would like to end by saying that *Aspiration is the First Step to Inspiration.*

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Thesis Introduction

In the field of industrial organisation, economists have developed strong theoretical models to understand how traditional markets function. These models have guided the creation of effective policies that promote competition, efficiency, and consumer welfare. However, digital markets created by digital platforms operate in ways that differ fundamentally from these traditional settings. Their unique characteristics, particularly network effects and multi-sided interactions, challenge the assumptions of traditional market theory and require new approaches to both analysis and regulation.

This thesis seeks to advance the understanding of digital platform markets by developing theoretical and simulation-based frameworks that explain how these markets function, compete, and affect user surplus. It explores how pricing structures, mergers, and advertising strategies determine outcomes for users and platforms. Across three interconnected chapters, the thesis develops models that bring coherence to a fragmented body of research and provide insights relevant to both academia and policy.

The first chapter lays the conceptual foundation by examining the economic characteristics that distinguish digital markets from traditional ones. It highlights the defining features of digital platforms and the coexistence of price-based and non-price-based models. While existing studies analyse these features in isolation, this chapter develops a *unified theoretical framework* that integrates price- and non-price-based market structures within a single analytical model. The framework captures interactions among multiple platforms, strategic pricing decisions, user behaviour, and the implications of competition and mergers. This unified model extends the traditional theory of multi-sided platforms by incorporating both direct and indirect network effects and allowing for hybrid monetisation strategies, such as paid and ad-supported systems. By doing so, it provides a comprehensive tool for analysing how digital platforms behave under varying market conditions, ranging from monopoly to highly competitive settings.

Building on the theoretical model developed in Chapter 1, the second chapter examines competition and mergers in price-based digital markets. These markets consist of platforms that connect different user groups, such as buyers and sellers or drivers and riders, while creating value through network effects. Mergers between digital platforms differ from those in traditional markets. While consolidation may strengthen network effects and enhance platform utility, it can also increase the price for users and reduce their surplus. The overall effect depends on the relative strength of the network effects versus the degree of market concentration. This chapter extends the model developed by Tan and Zhou (2021) to address these complexities. I investigated both symmetric and asymmetric platform structures and analysed how user surplus evolves as markets consolidate, from triopoly to duopoly, and from duopoly to monopoly. Through simulation-based analysis, I identified the conditions under which mergers enhance or reduce user surplus, considering factors such as stronger network effects, platform asymmetry, and market coverage. The results offer guidance for policymakers and regulators in assessing competition and mergers in digital markets, where traditional antitrust principles may no longer apply directly.

The final chapter turns to ad-based digital platform markets, where platforms offer free services to users while generating revenue through targeted advertising. Platforms such as Google and Meta exemplify this model, where user engagement drives ad effectiveness, revenue generation, and, ultimately, market dominance. In this chapter, I examined three interrelated issues: the factors driving advertising spending on digital platforms; the conditions under which *multi-homing*, users engaging with multiple platforms, affects user welfare; and how mergers among ad-based platforms alter user welfare when multi-homing exists. A formal model of an ad-based monopoly platform is developed to explain rising ad spending and the role of network effects in reinforcing market power. The analysis then extends to a multi-homing setting, incorporating platform substitutability and advertising nuisance to determine when users benefit or suffer from engaging with multiple platforms. Finally, the model explores how mergers between ad-based platforms influence user welfare under varying degrees of substitutability and network strength. The results contribute to a more accurate understanding of how digital advertising markets evolve and how user welfare depends on the balance between free access, advertising intensity, and competition.

By integrating these strands, the thesis contributes to the broader understanding of digital market economics and provides a systematic approach to evaluating competition, regulation, and welfare in the platform economy.

Chapter 1

The Economics of Digital Markets

1.1 Introduction

The 21^{st} century has witnessed a profound transformation in the global economy, driven by the exponential rise of digital platforms. Digital businesses now permeate nearly every sector, offering services that are frequently free to users, while monetising through alternative means such as subscription fees or advertising. This digital revolution has created vast new economic opportunities, from personalised user experiences and rapid service delivery to global scalability even for the smallest enterprises (Akman, 2019). Unlike traditional firms rooted in physical supply chains, digital firms operate through large user networks and ad-supported interactions, reshaping the way economic value is created and exchanged.

In this context, understanding the economics of digital platform markets requires a structured analytical approach. This chapter develops a unified theoretical framework that synthesises the diverse literature on digital platform markets. By adjusting a small set of parameters, the framework captures both price-based and non-price-based platforms. Rather than proposing separate models, it provides a common structure in which assumptions about pricing, advertising, network effects, and homing behaviour map to well-established theoretical settings under different market structures. This approach clarifies how different strands of the literature relate to one another and allows results from disparate models to be analysed within a single coherent framework. The main contribution of this chapter is to organise the literature around this unified framework, highlighting the connections between prior theoretical approaches and providing a basis for systematic comparison of platform monetisation strategies.

To ground this discussion, it is useful to define digital markets more precisely. *Digital markets are those in which companies develop and apply new technologies to existing businesses or create brand new services using digital capabilities.*[1] In other words, digital markets are created by digital platforms that facilitate the exchange of goods and services between agents (OECD, 2012). This represents a significant departure from what was commonly experienced two decades ago, as many of today's services, such as social media platforms, mobile applications, and various forms of online shopping, are entirely novel innovations.

A defining characteristic of digital markets is their reliance on *multi-sided platforms*, which function as intermediaries that facilitate interactions between two or more distinct user groups. These platforms are typically digital applications or software ecosystems that simultaneously serve multiple groups, for example, consumers, sellers, advertisers, and content creators, and generate value by enabling interactions among them. What distinguishes these platforms is that the value for one group is significantly dependent on the participation of the same and the other group of agents, a phenomenon known as *network effects* (ACCC, 2019).

The term *multi-sided* reflects this core functionality: each *side* represents a distinct group of agents whose participation enhances the overall utility from the platform. For instance, on a social media platform, one side might consist of end-users who create and consume content, while another consists of advertisers seeking to reach those users. Similarly, in a ride-hailing application, drivers and passengers constitute two sides of the market; each derives value from the presence of the other. These interactions are orchestrated through a common interface, the *digital platform*. This structure is in sharp contrast to the *traditional one-sided business model*, where firms typically operate through linear supply chains, where producers purchase inputs, engage

[1] https://www.gov.uk/government/publications/competition-and-markets-authoritys-digital-markets-strategy/the-cmas-digital-markets-strategy

in production, and sell directly to end consumers. Traditional firms operate through linear value chains. For example, a clothing firm designs, manufactures, and sells apparel to consumers. The firm earns revenue directly from product sales, and transactions primarily occur between the firm and its customers. Although intermediaries such as retailers may be involved, network effects are largely absent. The firm's main decisions concern product quality, pricing, and attracting buyers through marketing and brand reputation.

Thus, the multi-sidedness of digital platforms represents a structural innovation in market design. It enables scalable interaction, strengthens network effects, and transforms users from passive consumers to active participants in value creation.

1.1.1 Features of Digital Markets

Having established a foundational understanding of digital markets, it is essential to explore the distinct features that differentiate them from traditional markets. Among others, the following key features collectively define the digital market landscape.

(i) Multi-Sidedness: Digital platforms typically serve multiple user groups simultaneously, such as consumers, service providers, and advertisers. This *multi-sided* nature allows platforms to create interdependent relationships between these groups, often offering free or subsidised access to one side in order to attract participation from another.

For example, *Meta* provides free access to users and content creators while generating revenue from advertisers through targeted advertising based on user data. In this way, the participation of one group increases the value of the platform for others, strengthening user engagement and revenue potential.

(ii) Network Effects: A key feature of digital platforms is the presence of *network effects*, meaning that the value of the platform grows as more users join. There are two main types: *direct* and *indirect* network effects.

Direct network effects occur when each additional user directly increases the value of the service for other users on the same side. For instance, consider a social media platform like Meta. The more people who join, the more engaging and useful the platform becomes for each user. People can share updates, interact, and stay connected with a wider circle, making participation more rewarding as the network grows. Even holding individual preferences fixed, the platform becomes more useful as more users are present.

Indirect network effects arise when the value enjoyed by users on one side of a platform depends on participation on the other side. Consider a video game console such as the PlayStation. A larger base of players increases the value of the platform for game developers by expanding the potential audience for games, while a wider variety of available games increases the value of the console for players. In this case, the value is created through cross-side interactions, with each group's utility depending on the activity of the other, rather than directly on its own size.

(iii) Near-Zero Marginal Costs: In digital markets, *near-zero marginal costs* refer to the fact that once a digital product or service has been created, producing and distributing additional copies costs almost nothing. Unlike traditional goods, where each extra unit requires materials, labour and transportation, digital goods can be replicated and delivered instantly at negligible cost. For example, consider software or online music streaming. Developing an application or recording an album involves significant upfront (fixed) costs, such as design, coding, or production, but once the product exists, serving one more user costs very little. Distributing another software download or streaming one more song does not require additional manufacturing or physical distribution (Van Gorp and Honnefelder, 2015).

This cost structure has major implications for digital markets. It allows firms to scale rapidly because adding new users does not substantially increase total costs. It also supports business models that leverage low marginal costs, such as subscription-based models that generate recurring revenue, or free-access models that prioritise user growth to monetise through advertising. However, it can also lead to market concentration, as firms that succeed early can dominate by spreading their high fixed costs over a large number of users, making it difficult for new entrants to compete on price.

(iv) Economies of Scale and Scope: In digital markets, firms often experience strong *economies of scale* because digital goods and services typically involve high fixed costs but extremely low marginal costs. For example, developing a software platform or an online service requires substantial upfront investment in design, programming, and infrastructure. However, once the product is created, the cost of serving additional users, such as distributing software updates or streaming content, is negligible. The cost structure discussed above allows digital platforms to scale rapidly and reach large user bases without a proportional increase in production costs. Compared to traditional markets, where expanding output usually requires additional materials, labour, and logistics, digital businesses can grow with minimal incremental expenses. This cost structure allows large platforms to spread fixed costs over a large user base, lowering average costs per user. Combined with the network effects that make the platform more valuable as more users join, these factors create strong barriers to entry for new competitors, which can lead to market concentration over time.

Digital markets also exhibit strong *economies of scope*, as firms can use shared infrastructure, data, and technology to offer multiple products or services efficiently. For instance, *Google* leverages its search algorithms, data analytics, and cloud infrastructure to provide search, advertising, maps, email, and video content within a single integrated ecosystem. This ability to reuse and combine resources across services strengthens competitive advantages and creates high entry barriers for new firms (Parker et al., 2020; Baye and Prince, 2020).

(v) Market Concentration and Lock-In Effects: In digital markets, strong *network effects* are a major source of market concentration. As more users join a platform, its value to each user increases, making the platform more attractive relative to smaller rivals. This creates a self-reinforcing process in which large platforms continue to grow while smaller competitors struggle to attract users, even if they offer comparable services. As a result, entry and expansion by rivals become difficult because users anticipate a lower value from participating in a smaller network.

Lock-in effects further reinforce this concentration by increasing the cost of switching away from incumbent platforms. These switching costs may arise from the loss of personal data, digital assets, or established social connections, as well as from the time and effort required to learn a new system. When alternative platforms also have fewer users or a more limited ecosystem, switching becomes even less attractive. Together, network effects that favour large platforms and lock-in effects that discourage switching can lead to *winner-takes-most*, or even *winner-takes-all*, market outcomes (Van Gorp and Honnefelder, 2015).

(vi) Single vs Multi-homing: Another important feature of digital markets is the distinction between *single-homing* and *multi-homing*. *Single-homing* can arise for various reasons, including user preferences, platform design, or the relative benefits of concentrating activity on a single service provider. For example, many social media users remain on a single platform like *Meta* because their network of friends and communities exists only there.

In contrast, *multi-homing* refers to users engaging on multiple platforms simultaneously. For example, a restaurant might list its services on both *Uber Eats* and *Deliveroo* to reach more customers. Multi-homing can mitigate concerns about market concentration by lowering switching costs and reducing users' dependence on any single platform. When participants are able to multi-home, competing platforms can coexist more easily, as users are less constrained to commit exclusively to the dominant platform.

The balance between single-homing and multi-homing therefore, plays a crucial role in shaping competitive dynamics in digital markets. Strong network effects combined with widespread single-homing tend to strengthen incumbents and raise entry barriers, while greater scope for multi-homing can sustain competition by limiting the market power of dominant platforms.

In summary, these features represent a fundamental shift in the way value is created, captured, and distributed in digital markets. Large platforms can act as gatekeepers, controlling access to key user bases and shaping market outcomes. This raises policy concerns, such as anti-competitive behaviours: for instance, *killer acquisitions*, where dominant platforms buy potential rivals to limit competition, or platform self-preferencing, where a platform prioritises its own products over those of competitors (Lear, 2019). Understanding these peculiarities is essential for developing models that accurately reflect the conduct of the platform and for designing effective regulatory responses.

1.1.2 Illustration of the Digital Ecosystem

Building on the distinctive features outlined above, it becomes evident that digital platforms consist of multiple participant groups, such as users, riders, buyers, content creators, sellers, drivers, and advertisers. Interactions between these groups create a *digital market*, which can generally be classified into price- and non-price-based models. In a price-based market, users are required to pay transaction fees or subscribe to the platform, while in a non-price-based market, users gain free access, and the platform typically generates revenue from advertisers or other indirect monetisation strategies like data licensing, affiliate marketing, or partnerships. Importantly, pricing often differs across sides of the market. For example, on food delivery platforms such as *Deliveroo*, consumers typically access the platform for free, while restaurants pay commission fees. By contrast, on other platforms, all user groups can be charged directly; for example, both buyers and sellers can pay access or transaction fees. Some platforms therefore, operate under hybrid models, where different sides face different pricing structures, and both user fees and advertising revenues coexist.

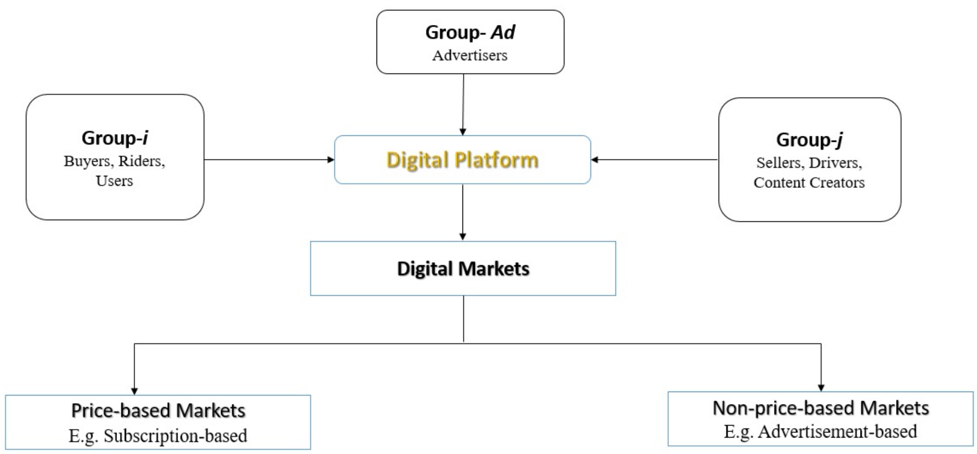


Figure 1.1: Digital Ecosystem

As illustrated in the diagram, the interactions within the platform ecosystem are shaped by both direct and indirect network effects. For example, a larger number of users increases the value of the platform for advertisers (indirect network effect), while more users within the same group can increase the value of the platform for each other (direct network effect). The resulting structure is highly integrated and can facilitate participation and interactions between multiple user groups, providing a flexible foundation for digital platform markets.

1.1.2.1 Price-Based Markets

In price-based digital markets, platforms earn revenue directly from users through payments for access, usage, or premium services. The relationship between the user (group-i, j) and the platform is transactional, where users pay for value-added content or uninterrupted service. This model typically includes the following:

(i) Subscription-based services: Users pay a recurring fee monthly or annually for continuous access to digital content or features. Examples include streaming platforms such as *Netflix*, *Spotify*, and *Amazon Prime*, where users subscribe for exclusive entertainment and ad-free content.

(ii) Pay-per-use or one-time purchases: Users make individual payments for specific products or services, such as purchasing an e-book, renting a movie, or buying a software license. Examples include *Microsoft Office*, *Adobe Acrobat*, and online gaming stores.

The key characteristic of price-based markets is the direct financial link between the platform and the user. The revenue of the platform depends on the number of paying users, user retention, and the perceived value of the service. Platforms in this category often focus on user experience, personalisation, and long-term engagement to reduce churn and sustain growth.

1.1.2.2 Non-Price-Based Markets

Non-price-based digital markets generate revenue indirectly rather than charging users for access. In these markets, users (group-i, j) often access services for free, while another group, typically advertisers (group-Ad), generates revenue for the platform. Value exchange is mediated through user attention or participation rather than direct payment. It includes:

Advertisement-based models: Platforms provide free access to users while displaying paid ads. Examples include *YouTube*, *Meta*, and *Instagram*, where advertising revenue depends on user engagement and reach. In these markets, the user base is crucial because larger audiences attract more advertisers, creating strong indirect network effects. However, excessive advertising reduces user satisfaction or engagement, so balancing between monetisation and user experience is essential.

In summary, price-based markets rely on direct payments from users for sustained revenue, while non-price-based markets depend primarily on advertisers. Both markets leverage network effects to increase user participation and overall platform value, but they differ fundamentally in how value and payments circulate within the digital ecosystem.

1.2 Unified Theoretical Framework for Digital Markets

Understanding the key differences in platform incentives and behaviour is essential for analysing competition and market outcomes in digital markets. Given how these markets differ from traditional settings, there is a clear need for formal economic models to guide intuition and provide insight into how platform monetisation strategies shape competition and welfare. Over the past two decades, a rich body of literature has emerged addressing various aspects of digital markets, from designing platform models for both price- and non-price-based (or advertisement) scenarios for analysing platform competition and mergers. However, these studies often take place in isolation, leaving the field fragmented. Hence, a unified theoretical framework is designed to capture the economic behaviour of digital platforms. The framework is flexible and can be extended to analyse different market structures. Importantly, the model does account for the zero-pricing constraint. In this unified framework, one side of the platform i.e., users group face a zero monetary price, consistent with non-price-based platform markets. Instead of charging users directly, the platform generates revenue from the other side of the market, such as advertisers. Participation on the user side is therefore driven by factors like network benefits rather than direct payments. While the framework can, in principle, accommodate negative prices, the analysis focuses on the zero-price case as a natural benchmark, which also aligns with common pricing structures observed in digital platform markets.

By incorporating multiple platforms, the model can capture competitive interactions, strategic pricing, user switching behaviour, and potential mergers. This versatility makes the framework a general analytical tool for representing a wide range of configurations in digital markets, from single dominant platforms to highly competitive multi-platform environments, while providing insights for both economic research and policy design.

This chapter aims to develop a unified theoretical framework that brings together these diverse strands of research. By doing so, it seeks to organise the existing literature in a systematic and rigorous way, helping to better understand the distinctive features of digital markets and the implications for economic policy. This divergence gives rise to a central research challenge:

How can a unified theoretical framework be constructed to analyse the functioning of multi-sided digital platforms across various market structures, integrating both price-based and non-price-based interactions?

The analysis begins by examining the fundamental structure and economic behaviour of digital platforms. A unified multi-sided digital platform model is constructed to capture the two segments of digital markets that are both *price-based* and *non-price-based markets*. The unified framework incorporates cross-side interactions, whereby agents' participation decisions are influenced by the presence of agents on the other side of the platform. For instance, sellers derive value from the number of buyers, thereby capturing indirect network effects.

However, the model abstracts from strategic interactions within each side, such as competition between sellers in pricing. These interactions are indeed important in many platform settings, as highlighted in the literature (e.g., (Karle et al., 2020)). The goal of this chapter is to provide a tractable and unified framework that focuses on platform-level pricing and participation decisions rather than detailed within-side competition. Incorporating such strategic interactions between users would significantly increase the complexity of the model and is therefore left as a potential extension. In the current framework, users are modelled as an atomless continuum, so that each individual user has a negligible impact on the platform as a whole, abstracting from strategic interactions at the user level. Nonetheless, the framework captures the primary indirect network effects that are central to many platform environments.

In this sense, seller-side competition is captured only indirectly through its effect on utility from joining the platform, rather than being modelled explicitly. More broadly, the current framework is intended as a foundational structure upon which richer models, incorporating within-side competition and strategic interactions, can be built.

1.2.1 Theoretical Model

Consider a **multi-sided digital platform A** (e.g., Uber, YouTube, Meta) that competes with other platforms to attract distinct agent groups that are as follows.

(i) **Group-i**: Consumers, such as riders, buyers, and users.

(iii) **Group-j**: Suppliers, such as drivers, sellers, and content creators.

(iii) **Group-Ad**: Advertisers.

1.2.2 Payoff Functions

The payoff function for users, advertisers, and platform A is given below. It is important to note that these functions assume a particular functional form, linear and additive, which simplifies the analysis and allows for the tractable derivation of equilibrium outcomes. While this specification captures the key interactions between platform participation, network effects, and ad exposure, alternative functional forms could introduce non-linearities or complementarities that may affect user or advertiser behaviour. Therefore, the choice of linear and additive utility is considered here as a modelling simplification that highlights the main economic drivers while remaining analytically tractable.

$$\begin{aligned}
U_i^A &= s_i^A + \alpha_i^A \cdot n_i^A + \beta_i^A \cdot n_j^A - \lambda_i^A \cdot n_{Ad}^A - p_i^A \\
U_j^A &= s_j^A + \alpha_j^A \cdot n_j^A + \beta_j^A \cdot n_i^A - \lambda_j^A \cdot n_{Ad}^A - p_j^A \\
U_{Ad}^A &= s_{Ad}^A + \beta_{Ad}^A(n_i^A + n_j^A) - p_{Ad}^A(n_i^A + n_j^A) \\
\pi^A &= n_i^A(p_i^A, p_j^A) \cdot p_i^A + n_j^A(p_j^A, p_i^A) \cdot p_j^A + n_{Ad}^A(p_{Ad}^A) \cdot p_{Ad}^A \cdot (n_i^A + n_j^A)
\end{aligned}$$

For a user in group-i, the utility (U_i^A) is made up of: s_i^A: stand-alone benefits that capture the intrinsic value or the direct utility of the platform. For example, basic access to platform features, adjusted upward by direct network effect: $\alpha_i^A n_i^A$: direct network benefits captured by α_i^A generated from each additional user of group-i, and n_i^A is the number of group-i users. Example: the value of joining Facebook increases if more of the users on the same side also join, and $\beta_i^A n_j^A$: indirect network effects, where the parameter β_i^A measures the indirect network benefits derived from the presence of users in the other group-j. For example, more content creators (group-j) increase user satisfaction in group-i. Adjusted downward by $\lambda_i^A n_{Ad}^A$: advertisement (ad) disutility, where λ_i^A measures the level of ad nuisance, and n_{Ad}^A is the number of advertisers, and p_i^A: price charged to group-i users for platform access.

For group-j users (e.g., content creators), the utility function (U_j^A) consists of: s_j^A: stand-alone benefits for group-j users (e.g., tools available for content creation). Adjusted upward by indirect network effects represented by $\beta_j^A n_i^A$, where β_j^A captures the indirect network benefits of having more group-i users (n_i^A). Example: more viewers make content creation more rewarding. Adjusted downward by $\lambda_j^A n_{Ad}^A$: advertisement disutility, and p_j^A: price charged to group-j users. Unlike users in group-i, members of group-j typically do not benefit from direct interactions with others in their own group. For instance, on a platform like Meta, content creators do not gain value simply because there are more creators, as additional creators may produce substitutable content. Hence, $\alpha_j^A n_j^A = 0$ in the current formulation. More generally, the effect of additional group-j users could even be negative if increased competition for attention or content redundancy reduces the utility of existing group-j users.

The utility for advertisers (U_{Ad}^A) is modelled as: s_{Ad}^A: stand-alone benefits of being present on the platform, reflecting factors such as brand visibility. Adjusted upward by ad reach $\beta_{Ad}^A(n_i^A + n_j^A)$: where β_{Ad}^A captures the advertising effectiveness and $(n_i^A + n_j^A)$ represents the total user base, since advertisers gain from reaching a larger pool of users. Lastly, adjusted downward by the total advertising payment $p_{Ad}^A(n_i^A + n_j^A)$ made by each advertiser. Here, p_{Ad}^A is the price charged per user view (or ad impression), and $(n_i^A + n_j^A)$ is the total number of users exposed to the advertisement.

The pay-off function (π^A) for the platform A captures all the main sources of the platform's revenue and accounts for fixed operating costs. The first term, $n_i^A(p_i^A, p_j^A) \cdot p_i^A$, reflects the total revenue generated from users of group-i, where the number of participating users n_i^A depends both on the price (p_i^A) they are charged and the presence of users of group-j. Similarly, the second term, $n_j^A(p_j^A, p_i^A) \cdot p_j^A$, represents the revenue obtained from users in group-j, whose participation also depends on cross-group pricing. The third term, $n_{Ad}^A(p_{Ad}^A) \cdot p_{Ad}^A \cdot (n_i^A + n_j^A)$, denotes advertising revenue. Here, $n_{Ad}^A(p_{Ad}^A)$ is the number of advertisers willing to pay the advertising price p_{Ad}^A, and $(n_i^A + n_j^A)$ is the total user base that advertisers can reach. In the current framework, the fixed cost C^A is intended to represent a sunk cost, which does not affect the platform's optimisation problem in the static setting considered here. As such, it does not play a role in the analysis, and is omitted from the profit function to avoid unnecessary notation. More broadly, fixed costs becomes economically relevant in other contexts. In particular, if fixed costs are not fully sunk, platform mergers may generate cost synergies by eliminating the duplication of fixed investments (e.g., infrastructure, technology, or platform development costs). This reduction in total costs could, in principle, affect incentives for entry, pricing, and overall market structure. While these considerations are not explicitly modelled here, they provide a natural and important direction for future research on platform mergers and efficiency gains. This interpretation of fixed costs applies throughout the subsequent chapters of the thesis.

Although the above profit formulation represents a monopoly setting, in a competitive environment, the number of participating agents on each side becomes a function of not only the platform's own prices but also the prices of rival platforms (k), which is given below:

$$\pi^A = n_i^A(p_i^A, p_j^A, p_i^k, p_j^k)\, p_i^A + n_j^A(p_j^A, p_i^A, p_j^k, p_i^k)\, p_j^A + n_{Ad}^A(p_{Ad}^A, p_{Ad}^k)\, p_{Ad}^A\, (n_i^A + n_j^A),$$

where $k \in \{B, C, \dots, n\}$.

Given these payoff functions, the interaction between users, advertisers, and the platform is modelled as a two-stage game. The timing of decisions reflects the strategic nature of platform competition, where pricing choices are made in anticipation of subsequent participation decisions by users and advertisers. This structure allows the analysis to capture how platforms internalise network effects and advertising externalities imposed on users when setting prices.

1.2.3 Two-Stage Game

The rationale for modelling the interaction as a two-stage game is that agents first observe the platform's prices and then decide whether to participate, based on the utilities they expect to receive. The agent's decision to join depends not only on his own preferences, but also on expectations about the participation of others, as these determine the strength of network effects.

1. The platform sets the prices for users and/or advertisers.
2. Users and advertisers decide whether to join or not.

The above game can be solved by *backward induction.* It begins with the second stage of the game, where users and advertisers observe the set of prices chosen by the platform and then make their participation decisions based on their respective utility functions. These decisions are not unilateral, but are determined in equilibrium: each agent takes as given the participation of others and chooses whether to join based on the resulting utility. This stage therefore, characterises the demand side of the platform, where equilibrium participation depends on stand-alone values, network effects, prices, and/or advertisement disutility.

Anticipating the participation of the agents, the platform makes its decision in the first stage by setting the optimal price. The platform strategy incorporates expectations about how changes in prices affect participation on each side of the market. Thus, the equilibrium outcome of the two-stage game in the case of a monopoly platform consists of (i) the platform's optimal price (or ad intensity) in the first stage and (ii) the corresponding Nash equilibrium participation levels in the second stage. Further, this framework enables us to distinguish between two cases depending on the market environment: price-based or non-price-based.

In the case of competing platforms, each platform anticipates not only the participation decisions of its own users and advertisers, but also how participation will be distributed across rival platforms. Users and advertisers observe the prices set by all platforms and form beliefs about the actual participation on each platform before deciding which platform to join. Therefore, an equilibrium is characterised by a set of prices and participation levels across platforms such that each agent's joining decision is optimal given the realised participation on all platforms and each platform's pricing decision is optimal given these participation outcomes.

1.2.4 Joining Decision

This section focuses on the joining decisions of agents in platform markets. It discusses both the monopoly platform model, where users decide whether to join a monopoly platform depending on whether the utility from participation is positive, and the platform competition model, where users choose between multiple platforms based on utility comparisons.

1.2.4.1 Monopoly Platform Model

When platform A is the monopoly in the market, an agent in group-i, j, Ad decides to join platform A if doing so yields positive utility:

$$U_i^A > 0, \quad U_j^A > 0, \quad U_{Ad}^A > 0$$

This can be interpreted as follows: An agent will join platform A if his net utility is positive. The monopolist aims to maximise his profit: if the price is set too high, then not all users will participate; if the price is set at the level where utility is positive, then all users with positive utility will join the platform.

1.2.4.2 Platform Competition Model

Platform A operates in a competitive environment in which it competes with other platforms to attract a distinct group of agents. Let n_i^A, n_j^A, n_{Ad}^A be the number of agents participating in the different groups- i, j and Ad on the platform A. The platform enables interactions between these groups while exhibiting both *direct* and *indirect network effects*. Agents can either single-home or multi-home, and the conditions governing these choices are outlined below.

Single-homing: An agent single-homes when the incremental benefit from joining a second platform is too small relative to the additional cost or advertisement disutility. Formally, an agent in each group prefers to single-home on platform A when-

$$U_i^A > U_i^k, \quad \forall k \in \{B, C, \ldots, n\},\ \forall i$$

$$U_j^A > U_j^k, \quad \forall k \in \{B, C, \ldots, n\},\ \forall j$$

$$U_{\text{Ad}}^A > U_{\text{Ad}}^k, \quad \forall k \in \{B, C, \ldots, n\},\ \forall \text{Ad}$$

This means that an agent compares his utility from A with the utility from each competing platform and joins A only when it provides the highest utility among all available options. Thus, in single-homing, an agent's decision to join a platform is determined by pairwise comparisons of utility across competing platforms.

Multi-homing: An agent multi-homes when cross-platform differentiation is strong or when the marginal utility from broader access outweighs redundancy in content or services, or the additional cost of joining another platform, exceeds the maximum of the utilities from single-homing on each platform:

$$U_i^{A+k} > \max\left\{U_i^A,\, U_i^k\right\}, \quad \forall k \in \{B, C, \ldots, n\},\ \forall i$$

$$U_j^{A+k} > \max\left\{U_j^A,\, U_j^k\right\}, \quad \forall k \in \{B, C, \ldots, n\},\ \forall j$$

$$U_{\text{Ad}}^{A+k} > \max\left\{U_{\text{Ad}}^A,\, U_{\text{Ad}}^k\right\}, \quad \forall k \in \{B, C, \ldots, n\},\ \forall \text{Ad}$$

This condition states that an agent chooses to multi-home, that is, to join both platforms (A) and (k), if and only if the total utility from participating on both platforms exceeds the highest utility the agent could obtain from joining either platform alone.

The expression captures the trade-off faced by the agent: multi-homing is attractive when access to multiple platforms provides sufficiently large additional benefits, such as broader reach, complementary content, or enhanced interactions, after accounting for any redundancy in services and the extra costs of participation. If these combined benefits exceed the utility from the best single-platform option, the agent optimally chooses to multi-home; otherwise, the agent single-homes on the platform that yields the highest utility.

Further, in reality, advertisers often multi-home because they benefit from broader audience reach, whereas users are more likely to single-home when network benefits are concentrated on one dominant platform or when the additional cost and/or redundancy of joining a second platform outweigh the gains.

Implications for platform competition: A platform's pricing and strategic decisions depend heavily on the pattern of single-homing and multi-homing:

(i) When users single-home but advertisers multi-home, competition for the single-homing side becomes intense. Platforms lower prices or subsidise users to gain a large user base, knowing that advertisers benefit from greater ad reach.

(ii) When agents on both sides single-home, platform competition becomes winner-takes-most, and small differences in utility lead to large differences in market shares due to strong network effects.

(iii) When agents on both sides multi-home, competition resembles differentiated product markets; price pressure is weaker, and platforms compete on quality, ad tools, or matching efficiency rather than subsidies.

The intuitive implications for platform competition described above are derived from the structure of the model and are consistent with insights from the existing literature Belleflamme and Peitz (2019).

1.2.5 Channels of Monetisation in Platform Markets

The utility expressions for the group-i, j defined above, it is evident that key parameters such as prices p_i^A and p_j^A, along with the advertisement-related disutility term $\lambda_{i,j}^A n_{Ad}^A$, serve as toggles that differentiate between price-based and non-price-based markets. Specifically, in a price-based market configuration, users pay a monetary fee, and in return, users do not encounter the disutility of ads, which implies $\lambda_{i,j}^A = 0$. Conversely, in non-price-based markets, users access the platform without paying the price, but experience ad disutility, which introduces a disutility captured by the term $\lambda_{i,j}^A n_{Ad}^A$. This switchability allows the unified framework of the multi-sided platforms to flexibly represent both types of digital markets. For instance, in a *pure price-based market* such as paid subscription software, the ad nuisance term disappears, the only cost users face is the subscription price, and their utility depends on the stand-alone value of the platform, as well as the direct and indirect network effects. In contrast, in a *pure ad-based market* such as a social media platform with free access but a heavy reliance on advertising, the price term is nil. Here, users do not pay for services, but instead bear the disutility of advertisements. In summary, the unified framework captures both segments of the digital markets.

Digital platform cost structures are typically characterised by high fixed costs and negligible marginal costs, reflecting substantial upfront investment in infrastructure and technology alongside near-zero costs of serving additional users. In the present framework, these fixed costs are treated as sunk and therefore do not affect the platform's optimisation problem. This cost structure underlies the importance of scale and network effects in digital markets, as platforms benefit from spreading fixed costs over a large user base.

Importantly, the pay-off function of the platform flexibly models both price-based and non-price-based digital markets through the switchability of revenue channels. In a non-price-based configuration, the platform relies solely on advertising revenue to generate profit. In contrast, in a price-based market, the platform generates revenue primarily from direct payments by users. This dual capability makes the unified framework of the platform model well-suited for analysing a broad range of digital platform strategies.

Illustrative Application of the Model: The unified framework discussed above can be applied to real-world cases such as Amazon, Meta, Spotify, and Uber. For illustration, however, the case of YouTube provides a representative example of the unified framework, as it supports both price-based and non-price-based access.

For most users (group-i), YouTube is free to use, implying $p_i^A = 0$. These users derive stand-alone benefits (s_i^A) from entertainment, education, and information. They benefit from direct network effects ($\alpha_i^A n_i^A$) by engaging with other users through likes, comments, and shares. The indirect network effects ($\beta_i^A n_j^A$) are significant, as the presence of more content creators (group-j) leads to a richer and more diverse video library, increasing the value of the platform for viewers. However, in this free access model, users are shown advertisements, which results in a disutility component $\lambda_{i,j}^A n_{Ad}^A$ in their utility function.

YouTube also offers a subscription-based model (e.g., YouTube Premium), where users can pay a price $p_i^A > 0$ to avoid advertisements. In this case, the ad disutility term $\lambda_{i,j}^A n_{Ad}^A$ is zero, representing a price-based market configuration. This illustrates how the same modelling framework can accommodate both ad-supported and paid-access formats, as well as platforms that combine the two within a single business model.

For content creators (group-j), the stand-alone benefit s_j^A arises from creative expression, community building, or personal branding. They benefit indirectly from the size of the viewer base ($\beta_j^A n_i^A$), which increases their visibility and monetisation potential. As creators typically do not gain utility from other creators, the direct network effect $\alpha_j^A n_j^A$ is nil. Some creators also subscribe to premium platform features or tools, as shown in p_j^A, and they experience ad disutility if their interface is sponsored by ads.

Advertisers on YouTube derive utility U_{Ad}^{A} from the aggregate reach of the platform. Their benefit increases with the total number of viewers and creators $(n_i^A + n_j^A)$ and is offset by the cost of advertising, $p_{Ad}^{A}(n_i^A + n_j^A)$. The effectiveness of ads is directly related to the size of the audience, aligned with the structure of the utility function of the advertiser.

Having established the unified framework and its components, the next step is to formalise how agents interact within it. The next section highlights the key trade-offs faced by platforms when setting prices in the presence of network effects, advertising demand, and agent participation incentives.

1.2.6 Platform Pricing and Participation Trade-offs

As described above, the analysis follows the logic of backward induction. In the second stage, given the set of platform prices, users and advertisers decide whether to participate based on their respective utility functions. These participation decisions determine how demand on each side of the platform responds to prices, network effects, and ad disutility. Anticipating these responses, the platform chooses its price structure in the first stage to balance higher margins against the effect of prices on participation across the users and advertisers' sides. The outcomes of this interaction characterise a set of prices and participation levels that are mutually consistent, highlighting the key trade-offs faced by platforms in setting prices in the presence of network effects and ad disutility.

Two distinct cases are considered: in case 1, optimal prices are determined for advertisers in non-price-based markets, while in case 2, optimal prices are determined for users in price-based markets.

1.2.6.1 Case 1: Non-Price-Based Market (Ad-Supported Model)

In the case of a non-price-based or ad-supported platform model, the platform generates its revenue entirely from advertisers. The platform sets the price (p_{Ad}^{A}) for advertisers, which affects the participation of advertisers and indirectly the utility of users due to the presence of the ad disutility term $\lambda_{i,j}^{A} n_{Ad}^{A}$. The advertisers decide to join the platform based on their payoff. The platform determines the optimal price by maximising the profit function

π^A with respect to p^A_{Ad}:

$$\pi^A = p^A_{Ad} \cdot n^A_{Ad}(p^A_{Ad}) \cdot (n^A_i + n^A_j)$$

Here, p^A_{Ad} represents the price charged to each advertiser per unit of user exposure to an advertisement (e.g., per view or impression), while $n^A_{Ad}(p^A_{Ad})$ denotes the number of advertisers, which decreases as the advertising price increases since advertisers are price sensitive. The term $(n^A_i + n^A_j)$ captures the total number of users in both groups i and j, reflecting the overall audience size reached by advertisements on the platform. A larger user base increases the value of joining for advertisers, so n^A_{Ad} depends not only on the advertising price p^A_{Ad} but also indirectly on n^A_i and n^A_j. Finally, C^A denotes the fixed cost of operating the platform.

When maximising profit with respect to the advertising price p^A_{Ad}, the fixed cost C^A drops out of the first-order condition because it does not depend on p^A_{Ad}. Fixed costs only shift the overall profit level, but do not affect the choice of the optimal price. Thus, the optimal p^A_{Ad} is determined entirely by the trade-off between the higher revenue per advertiser and the lower number of advertisers willing to join when the price increases, given the size of the user base $(n^A_i + n^A_j)$.

Advertiser Sensitivity and Network Effects: Setting a higher p^A_{Ad} reduces the number of participating advertisers, potentially lowering total revenue but also mitigating the disutility of ads that users experience. A lower ad disutility can improve user utility and participation, increasing the total audience size, and paradoxically, making the platform more attractive to advertisers despite the higher price. Conversely, if p^A_{Ad} is too low, too many advertisers can join, degrading the user experience via excessive ad exposure and causing user attrition. Thus, there is a delicate trade-off between advertising volume and user retention.

1.2.6.2 Case 2: Price-Based Market (Subscription Model)

In a price-based market, or subscription model, the platform generates revenue by charging prices to both user groups i and j. A user will join the platform if their stand-alone value and network benefits exceed the price charged. The platform determines the optimal user-side prices by maximising its profit function π^A with respect to p_i^A and p_j^A.

$$\pi^A = n_i^A(p_i^A, p_j^A) \cdot p_i^A + n_j^A(p_j^A, p_i^A) \cdot p_j^A$$

In the above profit function, p_i^A and p_j^A represent the prices charged to users in the group-i and group-j, respectively. The number of users $n_i^A(p_i^A, p_j^A)$ in the group i depends negatively on their own price (p_i^A), since higher prices reduce their participation, and also on the price (p_j^A) charged to group-j due to indirect network effects; for example, if fewer users join group-j, the platform becomes less attractive for group-i. Similarly, $n_j^A(p_j^A, p_i^A)$ represents the number of users in the group-j, which depends on both p_j^A and p_i^A for the same reasons. The terms $n_i^A(p_i^A, p_j^A) \cdot p_i^A$ and $n_j^A(p_j^A, p_i^A) \cdot p_j^A$ correspond to the total revenues collected from each group of users. Finally, the term C^A denotes the fixed cost of operating the platform, which does not vary with the number of users.

This formulation highlights two key characteristics of digital platforms. First, there is an interdependence between user groups: the demand from one group depends on the participation of the other. The reduction of p_i^A can attract more users in group i, which in turn can increase the participation of users in group j, and vice versa. Second, the fixed cost structure implies that since C^A is constant, the platform maximises profit with respect to the user-side prices p_i^A and p_j^A, the fixed cost C^A disappears from the first-order conditions because it does not depend on either price. In other words, C^A affects the overall level of profit but does not influence the choice of optimal prices. The optimal prices are therefore determined solely by the trade-off between higher revenue per user and the effect of prices on user participation, taking into account both direct and indirect network effects.

Although the optimal prices in cases 1 and 2 are derived for a monopoly platform, pricing in a competitive environment is inherently strategic. When multiple platforms operate in the market, each platform chooses its price vector based on the prices set by its rivals. In both price-based and non-price-based configurations, a platform's profit depends not only on its own prices but also on the prices set by competing platforms, which influence user and advertiser participation.

Formally, platform A selects prices to maximise its profit given the price vectors of rival platforms $k \in \{B, C, \ldots, n\}$. Since participation on each side depends on relative prices across platforms, any change in the pricing strategy of one platform alters the allocation of users and advertisers across all platforms. Rival platforms therefore, respond by adjusting their own prices, giving rise to a system of best-response functions. An equilibrium in the competitive setting is characterised by a set of prices for each platform such that no platform has an incentive to deviate unilaterally, given the prices of its competitors. At these prices, the participation levels on each side of every platform are mutually consistent with the utility-maximizing joining decisions of the agents and the profit-maximizing pricing strategies of the platforms. Consequently, the equilibrium of users and advertisers reflects the interaction between network effects and competitive pricing, including the possibility that users and advertisers shift between platforms in response to relative prices and network sizes.

After determining the optimal prices, they will be substituted into the second stage of the game to compute the participation number of users and advertisers.

User Price Sensitivity: Users are typically sensitive to price: higher prices reduce user utility, and consequently their participation. A user decides to join the platform if combined factors such as stand-alone benefits, direct network effects, and indirect network effects outweigh the price. If the platform sets the price too high, marginal users may opt out, leading to partial market coverage. This reduces the size of the network and diminishes both direct and indirect network effects for remaining users, which may trigger further exits. Conversely, if the platform sets the price sufficiently low, it can ensure that all users join, achieving full market coverage. This creates a feedback loop that reduces total participation and, ultimately, profit. Thus, the platform must balance the trade-off between extracting revenue per user and maintaining a sufficiently large network.

Strategic Insight: In both models, the platform's pricing strategy directly influences users and advertisers participation through utility-based thresholds. Backward induction allows the platform to anticipate these behavioural reactions and adjust its strategy to optimise profit, ensuring that the interdependence of platform pricing, network effects, and agents' decision-making is fully captured.

1.3 Organising the Literature within the Unified Framework

Having developed a unified framework, this section organises the framework within the existing literature on digital platform markets. The model is deliberately flexible: by adjusting a small set of parameters, it can represent both price-based and non-price-based platform environments, as well as different market configurations such as symmetric and asymmetric, studied in prior work.

Rather than proposing a new model for each market structure, the framework provides a common structure within which several strands of the literature can be organised. Specific assumptions about pricing, advertising intensity, network effects, or homing behaviour correspond to well-established theoretical settings in the literature. By mapping these assumptions into the unified framework, the section clarifies how existing models relate to one another and how they can be viewed as special cases of a broader platform structure. This approach highlights the connections between different branches of the platform literature and shows how results from price- and advertisement-based models can be analysed within a single coherent framework.

The stand-alone benefit parameter in the model captures the intrinsic value users derive from accessing a platform's content or service, even in the absence of other participants. This element unifies the treatment of intrinsic benefits across both price- and non-price-based markets. In Peitz et al. (2017), it appears as the baseline value of the platform that determines participation relative to the outside option. Belleflamme and Peitz (2019) similarly interpret it as the fundamental surplus from joining a platform, independent of others' participation, while in ad-supported settings, Zennyo (2020) models a comparable stand-alone benefit that motivates user engagement prior to accounting for advertising or pricing effects. Together, these formulations establish the stand-alone component as the core source of individual utility in platform participation.

The terms in the payoff that capture within-side or direct network effects (users valuing the presence or activity of other same-side users) are found in Belleflamme and Peitz (2019), who formalise such within-side effects and show how they interact with platform pricing and product variety. Hyun

(2016) models the direct network effects on the buyer's side and their demand-augmenting versus demand-sensitising consequences.

Indirect or cross-side network effects describe how participation on one side of the platform enhances the value for agents on the other side. This factor is at the core of the two-sided market literature, beginning with Rochet and Tirole (2003, 2006) and extended by Armstrong (2006), showing that pricing adjustments on one side influence participation on the other. Further refinements, such as strategic pricing and heterogeneity results in Weyl (2010) and Ambrus and Argenziano (2009) show how these cross-side effects shape optimal price allocation and equilibrium participation.

The explicit price terms in the payoff reflect the membership and usage fee structure emphasised by Rochet and Tirole (2003) and operationalised in subsequent work Caillaud and Jullien (2003), Armstrong (2006), and Weyl (2010). These papers show that it is the *structure* of prices across sides (who pays and how), not just the total price, that determines participation and platform welfare. Models that allow for richer heterogeneity and pricing instruments (fixed fees, per-transaction fees, two-part tariffs), such as Belleflamme and Peitz (2019), Jullien (2011), and Kind et al. (2016) motivate the way to include distinct price terms in the payoff functions.

Ad-based (non-price) platforms require an explicit ad-disutility term in the user payoff. The media-market formalisation of Anderson and Coate (2005) introduces the trade-off between ad intensity and viewer disutility and shows how ad levels can be inefficiently high or low depending on advertiser demand and viewer loss. Subsequent contributions Peitz et al. (2017), Kind et al. (2016), and Zennyo (2020) introduced the role of ad nuisance on the utility of users in their models. Reisinger (2012) and related work further show how ad nuisance interacts with competition for advertisers and user time allocation. In the unified framework, this effect is explicitly captured through the ad disutility term.

Using this unified framework, the literature can be organised according to market characteristics such as single-homing, multi-homing and price- and non-price-based platforms. By mapping studies to these dimensions, the chapter integrates fragmented theoretical approaches into a flexible structure that applies across different types of digital platforms and market settings, highlighting connections between models, clarifying how insights from one context inform another, and enabling analysis of competitive interactions and user switching behaviour.

The core insight of digital markets has evolved into a comprehensive body of theory explaining how multi-sided platforms coordinate interactions between distinct user groups. Under the restriction that platforms choose only prices, the unified framework nests the canonical models of two-sided markets. The formal foundations were established Rochet and Tirole (2003, 2006), Caillaud and Jullien (2003) and Armstrong (2006). These early papers provide basic frameworks for modelling network effects among customers between the two sides and for studying pricing schemes of both monopolistic and duopolistic platforms. In these models, platforms choose price structures rather than simple price levels, internalising cross-side externalities when setting fees.

Subsequent theoretical work extends this unified framework along several dimensions while preserving its basic logic. Weyl (2010) and Weyl et al. (2010); White and Weyl (2016) generalise preferences and competitive environments by allowing for rich heterogeneity in both membership benefits and interaction values. Their insulating tariff approach reframes platform pricing as a mechanism that stabilises user utility against fluctuations in participation on the opposite side. Similarly, Jullien and Pavan (2019) show how correlated preferences across sides affect equilibrium prices and welfare, illustrating how assumptions about preference correlation naturally map into the same underlying platform pricing structure.

A related strand examines platform competition and market concentration, where the models are structured in accordance with the unified framework. Correia-da Silva et al. (2019) analyse Cournot competition between platforms and show that changes in the number of platforms can benefit or harm users on both sides depending on the strength of cross-side externalities. These theoretical results resonate with empirical evidence from the media industries. Studies of newspapers Chandra and Collard-Wexler (2009), Fan (2013), radio Sweeting (2010), Jeziorski (2014), and television and magazines Song (2021) show that increased concentration does not systematically raise prices on both sides of the market. Instead, platforms often rebalance price structures, with prices on one side increasing while those on the other fall, consistent with the predictions of two-sided market theory.

Homing decisions by users constitute another central dimension of the unified framework. Rochet and Tirole (2006) and Caillaud and Jullien (2003) emphasise that multi-homing arises when platforms are incompatible, and users seek to maximise network benefits or minimise transaction costs. Armstrong (2006) formalises how asymmetric homing patterns give rise to competitive bottlenecks, whereby platforms compete intensely for single-homing users while extracting rents from multi-homing users. This pattern explains why the single-homing side is often subsidised and why price asymmetries may emerge even when network effects are symmetric. Additionally, Tan and Zhou (2021) focuses on price competition with symmetric platforms based on the assumptions of single-homing with full market coverage and subsidises a group with stronger network effects. Later contributions refine and sometimes challenge this conventional view. Belleflamme and Peitz (2019) and Anderson et al. (2019) demonstrate that multi-homing can flip the side of the market on which platforms compete most aggressively, affecting not only prices but also the distribution of merger and entry effects. Within a unified framework, these results follow from differences in demand elasticity and outside options rather than from fundamentally distinct market structures.

Under the restriction that platforms are ad-based, the unified framework nests the canonical models of digital markets by toggling ad disutility on the user side. Filistrucchi et al. (2012), who distinguished between *transaction* and *non-transaction* two-sided markets, aligns with the unified framework. This distinction clarified that ad-based platforms represent a distinct form of multi-sided market in which price mechanisms are largely replaced by indirect participation incentives. Foundational ad-based platform models, such as Armstrong (2006), Reisinger (2012), and Zennyo (2020) show how platforms subsidise users to attract advertisers when cross-group network effects are strong.

Further, the application of a unified model by combining both price- and non-price-based parameters is included in papers such as Wu and Huang (2024) and Zennyo (2020), which illustrates the trade-offs between free and paid tiers and the role of ad disutility in user retention and conversion, which can be explicitly modelled through tiered utility functions. Complementors and data network effects studied by Knorr et al. (2025) emphasise evolving indirect network effects, aligning with dynamic network models such as Sun et al. (2025) for long-term adoption and growth.

Overall, the literature on digital markets can be understood as a collection of special cases within a common platform structure. Differences across models arise from specific assumptions about pricing instruments, user behaviour, monetisation, and market structures. A unified framework makes these connections explicit, clarifying how seemingly disparate results relate to one another and providing a coherent foundation for analysing platform behaviour across a wide range of market environments.

1.4 Conclusion

This chapter provides conceptual groundwork for understanding the economics of digital markets through the construction of a unified theoretical framework. Beginning with a systematic exploration of the defining features of digital platforms, it distinguished digital markets from traditional market structures in both form and function. Particular emphasis was placed on how demand-side economies, cross-group interactions, and monetisation design shape the processes of value creation, user participation, and pricing strategies in digital ecosystems.

The synthesis of the prior literature highlighted key theoretical contributions that have shaped the understanding of two-sided platform markets, especially those related to network effects, advertising disutility, and complex pricing interactions, which often move in different theoretical directions. By identifying the absence of a cohesive framework that can accommodate both price-based (subscription, transaction) and non-price-based (ad-supported) market forms, this chapter establishes a clear rationale for developing a unified model that generalises across digital platform types and provides a foundation for both theoretical exploration and applied analysis. In doing so, the model organises the existing literature as an integrative structure that connects and extends previous approaches to digital platform markets.

The unified framework developed here is designed to be *switchable* in structure, allowing smooth transitions between price-driven and ad-driven environments. This switchability enables the model to flexibly capture the diversity of digital markets without the need to construct separate theoretical models for each specific case. It provides a coherent analytical environment where:

1. The role of policy interventions, such as advertising regulation or price caps, can be systematically compared across different types of markets

within the same framework.

2. The implications of platform design choices, such as whether to monetise primarily through subscription fees, transaction charges, or advertising revenues, can be examined through changes in the underlying utility and cost structure.

3. The model maintains analytical tractability while being broad enough to describe realistic platform strategies, user behaviour, and regulatory environments across the digital platform ecosystem.

In short, this unified and switchable setup captures the full spectrum of digital markets, from fully price-based to fully ad-supported and hybrid cases, within a single consistent theoretical structure. This makes the framework not only theoretically robust but also practically useful for analysing platform strategies and evaluating regulatory or welfare outcomes.

In the coming chapters, this unified framework is employed to analyse various aspects of digital platform markets, including competitive interactions, user surplus under single and multi-homing, and the effects of pricing and advertising strategies. It provides a consistent foundation for extending the analysis to symmetric and asymmetric platforms, as well as price- and ad-based mechanisms.

Chapter 2

Competition and Merger Analysis among Digital Platforms

2.1 Introduction

In price-based digital markets, digital platforms act as intermediaries connecting distinct groups of users and facilitating transactions, interactions, and value creation. These multi-sided digital platforms include user groups such as buyers and sellers, service providers and consumers, or content creators and viewers. Platforms generate value through *network effects*, where a user's benefit increases with the number of other users on the same side (direct network effects) or on the different/ cross-side (indirect network effects). Examples such as Amazon, Uber, Meta and PayPal illustrate how digital platforms thrive by coordinating user interactions to enhance overall engagement and platform value. Unlike traditional firms, digital platforms benefit from network effects, which shape pricing strategies and market structure. These factors directly influence user participation and user surplus, making market structure and pricing decisions central to welfare outcomes.

In contrast to traditional merger outcomes, where consolidation often leads to increased prices and reduced consumer welfare, mergers in digital markets, can have mixed effects on user welfare. When platforms merge, the resulting concentration of users can strengthen network effects, improving the value of participation and potentially offsetting some negative impacts of higher prices. However, whether these gains dominate depends on the strength of network effects relative to the market power of the merged platform. If a merger leads to a monopoly platform, the larger user base enhances connectivity and convenience; but a lack of competition increases the risk of price hikes and reduces user welfare overall. An illustrative real-world example is the competition between Uber and Lyft. If these platforms were to merge, users could benefit from improved matching efficiency, larger driver pools, and stronger network effects associated with reduced waiting times. However, the resulting dominant platform could also exercise greater market power, potentially leading to higher prices and reduced user welfare. Conversely, the presence of competing platforms disciplines pricing and preserves competitive pressure, allowing users to benefit from scale-related efficiencies without fully experiencing monopoly pricing.

This chapter draws on insights from Tan and Zhou (2021), who investigated symmetric platform competition and extended their work in several important directions. First, the original study incorporates a model and illustrates, using a particular numerical example, that the user surplus increases when the number of platforms decreases from three to two. They attribute this counterintuitive result to the strengthening of network effects after user consolidation. While they impose symmetry, it is important to investigate asymmetries, since real-world platforms are often asymmetric. Second, their study focuses on competitive market structures that expand from duopoly to triopoly and beyond, analysing user surplus when the number of platforms increases. However, it does not examine mergers from a duopoly that lead to a monopoly scenario. An extension of their work is to analyse how user surplus changes when a duopoly consolidates into a monopoly platform.

To address these gaps, this chapter examines the implications for user surplus of market consolidation under both symmetric and asymmetric competition. The analysis specifically investigates how user surplus evolves when moving from a triopoly to a duopoly and ultimately from duopoly to a monopoly. This extension is especially relevant given the growing prevalence of dominant platforms engaging in mergers and acquisitions, leading to increasingly concentrated market structures. This chapter also relies on numerical analysis, but allows for asymmetric platform structures to explore how such asymmetries influence competition in digital markets. Further, it incorporates different strengths of network effects, asymmetric user distributions, and varying platform sizes to provide a clearer understanding of digital platform markets.

Further, the monopoly scenario is examined because many digital platform markets are highly concentrated, with Big Tech such as Google, Meta, Amazon, and Apple operating in environments where their scale and user base give them near-monopoly power. These platforms command significant market power through their large user base and frequent acquisitions of emerging technologies, which, rather than diversifying competition, reinforce their dominant position in the market. However, not all users participate in these platforms; some are excluded due to pricing or a preference for alternative platforms. To capture this reality, the analysis incorporates partial market coverage, allowing a share of users who choose not to join the dominant platform. This provides a more realistic representation of digital markets, where network effects are strong but participation is incomplete. In light of these features, the study examines how such concentrated structures and partial coverage influence user welfare, competition, and policy outcomes.

The central objective of this chapter is to extend the theoretical understanding of competition and mergers in digital platform markets, focusing on both symmetric and asymmetric scenarios and their implications for user surplus. The core focus on user surplus is motivated by its central role in competition policy assessments of platform markets. In particular, the Competition and Markets Authority (CMA) places significant emphasis on consumer outcomes when evaluating mergers, including effects on prices. In this sense, focusing on user surplus allows the analysis to align closely with the criteria used in practice for evidence-based merger evaluation. This focus is not intended to suggest that platform profits or overall social welfare are unimportant. Rather, it reflects a deliberate choice to prioritise the dimension most directly relevant for policy. Extending the analysis to incorporate platform profits and total welfare would be a natural complement. Against this background, the chapter addresses the following research questions:

(i) In both symmetric and asymmetric settings, how does a merger from duopoly to monopoly affect user surplus, and is the impact necessarily negative?

(ii) Under what conditions does a merger from three symmetric platforms to two result in a higher user surplus?

(iii) In asymmetric platform markets, what factors determine whether a merger from three to two platforms increases or reduces user surplus?

By answering these questions, this chapter contributes to a deeper understanding of the complex relationship between network effects and user surplus in digital markets.

A simulation-based approach is adopted to investigate the conditions under which user surplus improves or deteriorates after a merger, with particular attention to the role of network effects, platform asymmetries, and market coverage assumptions (i.e., full vs. partial market coverage). Analytical solutions are generally intractable because each user's platform choice depends on the choices of all other users, making the problem highly interdependent and difficult to solve mathematically. Importantly, the analytical intractability of the model does not arise from platform competition per se, as symmetric multi-platform settings can often be solved in closed form (e.g. Tan and Zhou (2021)). Instead, the primary source of complexity is the introduction of asymmetries across platforms, combined with endogenous participation and cross-side network effects. These features jointly generate a high-dimensional system of interdependent equilibrium conditions, which are not amenable to closed-form solutions.

In line with the existing literature, the current framework assumes single-homing on both sides of the platform. Under this assumption, users choose the platform that yields the highest net utility and allocate fully to that platform. This modelling choice is consistent with Tan and Zhou (2021), who also adopt a single-homing framework. Allowing for multi-homing would be a natural extension, but it would substantially increase the complexity of the model by introducing partial participation and more involved adoption decisions across platforms. Given the focus of this chapter on the role of network effects in shaping market outcomes, the single-homing assumption is maintained to preserve tractability. Extending the framework to allow for multi-homing on one or both sides of the platform remains an important direction for future research.

This contrasts with parts of the literature that impose symmetry to obtain tractability. For example, Tan and Zhou (2021) derive analytical results under symmetric platforms, which reduces the dimensionality of the problem. Similarly, Jullien et al. (2021) highlight that introducing richer structures, such as asymmetries and endogenous participation, substantially increases analytical complexity in multi-sided platform models. The approach taken here deliberately departs from symmetry in order to capture heterogeneity across platforms, which is a central feature of many digital markets. While this comes at the cost of analytical tractability, it allows the model to study richer interaction patterns and more realistic competitive environments. As a result, the analysis relies on numerical methods to characterise equilibrium outcomes. This framework contributes to the literature by using a simulation-based approach to systematically evaluate welfare outcomes across different market structures, thus allowing exploration of scenarios that are difficult to analyse analytically. In addition, it provides regulators and policymakers with clearer guidance on when platform mergers are likely to harm or benefit users and under what structural conditions those effects arise.

To examine how network effects influence user surplus across different platform market structures, this study draws on a broad literature that investigates competition and pricing in two-sided digital platform markets. The review synthesises both theoretical and empirical contributions to identify gaps and inform the modelling framework of this chapter. Key strands of the literature include analyses of platform structure (monopoly and oligopoly), the role of symmetry versus asymmetry, the implications of mergers, and methodological approaches used to assess user surplus.

The foundational works of Rochet and Tirole (2003, 2006), Caillaud and Jullien (2003), and Armstrong (2006) established the core economic principles that govern two-sided platforms, particularly under monopoly conditions and indirect network effects. These studies examine how platforms optimise pricing strategies to balance user participation across both sides. Building on this foundation, the present study expands the analysis to consider heterogeneous users, direct network effects, and scenarios with full market coverage, allowing for a broader evaluation of welfare outcomes under varying platform conditions.

Perloff and Salop (1985) presented a traditional market model that provides a general framework to analyse consumer choice among differentiated products in a symmetric oligopoly. Their framework offers a methodological basis that can be adapted to study digital platform competition with network effects, even though the original model focuses on consumer preferences on differentiated products characterised by symmetric equilibrium and does not incorporate network effects directly.

Tan and Zhou (2021) developed an oligopoly model of symmetric digital platforms competing through pricing strategies. They demonstrate that under certain conditions, a merger from three to two platforms can increase user surplus due to enhanced network effects. Their analysis is grounded in a general theoretical framework, supported by numerical examples to illustrate key insights. However, their analysis does not examine user surplus in monopoly settings or in markets characterised by asymmetric platform structures.

The literature increasingly acknowledges that real-world platforms are rarely symmetric. Jullien (2011) investigates the stackelberg price competition game between two single-homing platforms, which illustrates how platform asymmetries influence pricing outcomes. The study introduces the "divide-and-conquer" strategy and shows how platforms secure participation by subsidising one side of the market. However, it finds that profitability, rather than welfare, is maximised and that a dominant position does not necessarily benefit consumers. This motivates a re-examination, in this chapter, of how price competition and platform dominance influence user surplus under asymmetric market structures.

More recent work by Jullien et al. (2021) extends the analysis to asymmetric platforms in oligopoly settings, incorporating heterogeneous user preferences and allowing for more realistic market scenarios. Their findings show that under single-homing, platforms can extract higher total fees due to locked-in users, leading to stronger market power. Although asymmetric competition may lead to price concessions on one side, these often result in reduced surpluses on the other, leaving the net welfare effect ambiguous. Importantly, their analysis raises concerns that mergers or platform exits can significantly harm welfare by further reducing competition and variety, especially when users have limited switching options.

A substantial body of empirical literature highlights the consumer harms associated with platform mergers. Chandra and Collard-Wexler (2009), studying the Canadian newspaper industry, and Sweeting (2010) in the radio industry, both find that mergers reduce product variety and increase prices. Fan (2013) extends this argument by showing that in US newspaper markets, ownership concentration reduces product variety and increases costs, particularly in already concentrated environments. These studies underscore the risks of reduced competition and demonstrate how consolidation can reduce consumer choice.

Structural models such as Jeziorski (2014) and Correia-da Silva et al. (2019) offer more detailed perspectives. Jeziorski (2014) finds that some efficiency gains from mergers may partially offset losses in product varieties and consumer welfare, especially when pre-merger competition is robust. Correia-da Silva et al. (2019), using a Cournot framework, suggest that horizontal mergers among multi-sided platforms can lower prices on some sides, even in the absence of explicit cost efficiencies, particularly in cases of asymmetry or low initial concentration. These insights highlight that merger outcomes depend on structural and behavioural conditions, such as market coverage and pricing power. In platform-specific contexts, Song (2021) investigates two-sided magazine markets and concludes that mergers amplify market power, allowing platforms to extract higher prices and reduce choices. These findings confirm that merger leads to higher prices, and reduced platform choice for users is central to understanding welfare outcomes in platform mergers.

These insights are particularly relevant to this chapter, which uses symmetric and asymmetric competition models to examine how mergers affect user surplus. Compared to previous studies, this chapter employs a simulation-based framework that allows for heterogeneous user preferences, asymmetric platforms, and full or partial (in the case of monopoly) market coverage, providing a way to explore complex interactions that are analytically challenging. Although existing models have not fully addressed how stronger network effects influence welfare after a merger, this study explicitly investigates the conditions under which such effects enhance user welfare in a consolidated market. The present study contributes to the literature by offering a numerical simulation-based investigation of merger effects in both symmetric and asymmetric digital platforms, with a particular focus on single-homing and full market coverage. Although simulation results are not analytical proofs, this approach allows exploration of complex interactions that are not tractable in analytical models. It directly addresses the research gaps left by Tan and Zhou (2021) by examining the welfare consequences of mergers leading to monopoly and incorporating heterogeneous user preferences and asymmetric platforms. This approach allows for a more comprehensive evaluation of how network effects interact with pricing strategies, competitiveness, and platform structure. This investigation therefore, adds to theoretical understandings of digital market consolidation, offering insights that are particularly relevant to regulators and policymakers concerned with preserving user welfare in increasingly concentrated platform ecosystems.

The rest of the chapter is organised as follows. Section 2.2 introduces the monopoly model, Section 2.3 develops the two platform model, and Section 2.4 extends the analysis to a three platform setting. From Section 2.5 onward, examine market outcomes and user surplus across these market structures, with numerical illustrations highlighting the impact of network effects and platform asymmetries.

2.2 Monopoly Platform Model

This section considers a single platform that serves multiple groups of users. The platform sets prices for each group, and users decide whether to join based on their individual valuations, the network effects, and the prices, joining only if their utility is positive. By determining the equilibrium prices and participation levels, this framework captures how the platform balances pricing and network effects to maximise its profit while affecting user surplus in digital platform markets.

2.2.1 Model Assumptions

Consider a monopoly platform A serving two different user groups, i and j (with $i \neq j$). Examples of these groups include riders and drivers, users and advertisers, etc., where interaction between users generates network effects for the other. Users on the same side of the platform generate direct (same side) network effects, where the value for each user increases as more users join their group. Interactions between the two groups create indirect (cross-side) network effects, where the participation of one group increases the value of the platform for the other.

The following assumptions define the setup and remain valid for the two- and three-platform model:

(i) The population of each group is normalised to 1.

(ii) Group-j users derive no direct network benefit. This is because advertisers and/or drivers do not directly benefit from interacting with each other.

(iii) Each user independently draws their stand-alone value of the platform from a continuous distribution .

(iv) The parameter that governs the indirect network benefit is the same between the two groups, and the same holds for the direct network benefits.

2.2.2 Two-stage game:

In order to model the interaction between the platforms and users, the market is represented as a two-stage game. In the first stage, platform A sets prices for the two user groups, anticipating how these prices will affect the participation of users. In the second stage, users decide whether to join the platform based on their utilities. Formally, the game proceeds as follows:

1. The platform A sets the prices simultaneously for both groups.

2. Users in both groups decide whether to join the platform A.

This game is solved by backward induction. In the second stage, for any given vector of platform prices, each user joins A only if it yields positive utility (or abstains if the utility is negative). The joining decision depends not only on the platform's price but also on the anticipated joining decisions of other users on both sides of the market. Each user's decision is interdependent with others' due to the participation loop, and equilibrium participation levels are determined where no user has an incentive to change their decision given others' choices, that is, a *Nash equilibrium in joining decisions.* The equilibrium participation levels for each group on the platform are then aggregated. In the first stage, anticipating these equilibrium participation responses, the platform simultaneously chooses its optimal price pair for the two user groups, taking into account how changes in the price of one side affect participation on the other side through network effects.

The rationale for adopting a two-stage game structure is that the platforms must commit to prices before users make joining decisions, reflecting how pricing typically precedes participation in reality. The backward-induction solution then yields the equilibrium prices and participation levels, which can be used to analyse comparative statics such as how changes in the strength of network effects or asymmetries across platforms alter equilibrium outcomes. Further, this approach captures the strategic interdependence inherent in multi-sided platforms, where decisions on one side affect outcomes on the other.

2.2.3 Payoff Functions

The utility function for both user groups follows the original framework introduced by Armstrong (2006) and Rochet and Tirole (2006), as well as the more recent adaptations in Tan and Zhou (2021) and Xie et al. (2021). While the current model uses different parameters, the functional form remains the same.

The utility of a user joining platform A is calculated as follows:

$$U_i^A = s_i^A + \alpha \cdot n_i^A + \beta \cdot n_j^A - p_i^A \qquad ; \quad U_j^A = s_j^A + \beta \cdot n_i^A - p_j^A$$

The utility function (U_i^A, U_j^A) is composed of stand-alone benefits (s_i^A, s_j^A) of the platform (i.e., the direct utility that the platform provides), adjusted upward by the direct and indirect network effects. The term $\alpha \cdot n_i^A$ captures *direct network effects*, where α is the direct network benefits parameter measuring the strength of same-group interactions and n_i^A is the number of group-i users on the platform. Similarly, $\beta \cdot n_j^A$ $(\beta \cdot n_i^A)$ represents *indirect network effects*, where β is the indirect network benefits parameter capturing different/cross-group interactions with n_j^A (n_i^A) as the number of users from the other group, i.e., j (i). The utility is adjusted downward by the price (p_i^A, p_j^A) charged by the platform to each group. Moreover, group-j users do not experience within-group interaction, so their utility excludes direct network effects (that is, $\alpha \cdot n_j^A = 0$ for group j).

To illustrate the above, on LinkedIn, job seekers and professionals are on one side, and recruiters/companies are on the other. The stand-alone benefits for job seekers include maintaining an online curriculum vitae (CV), building a personal profile, and accessing professional content or skills courses on the platform. Recruiters also have stand-alone benefits: a ready-made applicant tracking system and tools to post jobs or advertise their company, which are useful even with a pool of users.

Direct network effects are strong on the job seeker/professional side: as more professionals join, the platform becomes more valuable to other professionals for networking, endorsements, and information exchange. A larger professional network makes it easier to connect, share updates, and build credibility.

Indirect network effects link the two groups. A larger pool of professionals makes LinkedIn more valuable to recruiters by increasing the chance of finding qualified candidates. At the same time, more recruiters and job postings make LinkedIn more valuable to professionals because it improves access to opportunities.

Pricing reflects this structure. Professionals can use the platform for free (with ads and limited features) or pay for premium services such as InMail and advanced search. Recruiters pay subscription fees for LinkedIn Recruiter and advertising tools. Thus, LinkedIn displays: stand-alone benefits for both groups, strong direct network effects only on the professional side, and strong indirect network effects between professionals and recruiters.

Returning to the model, the payoff function for platform A is as follows:

$$\pi^A = n_i^A(p_i^A; p_j^A) \cdot p_i^A + n_j^A(p_j^A; p_i^A) \cdot p_j^A;$$

This payoff function represents the revenue generated by platform A from both user groups i and j, after incurring the fixed cost. The term $n_i^A(p_i^A, p_j^A) \cdot p_i^A$ denotes the revenue generated from group-i users, where n_i^A is the fraction of users in group-i who choose to join the platform and p_i^A is the price charged to each of them. Similarly, $n_j^A(p_j^A, p_i^A) \cdot p_j^A$ is the revenue from the group-j users, which also depends on the pricing and the participation of the users on the other side. The total revenue is the sum of these two components. As stated in Chapter 1, fixed costs are assumed to be sunk and therefore do not affect the platform's optimisation problem in the static setting considered here. For this reason, they are omitted from the profit function. While fixed costs may play an important role in settings such as entry or mergers through cost synergies, these aspects are not modelled here, even in the two- and three-platform model extensions considered within this chapter.

2.2.4 User Joining Behaviour

The pricing strategy adopted by a monopoly platform can lead to full or partial market coverage. Full market coverage occurs when the platform sets its price low enough so that every user derives positive utility and chooses to join. Alternatively, the platform can set a higher price to maximise profit, even though some marginal users will opt out because their utility is negative, resulting in partial coverage.

The exposition adopted here distinguishes between full and partial market coverage to highlight how participation constraints affect platform outcomes. While the underlying participation conditions are general, presenting the full coverage case separately allows for a transparent benchmark in which all users participate. In this setting, pricing depends directly on users with the lowest valuation and the strength of network effects, without the additional complexity introduced by endogenous participation. This benchmark helps clarify how network effects translate into pricing when participation is not binding.

By contrast, the partial coverage case introduces endogenous participation, where marginal users opt out and equilibrium allocations are determined through a fixed-point problem. This distinction is useful for understanding how results change once participation constraints become binding. The focus on both cases is therefore intentional: full coverage provides a tractable benchmark, while partial coverage captures the more realistic scenario of incomplete participation in platform markets.

Full Market Coverage: Full market coverage ($n_i^A = 1, n_j^A = 1$) occurs when the utility of every user joining platform A is non-negative for all possible values of s_i^A ($U_{i,j}^A \geq 0 \ \ \forall s_i^A$), ensuring that all users participate. This condition is satisfied if the utility of the user with the lowest s_i^A is zero, since ensuring non-negativity at the lowest value of s_i^A guarantees non-negativity for all users.

For any user in group-i, the participation condition is-

$$U_i^A = s_i^A + \alpha\, n_i^A + \beta\, n_j^A - p_i^A \geq 0$$

so, the price must satisfy the following:

$$p_i^A \leq s_i^A + \alpha\, n_i^A + \beta\, n_j^A \qquad \text{for all users who are joining the platform A}$$

Hence, to maximise revenue while keeping the intended set of users denoted by $\mathcal{I}_A$ on the platform, the platform optimally sets-

$$p_i^A = \min_{i \in \mathcal{I}_A} \left\{ s_i^A \right\} + \alpha\, n_i^A + \beta\, n_j^A$$

Because participation rates n_i^A, n_j^A generally depend on the chosen price, the equilibrium price and participation levels must be solved simultaneously as a fixed point.

In this case of full market coverage (the total measure of users equals one), all users participate on both sides, i.e., $n_i^A = n_j^A = 1$. The pricing condition then simplifies to-

$$p_i^A = \min_{i} \left\{ s_i^A \right\} + \alpha + \beta$$

Similarly, for group-j users, the participation condition is-

$$U_j^A = s_j^A + \beta\, n_i^A - p_j^A \geq 0$$

Thus, the price must satisfy-

$$p_j^A \leq s_j^A + \beta\, n_i^A \qquad \text{for all users who are joining the platform A}$$

To maximise revenue while keeping the intended set of group-j users represented by $\mathcal{J}_A$ on the platform A, the price is set at-

$$p_j^A = \min_{j \in \mathcal{J}_A} \left\{ s_j^A \right\} + \beta\, n_i^A$$

Under full market coverage, where all users on both sides participate ($n_i^A = n_j^A = 1$), this simplifies to the following:

$$p_j^A = \min_j \left\{ s_j^A \right\} + \beta$$

From the above, it is clear that full market coverage arises only when prices are low enough for users with the lowest stand-alone valuations to participate. When this condition is not met, some users opt out, and the analysis shifts to partial market coverage.

Partial Market Coverage: In the case of partial market coverage, where marginal users do not join platform A due to higher prices. The joining decision for a user in group-i is given as follows:

$$s_i^A + \alpha n_i^A + \beta n_j^A - p_i^A > 0$$

is rearranged as

$$s_i^A > p_i^A - \alpha n_i^A - \beta n_j^A \tag{1}$$

Similarly, users in group-j decide to join the platform A, if:

$$s_j^A + \beta n_i^A - p_j^A > 0$$

is rearranged as

$$s_j^A > p_j^A - \beta n_i^A \tag{2}$$

Inequalities (1) and (2) define the threshold valuations (i.e., the conditions for participation), which correspond to the equality case, users who are indifferent to joining. Each user draws their stand-alone value for platform A, denoted s_i^A for group-i and s_j^A for group-j, from a continuous distribution. Any user whose s_i^A or s_j^A exceeds the respective threshold (price minus network effects) will decide to join the platform A.

The proportion of users who join platform A is determined by the cumulative distribution functions (CDFs) of the user valuations, $G_i^A(\cdot)$ and $G_j^A(\cdot)$, for the groups i and j, respectively. A user joins if his valuation $s_{i,j}^A$ exceeds the threshold defined above.

For group-i, the proportion of users with s_i^A above the threshold is-

$$\Pr\big(s_i^A > \text{threshold}\big) = 1 - G_i^A\big(p_i^A - \alpha n_i^A - \beta n_j^A\big)$$

and for group-j-

$$\Pr\big(s_j^A > \text{threshold}\big) = 1 - G_j^A\big(p_j^A - \beta n_i^A\big)$$

Here, $G_{i,j}^A(\cdot)$ gives the probability that a user's valuation is below the threshold. Therefore, the probability that $s_{i,j}^A$ exceeds the threshold (i.e., that the user joins the platform A) is its complement, $1 - G_{i,j}^A(\cdot)$.

Then the proportion of users who join platform A is given by:

$$n_i^A = 1 - G_i^A\big(p_i^A - \alpha n_i^A - \beta n_j^A\big)$$
$$n_j^A = 1 - G_j^A\left(p_j^A - \beta n_i^A\right)$$

The above expressions define the total number of users $(n_{i,j}^A)$ in both groups who join the platform A, given the prices p_i^A, p_j^A. Further, these expressions define a system of interdependent equations, where n_i^A and n_j^A depend on each other. The right-hand side of each equation contains the left-hand side as part of a function, making the system non-linear and implicit. The interdependence of n_i^A and n_j^A implies that solving for equilibrium user allocations involves finding values of n_i^A and n_j^A that simultaneously satisfy both equations. This is a classic *fixed point problem*, where the equilibrium values are fixed points of a function that maps the user shares into themselves.

The equilibrium is therefore characterised as a fixed point of the system of participation equations. Under standard regularity conditions, existence of an equilibrium can be ensured. In particular, if the valuation distributions $G_i^A(\cdot)$ and $G_j^A(\cdot)$ are continuous, the mapping from $[0, 1]^2$ into itself is continuous and defined on a compact set, which guarantees the existence of a fixed point. Uniqueness is not imposed analytically in general, as it depends on the strength of the network effects. In the presence of strong network effects, multiple fixed points may in principle arise due to strategic complementarities in participation decisions. However, in the numerical implementation, parameter values are restricted to regions in which the fixed-point algorithm converges to a unique solution, ensuring well-defined equilibrium outcomes for all simulations reported.

Equilibrium Analysis: The users' equilibrium is determined by their independent stand-alone values, s_i^A and s_j^A, which are drawn from continuous distributions as specified in assumption (iii). In this analysis, these continuous distributions are assumed to be a uniform distribution (UD): $s_i^A \sim \text{UD}(l_i^A, h_i^A)$ for group-i and $s_j^A \sim \text{UD}(l_j^A, h_j^A)$ for group-j. The uniform distributions are defined over the intervals (l_i^A, h_i^A) and (l_j^A, h_j^A), respectively.

For a UD with $x \sim UD[l, h]$, the CDF is given by:

$$G(x) = \frac{x - l}{h - l}, \quad \text{for } x \in [l, h]$$

This CDF is used to calculate the proportion of users whose valuations exceed the thresholds for the platform A.

For the two user groups, the CDFs at the respective thresholds are-

$$G_i^A(\cdot) = \frac{p_i^A - \alpha n_i^A - \beta n_j^A - l_i^A}{h_i^A - l_i^A}; \quad G_j^A(\cdot) \quad = \frac{p_j^A - \beta n_i^A - l_j^A}{h_j^A - l_j^A}$$

On substitution, the equilibrium users are as follows:

$$n_i^A = 1 - \frac{p_i^A - \alpha n_i^A - \beta n_j^A - l_i^A}{h_i^A - l_i^A}$$
$$n_j^A = 1 - \frac{p_j^A - \beta n_i^A - l_j^A}{h_j^A - l_j^A}$$

On simplification of the above, the results are-

$$n_i^A = \frac{h_i^A - p_i^A + \alpha n_i^A + \beta n_j^A}{h_i^A - l_i^A}$$
$$n_j^A = \frac{h_j^A - p_j^A + \beta n_i^A}{h_j^A - l_j^A}$$

Solving the above equations simultaneously yields the equilibrium participation for the platform A:

$$n_i^A(p_i^A;p_j^A) = \frac{h_i^A l_j^A - \beta h_j^A - h_i^A h_j^A + \beta p_j^A + h_j^A p_i^A - l_j^A p_i^A}{2(\alpha h_j^A - \alpha l_j^A - h_i^A h_j^A + h_i^A l_j^A + h_j^A l_i^A - l_i^A l_j^A + \beta^2)}$$
$$n_j^A(p_j^A;p_i^A) = \frac{\alpha h_j^A - \beta h_i^A - h_i^A h_j^A - \alpha p_j^A + \beta p_i^A + h_j^A l_i^A + h_i^A p_j^A - l_i^A p_j^A}{2(\alpha h_j^A - \alpha l_j^A - h_i^A h_j^A + h_i^A l_j^A + h_j^A l_i^A - l_i^A l_j^A + \beta^2)}$$

For any pair of first-stage prices (p_i^A, p_j^A), the above expressions are the equilibrium number of users joining each side of the platform in the second stage. These equilibrium participation levels are then substituted into the platform's profit function to determine optimal prices:

$$\pi^A = n_i^A(p_i^A;p_j^A) \cdot p_i^A + n_j^A(p_j^A;p_i^A) \cdot p_j^A$$

To derive the equilibrium prices, the profit function is maximised twice using the prices of the respective groups. During the optimisation of the profit function with respect to $p_{i,j}^A$. Due to the uniform distribution of user valuations and the linear form of the participation functions, the profit function is quadratic in each price, and the cross-effects cancel, leading to the simple solution:

$$p_i^A = \frac{h_i^A}{2} \qquad ; \quad p_j^A = \frac{h_j^A}{2}$$

The equilibrium prices above take a "monopoly-like" form in the partial market coverage case, which may indeed appear surprising at first sight in the presence of network effects. The key intuition is that, under partial market coverage, network effects affect the level of demand (i.e. participation) but not its sensitivity to price. With uniform valuation distributions and linear network effects, equilibrium participation is linear in prices. As a result, network effects shift demand upward but do not change its slope with respect to price. Hence, while stronger network effects increase equilibrium demand and shift marginal revenue upward in level, they do not affect the marginal sensitivity of demand to price. Consequently, when solving the first-order conditions, the network effect terms cancel out, and optimal prices depend only on the support of the valuation distributions, yielding standard monopoly mark-ups. As a result, the optimal monopoly prices remain unchanged under partial market coverage.

In contrast, in the full market coverage case, all users participate, so increases in network effects directly raise willingness to pay at the margin, and therefore optimal prices increase with network strength. The distinction between the two regimes explains why network effects do not alter pricing under partial coverage but do so under full coverage.

After determining the equilibrium prices, they will be substituted to determine the equilibrium number of users. Then the equilibrium number of users is as follows:

$$n_i^A = \frac{h_i^A l_j^A - \beta h_j^A - h_i^A h_j^A}{2(\alpha h_j^A - \alpha l_j^A - h_i^A h_j^A + h_i^A l_j^A + h_j^A l_i^A - l_i^A l_j^A + \beta^2)}$$
$$n_j^A = \frac{\alpha h_j^A - \beta h_i^A - h_i^A h_j^A + h_j^A l_i^A}{2(\alpha h_j^A - \alpha l_j^A - h_i^A h_j^A + h_i^A l_j^A + h_j^A l_i^A - l_i^A l_j^A + \beta^2)}$$

2.2.5 User Surplus Calculation

After computing the equilibrium numbers of users and the corresponding prices, the user surplus for each group on the platform A is calculated under full and partial market coverage. In the full market coverage case ($n_{i,j}^A = 1$), the user surplus is determined assuming all users participate, while in the partial market coverage case, the surplus is based on the equilibrium participation levels $n_{i,j}^A$ derived above.

Case 1: Full market Coverage To ensure full market coverage, the monopoly platform A sets the price so that even a user with the lowest stand-alone valuation ($s_{i,j}^A$) will choose to join the platform. Since the stand-alone values are drawn from a uniform distribution, each value within this interval has an equal likelihood of being realised. Accordingly, the average user surplus (US) provides a meaningful measure of the expected utility for users in each group on platform A, which is given as below:

$$\text{Group-}i: \quad \bar{US}_i^A = \int_{l_i^A}^{h_i^A} U_i^A(s_i^A) \times \frac{1}{h_i^A - l_i^A} \times ds_i^A$$
$$\text{Group-}j: \quad \bar{US}_j^A = \int_{l_j^A}^{h_j^A} U_j^A(s_j^A) \times \frac{1}{h_j^A - l_j^A} \times ds_j^A$$

These expressions represent the average surplus per user ($\bar{US}_i^A$) in group-i, since the probability density term $\frac{1}{h_i^A - l_i^A}$ assigns the equal likelihood to all user types of s_i^A within the group. The term $U_i^A(s_i^A)$ denotes the utility of a user of type s_i^A, and integrating over the interval (l_i^A, h_i^A) gives the expected surplus for a user type in group-i. A similar interpretation applies to group–j.

After solving the above integrals, the average user surplus is:

$$\bar{US}_i^A = \frac{h_i^A + l_i^A}{2} + \alpha n_i^A + \beta n_j^A - p_i^A$$
$$\bar{US}_j^A = \frac{h_j^A + l_j^A}{2} + \beta n_i^A - p_j^A$$

Under full market coverage, all users participate on both sides, so $n_i^A = n_j^A = 1$. Consequently, the aggregate user surplus for each group simplifies to the average surplus:

$$US_i^A = n_i^A \times \bar{US}_i^A = \bar{US}_i^A$$
$$US_j^A = n_j^A \times \bar{US}_j^A = \bar{US}_j^A$$

Finally, the total user surplus on the monopoly platform A is the sum of the surpluses generated by both groups:

$$US^A = US_i^A + US_j^A$$

Case 2: Partial market Coverage In the case of partial market coverage, where not all users join the platform A, a cut-off stand-alone value represented by $\hat{s}_{i,j}^A$ is determined for marginal users who are indifferent between joining and not joining. This situation arises due to the relatively high price set by the monopoly platform, which makes participation unattractive for users with lower stand-alone values. These users participate only if their utility from joining is non-negative (i.e., $U_{i,j}^A = 0$ at the cut-off stand-alone value). These values are given as follows:

$$\hat{s}_i^A = p_i^A - \alpha n_i^A - \beta n_j^A$$
$$\hat{s}_j^A = p_j^A - \beta n_i^A$$

The average user surplus for each group is obtained by integrating the utility of those users who join the platform A, which is given by:

$$\text{Group-}i: \quad \bar{US}_i^A = \int_{\hat{s}_i^A}^{h_i^A} U_i^A(s_i^A) \times \frac{1}{h_i^A - \hat{s}_i^A}\, ds_i^A$$
$$\text{Group-}j: \quad \bar{US}_j^A = \int_{\hat{s}_j^A}^{h_j^A} U_j^A(s_j^A) \times \frac{1}{h_j^A - \hat{s}_j^A}\, ds_j^A$$

In the above, $\bar{US}_i^A$ and $\bar{US}_j^A$ measure the average user surplus generated by a user in each group who joins the platform A. For group-i, the integration is performed on all users whose s_i^A is above the cut-off $\hat{s}_i^A$, that is, $s_i^A \geq \hat{s}_i^A$, since only those users join the platform A. The term $U_i^A(s_i^A)$ represents the utility of an individual user of type s_i^A. While the density $\frac{1}{h_i^A - \hat{s}_i^A}$ reflects the conditional distribution of user types *given that they join the platform*. Since only users with valuations above the threshold $\hat{s}_i^A$ participate, this density assigns equal likelihood to all joining types within the interval $[\hat{s}_i^A, h_i^A]$. Integrating the individual surplus over this interval and normalising by $\frac{1}{h_i^A - \hat{s}_i^A}$ yields the *average surplus per joining user*. The same interpretation follows for group-j.

It is useful to clarify the distinction between total user surplus and surplus per participating user. In the partial market coverage case, not all users participate due to the presence of a cut-off stand-alone value, implying that both the surplus of participating users and the number of participating users matter for total user surplus. The analysis first computes the average user surplus conditional on participation, obtained by integrating utility over the set of users who join the platform and normalising by the mass of participants, thereby isolating the surplus generated for an individual joining user.

However, for evaluating total welfare effects (including mergers), total user surplus, which also accounts for changes in the number of participating users, is the relevant object. In the partial coverage case, total consumer surplus can be obtained by multiplying the average surplus per user by the equilibrium measure of participating users, so both the intensive and extensive margins are fully captured in the welfare analysis.

After solving the above integrals, the average user surplus is as follows:

$$\bar{US}_i^A = \frac{h_i^A + \hat{s}_i^A}{2} + \alpha n_i^A + \beta n_j^A - p_i^A$$
$$\bar{US}_j^A = \frac{h_j^A + \hat{s}_j^A}{2} + \beta n_i^A - p_j^A$$

The aggregate user surplus for each group is obtained by multiplying the average surplus by the equilibrium number of users (n_i^A/n_j^A) for each group.

$$US_i^A = n_i^A \times \bar{US}_i^A$$
$$US_j^A = n_j^A \times \bar{US}_j^A$$

Finally, the total user surplus on the monopoly platform A is the sum of the surpluses generated by both groups:

$$US^A = US_i^A + US_j^A$$

The monopoly framework provides a baseline for understanding platform behaviour in the absence of competitive pressure, including how prices are set and how users participate. The next step is to analyse the competition between the two platforms and examine how this changes market outcomes. Moving from a monopoly to a two-platform model highlights how user choices, network effects, and pricing strategies evolve when users can choose between alternatives.

2.3 Two Platform Model

This section considers the competition between two platforms that serve the two user groups. Each platform sets prices for its respective user groups, and users choose the platform that provides the highest utility. That is, each user joins the platform for which their individual utility exceeds the utility they would obtain from the other platform. By modelling the strategic interaction between the platforms' pricing decisions, this framework captures how competition influences user participation, platform profits, and user surplus in the two-sided market.

2.3.1 Model Setup

In this case, consider two platforms, A and B, with $k \in \{A, B\}$, each competing to attract users from two distinct groups, i and j. Each group is assumed to have a total mass of 1. Let n_i^k and n_j^k denote the number of users in group i and j, respectively, on the platform k.

Users derive heterogeneous *stand-alone benefits* from each platform, with each valuation independently drawn from a platform-specific distribution. This allows users to distinguish between platforms based on their individual valuations. The analysis assumes full market coverage and single-homing, meaning that each user joins exactly one platform in equilibrium. These assumptions are consistent with the literature Tan and Zhou (2021), White and Weyl (2016), and **?**. Following Perloff and Salop (1985), the framework for user joining decisions is based on the product differentiation model, extended here to incorporate network effects in multi-sided digital platforms.

2.3.2 Two-stage game:

The interaction between platforms and users can be represented as a two-stage game. In the first stage, the platforms set their prices, anticipating the participation decisions of the users. In the second stage, users on both sides decide which to join, given the set of prices and expected network effects. The sequence of decisions is summarised as follows:

1. Platforms simultaneously fix prices for both groups.

2. Users in both groups decide whether to join the platform.

The above game is solved using backward induction, i.e., determining the user joining decisions on platforms based on the given prices. The resulting participation levels form a *Nash equilibrium in joining decisions*, where no user has an incentive to switch platforms. In the first stage, anticipating these equilibrium participation responses, both platforms simultaneously choose their optimal prices for the two user groups. The outcome of this two-stage game constitutes a *subgame-perfect Nash equilibrium*, since the pricing decisions are optimal given the subsequent equilibrium joining decisions of the users.

2.3.3 Payoff Functions

The utility functions for users are the following:

$$U_i^k = s_i^k + \alpha \cdot n_i^k + \beta \cdot n_j^k - p_i^k$$
$$U_j^k = s_j^k + \beta \cdot n_i^k - p_j^k$$

Here: s_i^k, s_j^k are individual-specific stand-alone values, $\alpha \cdot n_i^k$ is the direct network effects of the same group, $\beta \cdot n_{i,j}^k$ are indirect network effects of the cross-group, and p_i^k, p_j^k is the price paid by each group on the platform k.

As stated earlier, under the assumption of full market coverage, each user joins exactly one platform in equilibrium. Since each group has a unit mass of users, the number of users on a platform is equal to the proportion of the group that chooses to join it:

$$n_i^A, n_i^B \in [0,1], \quad \text{with} \quad n_i^A + n_i^B = 1$$

$$n_j^A, n_j^B \in [0,1], \quad \text{with} \quad n_j^A + n_j^B = 1$$

These expressions simply state that with all users active in the market, each user must choose either platform A or platform B. The participation shares therefore, reflect how users sort between the two platforms in equilibrium. The participation of users n_i^k and n_j^k depends on the full vector of prices set by both platforms, reflecting the strategic interaction in a two-sided market.

The payoff functions for the platforms are as follows:

$$\pi^A = p_i^A \cdot n_i^A(p_i^A, p_i^B; p_j^A, p_j^B) + p_j^A \cdot n_j^A(p_j^A, p_j^B; p_i^A, p_i^B)$$
$$\pi^B = p_i^B \cdot n_i^B(p_i^B, p_i^A; p_j^B, p_j^A) + p_j^B \cdot n_j^B(p_j^B, p_j^A; p_i^B, p_i^A)$$

The payoff functions represent the profits of platforms A and B, accounting for the revenues from both user groups and associated fixed costs. The number of users joining each platform is an endogenous outcome of user decisions and depends on the entire vector of prices offered by all platforms. The profit of each platform is the sum of the price charged to each group multiplied by the equilibrium number of users of that group. This structure captures the strategic interdependence between platforms: the demand faced by one platform is influenced not only by its own pricing decisions but also by the pricing strategies of its competitors. Platforms must therefore choose prices strategically to balance revenue generation from both sides of the market while accounting for user reactions on both sides and fixed operating costs.

2.3.4 User Joining Behaviour

The second stage of the game relates to users' participation decisions. Each user decides whether to join a platform based on the utility they would obtain, which depends not only on prices but also on the number of other users who join. Because utility is influenced by network effects, users anticipate the participation decisions of others. The equilibrium number of users on each platform is therefore determined at the point where these expectations are consistent with actual participation, that is, at the fixed point of this interaction.

To illustrate the logic of user choice in this model, consider a simple thought experiment. Suppose a user is deciding between two competing platforms A and B. Each platform provides a certain level of utility determined by three main factors: the user's stand-alone benefits of the platform, the direct and indirect network effects, and the price charged for access. A user will decide to join the platform A only if the utility obtained from it exceeds that of the platform B, that is, $U_i^A > U_i^B$. This condition can be written as-

$$s_i^A + \alpha \cdot n_i^A + \beta \cdot n_j^A - p_i^A > s_i^B + \alpha \cdot n_i^B + \beta \cdot n_j^B - p_i^B$$

is rearranged as

$$s_i^B < s_i^A + \alpha(n_i^A - n_i^B) + \beta(n_j^A - n_j^B) - (p_i^A - p_i^B) \tag{3}$$

This inequality defines the threshold for a group-i user's choice: given a stand-alone value for the platform A, only those whose valuation of the platform B lies below this threshold will prefer A. Intuitively, imagine that each user draws their stand-alone value from a distribution of the platform A; then the question becomes, what proportion of users will actually choose A? The answer depends on the probability that the above inequality holds. Let $G_i^B(\cdot)$ denote the cumulative distribution function (CDF) of s_i^B, representing the probability that a user's valuation of B falls below the threshold.

The same reasoning applies to users on the other side of the market, group-j, who will choose platform A if $U_j^A > U_j^B$. This can be written as-

$$s_j^A + \beta n_i^A - p_j^A > s_j^B + \beta n_i^B - p_j^B$$
$$\text{is rearranged as}$$
$$s_j^B < s_j^A + \beta(n_i^A - n_i^B) - (p_j^A - p_j^B) \tag{4}$$

Together, inequalities (3) and (4) define the decision boundaries for users in groups i and j. The probability that a user from each group chooses the platform A can thus be expressed as-

$$\Pr(\text{user } i \text{ chooses } A \mid s_i^A) = G_i^B\left(s_i^A + \alpha(n_i^A - n_i^B) + \beta(n_j^A - n_j^B) - (p_i^A - p_i^B)\right)$$
$$\Pr(\text{user } j \text{ chooses } A \mid s_j^A) = G_j^B\left(s_j^A + \beta(n_i^A - n_i^B) - (p_j^A - p_j^B)\right)$$

The above expressions represent the probability that a user from group-i or j chooses the platform A over B, conditional on their stand-alone value for A. These probabilities are computed using the CDFs of the corresponding stand-alone values for platform B. For group-i, the probability reflects the adjusted difference in utility between A and B, including network effects and price differences. Similarly, for group-j, the probability incorporates indirect network effects from group-i and relative pricing. These expressions capture how user platform choices depend not only on their individual valuations but also on the strategic decisions of both platforms and the resulting user distributions.

Returning to the thought experiment, once the individual decision rules are established, the next step is to determine what fraction of users actually join the platform A. Since each user's stand-alone value s_i^A (or s_j^A) is drawn from a continuous distribution with probability density function (PDF) $g_i^A(s_i^A)$ (or $g_j^A(s_j^A)$), the overall proportion of users choosing the platform A corresponds to the probability-weighted share of users whose valuations satisfy the choice condition ($U_i^A > U_i^B$). This means that a user joins the platform A only if his stand-alone value for B, denoted s_i^B, falls below the threshold determined by the difference in utilities between the two platforms specifically. The integral therefore sums, over all possible stand-alone values ($s_i^A \in [0, 1]$), the probability that this condition holds. In this context, the CDF $G_i^B(\cdot)$ (or $G_j^B(\cdot)$) gives the probability that a user's valuation for platform B is below the threshold implied by the utility comparison, reflecting the likelihood of choosing the platform A. Hence, the proportion of users joining platform A is given by:

$$n_i^A = \int_0^1 G_i^B(\cdot) \times g_i^A(s_i^A) \times ds_i^A$$
$$n_j^A = \int_0^1 G_j^B(\cdot) \times g_j^A(s_j^A) \times ds_j^A$$

The above expressions yield the share of users in groups i and j who join the platform A. Each user's stand-alone value is independently drawn from a continuous distribution on $[0, 1]$, with $g_i^A(s_i^A)$ and $g_j^A(s_j^A)$ denoting the corresponding PDFs, and $G_i^B(\cdot)$ and $G_j^B(\cdot)$ the CDFs for platform B. The term $G_i^B(\cdot)\, g_i^A(s_i^A)$ (or $G_j^B(\cdot)\, g_j^A(s_j^A)$) captures the probability that a user with a given stand-alone value for A prefers it to B, weighted by the likelihood of that valuation. Integrating over all possible values yields the overall proportion of users in each group who choose the platform A.

Finally, those users who do not join A will join the platform B under the assumption of full market coverage and are given by:

$$n_i^B = 1 - n_i^A, \quad n_j^B = 1 - n_j^A$$

Equilibrium Analysis: To compute the equilibrium shares of users joining each platform, it is assumed that the stand-alone values $s_{i,j}^k$ are independently drawn from uniform distributions with support $[l_{i,j}^A, h_{i,j}^A]$. In both the two and three platform models, the platforms are considered *symmetric* if their stand-alone values are drawn from the same uniform distribution; otherwise, they are *asymmetric*. Under this assumption, the CDFs and PDFs of the platform

A are given by:

$$G_i^B(\cdot) = \frac{s_i^A + \alpha(n_i^A - n_i^B) + \beta(n_j^A - n_j^B) - (p_j^A - p_B^A) - l_i^B}{h_i^B - l_i^B}$$

$$G_j^B(\cdot) = \frac{s_j^A + \beta(n_i^A - n_i^B) - (p_j^A - p_j^B) - l_j^B}{h_j^B - l_j^B}$$

$$g_i^A(s_i^A) = \frac{1}{h_i^A - l_i^A}; \qquad g_j^A(s_j^A) = \frac{1}{h_j^A - l_j^A}$$

The CDFs are evaluated at the thresholds implied by the utility comparison $U_i^A > U_i^B$ and $U_j^A > U_j^B$. These functional forms are then substituted into the general expressions for the proportions of users in each group who join the platform A.

$$n_i^A = \int_{l_i^A}^{h_i^A} \frac{s_i^A + \alpha(n_i^A - n_i^B) + \beta(n_j^A - n_j^B) - (p_i^A - p_i^B) - l_i^B}{(h_i^B - l_i^B)(h_i^A - l_i^A)} \times \frac{1}{h_i^A - l_i^A} \times ds_i^A$$

$$n_j^A = \int_{l_j^A}^{h_j^A} \frac{s_j^A + \beta(n_i^A - n_i^B) - (p_j^A - p_j^B) - l_j^B}{(h_j^B - l_j^B)(h_j^A - l_j^A)} \times \frac{1}{h_j^A - l_j^A} \times ds_j^A$$

$$n_i^B = 1 - n_i^A, \quad n_j^B = 1 - n_j^A$$

The above expressions describe a system of fixed-point conditions, as the participation levels themselves appear on both sides of the equations. These will be solved simultaneously to arrive at $n_i^A(p_i^A, p_i^B; p_j^A, p_j^B)$, $n_j^A(p_j^A, p_j^B; p_i^A, p_i^B)$, $n_i^B(p_i^B, p_i^A; p_j^B, p_j^A)$, and $n_j^B(p_j^B, p_j^A; p_i^B, p_i^A)$, the results are as follows:

$$n_i^A = \frac{\alpha h_j^B - 2\beta h_j^A + 2\beta h_j^B - 2\alpha l_j^B - 2\beta l_j^A + 2\beta l_j^B - h_i^A h_j^B + 4\beta p_j^A - 4\beta p_j^B + h_i^A l_j^B - h_j^B l_i^A + 2h_j^B l_i^B + l_i^A l_j^B - 2l_i^B l_j^B + 2h_j^B p_i^A - 2h_j^B p_i^B - 2l_j^B p_i^A + 2l_j^B p_i^B + 4\beta^2}{2\left(2\alpha h_j^B - 2\alpha l_j^B - h_i^B h_j^B + h_i^B l_j^B + h_j^B l_i^B - l_i^B l_j^B + 4\beta^2\right)}$$

$$n_j^A = \frac{2\alpha h_j^A - 2\beta h_i^A + 2\beta h_i^B + 2\alpha l_j^A - 4\alpha l_j^B - 2\beta l_i^A + 2\beta l_i^B - h_i^B h_j^A - 4\alpha p_j^A + 4\alpha p_j^B - h_i^B l_j^A + h_j^A l_i^B + 2h_i^B l_j^B + 2h_i^B p_j^A - 2h_i^B p_j^B + l_i^B l_j^A - 2l_i^B l_j^B + 4\beta p_i^A - 4\beta p_i^B - 2l_i^B p_j^A + 2l_i^B p_j^B + 4\beta^2}{2\left(2\alpha h_j^B - 2\alpha l_j^B - h_i^B h_j^B + h_i^B l_j^B + h_j^B l_i^B - l_i^B l_j^B + 4\beta^2\right)}$$

$$n_i^B = \frac{2\alpha h_j^B + 2\beta h_j^A - 2\beta h_j^B - 2\alpha l_j^B + 2\beta l_j^A - 2\beta l_j^B + h_i^A h_j^B - 2h_i^B h_j^B - 4\beta p_j^A + 4\beta p_j^B - h_i^A l_j^B + h_j^B l_i^A + 2h_i^B l_j^B - l_i^A l_j^B - 2h_j^B p_i^A + 2h_j^B p_i^B + 2l_j^B p_i^A - 2l_j^B p_i^B + 4\beta^2}{2\left(2\alpha h_j^B - 2\alpha l_j^B - h_i^B h_j^B + h_i^B l_j^B + h_j^B l_i^B - l_i^B l_j^B + 4\beta^2\right)}$$

$$n_j^B = \frac{2\alpha h_j^A - 4\alpha h_j^B - 2\beta h_i^A + 2\beta h_i^B + 2\alpha l_j^A - 2\beta l_i^A + 2\beta l_i^B - h_i^B h_j^A + 2h_i^B h_j^B - 4\alpha p_j^A + 4\alpha p_j^B - h_i^B l_j^A + h_j^A l_i^B - 2h_j^B l_i^B + 2h_i^B p_j^A - 2h_i^B p_j^B + l_i^B l_j^A + 4\beta p_i^A - 4\beta p_i^B - 2l_i^B p_j^A + 2l_i^B p_j^B - 4\beta^2}{2\left(2\alpha l_j^B - 2\alpha h_j^B + h_i^B h_j^B - h_i^B l_j^B - h_j^B l_i^B + l_i^B l_j^B - 4\beta^2\right)}$$

The above expressions represent the equilibrium user levels for any given set of prices, making it possible to analyse how user participation responds to changes in prices across groups.

Next comes the first stage of the game, which involves determining the equilibrium prices for both groups. The equilibrium prices are obtained by maximising each platform's profit function twice with respect to its own prices of the respective groups, given the prices of the other platform, which yields the best response functions. The equilibrium is then found at the fixed point where pricing strategies of both platforms are the mutual best responses.

$$\pi^A = n_i^A(p_i^A, p_i^B; p_j^A, p_j^B) \cdot p_i^A + n_j^A(p_j^A, p_j^B; p_i^A, p_i^B) \cdot p_j^A;$$
$$\pi^B = n_i^B(p_i^B, p_i^A; p_j^B, p_j^A) \cdot p_i^B + n_j^B(p_j^B, p_j^A; p_i^B, p_i^A) \cdot p_j^B;$$

Each platform chooses prices to maximise its profit, taking the rival platform's prices as given. Differentiating the profit functions π^A and π^B with respect to their respective prices p_i^A, p_j^A, p_i^B, and p_j^B yields the first-order conditions for profit maximisation. Solving these conditions gives the best response functions for each platform. During the optimisation of the profit function with respect to the prices of each group of both platforms $(p_i^A, p_j^A, p_i^B, p_j^B)$. The Nash equilibrium prices are obtained at the point where both platforms' best responses coincide. Substituting the equilibrium user shares into these first-order conditions and simplifying yields the following equilibrium prices:

$$p_i^A = \frac{h_i^A}{6} - \beta - \alpha + \frac{h_i^B}{3} + \frac{l_i^A}{6} - \frac{2l_i^B}{3}$$
$$p_j^A = \frac{h_j^A}{6} - \beta + \frac{h_j^B}{3} + \frac{l_j^A}{6} - \frac{2l_j^B}{3}$$
$$p_i^B = \frac{2h_i^B}{3} - \beta - \frac{h_i^A}{6} - \alpha - \frac{l_i^A}{6} - \frac{l_i^B}{3}$$
$$p_j^B = \frac{2h_j^B}{3} - \frac{h_j^A}{6} - \beta - \frac{l_j^A}{6} - \frac{l_j^B}{3}$$

After determining the prices, they will be substituted in the second stage of the game to determine the equilibrium number of users, and are as follows:

$$n_i^A = \frac{6\alpha(h_j^B - l_j^B) + 2\beta(h_j^B - h_j^A + l_j^B - l_j^A) + 12\beta^2 + h_i^A(l_j^B - h_j^B) + h_i^B(2l_j^B - 2h_j^B) + l_i^A(l_j^B - h_j^B) + l_i^B(4h_j^B - 4l_j^B) + l_i^A l_j^B - 4l_i^B l_j^B}{6\left(2\alpha(h_j^B - l_j^B) - h_i^B(h_j^B - l_j^B) + h_j^B l_i^B - l_i^B l_j^B + 4\beta^2\right)}$$

$$n_j^A = \frac{2\alpha(h_j^A + 2h_j^B + l_j^A - 4l_j^B) + 2\beta(h_i^B - h_i^A + l_i^B - l_i^A) - h_i^B(h_j^A + l_j^A + 2h_j^B - 4l_j^B) + l_i^B(h_j^A + 2h_j^B + l_j^A - 4l_j^B) + 12\beta^2}{6\left(2\alpha(h_j^B - l_j^B) - h_i^B(h_j^B - l_j^B) + h_j^B l_i^B - l_i^B l_j^B + 4\beta^2\right)}$$

$$n_i^B = \frac{6\alpha(h_j^B - l_j^B) + 2\beta(h_j^A + l_j^A - l_j^B) - 2bh_j^B - 6al_j^B + 12\beta^2 + h_i^A(h_j^B - l_j^B) + h_i^B(4l_j^B - 4h_j^B) + l_i^A(h_j^B - l_j^B) + l_i^B(2h_j^B - 2l_j^B) - l_i^A l_j^B - 2l_i^B l_j^B}{6\left(2\alpha(h_j^B - l_j^B) - h_i^B(h_j^B - l_j^B) + h_j^B l_i^B - l_i^B l_j^B + 4\beta^2\right)}$$

$$n_j^B = \frac{2\alpha(h_j^A - 4h_j^B + l_j^A + 2l_j^B) + 2\beta(h_i^B - h_i^A + l_i^B - l_i^A) - 12\beta^2 + h_i^B(h_j^A + l_j^A - 4h_j^B - 2l_j^B) + l_i^B(h_j^A + l_j^A - 4h_j^B - 2l_j^B) + 2l_i^B l_j^B}{6\Big(h_i^B(h_j^B - l_j^B) - 2\alpha(h_j^B - l_j^B) - h_j^B l_i^B + l_i^B l_j^B - 4\beta^2\Big)}$$

As seen above, the expressions for the equilibrium number of users are indeed complex because they arise from a two-sided fixed-point system in which each participation share depends linearly on all others. The equilibrium prices are derived in a subsequent step from the platforms' profit maximisation problems and depend on these participation shares, thereby inheriting the same interdependence. These expressions are not intended to be interpreted directly in isolation. Rather, they are the algebraic solution to the system of participation conditions. Given the linear structure of utility and the assumption of uniform heterogeneity, the participation levels can be solved in closed form, but the resulting expressions are necessarily lengthy due to simultaneous feedback effects across groups and platforms.

The economically meaningful content of the model is most clearly seen at the pricing stage. Once the participation functions are substituted into the platforms' profit maximisation problems, the resulting first-order conditions yield linear best-response functions and closed-form equilibrium prices. These equilibrium prices admit clearer interpretation and allow for transparent comparative statics.

After the computation of the equilibrium number of users and their prices, the surplus for each group will be calculated.

2.3.5 User Surplus Calculation

Following the logic of surplus calculation in the monopoly model, each user draws their stand-alone value from a uniform distribution over the support, so all values are equally likely. The uniform distribution allows the integration, weighted by the density, to average the individual utility across all possible stand-alone values, providing a meaningful measure of expected utility for a representative user in the monopoly setting. This approach forms the foundation for extending the analysis to more complex environments, such as duopoly or multi-platform markets, where user interactions and joining thresholds become intricate. Once the average user surplus is determined, the *aggregate user surplus* can be calculated by multiplying the average surplus by the number of users on each side of the platform.

In the duopoly setting, the joining thresholds for each platform are not easily derived because a user's decision to join the platform A depends on the independently and randomly drawn stand-alone value s_i^B of the platform B. Because these values are drawn independently, users who do not join the platform A are effectively spread uniformly across the support of s_i^A. The same logic applies in reverse: whether a user joins platform B depends on the independently drawn value for the other platform, so non-joining users are likewise uniformly distributed over their respective support. This allows the average user surplus to be computed over the entire support of s_i^A, which represents the expected utility of a representative user.

$$\bar{US}_i^k = \int_{l_i^k}^{h_i^k} U_i^k(s_i^k) \times \frac{1}{h_i^k - l_i^k} \times ds_i^k, \quad k \in \{A, B\}$$
$$\bar{US}_j^k = \int_{l_j^k}^{h_j^k} U_j^k(s_j^k) \times \frac{1}{h_j^k - l_j^k} \times ds_j^k, \quad k \in \{A, B\}$$

These expressions represent the average surplus per user ($\bar{US}_{i,j}^A$) for each group, $U_i^k(s_i^k)$ and $U_j^k(s_j^k)$ represent the individual utilities of users from groups i and j on platform k. The terms l_i^k, h_i^k and l_j^k, h_j^k define the support of the uniform distribution of user valuations for each group on that platform. The factor $1/(h_i^k - l_i^k)$ (or $1/(h_j^k - l_j^k)$) is the probability density under the uniform distribution, ensuring that the integral captures the average utility of a user in the group.

After solving the above integrals, the average user surplus is:

$$\bar{US}_i^k = \frac{h_i^k + l_i^k}{2} + \alpha n_i^k + \beta n_j^k - p_i^k, \quad k \in \{A, B\}$$
$$\bar{US}_j^k = \frac{h_j^k + l_j^k}{2} + \beta n_i^k - p_j^k, \quad k \in \{A, B\}$$

Since the total population is normalised to 1, the above expression represents the *average user surplus* for each group on the platform k. Recall that in the monopoly model, the average user surplus is defined as the integral of individual surpluses over a unit mass of users, and the same interpretation applies in the duopoly setting. The probability density function $g_{i,j}^k(s_{i,j}^k) = 1/(h_{i,j}^k - l_{i,j}^k)$

weights the contribution of each user in the population. Multiplying by n_i^k or n_j^k accounts for the share of users who actually join each platform, ensuring that the calculation of aggregate surplus remains consistent with the monopoly framework. Consequently, the aggregate user surplus for each group is then obtained by multiplying the average surplus by the equilibrium number of users, n_i^k or n_j^k, on each platform:

$$US^k = n_i^k \times US_i^k + n_j^k \times US_j^k, \quad k \in \{A, B\}$$

where n_i^k and n_j^k denote the shares of users from groups i and j who choose platform k (so that $n_i^A + n_i^B = 1$ and $n_j^A + n_j^B = 1$).

The total user surplus under the duopoly is then given by the sum of the surpluses across both platforms:

$$US_{\text{total}} = US^A + US^B$$

Building on the duopoly framework, the next step is to analyse the competition among the three platforms. Moving from two to three platforms allows for examination of how user choices, network effects, and pricing strategies adapt when additional alternatives are available. This extension highlights how market outcomes, such as user surplus, participation shares, and platform pricing, are influenced by the presence of an additional platform, and provides a foundation for understanding more general multi-platform markets.

2.4 Three Platform Model

The analysis is extended to a market with three platforms to capture richer competitive interactions, including asymmetric platforms. This three-platform model builds on the framework developed for duopoly and monopoly settings, generalising the equilibrium analysis of prices and user surplus. The addition of a third platform allows examination of how market consolidation, platform heterogeneity, and strategic interactions influence overall welfare and platform strategies. It also provides a foundation for analysing the effects of mergers or entry into more complex digital platform ecosystems.

2.4.1 Model setup

Consider three digital platforms, denoted A, B, and C, where $k \in \{A, B, C\}$. Each platform simultaneously serves two distinct groups: group i and group j, where $i \neq j$. Each user independently draws their stand-alone value from each platform's distribution and decides to join the platform that provides the highest individual utility. Let n_i^k and n_j^k denote the aggregate number of users of the groups i and j, respectively, who choose the platform k.

2.4.2 Two-stage game:

The interaction between platforms and users is modelled as a two-stage game. Formally, the sequence of decisions is as follows:

1. Platforms simultaneously fix prices for both groups.

2. Users in both groups decide whether to join the platform.

The above game is solved by using backward induction, i.e., determining the platform joining decisions of users based on the given prices. Once users have joined, the platform sets the price for both groups. The equilibrium of this game is a *Nash equilibrium*, where each platform's pricing strategy is the best response to the other platform's strategy, and no user can improve their utility by unilaterally switching platforms given the prices.

2.4.3 Payoff Functions

The payoff functions for the users are given by:

$$U_i^k = s_i^k + \alpha \cdot n_i^k + \beta \cdot n_j^k - p_i^k \qquad ; \quad U_j^k = s_j^k + \beta \cdot n_i^k - p_j^k$$

The utility function $(U_{i,j}^k)$ is made up of the stand-alone benefits $(s_{i,j}^k)$ of the platform adjusted upward by the network effects: direct network effects represented by $\alpha \cdot n_i^k$, and indirect network effects denoted by $\beta \cdot n_j^k$, and adjusted downward by the prices $(p_{i,j}^k)$ charged by the platform to each user group.

The payoff functions for the three platforms are as follows:

$$\begin{aligned}
\pi^A &= n_i^A(p_i^A, p_i^B, p_i^C; p_j^A, p_j^B, p_j^C) \cdot p_i^A + n_j^A(p_j^A, p_j^B, p_j^C; p_i^A, p_i^B, p_i^C) \cdot p_j^A; \\
\pi^B &= n_i^B(p_i^B, p_i^A, p_i^C; p_j^B, p_j^A, p_j^C) \cdot p_i^B + n_j^B(p_j^B, p_j^A, p_j^C; p_i^B, p_i^A, p_i^C) \cdot p_j^B; \\
\pi^C &= n_i^C(p_i^C, p_i^A, p_i^B; p_j^C, p_j^A, p_j^B) \cdot p_i^C + n_j^C(p_j^C, p_j^A, p_j^B; p_i^C, p_i^A, p_i^B) \cdot p_j^C;
\end{aligned}$$

In the above, each platform earns revenue by charging prices to users on both sides. The revenue for a platform depends not only on the prices it sets for both user groups and the number of users from each group who choose to join, but also on the prices set by the competing platforms, as these influence user choices.

2.4.4 User Joining Behaviour

The joining decision of the users among different platforms closely resembles that faced by consumers in the product differentiation model, the key distinction being the presence of network effects. Consequently, the framework developed by Perloff and Salop (1985) for analysing product differentiation is adapted to fit this context. In their model, consumers choose among the n available brands based on which option maximises their individual surplus. While this original formulation does not consider network effects and is primarily suited for single-sided markets, the present analysis extends it to incorporate network effects relevant to multi-sided digital platforms.

When users are faced with multiple competing platforms, each user evaluates the utility of joining each platform and joins the platform that provides the highest utility. In this setting, there are three platforms A, B, and C. All users belonging to either group-i or group-j, choose to join exactly one platform that provides the highest utility over the others, consistent with the full market coverage assumption. Once individual user preferences are established, they can be aggregated to obtain the total number of users on each platform.

Consider a thought experiment in which a user compares his utility from platform A with that from B, and again from A with that from C. A user decides to join the platform A only if it provides the highest utility relative to both B and C for a given draw from the distribution. Formally, a user joins A

over B when his stand-alone value from B falls below a certain threshold, and similarly joins A over C when his draw from C is below the corresponding threshold.

These comparisons imply that some users prefer the platform A over B, others prefer A over C, and analogous pairwise preferences arise for platforms B and C. This establishes a pairwise joining decision structure across platforms: A versus B and A versus C; B versus A and B versus C; and C versus A and C versus B. A user joins the platform A only if both relevant pairwise conditions are satisfied simultaneously, that is, if A is preferred to both B and C. The same logic applies symmetrically to the other platforms.

The aggregate number of users joining a platform is then obtained as the product of the relevant pairwise user counts, reflecting the fact that users must pass all pairwise preference comparisons simultaneously. If the distributions of stand-alone values are identical across platforms, the equilibrium is symmetric; if they differ, each platform attains an asymmetric equilibrium.

To determine the number of users on each platform, users' joining decisions are modelled through pairwise comparisons between platforms. For group-i: Let q_i^A be the number of users in the group-i who decide to join the platform A over platform B. Let q_i^B be the number of users in group-i who decide to join the platform B to C. Let q_i^C be the number of users in the group-i who decide to join the platform C to A. Let r_i^A be the number of users in the group-i who decide to join the platform A to C. Let r_i^B be the number of users in the group-i who decide to join the platform B to the platform A. Let r_i^C be the number of users in the group-i who decide to join the platform C to platform B.

A user in group-i joins the platform A if the utility from A exceeds the utility from both competing platforms B and C. The same logic applies to platforms B and C. Under these joining conditions, the pairwise numbers of users are computed by solving them simultaneously. The aggregate number of group-i users on each platform is then obtained as the product of the relevant pairwise user counts. Specifically, the total number of users in the group-i joining platform A depends on q_i^A and r_i^A; those joining platform B depend on q_i^B and r_i^B; and those joining platform C depend on q_i^C and r_i^C.

Similarly, for group-j: Let q_j^A be the number of users in the group-j who decide to join the platform A to B. Let q_j^B be the number of users in the group-j who decide to join the platform B to C. Let q_j^C be the number of users in the group-j who decide to join the platform C to A. Let r_j^A be the number of users in the group-j who decide to join the platform A to C. Let r_j^B be the number of users in the group-j who decide to join the platform B to A. Let r_j^C be the number of users in the group-j who decide to join the platform C to B.

A user in group-j joins the platform A if it provides higher utility than both competing platforms B and C; the same condition applies to the platforms B and C. Based on these joining decisions, the pairwise numbers of users are computed by solving them simultaneously. Analogously to group-i, the aggregate number of group-j users on each platform is then computed as the product of the relevant pairwise user counts. Specifically, the total number of users in the group-j joining platform A depends on q_j^A and r_j^A; those joining platform B depend on q_j^B and r_j^B; and those joining platform C depend on q_j^C and r_j^C.

The pairwise joining decision structure can be summarised in the table 2.1 and 2.2 below, which shows how the number of users joining one platform over another contributes to the aggregate number of users on each platform. The pairwise comparison approach follows the framework of Perloff and Salop (1985), where consumers choose the option that yields the highest surplus. In the present extension to a multi-platform environment, users compare all available platforms and select the one that provides the highest utility. The pairwise comparisons therefore serve as a tractable device to represent these global comparisons.

The approach considered above is intended to provide a step-by-step exposition of the pairwise decision structure underlying the multi-platform choice problem, as also represented in the table for ease of interpretation. While the method is presented in a pairwise-comparison form for clarity and tractability, it is equivalent to a standard discrete choice formulation under full market coverage.

An alternative modelling strategy would be to adopt a Salop-circle framework. However, the current approach already delivers closed-form expressions while preserving the role of network effects in a transparent manner. Extending the framework to alternative spatial representations is a natural direction for future research.

Table 2.1: Pairwise User Numbers and Aggregate Participation for group-i

Platform	**Pairwise Numbers**	**Aggregate Users**
A	q_i^A (A over B), r_i^A (A over C)	$n_i^A = q_i^A \times r_i^A$
B	q_i^B (B over C), r_i^B (B over A)	$n_i^B = q_i^B \times r_i^B$
C	q_i^C (C over A), r_i^C (C over B)	$n_i^C = q_i^C \times r_i^C$

Table 2.2: Pairwise User Numbers and Aggregate Participation for group-j

Platform	**Pairwise Numbers**	**Aggregate Users**
A	q_j^A (A over B), r_j^A (A over C)	$n_j^A = q_j^A \times r_j^A$
B	q_j^B (B over C), r_j^B (B over A)	$n_j^B = q_j^B \times r_j^B$
C	q_j^C (C over A), r_j^C (C over B)	$n_j^C = q_j^C \times r_j^C$

2.4.4.1 Joining Decision for Platform A

Building on the pairwise comparison logic, a user in the group-i will join the platform A if the utility derived from A exceeds the utility from both competing platforms B and C. Formally, this captures the outcome of the thought experiment described above: a user compares his utility from A against those from B and C. The condition for choosing platform A is therefore given by-

$$U_i^A > U_i^B \quad \text{and} \quad U_i^A > U_i^C$$

The expressions are as follows:

$$s_i^A + \alpha \cdot q_i^A + \beta \cdot q_j^A - p_i^A > s_i^B + \alpha \cdot q_i^B + \beta \cdot q_j^B - p_i^B$$
$$\text{and}$$
$$s_i^A + \alpha \cdot r_i^A + \beta \cdot r_j^A - p_i^A > s_i^C + \alpha \cdot r_i^C + \beta \cdot r_j^C - p_i^C$$

Similarly, a user in group j will join A if:

$$U_j^A > U_j^B \quad \text{and} \quad U_j^A > U_j^C$$

$$s_j^A + \beta \cdot q_i^A - p_j^A > s_j^B + \beta \cdot q_i^B - p_j^B$$
$$\text{and}$$
$$s_j^A + \beta \cdot r_i^A - p_j^A > s_j^C + \beta \cdot r_i^C - p_j^C$$

In the above, a user evaluates their utility on each platform and joins A if both inequalities are satisfied simultaneously. That is, the utility of A must exceed the utility of *each* competing platform. The first inequality compares the utility of platform A with that of platform B ($A \succ B$), while the second compares platform A to platform C ($A \succ C$).

After the rearrangement, the inequalities are as follows:

$$s_i^B < s_i^A + \alpha\,(q_i^A - q_i^B) + \beta\,(q_j^A - q_j^B) - (p_i^A - p_i^B) \tag{5}$$

and

$$s_i^C < s_i^A + \alpha\,(r_i^A - r_i^C) + \beta\,(r_j^A - r_j^C) - (p_i^A - p_i^C) \tag{6}$$

$$s_j^B < s_j^A + \beta\,(q_i^A - q_i^B) - (p_j^A - p_j^B) \tag{7}$$

and

$$s_j^C < s_j^A + \beta\,(r_i^A - r_i^C) - (p_j^A - p_j^C) \tag{8}$$

Inequalities (5), (6), (7) and (8) describe, given $s_{i,j}^A$, the inequalities that define the threshold values for $s_{i,j}^B$ and $s_{i,j}^C$, below which platform A is strictly preferred. The terms $q_{i,j}^A$ and $r_{i,j}^A$ represent pairwise comparison outcomes, appearing in these expressions should be interpreted as outcomes of a thought experiment: they represent the number of users who would join one platform over another when platforms are compared pairwise, keeping the user's draw for the third platform fixed. They are not the actual market shares ($n_{i,j}^A$) of platform A; rather, they are intermediate objects used to express the utility differences in a tractable way. To clarify the role of the pairwise comparison terms, it appears that users consider only pairwise subsets of platforms (e.g., through the q's and r's) this is a matter of exposition rather than substance. These terms are not interpreted as final market shares but as intermediate objects used to express utility differences between platforms. The final participation levels are obtained by imposing the condition that a platform is chosen only if it delivers the highest utility relative to all alternatives simultaneously. Hence, the approach remains fully consistent with global choice over multiple platforms and does not rely on incomplete pairwise optimisation. The same method is followed for the other two platforms, and these terms are then solved simultaneously to arrive at the market shares for each platform.

In the above formulation, the terms $(q_i^A - q_i^B)$, $(r_i^A - r_i^C)$, $(q_j^A - q_j^B)$, and $(r_j^A - r_j^C)$ capture the net number of users in each group who join the platform A in the relevant pairwise comparisons. Specifically, $(q_i^A - q_i^B)$ represents the net number of group-i users who join the platform A over B, based on the pairwise comparison between A and B. Similarly, $(r_i^A - r_i^C)$ reflects the net number of group-i users who join the platform A over C, while $(q_j^A - q_j^B)$ and $(r_j^A - r_j^C)$ describe the corresponding net number of group-j users who join the platform A over B and C. The same strategy is followed for the other two platforms, and these differences effectively summarise the outcome of the pairwise comparisons, which are then used to determine the aggregate number of users joining each platform.

Following the logic of Perloff and Salop (1985), each user compares the utility of all available platforms and joins the one that yields the highest payoff. The likelihood that a user in group-i joins A is the joint probability that both inequalities (5) and (6) hold. This is represented as:

$$\Pr(A \succ B \mid s_i^A) = G_i^B\big(s_i^A + \alpha(q_i^A - q_i^B) + \beta(q_j^A - q_j^B) - (p_i^A - p_i^B)\big)$$

and

$$\Pr(A \succ C \mid s_i^A) = G_i^C\big(s_i^A + \alpha(r_i^A - r_i^C) + \beta(r_j^A - r_j^C) - (p_i^A - p_i^C)\big)$$

where $G_i^B(\cdot)$ and $G_i^C(\cdot)$ are the CDF of s_i^B and s_i^C, respectively. These expressions rely on the pairwise comparison, which captures the net number of users in each group who join one platform over another in the relevant comparison. Assuming independent draws of s_i^B and s_i^C, the joint probability that platform A offers higher utility than both B and C is the product of the two pairwise probabilities:

$$\Pr(A \text{ is preferred by user } i) = G_i^B(\cdot) \cdot G_i^C(\cdot)$$

This joint probability captures the likelihood that a user in group-i joins the platform A over both competing platforms B and C for a given stand-alone value s_i^A. It depends on the relative number of users $(q_i^A, q_i^B, r_i^A, r_i^C)$ and $(q_j^A, q_j^B, r_j^A, r_j^C)$ who join A over the other platforms through their influence on network effects, cross-group interactions, and differences in price. Therefore, it is not merely the net utility gaps between two platforms in isolation. Instead, the total number of users joining the platform A corresponds to the probability that A provides the highest utility among competing platforms B and C, consistent with the Perloff and Salop (1985) framework.

Similarly, for a user in the group-j, the joining decision thresholds with a given draw of a stand-alone value s_j^A are as follows:

$$\Pr(A \succ B \mid s_j^A) = G_j^B \left(s_j^A + \beta(q_i^A - q_i^B) - (p_j^A - p_j^B)\right)$$

and

$$\Pr(A \succ C \mid s_j^A) = G_j^C \left(s_j^A + \beta(r_i^A - r_i^C) - (p_j^A - p_j^C)\right)$$

Assuming independent draws of s_j^B and s_j^C, the joint probability that platform A offers higher utility than both B and C is given by the product of these two CDF evaluations:

$$\Pr(A \text{ is preferred by user } j) = G_j^B(\cdot) \cdot G_j^C(\cdot)$$

Analogously to group-i, the joint probability determines the likelihood that a user in group-j joins the platform A over both competing platforms B and C for a given stand-alone value s_j^A, the relative number of users $(q_i^A, q_i^B, r_i^A, r_i^C)$ who join the platform A over the other platforms through their influence on cross-group interactions and differences in price.

Aggregate Participation: If every user in group-i had a stand-alone value s_i^A, the probability that they join platform A over both B and C is given by the product of the two CDFs, multiplied by the corresponding probability density function (PDF). Hence, the aggregate number of group-i users who join the platform A is obtained by integrating this probability over the entire support of s_i^A, where s_i^A is drawn from a distribution with density $g_i^A(s_i^A)$ on $[0, 1]$. The expressions are given below:

$$\begin{aligned} n_i^A = \int_0^1 & G_i^B \left(s_i^A + \alpha(q_i^A - q_i^B) + \beta(q_j^A - q_j^B) - (p_i^A - p_i^B)\right) \\ & \times G_i^C \left(s_i^A + \alpha(r_i^A - r_i^C) + \beta(r_j^A - r_j^C) - (p_i^A - p_i^C)\right) \\ & \times g_i^A(s_i^A) \times ds_i^A \end{aligned}$$

The same procedure applies to group-j and to all other platforms. This approach ensures that aggregate participation reflects the joining decisions of individual users on all competing platforms.

If every user in group-j had a stand-alone value s_j^A, then the probability that a user joins the platform A over both B and C is given by the product of the two CDFs, multiplied by the corresponding PDF. By integrating this probability over the distribution $g_j^A(s_j^A)$, the aggregate number of group-j users joining the platform A is obtained:

$$
\begin{aligned}
n_j^A = \int_0^1 & G_j^B \left(s_j^A + \beta(q_i^A - q_i^B) - (p_j^A - p_j^B)\right) \\
& \times G_j^C \left(s_j^A + \beta(r_i^A - r_i^C) - (p_j^A - p_j^C)\right) \\
& \times g_j^A(s_j^A) \times ds_j^A
\end{aligned}
$$

The above integrals represent the aggregate participation on the platform A that depends on the number of users $(q_{i,j}^A, q_{i,j}^B, r_{i,j}^A, r_{i,j}^C)$ in each group who join the platform A over the other two platforms in pairwise comparisons. Hence, the equilibrium values of n_i^A and n_j^A are obtained endogenously by solving the system of equations that relate these pairwise joining counts $(q's, r's)$ to total participation on the platform A. Specifically, the numbers of users who join the A over B and A over C are computed simultaneously to determine the equilibrium values of $(q_i^A, q_i^B, r_i^A, r_i^C)$ and $(q_j^A, q_j^B, r_j^A, r_j^C)$. The same procedure is applied symmetrically for the platforms B and C. Once the pairwise number of users in each group and for each platform is determined, they are then substituted into the integrals to compute the aggregate number of users who actually join each platform. This ensures that the aggregate participation reflects the number of users who join one platform over the others in the relevant comparisons.

2.4.4.2 Joining Decision for Platform B

The joining decision rule for platform B mirrors that of platform A. Each user draws their stand-alone value $(s_{i,j}^B)$ and joins the platform B if it offers greater utility than both competitors A and C.

A group-i user joins the platform B if:

$$U_i^B > U_i^A \quad \text{and} \quad U_i^B > U_i^C$$

which yields:

$$s_i^A < s_i^B + \alpha(r_i^B - r_i^A) + \beta(r_j^B - r_j^A) - (p_i^B - p_i^A) \tag{9}$$

and

$$s_i^C < s_i^B + \alpha(q_i^B - q_i^C) + \beta(q_j^B - q_j^C) - (p_i^B - p_i^C) \tag{10}$$

Similarly, a user in the group-j joins the platform B only if:

$$U_j^B > U_j^A \quad \text{and} \quad U_j^B > U_j^C$$

which implies:

$$s_j^A < s_j^B + \beta(r_i^B - r_i^A) - (p_j^B - p_j^A) \tag{11}$$

and

$$s_j^C < s_j^B + \beta(q_i^B - q_i^C) - (p_j^B - p_j^C) \tag{12}$$

The above inequalities (9)–(12) determine the thresholds for the number of users who join the platform B to A and C. For a given draw of $s_{i,j}^B$, they define the conditions under which alternative platforms provide lower utility.

Let, $G_i^A(\cdot)$ and $\tilde{G}_i^C(\cdot)$ denote the CDF of s_i^A and s_i^C, the probabilities that group-i users join the platform B are:

$$\Pr(B \succ A \mid s_i^B) = G_i^A\left(s_i^B + \alpha(r_i^B - r_i^A) + \beta(r_j^B - r_j^A) - (p_i^B - p_i^A)\right)$$

and

$$\Pr(B \succ C \mid s_i^B) = \tilde{G}_i^C\left(s_i^B + \alpha(q_i^B - q_i^C) + \beta(q_j^B - q_j^C) - (p_i^B - p_i^C)\right)$$

Similarly, let $G_j^A(\cdot)$ and $\tilde{G}_j^C(\cdot)$ denote the CDF of s_j^A, and s_j^C, the probabilities that group-j users join the platform B are:

$$\Pr(B \succ A \mid s_j^B) = G_j^A\left(s_j^B + \beta(r_i^B - r_i^A) - (p_j^B - p_j^A)\right)$$

and

$$\Pr(B \succ C \mid s_j^B) = \tilde{G}_j^C\left(s_j^B + \beta(q_i^B - q_i^C) - (p_j^B - p_j^C)\right)$$

The joint probability that a user in the group-i or group-j joins the platform B to both A and C is given by the product of these two CDF evaluations:

$$\Pr\big(B \text{ is preferred by user } i\big) = G_i^A(\cdot) \cdot \tilde{G}_i^C(\cdot)$$
$$\Pr\big(B \text{ is preferred by user } j\big) = G_j^A(\cdot) \cdot \tilde{G}_j^C(\cdot)$$

Aggregate Participation: The aggregate number of users with stand-alone value s_i^A and s_j^A who join the platform B is obtained by integrating the corresponding joint CDFs over the distributions of s_i^B and s_j^B, using the density functions $g_i^B(s_i^B)$ and $g_j^B(s_j^B)$ on the support $[0, 1]$, as given below:

$$\begin{aligned}
n_i^B = \int_0^1 & G_i^A\big(s_i^B + \alpha(r_i^B - r_i^A) + \beta(r_j^B - r_j^A) - (p_i^B - p_i^A)\big) \\
& \times \tilde{G}_i^C\big(s_i^B + \alpha(q_i^B - q_i^C) + \beta(q_j^B - q_j^C) - (p_i^B - p_i^C)\big) \\
& \times g_i^B(s_i^B) \times ds_i^B \\
n_j^B = \int_0^1 & G_j^A\big(s_j^B + \beta(r_i^B - r_i^A) - (p_j^B - p_j^A)\big) \\
& \times \tilde{G}_j^C\big(s_j^B + \beta(q_i^B - q_i^C) - (p_j^B - p_j^C)\big) \\
& \times g_j^B(s_j^B) \times ds_j^B
\end{aligned}$$

2.4.4.3 Joining Decision for Platform C

The joining decision rule for platform C is the same as that of platforms A and B. Each user draws his stand-alone value $(s_{i,j}^C)$ and chooses the platform C if it offers greater utility than both competitors A and B. A group-i user joins the platform C if:

$$U_i^C > U_i^A \quad \text{and} \quad U_i^C > U_i^B$$

which yields:

$$s_i^A < s_i^C + \alpha(q_i^C - q_i^A) + \beta(q_j^C - q_j^A) - (p_i^C - p_i^A) \tag{13}$$

and

$$s_i^B < s_i^C + \alpha(r_i^C - r_i^B) + \beta(r_j^C - r_j^B) - (p_i^C - p_i^B) \tag{14}$$

Similarly, platform C is chosen by a group-j user if:

$$U_j^C > U_j^A \quad \text{and} \quad U_j^C > U_j^B$$

which implies:

$$s_j^A < s_j^C + \beta(q_i^C - q_i^A) - (p_j^C - p_j^A) \tag{15}$$

and

$$s_j^B < s_j^C + \beta(r_i^C - r_i^B) - (p_j^C - p_j^B) \tag{16}$$

The above inequalities (13)–(16) determine the thresholds for the number of users who join the platform C to A and B. For a given draw of $s_{i,j}^C$, they define the conditions under which alternative platforms provide lower utility.

Let, $\tilde{G}_i^A(\cdot)$ and $\tilde{G}_i^B(\cdot)$ denote the CDF of s_i^A and s_i^B, the probabilities that group-i users join the platform C are:

$$\Pr(C \succ A \mid s_i^C) = \tilde{G}_i^A\left(s_i^C + \alpha(q_i^C - q_i^A) + \beta(q_j^C - q_j^A) - (p_i^C - p_i^A)\right)$$

and

$$\Pr(C \succ B \mid s_i^C) = \tilde{G}_i^B\left(s_i^C + \alpha(r_i^C - r_i^B) + \beta(r_j^C - r_j^B) - (p_i^C - p_i^B)\right)$$

Similarly, let $\tilde{G}_j^A(\cdot)$, and $\tilde{G}_j^B(\cdot)$ denote the CDF of s_j^A, and s_j^B, the probabilities that group-j users prefer the platform C are:

$$\Pr(C \succ A \mid s_j^C) = \tilde{G}_j^A\left(s_j^C + \beta(q_i^C - q_i^A) - (p_j^C - p_j^A)\right)$$

and

$$\Pr(C \succ B \mid s_j^C) = \tilde{G}_j^B\left(s_j^C + \beta(r_i^C - r_i^B) - (p_j^C - p_j^B)\right)$$

The joint probability that a user in the group-i or group-j joins the platform C to both A and B is given by the product of these two CDF evaluations:

$$\Pr\big(C \text{ is preferred by user } i\big) = \tilde{G}_i^A(\cdot) \cdot \tilde{G}_i^B(\cdot)$$

$$\Pr\big(C \text{ is preferred by user } j\big) = \tilde{G}_j^A(\cdot) \cdot \tilde{G}_j^B(\cdot)$$

Aggregate Participation: The aggregate number of users with stand-alone value s_i^C and s_j^C who join the platform C is obtained by integrating the corresponding joint CDFs over the distributions of s_i^C and s_j^C, using the density functions $g_i^C(s_i^C)$ and $g_j^C(s_j^C)$ on the support $[0, 1]$, as given below:

$$
\begin{aligned}
n_i^C = \int_0^1 & \tilde{G}_i^A\big(s_i^C + \alpha(q_i^C - q_i^A) + \beta(q_j^C - q_j^A) - (p_i^C - p_i^A)\big) \\
& \times \tilde{G}_i^B\big(s_i^C + \alpha(r_i^C - r_i^B) + \beta(r_j^C - r_j^B) - (p_i^C - p_i^B)\big) \\
& \times g_i^C(s_i^C) \times ds_i^C \\
n_j^C = \int_0^1 & \tilde{G}_j^A\big(s_j^C + \beta(q_i^C - q_i^A) - (p_j^C - p_j^A)\big) \\
& \times \tilde{G}_j^B\big(s_j^C + \beta(r_i^C - r_i^B) - (p_j^C - p_j^B)\big) \\
& \times g_j^C(s_j^C) \times ds_j^C
\end{aligned}
$$

The aggregate number of users who join the platform C over platforms A and B, computed above, allows the full system of equations such as $(q's, r's)$ to be solved simultaneously. These solutions can then be substituted to obtain the equilibrium number of users on each platform.

Alternatively, the equilibrium number of users on the platform C, that is, any user who does not join the platform A or B must join platform C under the full market coverage assumption, is given below:

$$n_i^C = 1 - n_i^A - n_i^B; \quad n_j^C = 1 - n_j^A - n_j^B$$

Both approaches for calculating the equilibrium share of users for the platform C require solving for $(q's, r's)$, and the alternative expressions stated above provide an equivalent representation of the participation levels for the platform C once those values are determined.

The above formulations capture users' joining decisions on a platform under the assumptions of full market coverage and rational utility-maximising behaviour.

2.4.5 Equilibrium Analysis

Recall the thought experiment, the equilibrium analysis begins with the calculation of the number of users in each group for all platforms, i.e., $q^k_{i,j}$ and $r^k_{i,j}$. The CDFs defined above represent the probabilities that a group-i or group-j user with a given draw of s^k_i or s^k_j joins one platform over another. These CDF-based probabilities can be interpreted as participation numbers for each platform. In other words, the functions $q^k_{i,j}(s^k_i)$ and $r^k_{i,j}(s^k_i)$ represent the evaluations of the corresponding CDFs with respect to their relevant arguments:

$$
\begin{aligned}
q^A_i(s^A_i) &\equiv G^B_i\big(s^A_i + \alpha(q^A_i - q^B_i) + \beta(q^A_j - q^B_j) - (p^A_i - p^B_i)\big)\\
r^A_i(s^A_i) &\equiv G^C_i\big(s^A_i + \alpha(r^A_i - r^C_i) + \beta(r^A_j - r^C_j) - (p^A_i - p^C_i)\big)\\
q^B_i(s^B_i) &\equiv \tilde{G}^C_i\big(s^B_i + \alpha(q^B_i - q^C_i) + \beta(q^B_j - q^C_j) - (p^A_i - p^C_i)\big)\\
r^B_i(s^B_i) &\equiv G^A_i\big(s^B_i + \alpha(r^B_i - r^A_i) + \beta(r^B_j - r^A_j) - (p^B_i - p^A_i)\big)\\
q^C_i(s^C_i) &\equiv \tilde{G}^A_i\big(s^C_i + \alpha(q^C_i - q^A_i) + \beta(q^C_j - q^A_j) - (p^C_i - p^A_i)\big)\\
r^C_i(s^C_i) &\equiv \tilde{G}^B_i\big(s^C_i + \alpha(r^C_i - r^B_i) + \beta(r^A_j - r^C_j) - (p^C_i - p^B_i)\big)
\end{aligned}
$$

$$
\begin{aligned}
q^A_j(s^A_j) &\equiv G^B_j\big(s^A_j + \beta(q^A_i - q^B_i) - (p^A_j - p^B_j)\big)\\
r^A_j(s^A_j) &\equiv G^C_j\big(s^A_j + \beta(r^A_i - r^C_i) - (p^A_j - p^C_j)\big)\\
q^B_j(s^B_j) &\equiv \tilde{G}^C_j\big(s^B_j + \beta(q^B_i - q^C_i) - (p^B_j - p^C_j)\big)\\
r^B_j(s^B_j) &\equiv G^A_j\big(s^B_j + \beta(r^B_i - r^A_i) - (p^B_j - p^A_j)\big)\\
q^C_j(s^C_i) &\equiv \tilde{G}^A_i\big(s^C_j + \beta(q^C_i - q^A_i) - (p^C_j - p^A_j)\big)\\
r^C_j(s^C_i) &\equiv \tilde{G}^B_i\big(s^C_j + \beta(r^C_i - r^B_i) - (p^C_j - p^B_j)\big)
\end{aligned}
$$

When each user's stand-alone value is assumed to be drawn from a uniform distribution over the interval (l^k_i, h^k_i) for the group-i and (l^k_j, h^k_j) for the group-j, the CDF is given by:

$$G^k_{i,j}(x) = \frac{x - l^k_{i,j}}{h^k_{i,j} - l^k_{i,j}}, \quad \text{for } x \in [l^k_{i,j}, h^k_{i,j}]$$

where x is a point of evaluation of the CDF.

Hence, the number of users in each group for each platform is determined as follows:

$$q_i^A(s_i^A) = \left\{ q_i^A : q_i^A = \frac{s_i^A + \alpha(q_i^A - q_i^B) + \beta(q_j^A - q_j^B) - (p_i^A - p_i^B) - l_i^B}{h_i^B - l_i^B} \right\}$$

$$q_j^A(s_j^A) = \left\{ q_j^A : q_j^A = \frac{s_j^A + \beta(q_i^A - q_i^B) - (p_j^A - p_j^B) - l_j^B}{h_j^B - l_j^B} \right\}$$

$$q_i^B(s_i^B) = \left\{ q_i^B : q_i^B = \frac{s_i^B + \alpha(q_i^B - q_i^C) + \beta(q_j^B - q_j^C) - (p_i^B - p_i^C) - l_i^C}{h_i^C - l_i^C} \right\}$$

$$q_j^B(s_j^B) = \left\{ q_j^B : q_j^B = \frac{s_j^B + \beta(q_i^B - q_i^C) - (p_j^B - p_j^C) - l_j^C}{h_j^C - l_j^C} \right\}$$

$$q_i^C(s_i^C) = \left\{ q_i^C : q_i^C = \frac{s_i^C + \alpha(q_i^C - q_i^A) + \beta(q_j^C - q_j^A) - (p_i^C - p_i^A) - l_i^A}{h_i^A - l_i^A} \right\}$$

$$q_j^C(s_j^C) = \left\{ q_j^C : q_j^C = \frac{s_j^C + \beta(q_i^C - q_i^A) - (p_j^C - p_j^A) - l_j^A}{h_j^A - l_j^A} \right\}$$

$$r_i^A(s_i^A) = \left\{ r_i^A : r_i^A = \frac{s_i^A + \alpha(r_i^A - r_i^C) + \beta(r_j^A - r_j^C) - (p_i^A - p_i^C) - l_i^C}{h_i^C - l_i^C} \right\}$$

$$r_j^A(s_j^A) = \left\{ r_j^A : r_j^A = \frac{s_j^A + \beta(r_i^A - r_i^C) - (p_j^A - p_j^C) - l_j^C}{h_j^C - l_j^C} \right\}$$

$$r_i^B(s_i^B) = \left\{ r_i^B : r_i^B = \frac{s_i^B + \alpha(r_i^B - r_i^A) + \beta(r_j^B - r_j^A) - (p_i^B - p_i^A) - l_i^A}{h_i^A - l_i^A} \right\}$$

$$r_j^B(s_j^B) = \left\{ r_j^B : r_j^B = \frac{s_j^B + \beta(r_i^B - r_i^A) - (p_j^B - p_j^A) - l_j^A}{h_j^A - l_j^A} \right\}$$

$$r_i^C(s_i^C) = \left\{ r_i^C : r_i^C = \frac{s_i^C + \alpha(r_i^C - r_i^B) + \beta(r_j^C - r_j^B) - (p_i^C - p_i^B) - l_i^B}{h_i^B - l_i^B} \right\}$$

$$r_j^C(s_j^C) = \left\{ r_j^C : r_j^C = \frac{s_j^C + \beta(r_i^C - r_i^B) - (p_j^C - p_j^B) - l_j^B}{h_j^B - l_j^B} \right\}$$

The number of users $(q_{i,j}^k, r_{i,j}^k)$ who join one platform over another appears on both sides of the equation. Thus, it constitutes a **fixed point problem** at which user expectations about platform participation are exactly met by the realised outcomes. This setup therefore, defines a system of simultaneous equations, which must be solved together to find the equilibrium aggregate number of users for each platform.

Equilibrium Participation: After computing the number of users $(q^k_{i,j}, r^k_{i,j})$, these values will be substituted to determine the equilibrium aggregate number of users on each platform. The equilibrium will be calculated by aggregating these numbers of users, using the corresponding density function $g^k_i(s^k_i)$, which yields the total number of group-i users on each platform. The same construction applies to users in group-j. The user equilibria for each platform are as follows:

$$n^A_i = \int_{l^A_i}^{h^A_i} q^A_i(s^A_i) \times r^A_i(s^A_i) \times \frac{1}{h^A_i - l^A_i} \times ds^A_i$$

$$n^A_j = \int_{l^A_j}^{h^A_j} q^A_j(s^A_j) \times r^A_j(s^A_j) \times \frac{1}{h^A_j - l^A_j} \times ds^A_j$$

$$n^B_i = \int_{l^B_i}^{h^B_i} q^B_i(s^B_i) \times r^B_i(s^B_i) \times \frac{1}{h^B_i - l^B_i} \times ds^B_i$$

$$n^B_j = \int_{l^B_j}^{h^B_j} q^B_j(s^B_j) \times r^B_j(s^B_j) \times \frac{1}{h^B_j - l^B_j} \times ds^B_j$$

$$n^C_i = \int_{l^C_i}^{h^C_i} q^C_i(s^C_i) \times r^C_i(s^C_i) \times \frac{1}{h^C_i - l^C_i} \times ds^C_i \quad \text{OR} \quad n^C_i = 1 - n^A_i - n^B_i$$

$$n^C_j = \int_{l^C_j}^{h^C_j} q^C_j(s^C_j) \times r^C_j(s^C_j) \times \frac{1}{h^C_j - l^C_j} \times ds^C_j \quad \text{OR} \quad n^C_j = 1 - n^A_j - n^B_j$$

The above formulation closes the loop between the CDF-based probabilistic user behaviour and the aggregate platform participation, providing a consistent framework for the distribution of users across platforms. Further, the above expressions will be solved simultaneously to arrive at $n^A_i(p^A_i, p^B_i, p^C_i; p^A_j, p^B_j, p^C_j)$, $n^A_j(p^A_j, p^B_j, p^C_j; p^A_i, p^B_i, p^C_i)$, $n^B_i(p^B_i, p^A_i, p^C_i; p^B_j, p^A_j, p^C_j)$, $n^B_j(p^B_j, p^A_j, p^C_j; p^B_i, p^A_i, p^C_i)$, $n^C_i(p^C_i, p^A_i, p^B_i; p^C_j, p^A_j, p^B_j)$, and $n^B_j(p^C_j, p^A_j, p^B_j; p^C_i, p^A_i, p^B_i)$.

After determining the equilibrium number of users for each platform at a given set of prices, then comes the first stage of the games, i.e., the computation of the equilibrium prices for both groups. Each platform chooses prices for both user groups to maximise its profit, taking the prices set by the other platforms as given. This optimisation yields the *best response functions* for each platform.

$$\pi^A = n_i^A(p_i^A, p_i^B, p_i^C; p_j^A, p_j^B, p_j^C) \cdot p_i^A + n_j^A(p_j^A, p_j^B, p_j^C; p_i^A, p_i^B, p_i^C) \cdot p_j^A;$$
$$\pi^B = n_i^B(p_i^B, p_i^A, p_i^C; p_j^B, p_j^A, p_j^C) \cdot p_i^B + n_j^B(p_j^B, p_j^A, p_j^C; p_i^B, p_i^A, p_i^C) \cdot p_j^B;$$
$$\pi^C = n_i^C(p_i^C, p_i^A, p_i^B; p_j^C, p_j^A, p_j^B) \cdot p_i^C + n_j^C(p_j^C, p_j^A, p_j^B; p_i^C, p_i^A, p_i^B) \cdot p_j^C;$$

During the optimisation of the profit function with respect to the prices of each group of three platforms $(p_i^A, p_j^A, p_i^B, p_j^B, p_i^C, p_j^C)$, only the revenue components depend on prices and therefore determine the optimisation outcome. The best response functions describe the profit-maximising price choices of each platform given the prices of the competitors.

The equilibrium prices are then obtained as the *mutually consistent best responses*, i.e., the price combination $(p_i^A, p_j^A, p_i^B, p_j^B, p_i^C, p_j^C)$ where each platform's choice is to maximise its profit given the choices of the others. Once these equilibrium prices are determined, they are substituted back into the second stage to compute the equilibrium number of users, yielding the full set of equilibrium outcomes for both prices and participation.

After the computation of the equilibrium number of users and their prices, the surplus for each group will be calculated.

2.4.6 User Surplus Calculation

Following the logic of the user surplus calculation in the duopoly model, the average user surplus is defined as the expected utility of a user, given that the total population is normalised to 1. The uniform distribution assumption is carried forward to the triopoly setting, where the users are uniformly spread over the distribution. The threshold value s_i^A that determines whether a user joins the platform A depends on the independently and randomly drawn stand-alone values s_i^B and s_i^C. Because these values are drawn independently, users who do not join the platform A are effectively spread uniformly across the support of s_i^A. The same logic applies in reverse: whether a user joins platforms B or C depends on the independently drawn values for the other platforms, so non-joining users are likewise uniformly distributed over their respective supports. This allows the average user surplus to be computed over the entire support of s_i^A, which represents the expected utility of a representative user. Consequently, all user shares and integrals are interpreted relative to this unit mass, and the integrals are later adjusted by the proportion of users who actually join each platform.

For each group i and j on platform $k \in \{A, B, C\}$, the average user surplus $\bar{US}_{ij}^k$ is computed as:

$$\bar{US}_i^k = \int_{l_i^k}^{h_i^k} U_i^k(s_i^k) \times \frac{1}{h_i^k - l_i^k} \times ds_i^k, \quad k \in \{A, B, C\}$$
$$\bar{US}_j^k = \int_{l_j^k}^{h_j^k} U_j^k(s_j^k) \times \frac{1}{h_j^k - l_j^k} \times ds_j^k, \quad k \in \{A, B, C\}$$

Here, $U_i^k(s_i^k)$ and $U_j^k(s_j^k)$ are the individual utilities of users from groups i and j on platform k, and $[l_i^k, h_i^k]$ and $[l_j^k, h_j^k]$ define the support of the uniform distribution of valuations. The factor $1/(h_i^k - l_i^k)$ (or $1/(h_j^k - l_j^k)$) is the probability density under the uniform distribution, ensuring the integral gives the expected utility over the group.

After solving the above integrals, the average user surplus is:

$$\bar{US}_i^k = \frac{h_i^k + l_i^k}{2} + \alpha \cdot n_i^k + \beta \cdot n_j^k - p_i^k \quad k \in \{A, B, C\}$$
$$\bar{US}_j^k = \frac{h_j^k + l_j^k}{2} + \beta \cdot n_i^k - p_j^k \quad k \in \{A, B, C\}$$

Since the total population is normalised to 1, these integrals directly yield the *average user surplus* for each group on platform k. To compute the aggregate user surplus for the actual users on the platform k, the result is multiplied by the corresponding user shares:

$$US^k = n_i^k \times \bar{US}_i^k + n_j^k \times \bar{US}_j^k, \quad k \in \{A, B, C\},$$

where n_i^k and n_j^k denote the fractions of users from group-i and group-j joining the platform k (under the full market coverage assumption, $n_i^A + n_i^B + n_i^C = 1$ and $n_j^A + n_j^B + n_j^C = 1$).

Finally, the total user surplus in the triopoly setting is obtained by summing across all platforms:

$$US_{\text{total}} = US^A + US^B + US^C$$

The above formulation is used for the surplus computation in the numerical analysis within each market structure considered, including the monopoly, duopoly, and triopoly cases.

Due to the presence of multiple unknown and interdependent variables in the equilibrium expressions for users and prices, a closed-form analytical solution is not tractable. Accordingly, numerical simulations are implemented in MATLAB to explore how the equilibrium and properties of the model depend on various parameters. This approach accommodates both symmetric and asymmetric settings, allowing for a systematic examination of user participation and pricing outcomes under different upper and lower bounds of the UD and the network benefits parameters. Employing this simulation framework also facilitates the derivation of simplified expressions for comparative static analysis. The MATLAB code used in these simulations is included at the end of this thesis for reference.

Based on the outputs obtained from the MATLAB simulations, the following analysis examines user surplus (US) within a framework that incorporates the interplay of network effects central to digital platforms. The strength of the network effects is captured by the parameters α (direct network benefits) and β (indirect network benefits). By varying the values of these parameters, the analysis examines how changes in network benefits strength affect equilibrium under different market structures. The upper and lower bounds of the uniform distributions are kept fixed because they define the baseline stand-alone values of the users. Allowing these bounds to vary would change the degree of heterogeneity and confound the impact of network benefits. Holding them constant ensures that any change in the equilibrium outcomes is driven solely by network effects rather than by shifts in the intrinsic preferences of users. Hence, the core objective of this investigation is to understand how the impact of mergers varies with the strength of these network effects and, importantly, with platform symmetries and asymmetries.

The parameter values used in the numerical examples are chosen to provide a transparent and tractable benchmark, in particular by assuming uniform distributions with fixed bounds. This allows the analysis to isolate the impact of key parameters such as network effects and ad nuisance (in the third chapter) without introducing additional complexity from distributional assumptions. While alternative parameter specifications are not explicitly explored, the benchmark is selected to be representative of a broad class of distributions, so that the qualitative insights are expected to carry over beyond the specific functional form used. A systematic exploration of alternative distributions is undertaken in Chapter 2, and constitutes a natural direction for further work in the extended setting of Chapter 3.

To assess robustness, I have explored alternative parameter values within economically reasonable ranges in Chapter 2. The qualitative patterns reported in the simulations, particularly the ability of a monopoly platform to appropriate gains from stronger network effects, remain unchanged across these variations. For consistency and clarity of exposition, a single benchmark specification is presented in the text, particularly in Simulation Findings 1, 3, and 4.

At the same time, the results are not universally signed and depend on the underlying parameterisation. In particular, the impact of mergers on user surplus reflects the balance between stronger network effects and increased market power. The difference between the triopoly-to-duopoly and duopoly-to-monopoly cases arises because, in the former, competitive pressure remains and can induce price reductions that benefit users, whereas in the latter, the absence of competition allows the platform to appropriate these gains through pricing.

The analysis first examines the merger of duopoly platforms into a monopoly and then considers mergers from triopoly to duopoly.

2.5 Network Effects and Market Consolidation: From Duopoly to Monopoly through Merger

In traditional merger theory, particularly in Cournot models, a merger does not always increase the profits of the merged firm if other competitors remain in the market. This occurs because non-merging firms typically respond by expanding output, which can offset any price or cost advantages gained by the merged firm. This outcome is known as the *merger paradox*, where a merger may fail to increase profits unless it eliminates all remaining competition. Considering this phenomenon in the context of digital platforms helps illustrate how network effects and user surplus behave when competition disappears and highlights that accounting for competitors' responses to a merger scenario is crucial in understanding the effect of the merger in the new equilibrium that emerges.

When a merger between two platforms results in a monopoly, the competitive landscape changes fundamentally. This raises a critical question regarding the welfare implications of a monopoly platform market structure characterised by network effects. In the duopoly and triopoly settings, the analysis imposed full market coverage, so every user joined one of the available platforms in equilibrium. For the monopoly case, both full and partial market coverage are considered. This is not because partial coverage is unique to a monopoly, duopolists, or triopolists could also set prices that leave some users out, but because the earlier parts of the model abstracted from this possibility. Including both cases in the monopoly benchmark serves two purposes: full market coverage allows a like-for-like comparison with the multi-platform settings, while partial market coverage captures the possibility that a monopolist, having greater pricing power, may optimally choose prices that exclude some users.

A monopoly platform achieves the highest possible concentration of users, potentially strengthening network effects, but it also has full pricing power. If prices are set too high, some users may be excluded, leading to *partial market coverage*; if prices are set so that all users derive positive utility, *full market coverage* results. In the partial coverage case, total participation declines, which can reduce user surplus despite stronger network effects. To examine these outcomes, the analysis will consider two cases of market coverage.

This analysis is important because it presents a trade-off: on the one hand, user utility may increase due to maximal network effects in a consolidated user base; on the other, higher prices may reduce surplus, and possibly reduce participation. By simulating both partial and full market coverage scenarios, it is possible to examine whether the benefits of a consolidated user base outweigh the adverse effects of higher monopoly prices. This step extends the literature, including Tan and Zhou (2021), by providing a direct assessment of the welfare implications of a duopoly merger-induced monopoly in digital platform markets.

2.5.1 *Simulation Finding 1: Platform Merger from Duopoly to Monopoly*

This section is divided into two parts. The first examines how market outcomes change with different strengths of network effects. The second analyses how these network effects shape the incentives for platform consolidation.

2.5.1.1 *Impact of Changes in α and β on Market Outcomes*

In this investigation, both network benefit parameters, α and β, are increased simultaneously to illustrate their combined effect on the market outcomes. Consider a numerical example for monopoly platform A, for which upper and lower bounds are given by $h_{i,j}^{A} = 60$ and $l_{i,j}^{A} = 20$. The network benefit parameters α and β are examined within the range $[0, 5]$.

For the symmetric duopoly setting, both platforms have identical uniform distributions with $h_{i,j}^{A,B} = 60$ and $l_{i,j}^{A,B} = 20$. For the asymmetric duopoly setting, the uniform distributions of the platforms differ: $h_{i,j}^{A} = 60$, $l_{i,j}^{A} = 20$, $h_{i,j}^{B} = 50$, and $l_{i,j}^{B} = 30$. In both symmetric and asymmetric cases, the network benefit parameters α and β are examined within the range $[0, 5]$.

The analysis considers two distinct scenarios. The first scenario is *partial market coverage*, where some users are excluded because prices are high enough that they obtain non-positive utility. The second scenario is *full market coverage*, where all users participate and derive positive utility from joining the platform. In what follows, both scenarios are examined: the first allows the monopolist to freely set prices to maximise profit, even if that excludes some users, and the second assumes full market coverage. The choice between these approaches depends on the market context. Full coverage is appropriate in environments where participation by the entire user base is essential for value creation, particularly when strong network effects make exclusion unlikely. By contrast, partial coverage is more realistic in markets where platforms exercise pricing power or optimise profits by setting prices that intentionally exclude some users. Additionally, the impact of strengthening network effects is explored by systematically varying the direct (α) and indirect (β) network benefits parameters, allowing an assessment of how increasing network benefits influence user surplus and platform outcomes in both coverage scenarios. The primary objective is to compare user surplus across these two competitive structures, accounting for the strength of network effects and pricing behaviour of the monopoly platform. Further, in the numerical analysis, the parameters are varied in three ways: (i) jointly, by setting $\alpha = \beta$; (ii) by varying α while holding β fixed; and (iii) by varying β while holding α fixed. The joint variation provides a baseline that captures the overall strength of network effects, while the separate variations are used to isolate the individual contributions of direct and indirect network benefits. This combined approach ensures that the analysis both captures the aggregate impact of strengthening network effects and allows for a clearer interpretation of their separate roles in shaping the outcomes.

In the asymmetric duopoly scenario, the two platforms have different uniform distributions, but their average (mean) stand-alone value is the same. This mean-preserving approach ensures that differences in user outcomes are driven solely by the distribution spread rather than differences in average user valuations. By keeping the mean constant, it becomes possible to compare the asymmetric and symmetric duopoly with the monopoly cases, isolating the impact of changes in the network benefit parameters on equilibrium prices, participation, and user surplus.

In the case of partial market coverage, the equilibrium price remains unchanged even as network benefits are strengthened. Mathematically, this occurs because the contributions of the network benefit terms cancel out when solving the equilibrium conditions for both groups of users, leaving a term that depends only on the upper bound of the uniform distributions, which determines the price. Intuitively, stronger network effects make the platform more attractive, encouraging additional users to join and increasing overall participation without altering the price. As the value of interacting with other users increases, more users decide to join, raising participation levels on both sides of the platform. The platform benefits from higher total engagement and can earn more profit without increasing prices, while users enjoy greater surplus from being part of a larger network. In other words, the price remains fixed, but the overall activity and welfare on the platform increase as network effects strengthen. In the full market coverage case, stronger network effects lead the monopoly platform to raise prices for both groups because the increased user value is fully internalised in its pricing decision. Since all users continue to participate, changes in these parameters affect prices rather than participation. As a result, the user surplus remains unchanged as network benefits rise, while the platform's profit increases due to higher prices. Compared to the duopoly case, a monopoly consistently generates higher profits, reflecting the combined effect of consolidated user participation and network effects.

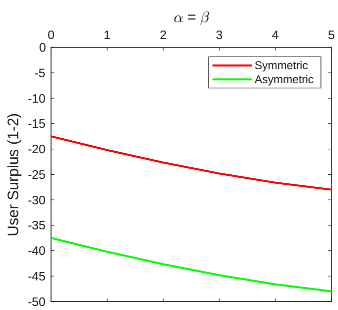
2.5.1.2 *User Surplus Comparison between Monopoly and Duopoly: Partial vs. Full Market Coverage*

After examining how changes in network effects influence market outcomes, this section considers the welfare implications of platform consolidation from a duopoly to a monopoly, using the numerical example above. This corresponds to a reduction in the number of competing platforms from two to one.

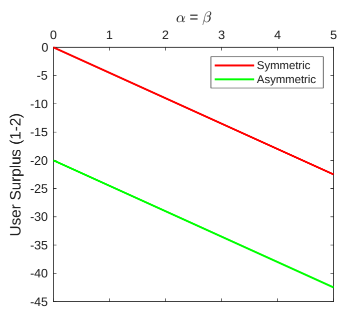
When the network benefits are strengthened, for group-i users, both α and β contribute to their overall utility, so increasing them together amplifies the total network effects for group-i relative to group-j. The equilibrium outcomes adjust: as more users join the platform, larger participation increases the value users receive, which in turn places downward pressure on equilibrium prices as the platform balances network benefits against its pricing incentives. This, in turn, raises user surplus for both groups. By considering the joint effect of α and β, this approach highlights how stronger network effects across direct and indirect parameters influence user participation, pricing, and their welfare in the platform market.

The user surplus is compared between the duopoly and the monopoly under partial market coverage. The analysis reveals that the user surplus is consistently lower in the monopoly setting compared to the duopoly, regardless of whether the platform competition is symmetric or asymmetric. This outcome is driven by the monopolist's ability to exercise greater pricing power, leading to setting a higher price for both groups. Although network effects become stronger, and it would otherwise contribute to higher user surplus, the monopolist captures the benefit of these effects by setting higher prices compared to the duopoly. Consequently, users do not fully benefit from the increased concentration of other users on the platform, resulting in a net decline in surplus relative to the duopoly setting. In the monopoly under full market coverage, as network effects strengthen, an increase in user surplus is anticipated due to increased user concentration and utility of the platform. However, the monopolist again internalises these gains through pricing: the price is adjusted upward as the network effect parameters increase, effectively absorbing the incremental utility that users would have gained. As a result, the increase in utility from stronger network effects is fully captured by the higher prices set by the monopolist, so the actual user surplus does not increase and remains constant despite stronger network effects. The net impact is that, even under full market coverage, the user surplus under monopoly does not improve with stronger network effects. Compared with the duopoly case, where competition can lead to lower prices and more favourable user surplus outcomes, the monopoly consistently underperforms in terms of user welfare.

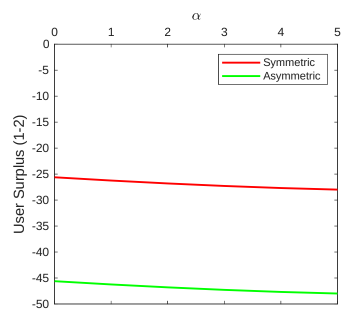
Figure 2.1(a) to (f) illustrate the differences in user surplus between the monopoly and duopoly, showing that these differences are substantially larger in the asymmetric compared to the symmetric setting. This is primarily because, in the asymmetric setting, equilibrium prices are lower than in the symmetric case, allowing users to capture a larger share of the platform's value, which increases user surplus. Building on the variations in user surplus between the monopoly under partial and full market coverage and the duopoly under asymmetric and symmetric market settings, the asymmetric duopoly configuration demonstrates that the change in user surplus resulting from a merger to a monopoly is more pronounced than in the symmetric setting. When network benefits are strengthened, this effect is further amplified, as the lower prices in the asymmetric case magnify the additional utility from network effects. Consequently, user surplus tends to be higher in the asymmetric setting, even though the mean value of the distributions is



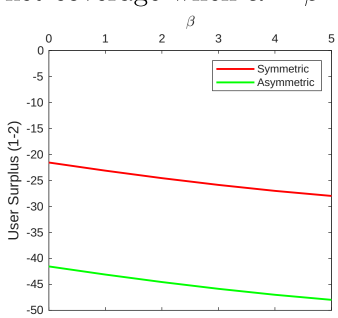
(a) US under partial market coverage when $\alpha = \beta$



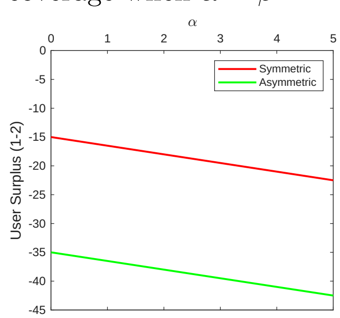
(b) US under full market coverage when $\alpha = \beta$



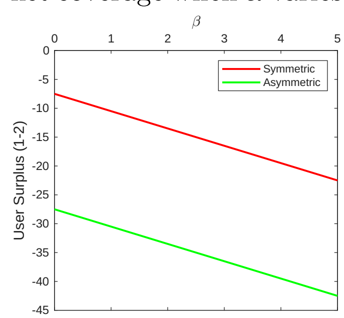
(c) US under partial market coverage when α varies



(d) US under partial market coverage when β varies



(e) US under full market coverage when α varies



(f) US under full market coverage when β varies

Figure 2.1: Differences in user surplus (US) between monopoly and duopoly scenarios under symmetric and asymmetric platform mergers as α and β increase over a given range (panels a and b). Panels (c) and (e) vary α while holding β fixed, whereas panels (d) and (f) vary β while holding α fixed, under both partial and full market coverage. Negative values indicate that user surplus is lower under monopoly, implying that a merger to monopoly is welfare-reducing for users.

the same. This highlights that platform asymmetry, together with stronger network effects, can significantly influence user welfare outcomes following a merger.

Based on the above analysis, a response to the first research question, *In both symmetric and asymmetric settings, how does a merger from duopoly to monopoly affect user surplus, and is the impact necessarily negative?* can be stated as follows. Based on the numerical simulation, the user surplus is consistently lower in the monopoly platform compared to the duopoly, both for symmetric and asymmetric platform configurations. When two competing platforms consolidate into a single dominant entity with no remaining competition, the resulting increase in market concentration reduces user surplus. Although consolidation increases network effects through the aggregation of a larger user base, these potential benefits are outweighed by the monopoly platform's ability to extract higher profits by setting higher prices under both full and partial market coverage assumptions. Furthermore, similar patterns were observed across different values of the network benefits parameters α and β, and no simulation cases were identified in which the user surplus under monopoly exceeded that of the duopoly. While this suggests a robust tendency for user surplus to decline under monopoly, the conclusion is limited to the parameter ranges explored in the simulations.

The simulation findings indicate that, despite stronger network effects, which theoretically should increase user surplus, the monopolist captures these gains through price adjustments. In the monopoly **full market coverage** scenario, prices increase in line with stronger network effects, thereby offsetting any improvement in user surplus. In the **partial market coverage** scenario, prices remain constant even as the parameters governing the network effects strengthen. This implies that although stronger network effects enhance the potential value users derive from participation, the monopoly platform does not adjust its prices in response. As a result, the improvement in network effects raises user surplus without any change in prices. Consequently, even though user participation rises on both sides of the platform, this does not translate into welfare gains under monopoly due to the monopolist's ability to appropriate these benefits through pricing strategies. Furthermore, while monopoly platform extracts significant profits due to their pricing power, competitive environments ensure a more balanced distribution of surplus, often favouring end-users.

The above insights have crucial policy implications: post-merger competition is particularly important because it limits the ability of any monopoly platform to exploit network effects solely for profit extraction. In a competitive duopoly or triopoly, platforms must balance pricing to attract and retain users, keeping prices lower and participation higher. In a competitive setting, equilibrium prices reflect both users' network effects and competitive pressure, which can result in lower prices and greater user participation among platforms compared to a monopoly, thereby enhancing user surplus. Consequently, even after a merger, maintaining multiple competing platforms helps ensure that network effects benefit users rather than being fully captured by the platform, promoting broader market participation and preventing excessive concentration of economic surplus in the hands of a single entity.

Further, this investigation complements the work of Tan and Zhou (2021), who primarily focused on competition between multiple platforms and concluded that platform consolidation from three to two platforms may not be harmful to user surplus, as long as some competition remains. They did not, however, explicitly address the welfare implications of monopoly formation. In contrast, the present study examines the welfare implications of full consolidation from a duopoly into a monopoly. The analysis reveals that user surplus declines when the number of competitors falls to one, highlighting that the U-shaped user surplus (US) pattern identified by Tan and Zhou (2021) is highly sensitive to the presence of remaining competition. This indicates that further consolidation to a monopoly does reduce user surplus, even when previous results suggested that limited consolidation might be benign. These results are consistent with empirical findings by Chandra and Collard-Wexler (2009), who demonstrate that mergers often lead to price increases, and Song (2021), who notes that mergers in platform markets tend to amplify market power, reduce product variety, and diminish consumer welfare. However, this study provides a more granular account of how monopoly platforms exercise market power, how changes in network benefits parameters affect pricing, and the broader implications for user surplus. By incorporating simulations of symmetric and asymmetric mergers, this study contributes to a more comprehensive understanding of how platform market structures impact user outcomes. It underscores the structural risks associated with the formation of a monopoly in digital platform markets and emphasises the critical role of competition in ensuring favourable outcomes for users.

After the investigation under the monopoly scenario, the next section applies the numerical simulation framework to examine how changes in oligopolistic market structure, specifically, when competition shifts from three platforms to two, affect user surplus and platform behaviour. It explores how stronger network effects influence outcomes as the market becomes more concentrated following a merger.

2.6 *Network Effects and Competitive Structure: From Triopoly to Duopoly*

This section presents simulation results examining how changes in market structure from triopoly to duopoly affect user surplus and broader platform outcomes under different levels of network benefits. The analysis considers both symmetric and asymmetric platform settings to capture how differences in size, pricing, or user composition influence the impact of mergers. The main object of interest is the change in user surplus when moving from three to two platforms, denoted $US(2-3)$, for given values of the network benefit parameters α and β. By systematically varying α and β, the analysis illustrates how the effect of consolidation on user surplus depends on the strength of network effects. When network effects are weak, mergers can reduce user surplus; when network effects are strong, mergers may increase user surplus. The subsequent Simulation Findings 2–4 examine these effects in detail, along with resulting changes in competition, prices, user participation, and profits, highlighting how outcomes differ between symmetric and asymmetric market configurations.

2.6.1 *Simulation Finding 2: Platform Merger from Triopoly to Duopoly in Symmetric Settings*

This section is divided into two parts. The first part examines how market outcomes in both the duopoly and triopoly change when one network benefit parameter is held fixed while the other is varied, and vice versa. The second part considers how these network effects influence the consequences of platform consolidation, showing that when network effects are weak, mergers can reduce user surplus, whereas when they are strong, consolidation can raise it.

2.6.1.1 *Impact of Changes in α and β on Market Outcomes*

This section examines how network effects influence platform consolidation from three to two platforms, showing that mergers can reduce user surplus when network effects are weak, but increase user surplus when network effects are strong.

To illustrate how changes in the strength of network effects impact outcomes in duopoly and triopoly platform markets, consider a numerical example with symmetric uniform distributions on the three platforms A, B, and C. The parameters are specified as follows: the upper and lower values of the distribution are $h_{i,j}^{A,B,C} = 14$ and $l_{i,j}^{A,B,C} = 2$, respectively; the direct and indirect network benefit parameters α and β are considered over the range $[0, 2]$.

The change in parameter values in this case reflects the use of a different distributional specification, rather than an arbitrary choice. The purpose is to examine whether the qualitative results depend on the underlying uniform distribution. By considering an alternative specification in Simulation Finding 2, the analysis tests the robustness of the results beyond the benchmark case. While the benchmark specification is maintained in other simulations for consistency and clarity, this variation is introduced deliberately to show that the key insights are not driven by a particular functional form.

There are two scenarios: (1) changing the parameter of direct network benefits α while keeping the value of β fixed, and (2) changing the parameter of indirect network benefits β while keeping the value of α fixed. The analysis focuses on symmetric platforms and examines how network effects influence price and user surplus when the market structure changes from a triopoly (A, B, C) to a duopoly (A, B). This structural change reflects either the entry of the platform C when A and B on the market or a merger involving the platform C with either A or B, resulting in only two platforms remaining on the market, where the platform C is essentially closed down.

In symmetric platform markets, users draw their stand-alone values from identical uniform distributions independently . While individual users prefer different platforms based on their realised draws, the platforms are identical in expectation. As a result, equilibrium prices and user shares are the same across platforms. The simulation results (see Proof for Simulation Finding 2 in Appendix) illustrate how user surplus responds to changes in direct and indirect network effects in symmetric markets following a merger. Further, depending on the strength of these network effects, equilibrium prices and user surplus vary differently under duopoly and triopoly settings.

The following summarises how changes in α and β influence equilibrium prices, participation, user surplus, and platform profits in a symmetric duopoly and triopoly. In both settings, all platforms are identical in their valuation distributions, cost structures, and strategic environments, so the equilibrium prices and user shares are the same across platforms.

Effect of Strengthening Direct Network Benefits

Duopoly: When α increases, group-i users gain a stronger direct network effects from a larger user base on their own side. Platforms respond symmetrically by reducing the price charged to group-i in order to attract more of these users. This expansion on group-i increases the platform's total user base, which indirectly raises the value for group-j users as well, even though their utility does not depend directly on α. Because the indirect network effect toward group-j strengthens through the increase in the number of group-i users, platforms have no incentive to raise or lower the price for group-j; the equilibrium price remains unchanged. As participation rises on both sides through this indirect channel, group-i user surplus increases, and overall surplus grows. Platform profits decline because the price reduction on group-i dominates the additional revenue gained from the larger user base on both sides.

Triopoly: In the symmetric triopoly, an increase in α leads all platforms to lower prices for group-i users, but the reduction is smaller than in the duopoly because the users are spread across three platforms, so the impact of stronger direct network effects on equilibrium prices is muted. For group-j, prices rise slightly because platforms compensate for the revenue reduction on group-i by adjusting prices on group-j. Since indirect network effects strengthen when α increases, group-j users become less sensitive to price, which allows platforms to raise the prices for group-j without losing their participation. As a result, group-i user surplus increases, though less sharply than in the duopoly case, while group-j surplus declines slightly due to higher prices. Platform profits initially decline as the price for group-i users falls, but additional revenue from group-j users gradually compensates for this loss. When α reaches approximately 1.5, the total profit begins to increase as the gain from group-j exceeds the loss from group-i. Symmetric equilibrium across the three platforms ensures that all platforms exhibit similar pricing and surplus patterns.

Duopoly vs. Triopoly: Based on the numerical example considered above, the increase in total user surplus is larger in duopoly than in triopoly because stronger direct network effects yield greater price reductions when user concentration is higher (two platforms instead of three).

Effect of Strengthening Indirect Network Benefits

Duopoly: When β increases, both groups benefit from stronger indirect network effects. The equilibrium prices for both groups fall and are symmetric across the platforms. The resulting reduction in prices substantially increases the user surplus for both groups. Platform profits decrease because lower prices are charged to the users in both groups.

Triopoly: In triopoly, the higher β leads all platforms to reduce prices for both groups, but the reduction is smaller than in a duopoly. Prices fall steadily to a moderate level of β, after which the rate of decline slows. Consequently, user surplus rises for both groups, though less sharply than in duopoly. Platform profits initially decrease as prices fall, but this decline is partially offset once β reaches approximately 2. Beyond this value, platforms begin raising prices for both groups because further price reductions would generate losses, and higher prices help recover some of the revenue lost at lower levels of β.

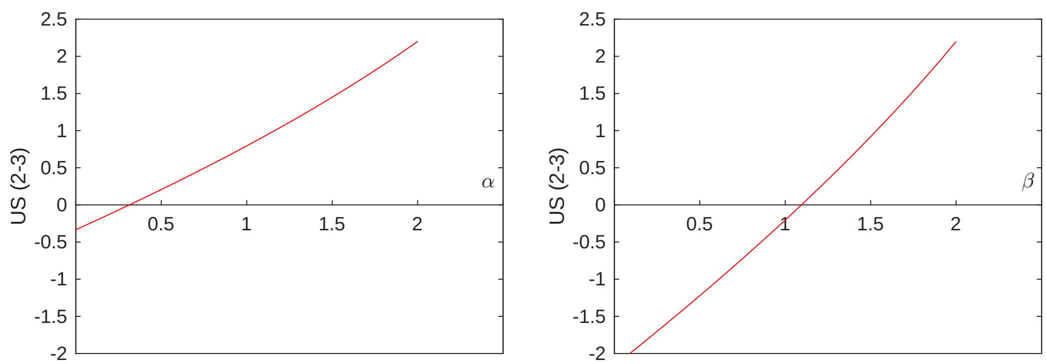
Duopoly vs. Triopoly: For the values of β considered in the numerical example above, the duopoly delivers a higher total user surplus than the triopoly because of weaker competitive pressure, and user concentration strengthens the impact of indirect network effects. With three platforms, the same network benefit is diluted across more competitors, reducing its effect on equilibrium prices.

2.6.1.2 *User Surplus Comparison between Duopoly and Triopoly*

This section examines how network effects influence platform consolidation from three to two platforms, showing that mergers can reduce user surplus when network effects are weak, but increase user surplus when network effects are strong, based on the numerical example considered above.

The simulation results directly address the second research question by identifying the conditions under which a merger from three to two symmetric platforms increases user surplus. Specifically, the merger enhances the user surplus *only when both direct and indirect network effects are sufficiently strong*. When network effects are weak, the competitive benefits and greater platform variety in a triopoly outweigh the concentration advantages of a duopoly, resulting in a higher user surplus under the triopoly settings. This analysis compares the user surplus across the duopoly and triopoly market structures as the strength of network effects increases. In the numerical example, the analysis identifies the values of the direct (α) and indirect (β) network benefit parameters at which the behaviour of user surplus shifts, and then examines how surplus evolves once these values are exceeded. The results show that stronger network effects improve user surplus, but this improvement becomes *more pronounced under a duopoly* once network effects reach sufficiently high levels.

To clearly understand the variations in network benefits on the user surplus, in the first scenario, when the value of α increases while the value of β remains constant, the effects differ between market structures, as represented in Figure 2.2(a). Under a *duopoly*, users in the group-i, who directly benefit from interactions within the group, experience lower equilibrium prices and a higher user surplus. Users in group-j, who do not benefit directly from the impact of stronger direct network effects, do not face a change in their prices, leaving their surplus largely unaffected. Thus, price adjustments primarily favour group-i, even though user allocations remain symmetric. In contrast, in a *triopoly*, stronger direct network effects influence equilibrium prices for both user groups. As α increases, group-i users again experience lower prices and higher surplus, but group-j users face higher prices as platforms partially shift the added value on group-i through interactions between groups. Although user allocations remain symmetric across platforms, opposing price movements result in differing surplus effects across groups. The comparison between the two market structures shows that when α is approximately 0.4, the concentration of users in a duopoly amplifies the stronger network effects, resulting in a substantially larger decline in prices compared to triopoly settings, which in turn leads to higher user surplus. Thus, the threshold identifies the point at which direct network effects become strong enough to outweigh the benefits of maintaining three competing platforms. Below this threshold, competition in a triopoly delivers higher user surplus; above it, stronger network effects make consolidation beneficial for users, so a merger from triopoly to duopoly increases user surplus.



(a) A numerical example of the difference in user surplus when network benefits parameters (β) is constant and (α) varies

(b) A numerical example of the difference in user surplus when network benefits parameters (α) is constant and (β) varies

Figure 2.2: Differences in user surplus (US) between two and three-platforms (2 - 3) scenario in symmetric platform competition. Positive values indicate that user surplus is higher under a duopoly compared to a triopoly so it is represented as (2 - 3), which means a merger is surplus enhancing.

In the second scenario, when the indirect network benefit parameter (β) increases while the direct parameter (α) is held constant, both market structures experience adjustments in equilibrium prices and user surplus, as represented in Figure 2.2(b). For values of β approximately 1.1, stronger indirect network effects intensify interactions between groups, leading to higher price responses in the duopoly than in the triopoly. Beyond this threshold, the concentration of users on two platforms enhances cross-side participation benefits, which outweigh the competitive advantages of having three platforms. As a result, the user surplus becomes higher under the duopoly once the indirect network effects are sufficiently strong. In both market structures, stronger indirect benefits raise user surplus for both groups, but the improvement is more pronounced in the duopoly, where price adjustments are sharper, and network feedback effects are stronger. Overall, these simulations demonstrate that mergers from triopoly to duopoly can improve user surplus only when there are stronger direct and indirect network effects.

Further, these findings are consistent with those reported by Tan and Zhou (2021), who demonstrated that user surplus can increase when the market shifts from triopoly to duopoly due to stronger network effects. Although their analysis illustrates this conclusion using an example, the contribution of this study lies in systematically identifying the precise conditions under which such a surplus improvement occurs. This analysis further shows that the network benefits must be sufficiently strong for this effect to hold, as demonstrated through numerical simulations.

Although mergers in traditional markets are typically welfare-reducing due to the increase in price after consolidation, the analysis by Tan and Zhou (2021) shows that mergers from triopoly to duopoly can enhance user surplus. However, this outcome holds only when the network effects are sufficiently strong. Specifically, this simulation-based analysis advances the literature by quantifying the threshold values of the network benefits parameters, direct (α) and indirect (β), beyond which a merger of three to two symmetric platforms leads to higher user surplus. These threshold values are not intended to be universal, but rather are conditional on the parameter space considered in the simulations. The purpose is to illustrate how the relationship between market structure and user surplus depends on the strength of network effects, rather than to claim that these thresholds are invariant across all parameterisations. Within the model, the key network benefits parameters (α and β) are varied systematically while holding other components fixed, allowing the identification of threshold values within a consistent framework. In addition, in certain cases, particularly in Simulation Finding 4, the parameter bounds are adjusted to explore a wider range of outcomes and ensure a more complete analysis of the model's behaviour. These thresholds therefore represent boundaries within the considered parameter space rather than universal values.

Importantly, the qualitative findings remain consistent across these variations. In particular, even when the parameter space is expanded, mergers from triopoly to duopoly can lead to improvements in user surplus in asymmetric settings. This result is driven by changes in stand-alone values, which alter participation incentives and can offset the loss of competition. This reinforces that the main insights of the model are not sensitive to a specific parameterisations but reflect more general underlying factors. Similar considerations apply to the other results in this section. The findings are based on systematic numerical exploration of the parameter space, rather than on a single arbitrary calibration. While a benchmark specification is used for clarity and consistency, the qualitative patterns are robust within economically relevant ranges of the parameters. At the same time, the results are not universal and may vary under alternative parameter values or modelling assumptions. Further I emphasised that the findings are interpreted as conditional on the chosen framework and parameterisations.

Therefore, this investigation not only reinforces the core insight of the Tan and Zhou (2021) study, but also extends it by offering a fine-grained, parameter-specific understanding of the relationship between market concentration and user welfare. This deeper insight is particularly relevant for policy discussions around digital platform mergers, as it provides a more detailed basis for evaluating the consumer welfare implications of market consolidation.

Although the above findings are derived under the assumption of symmetric platforms, they prompt further investigation into asymmetric platform settings to assess whether the observed effects on user surplus persist or exhibit notable differences.

2.6.2 *Simulation Finding 3: Platform Merger from Triopoly to Duopoly in Asymmetric Settings*

Analogous to the symmetric setting, this section is divided into two parts. The first part examines how market outcomes in both the duopoly and triopoly respond when one network benefit parameter is held fixed while the other varies. The second part analyses how the strength of network effects shapes the user surplus consequences of platform consolidation. In particular, it shows that when network effects are weak, consolidation from a triopoly to a duopoly can reduce user surplus due to the loss of competition, whereas when network effects are sufficiently strong, consolidation can increase user surplus by enhancing the value generated through user coordination on two platforms.

2.6.2.1 *Impact of Changes in α and β on Market Outcomes*

To illustrate the interaction between network effects on market outcomes in asymmetric settings, consider a numerical example with three platforms A, B, and C, each characterised by distinct uniform distributions. The upper and lower bounds of the uniform distributions (UDs) are specified as follows: $h^A_{i,j} = 60$, $l^A_{i,j} = 20$; $h^B_{i,j} = 50$, $l^B_{i,j} = 16$; and $h^C_{i,j} = 40$, $l^C_{i,j} = 14$. The direct and indirect network benefit parameters, α and β, are varied over the range $[0, 6]$.

The different UDs imply that, in equilibrium, platform A attracts a larger user base and enjoys stronger network effects, while platforms B and C are progressively smaller. This asymmetry generates systematic differences in equilibrium prices and user surplus even before any merger occurs. Larger platforms benefit from stronger participation effects, whereas smaller platforms face weaker network reinforcement and must compete more aggressively on price to attract users. These structural differences mean that the gains or losses from a merger depend critically on the relative sizes of the platforms: whether a merger raises user surplus is sensitive to the degree of asymmetry and the strength of induced network effects. This highlights the importance of accounting for platform heterogeneity when evaluating merger outcomes.

In an asymmetric triopoly, the variation in uniform distribution (UD) determines each platform's inherent attractiveness. Platforms with the superior UD attract more users, while those with less superior UD face greater competitive pressure and adjust prices more aggressively to retain or expand their user base.

The analysis proceeds in two scenarios: first, changing the direct network effect parameter α value while keeping the β value constant, and second, changing β value while keeping the α value fixed. The goal is to examine how these network effects influence equilibrium prices and user surplus as the market structure transitions from a triopoly (A, B, C) to various asymmetric duopolies.

Specifically, three duopoly 3 cases are considered:

(i) Platform A competes with B, where platform C merges with A,

(ii) Platform A competes with C, where platform B merges with A, and

(iii) Platform B competes with C, where platform A merges with B.

In all three cases, platform A is the largest platform, platform B is the second largest, and platform C is the smallest. The rationale for considering these three cases lies in how the relative size of the merging platforms shapes post-merger market outcomes.

In cases (i) and (ii), the large platform A acquires the smaller platform C in case (i) and the medium-sized platform B in case (ii). In both cases, the user base of the acquired platform may be absorbed into the larger platform, which amplifies network effects through increased user participation. However, these mergers improve user surplus only when network effects are sufficiently strong to offset the loss of competitive pressure. When network effects are weak, the reduction in competition dominates, leading to lower user surplus following the merger.

By contrast, case (iii) considers a merger between platforms A and B, after which platform A disappears, and the merged entity competes with platform C. Although a merger in which the largest platform exits the market is less typical, it is included for completeness. This case allows an examination of a boundary scenario in which a major platform disappears, offering insights into the limits of network effect-driven gains and how changes in market structure influence post-merger user surplus. The resulting concentration substantially reduces competition, but the merged platform may benefit from a significantly larger combined user base. When network effects are sufficiently strong, this expansion in participation enhances user surplus; otherwise, users face higher prices and reduced choice.

The following paragraph summarises how changes in direct (α) and indirect (β) network benefit parameters affect equilibrium prices, user participation, user surplus, and platform profits in an *asymmetric* duopoly and triopoly. Because platforms differ in their valuation distributions, the resulting equilibrium prices and user allocations are asymmetric across platforms in both market structures.

Effect of Strengthening Direct Network Benefits

Duopoly: An increase in α strengthens same-side network effects for group-i users. As α rises, platforms reduce their group-i prices in order to attract additional users on that side. The prices for group-j remain unchanged because α does not affect their utility. The reduction in their prices raises group-i user surplus and increases total surplus through greater participation in the platform with superior UD. Platform profits fall because the drop in price dominates the revenue gained from attracting marginal users.

Triopoly: In an asymmetric triopoly, an increase in α also leads all platforms to lower prices for group-i users, but the reduction is smaller than in the duopoly because the competitive response is diluted across three heterogeneous competing platforms. The prices for group-j increase slightly as platforms partially compensate for the revenue loss from group-i, reflecting both indirect network effects and strategic adjustments under asymmetry. Group-i surplus rises, though more modestly than in the duopoly, while group-j surplus falls due to the increase in their price. The dominant platform's profit decreases as its group-i price falls. The two smaller platforms initially lose revenue because they reduce prices to compete for users. However, as α becomes sufficiently large, particularly when α is approximately 5, the strengthened direct network effects allow these platforms to attract some users away from the dominant platform. Once they have secured a some additional users, their revenues stabilise and eventually begin to recover. At this point, the smaller platforms are able to raise their group-i prices slightly without losing users, allowing their profits to return to a sustainable level after the initial decline. Overall profit in the triopoly decreases at low and moderate α, rising only once the smaller platforms adjust their prices upward after gaining users.

Duopoly vs. Triopoly: Based on the numerical example considered above, the total user surplus gains are larger in the duopoly because user concentration strengthens the effect of direct network benefits on prices. With three platforms, the same increase in α is spread more thinly across competitors, leading to weaker price adjustments.

Effect of Strengthening Indirect Network Benefits

Duopoly: When β increases, both groups experience stronger cross-side network effects. The equilibrium prices fall for both groups, though asymmetrically, because the two platforms differ in their valuation distributions. Stronger indirect network effects increase participation on both sides, with the dominant platform gaining proportionally more users. User surplus rises for both groups. Platform profits decline because price reductions outweigh the additional revenue from larger participation.

Triopoly: In the asymmetric triopoly, higher β lowers the equilibrium prices for both groups across all platforms, but the decline is smaller than in the duopoly due to weaker competitive pressure. Prices fall for both groups on the dominant platform, prompting the two smaller platforms to also reduce their prices. After attracting some users from the dominant platform, these smaller platforms begin to raise their prices marginally to recover earlier revenue losses and sustain profitability, particularly when $\beta \approx 4$ for the platform C and $\beta \approx 5$ for the platform B. As a result, user surplus rises for both groups, though at a slower rate than in the duopoly case.

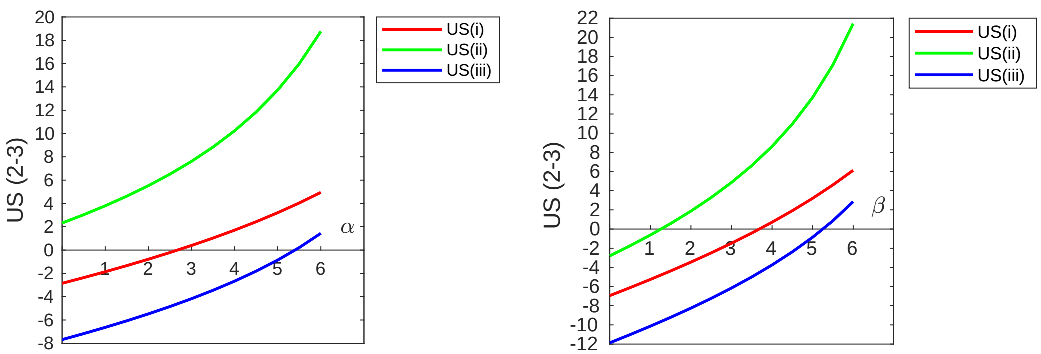
The dominant platform's profit continues to decline as both its group prices fall. The two smaller platforms initially face reduced profits, but later experience partial recovery by adjusting prices upward after capturing additional users. Nevertheless, the sum of profits from all platforms continues to decline overall.

Duopoly vs. Triopoly: For the values of β considered in the numerical example above, the duopoly generates a higher total user surplus than the triopoly. With two platforms, user concentration amplifies the stronger indirect network effects on equilibrium prices. With three platforms, the same increase in β is diluted due to competition pressure between them, resulting in smaller surplus gains.

2.6.2.2 *User Surplus Comparison between Duopoly and Triopoly*

Based on the numerical example considered above, in the case of triopoly to duopoly asymmetric platform merger, strengthening direct network effects leads to a reduction in equilibrium prices for both user groups, resulting in an improvement in user surplus, as illustrated in Figure 2.3(a). A similar pattern is observed when indirect network effects are strengthened, equilibrium prices fall across both groups, resulting in an increase in user surplus, as illustrated in Figure 2.3(b). In both cases, the user surplus increases more significantly under the duopoly compared to the triopoly when the network effects are sufficiently strong and the concentration of users is higher.

This finding shows that the strength of network effects required for a merger to be beneficial depends on both the size and composition of the merging platforms and the nature of asymmetry. Strong network effects are necessary for mergers involving dominant platforms to enhance user surplus, while mergers between asymmetric platforms can generate surplus gains at lower levels of network strength due to greater realisation of participation benefits.



(a) A numerical example of the differences in user surplus when (β) is constant and (α) varies

(b) A numerical example of the differences in user surplus when (α) is constant and (β) varies

Figure 2.3: Differences in user surplus (US) between two and three-platforms (2 - 3) scenarios: $US(i) : US_{A,B} - US_{A,B,C}, US(ii) : US_{A,C} - US_{A,B,C}$ and $US(iii) : US_{B,C} - US_{A,B,C}$ in asymmetric platform competition, where (2–3) indicates the change from triopoly to duopoly (positive values reflect higher surplus under duopoly).

The numerical analysis indicates that the asymmetry in platform sizes within the resulting duopoly creates uneven user distributions, with one platform typically maintaining a significantly larger user base. As network effects become stronger, the disparity in user surplus between the two platforms increases, reflecting the competitive advantage of the dominant platform. These findings extend the analysis of Tan and Zhou (2021) by explicitly incorporating asymmetric platform structures, a scenario that was not considered in their study. Examining asymmetries provides new insights into how heterogeneity among platforms influences equilibrium outcomes, highlighting a clear contribution of this work. Furthermore, the analysis addresses the third research question by explicitly identifying the conditions under which user surplus increases following a merger that reduces the number of competing platforms from three to two in asymmetric settings. In particular, it shows that the strength of network effects under asymmetric platform mergers is a critical factor in determining whether user surplus is enhanced, that is, in the first stage, when the value of α is approximately 3 in $US(i)$, 0 in $US(ii)$ and 5.5 in $US(iii)$ cases, and in the second stage, when the value of β is approximately 4 in $US(i)$, 1.5 in $US(ii)$ and 5.5 in $US(iii)$ cases. This directly answers the third research question by showing through a numerical example how the strength of network effects under asymmetric platform mergers increases user surplus.

The extent of platform asymmetry plays a central role in shaping how mergers affect user surplus. When asymmetries across platforms are limited, user bases and equilibrium prices remain relatively balanced, and consolidation from triopoly to duopoly has only modest welfare effects. By contrast, as asymmetry increases and one platform becomes significantly larger than its rivals, mergers generate more pronounced changes in equilibrium outcomes. The numerical analysis shows that asymmetric platform structures lead to uneven user distributions in the post-merger duopoly, with one platform typically maintaining a substantially larger user base. As network effects strengthen, this asymmetry magnifies differences in equilibrium prices and user surplus across platforms, reflecting the competitive advantage of the dominant platform. In such settings, consolidation can enhance user surplus because stronger user concentration intensifies network benefits and places downward pressure on prices, particularly on the larger platform. Therefore, understanding how varying levels of asymmetry influence post-merger outcomes is essential for assessing the welfare implications of platform mergers.

The implications of platform asymmetries are illustrated in the three asymmetric merger cases considered above. When the medium-sized platform B merges with the large platform A, the resulting duopoly, in which platform A competes with the smaller platform C, generates higher user surplus than the triopoly relative to the other two merger cases. This outcome reflects the uneven distribution of users across platforms induced by asymmetry: the merged platform inherits a substantially larger user base, which strengthens network effects and allows equilibrium prices to adjust more sharply. More generally, platform asymmetry magnifies the welfare consequences of consolidation. When platforms differ in size, mergers lead to a more concentrated user base on the dominant platform, intensifying both direct and indirect network effects. This concentration alters pricing incentives more strongly than in symmetric settings, resulting in larger changes in equilibrium prices and, consequently, user surplus. However, this effect is not universal and depends critically on the strength of network effects. The numerical results identify threshold values of the network benefit parameters beyond which consolidation becomes welfare-enhancing.

The above analysis reveals several important insights about asymmetric platforms.

(i) While the surplus-improving effect of mergers holds in both symmetric and asymmetric settings, the threshold values of the direct and indirect network effect parameters differ depending on which platforms merge. And therefore, for given network parameters, whether the merger is beneficial or not depends on the asymmetric nature of the market, and therefore, accounting for asymmetries is important.

(ii) In asymmetric markets, the resulting duopoly does not produce an even split of users across platforms; one platform typically becomes dominant while the other remains relatively small. This uneven distribution affects realised network effects: users on the larger platform experience stronger network effects than those on the smaller one, making the average user surplus a composite of very different individual outcomes. This pattern reflects a general tendency under asymmetric platform sizes rather than a universal outcome independent of the merger configuration. The final distribution of users depends on which platforms merge and on the initial asymmetries in platform size and attractiveness. In particular, as the analysis considers different merger configurations (A–B, A–C, and B–C), the results show that post-merger market shares vary systematically with the identity of the merging platforms. Mergers involving platforms of different sizes (e.g., A–C or B–C) can lead to more pronounced asymmetries in post-merger allocations than mergers between similarly sized platforms.

Thus, the emergence of a dominant platform is a typical outcome in asymmetric settings, but not a general result that holds uniformly across all merger combinations. This is precisely why multiple merger configurations are analysed in the chapter, to capture how both initial asymmetries and merger composition jointly determine post-merger market shares and user surplus.

(iii) The change in user surplus is not uniform across user groups. Some groups, especially those directly benefiting from network effects, gain disproportionately on the larger platform, while others experience small improvement. The choice of merger itself matters in asymmetric settings (e.g., platforms remain in competition after merger A, B vs. A, C vs. B, C), because different post-merger market shares lead to different surplus outcomes. This additional layer of heterogeneity highlights that, unlike in symmetric markets, both the strength of network effects and the composition of the merger jointly determine how the user surplus changes following consolidation.

Based on Simulation Findings 2 and 3, it can be concluded that user surplus increases when the market transitions from three platforms to two, provided that network effects are sufficiently strong. This result holds under both symmetric and asymmetric platform competition and is consistent with the findings of Tan and Zhou (2021). However, this investigation adds an important nuance by explicitly accounting for platform asymmetry. When platforms differ in size, consolidation creates uneven user distributions and pricing incentives across platforms, which magnifies the impact of network effects after a merger. As a result, mergers involving platforms of unequal size can generate larger changes in user surplus than mergers between similarly sized platforms. This analysis therefore contributes by identifying the specific conditions under which platform consolidation enhances user surplus and by highlighting the role of asymmetry in shaping post-merger welfare outcomes.

The numerical simulations show that the user surplus responds differently to changes in market structure depending on the strength of the network effects. When the direct or indirect network effect parameters exceed certain threshold values, users are better off in a duopoly because the greater concentration of users amplifies network benefits. Below these thresholds, however, the opposite holds: users benefit more from a triopoly, where increased competition leads to lower prices despite weaker network effects. This contrast naturally raises the question of how user surplus behaves when network effects are not sufficiently strong, motivating a closer examination of competition under weaker network effects and how market concentration interacts with platform asymmetry and user allocation.

Across both asymmetric and symmetric markets, the introduction of a third platform reduces equilibrium prices and increases the user surplus when network effects are moderate. In asymmetric markets, users are allocated proportionally to platform size, whereas in symmetric markets, they are evenly distributed. These findings are novel in systematically characterising when increased competition enhances user welfare, highlighting how network strength and market structure jointly determine outcomes. Finally, these competitive benefits persist only until network effects reach a sufficiently high level, as evident from the above Figure 2.2 and 2.3 (b) and partially (a), where the user surplus (2 - 3) is negative, i.e., below the origin. When network effects are strong, the user concentration in the duopoly, generates a higher surplus than the triopoly. Thus, the value of increased competition is highest when network effects are weak or moderate, but diminish and ultimately reverse when network effects are strengthened.

Examining both asymmetric and symmetric cases is important because real-world platform markets rarely operate under perfect symmetry. Differences in platform size, user composition, and competitive strength create uneven shifts in user surplus and pricing when market structure changes. In symmetric markets, all platforms adjust in the same way, so the welfare effects of added competition or consolidation are uniform. In asymmetric markets, however, the same structural change can produce very different outcomes across platforms, since user allocation and network benefits respond differently for large and small firms. These observations highlight that the welfare impact of competition depends critically on the degree of asymmetry present in the market. Motivated by this, the next investigation compares user surplus outcomes in asymmetric and symmetric markets, allowing a clearer assessment of how platform heterogeneity shapes user surplus, including in the context of platform consolidation.

2.6.3 *Simulation Finding 4: Comparative Analysis of User Surplus under Asymmetric and Symmetric Settings including Mergers from Triopoly to Duopoly*

The main aim of this section is to compare how user surplus varies between asymmetric and symmetric market structures, considering changes in network benefit parameters, adjustments to the upper and lower bounds of uniform distributions while keeping network benefits constant, and platform consolidation under strong network effects. While Simulation Findings 2 and 3 already consider transitions from three platforms to two under symmetric and asymmetric settings, the numerical examples considered there differ across cases, making it less transparent when market equilibria are driven by symmetry versus asymmetry. By contrast, this section employs a single numerical example that nests both symmetric and asymmetric configurations. This approach aligns with the standard practice in industrial organisation models of starting from symmetry, while recognising that real-world platform markets are typically asymmetric. Doing so allows a clearer comparison of market outcomes and user surplus across symmetric and asymmetric structures and strengthens the contribution to the literature by explicitly highlighting how platform heterogeneity alters the welfare effects of consolidation.

This analysis contributes to the literature by systematically highlighting the role of platform asymmetry in shaping user surplus. Three scenarios are central to this analysis: (i) comparison of user surplus in asymmetric and symmetric triopoly market configurations when the network benefit parameters are modified, (ii) Comparison of user surplus in asymmetric and symmetric triopoly market configurations and following a transition from triopoly to duopoly after a merger, as the upper and lower bounds of the uniform distributions are adjusted to gradually reduce platform differences and make the market symmetric, while the network benefit parameters are held constant, and (iii) comparison of user surplus from triopoly to duopoly mergers under asymmetric and symmetric market settings, as network effects strengthen. While Simulation Findings 2 and 3 compare user surplus when the market transition from triopoly to duopoly by varying either α or β while keeping the other parameter fixed, here in the third scenario, a numerical case is considered in which the mean value of the platform's uniform distribution remains the same in both symmetric and asymmetric settings, and both α and β vary simultaneously. Further, the motivation for the third scenario comes from real-world consolidation patterns, such as a dominant platform like Google acquiring an equal-sized rival such as Yahoo or Bing. After such a merger, the market becomes an asymmetric duopoly, with one large platform and one smaller competitor. By contrast, a merger between two similarly sized platforms in a symmetric triopoly produces a symmetric duopoly, where both remaining firms operate at comparable scale.

Given the complexity introduced by differences in platform size, the analysis relies on numerical examples. In the first scenario of the analysis, the parameters α and β are varied to examine how the strength of network effects influences outcomes under asymmetric and symmetric market settings. In the second scenario, the upper and lower bounds of the uniform distributions for the two smaller platforms are adjusted to vary the degree of asymmetry while keeping the network benefit parameters fixed. In the third scenario, user surplus is compared across triopoly to duopoly mergers in both asymmetric and symmetric settings when network effects are strengthened, allowing a direct assessment of how platform heterogeneity shapes post-merger outcomes. In all three scenarios, the stand-alone values drawn from the uniform distributions are kept consistent between the symmetric and asymmetric cases to ensure a fair comparison and to isolate the role of platform heterogeneity.

To investigate the first scenario, consider a numerical example: In asymmetric settings, $h^{A}_{i,j} = 60$, $l^{A}_{i,j} = 20$; $h^{B,C}_{i,j} = 50$, $l^{B,C}_{i,j} = 30$. In symmetric settings, all three platforms are identical with $h^{A,B,C}_{i,j} = 60$, $l^{A,B,C}_{i,j} = 20$. The network benefit parameters α and β are examined within the range $[0, 5]$.

2.6.3.1 Comparison of Market Outcomes under Asymmetric and Symmetric Settings with Varying Network Benefit Parameters

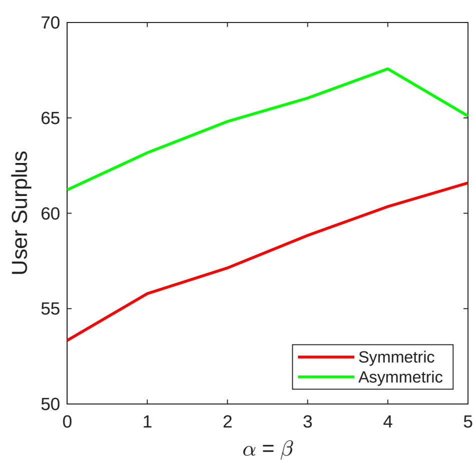
When network effects are strengthened, the market behaves differently in symmetric and asymmetric settings. In the symmetric setting, equilibrium prices fall in a smooth and predictable way because all platforms react identically: each has the same user base, the same stand-alone values, and the same incentive to lower prices when network effects become stronger.

In the asymmetric setting, prices fluctuate rather than decline steadily. This arises from two forces: how users are allocated across platforms, and how strongly network effects feed back into pricing. In the numerical example considered, the lower bounds of the uniform distributions for the two equal-sized platforms are superior to those of the dominant platform. This implies that, on one hand, users who are indifferent between the dominant platform and the two equal-sized platforms will join the smaller platforms whenever their drawn stand-alone value falls below the lower bound of the dominant platform's uniform distribution. As a result, these platforms initially set low prices to attract users located in this lower valuation bound. Because they share identical distributions, they end up with similar prices and similar user shares.

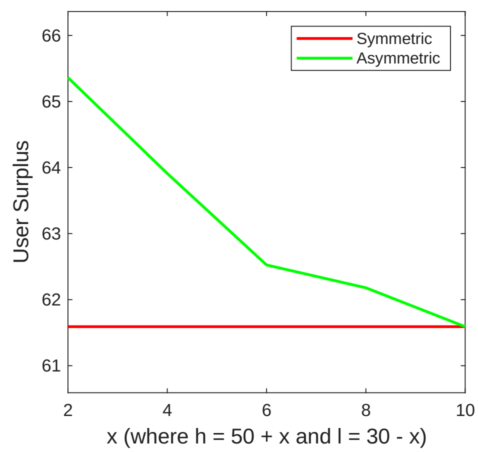
On the other hand, users who are indifferent between the dominant platform and the smaller platforms will choose the dominant platform if their drawn stand-alone value exceeds the upper bound of the uniform distribution of the smaller platforms. This is because the dominant platform offers higher stand-alone values to users due to the higher upper bounds of its uniform distribution compared to those of the smaller platforms. This allows the dominant platform to charge higher prices. If those users are more sensitive to price and network effects. Because the two smaller platforms are symmetric in size and price, they reduce their prices equally to attract these upper-bound users, causing the dominant platform A to lose users. As users reallocate to smaller platforms in response to these pricing incentives, the user surplus in the asymmetric scenario becomes larger than in the symmetric case, although the platform profits remain higher under symmetry (Figure 2.4 (a)). When network effects become strong, the pricing incentives shift. The two equal-sized platforms begin to raise their prices when α and β are approximately 4, which leads to a reduction in overall user surplus, and also causes them to lose part of their user base. The dominant platform, by contrast, lowers its price and ultimately attracts a larger share of users as the network effects become more pronounced, i.e., when α and β are approximately 5.

Insights: In this analysis, the interaction between platform asymmetry and strengthening network effects shows distinct patterns in user allocation and pricing. Smaller platforms initially attract users with lower stand-alone valuations due to higher lower bounds in their UDs, while the dominant platform captures users with higher stand-alone valuations, resulting in a higher overall user surplus in the asymmetric scenario compared to a symmetric market. Prices fluctuate more under asymmetry because platforms adjust differently in response to the strength of network effects, whereas symmetric platforms experience smoother price declines. This scenario also highlights a trade-off: user surplus can increase under asymmetry even as platform profits are higher in the symmetric case. Competition is concentrated at the extremes of the user valuation distribution, emphasising how heterogeneity shapes equilibrium outcomes.

Although the previous scenario illustrates market outcomes under varying network benefit parameters, it motivates a further investigation comparing user surplus as platform asymmetries transition to symmetries, while keeping network benefit parameters constant.



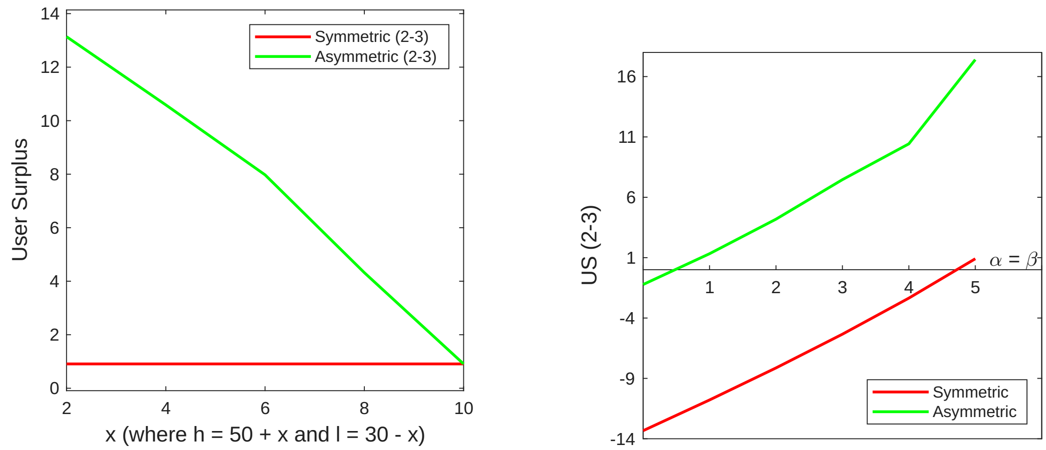
(a) Comparison of user surplus between asymmetric and symmetric platform settings when the network benefits get stronger



(b) Comparison of user surplus between asymmetric and symmetric platform settings when the upper and lower bounds of the distribution changes with the same value

Figure 2.4: Comparison of user surplus (US) under asymmetric and symmetric platform configurations.

To investigate the second scenario, consider a numerical example for the asymmetric case $h^A_{i,j} = 60$, $l^A_{i,j} = 20$; $h^{B,C}_{i,j} = 50 + x$, $l^{B,C}_{i,j} = 30 - x$ where $x = 2, 4, 6, 8, 10$. For the symmetric scenario, all three platforms are identical



(a) Comparison of user surplus between asymmetric and symmetric platform settings when the upper and lower bounds of the distribution change by the same value as the market moves from three to two platforms ((2-3), corresponding to a merger that is surplus-enhancing)

(b) Comparison of user surplus between asymmetric and symmetric platforms when moving from three to two platforms ((2 - 3), which means a merger is surplus enhancing) when the values of network benefit parameters are set equal and increased over a range of values

Figure 2.5: Comparison of user surplus (US) under asymmetric and symmetric platform consolidation.

with $h_{i,j}^{A,B,C} = 60$, $l_{i,j}^{A,B,C} = 20$. The network benefit parameters are set in both cases $\alpha = \beta = 5$. Following a merger, the market structure shifts from a triopoly to a duopoly, with platform C exiting and platforms A and B remaining in competition.

2.6.3.2 Comparison of Market Outcomes as Platforms Transition from Asymmetric to Symmetric

Platform competition: As the two smaller platforms become more similar to the dominant one, several patterns emerge. The equilibrium price on the dominant platform initially increases while the platforms remain relatively asymmetric (up to approximately $x = 6$). Beyond this point, the dominant platform's price begins to fall as the system approaches symmetry. For the two smaller platforms, prices fall when the asymmetry is moderate (up to around $x = 4$) but increase again as the platforms become nearly symmetric. Once full symmetry is reached, all three platforms converge to identical equilibrium prices and user shares, as expected.

A key intuition is that as x changes, it affects both the upper and lower bounds of the uniform distributions (UDs). In the given numerical example, the lower bounds of the uniform distributions for the two smaller platforms are higher than the lower bound of the dominant platform. This implies that users whose stand-alone values fall below the lower bounds of the dominant platform are more likely to join platforms B or C rather than A, as these smaller platforms provide relatively higher utility for low-valuation users. On the other hand, users with higher stand-alone values are more likely to be attracted to the dominant platform A, which offers higher maximum stand-alone values. The price adjustments influence user allocation across platforms. When asymmetry is reduced, that is, as x increases, the uniform distributions of the smaller platforms gradually converge to that of the dominant platform. users initially shift from the dominant platform A to the smaller platforms B and C, due to the combination of lower prices and more competitive UDs. The platform A begins to regain users only once its price starts to fall, which occurs when x reaches roughly 8. The resulting user surplus exhibits a clear pattern in Figure 2.4 (b) as the market moves from an asymmetric structure to a symmetric structure; the user surplus decreases steadily. In contrast, platform profits increase monotonically as the asymmetry is reduced, reaching their highest levels when the platforms are fully symmetric.

Platform merger: When a merger occurs, platform C exits the market and the structure changes from a triopoly to a duopoly, both in the asymmetric and symmetric cases, while the network benefit parameters are held fixed. As the value of x increases, the user-surplus difference between the duopoly and triopoly $(2 - 3)$ declines as the market moves from an asymmetric to a symmetric configuration.

Although the two platforms have identical equilibrium prices in the asymmetric case, users are evenly split between them. This occurs because changes in x affect both the upper and lower bounds of the uniform distributions. Platform A, which has a higher upper bound, competes more aggressively to attract users with high stand-alone value, while platform B, which has a higher lower bound, competes more aggressively for users with low stand-alone-value. These opposing competitive forces lead both platforms to set the same equilibrium price, resulting in an equal division of users. As x increases, equilibrium prices rise, which reduces user surplus, eventually reaching the symmetric merger configuration, as shown in Figure 2.5(a). Comparing asymmetric and symmetric mergers, users are better off under asymmetric settings because price adjustments in response to the merger are larger due to asymmetries, leading to higher gains in surplus relative to the symmetric case. Further, due to the increase in prices, platform profits exhibit an upward trend.

Insights: The analysis highlights the important role of platform asymmetry in shaping market outcomes. In the triopoly, when smaller platforms gradually approach the size of the dominant platform, pricing behaviour and user allocation shift in response to both the structure of the uniform distributions and competitive pressure. Initially, the dominant platform can raise prices without losing many users, but when smaller platforms become more comparable, they attract users by setting lower prices, reducing the dominant platform's share. In the duopoly, although both platforms have the same equilibrium outcomes, user surplus gradually declines as the platforms transition from asymmetric to symmetric due to an increase in prices. This transition causes the overall user surplus to decline as the market becomes more symmetric under both competition and merger scenarios, while platform profits rise, peaking under full symmetry. These results demonstrate that asymmetry can temporarily benefit users by creating price competition and diverse options, whereas symmetric structures favour platform profitability at the expense of user surplus.

Building on the previous analysis, the third scenario compares user surplus outcomes under platform consolidation from a triopoly to a duopoly in both asymmetric and symmetric markets, highlighting how differences in platform size affect post-merger results as network effects strengthen.

To investigate the third scenario, consider a numerical example: In asymmetric settings, $h_{i,j}^{A} = 60$, $l_{i,j}^{A} = 20$; $h_{i,j}^{B,C} = 50$, $l_{i,j}^{B,C} = 30$. In symmetric settings, all three platforms are identical with $h_{i,j}^{A,B,C} = 60$, $l_{i,j}^{A,B,C} = 20$. After the merger, platforms A and B remain in the market in both settings. The network benefit parameters α and β are examined within the range $[0, 5]$.

2.6.3.3 Comparison of Market Outcomes of Platform Consolidation in Asymmetric and Symmetric Settings

In the *symmetric setting*, when a merger occurs from a triopoly to a duopoly, stronger network effects lead to higher user surplus in the duopoly compared to the triopoly. This outcome arises because price reductions are more pronounced in the duopoly, even though stronger network effects push prices down in both market structures. In this scenario, equilibrium prices and user allocations remain identical across the platforms. Platform profits decrease in both triopoly and duopoly due to lower prices, but the merger benefits users by consolidating the user base, which strengthens network effects and increases user surplus .

In the *asymmetric setting*, the results are qualitatively similar when the platform consolidates to a duopoly, including the trend in platform profits. Since the stand-alone values of the uniform distributions remain the same across settings, user surplus comparisons are meaningful. The user surplus is higher in the asymmetric duopoly than in the symmetric case. This is driven by price adjustments in the asymmetric duopoly, as prices fall when network effects become stronger, which increases the user surplus. After the merger, the resulting duopoly platforms achieve an equal split of users and converge to symmetric equilibrium prices, even though the UDs of the two platforms are asymmetric. This occurs because users with lower valuations prefer platform B due to superior lower bounds, while users with higher valuations prefer platform A, leading to a balanced user allocation across the duopoly.

Insights: In symmetric markets, equilibrium user allocations remain evenly distributed across platforms. However, when the market consolidates from a triopoly to a duopoly under stronger network effects, user surplus increases because of greater user concentration on two platforms. In asymmetric markets, the effects are more pronounced: price reductions are more in a duopoly when compared to a triopoly when the network effects get stronger, and also the redistribution of users post-merger leads to higher user surplus compared to the symmetric case. These results highlight that consolidation can enhance user surplus through stronger network effects and price adjustments, and that these gains are larger in asymmetric markets, as shown in Figure 2.5(b), where differences in platform size and pricing intensify the impact of consolidation compared to symmetric settings. It is important to note that these findings are specific to the given numerical example and should not be generalised without further analysis.

These finding suggests that mergers involving asymmetric platforms yield larger gains for users in the form of lower prices and stronger network effects than mergers between two symmetric large platforms. However, these gains depend on the strength of the network effects. If network effects are weak, users are better off with more platforms rather than fewer. Regulators should therefore not treat all mergers as equal, but explicitly consider platform size differences and network effect strengths when assessing whether a merger will harm or benefit users. Over time, as platforms become more symmetric (e.g., through catch-up innovation or entry), the surplus-enhancing effect of mergers may diminish, making consolidation less beneficial to users. However, the presence of a competitor remains important, as the analysis of mergers to monopoly suggests, because without competition, higher prices can erode the potential gains from network effects.

Overall, this study systematically examines how platform asymmetry and network effects shape user-surplus outcomes in three related scenarios. In the first and third analyses, the impact of changes in network benefits parameters and in the second, the role of platform heterogeneity, were evaluated in competition and merger scenarios separately, keeping network effects constant in order to isolate the influence of asymmetry. This allowed a direct comparison between asymmetric and symmetric platforms, demonstrating that user surplus is higher in asymmetric settings due to differentiated price responses and strategic user allocation. Particularly, in the second scenario, when the market consolidates from a triopoly to a duopoly under stronger network effects, the numerical results show that consolidation can further enhance user surplus, especially when platforms are initially asymmetric, through larger price adjustments and subsequent declines as the market becomes symmetric. These findings contribute to the literature by highlighting the importance of asymmetry, a factor that is largely overlooked in previous studies that focus on symmetric platform mergers, and by providing numerical evidence of how heterogeneity interacts with network effects to influence market outcomes. From a policy perspective, the results suggest that regulators should consider not only the number of platforms and the strength of network effects, but also the relative sizes of merging platforms, as asymmetric mergers can produce different welfare effects than mergers between similarly sized competitors. The analysis provides example-based guidance for assessing mergers in digital platform markets, emphasising that the welfare impact is context-specific, sensitive to platform characteristics, and depends on preserving competition.

2.7 Conclusion

This chapter has undertaken a comprehensive analysis of how network effects and market structure interact to influence user welfare in digital platform markets. Using a simulation-based approach grounded in a mathematical model, the study investigates how the user surplus changes as the number of platforms shifts from two to one (duopoly to monopoly) and from three to two (triopoly to duopoly). The analysis considers both symmetric and asymmetric market scenarios and incorporates direct and indirect network effects as endogenous drivers of platform utility and user behaviour.

The study yields several important findings. First, the shift from duopoly to monopoly consistently results in a lower user surplus. Despite the potential gains in network effects from a fully consolidated user base, these gains are largely offset by the monopolist's ability to set higher prices and capture a greater share of the surplus. This holds both for partial and full market coverage scenarios. Even when user participation increases due to enhanced network effects, higher prices neutralise any potential improvement in welfare. These results underscore the structural risks associated with monopolisation in digital markets and highlight the critical role that remaining competition plays in maintaining user surplus after a merger.

Second, in contrast to the above, the analysis shows that when platforms are symmetric and asymmetric, merging three platforms into a duopoly can generate user surplus gains. A dominant platform attracts more users, which strengthens network effects and increases overall surplus, provided that prices do not increase too much. As platforms become more similar, these welfare gains shrink because user concentration and the resulting network effects weaken. This highlights the importance of platform heterogeneity in shaping how mergers affect user welfare, suggesting that differences in platform characteristics should be explicitly considered when evaluating the potential benefits or harms of proposed mergers.

Third, when transitioning from triopoly to duopoly, the user surplus can increase, but only under certain conditions. Specifically, this welfare-enhancing effect occurs when the network effect parameters, direct (α) or indirect (β), exceed certain thresholds in a numerical example. Below these thresholds, users benefit more from the competitive pricing that arises when three platforms operate in the market. This result holds across both symmetric and asymmetric platform structures and aligns with, while extending, the findings of Tan and Zhou (2021) by identifying the exact parameter values that influence the direction of welfare change.

This chapter makes the following key contributions: It develops and implements a rigorous simulation model to explore how user surplus responds to changes in market structure and network effect strength under both symmetric and asymmetric platform competition. The specific conditions under which platform consolidation from triopoly to duopoly improves user welfare are a significant extension of earlier studies. It offers a novel examination of the monopoly scenario, showing that further consolidation leads to welfare losses, even when the network effects are strong, which is not explored by Tan and Zhou (2021). In addition, it incorporates the construction of an asymmetric three-platform model into the analysis, which is a key contribution, as it better reflects the heterogeneity observed in real-world digital markets and enhances the external validity of the findings.

These findings result in several policy-relevant insights:

(i) Preserving Competition is Essential: Consistent welfare losses observed under the monopoly, mainly due to higher prices, compared with the duopoly case, underscore the importance of maintaining a competitive environment. Even in cases where consolidation could theoretically improve the user experience through stronger network effects, the simulation result is often offset by higher prices and reduced consumer surplus.

(ii) Merger Assessment Should be Contextual: Not all platform mergers are detrimental. When network effects are sufficiently strong, moving from three to two platforms can increase user welfare. Thus, regulators should evaluate mergers based on quantitative thresholds of network effect strength rather than adopt a blanket stance for or against consolidation.

(iii) Platform Asymmetry Matters: Given that asymmetric platforms yield different welfare outcomes compared to symmetric ones, policy assessments must account for heterogeneity in platform size and user distribution. Failing to consider these structural differences can lead to misguided merger decisions.

While this study provides a robust framework for analysing platform consolidation, several avenues remain open for future research. Incorporating dynamic user behaviour, considering platform differentiation in services, and extending the model to allow multi-homing could yield richer insights. Additionally, empirical validation using real-world data from platform markets would further strengthen the relevance of these theoretical findings.

In summary, this chapter contributes to a deeper understanding of how network effects and market structure jointly determine user welfare in digital platform markets. It offers both analytical rigour and policy-relevant insights, supporting a more evidence-based approach to platform merger assessments.

Chapter 3

Advertising Spending and User Welfare in Digital Platform Markets

3.1 Introduction

Advertisement-based digital platforms have emerged as dominant players in the modern digital economy, offering *free* services to users while generating revenue through targeted advertising. These platforms act as intermediaries between users and advertisers, where user engagement is critical for the platforms. Such platforms match advertisements (ads) with user preferences, thus increasing the effectiveness of ad campaigns and enhancing platform revenue[1]. Prominent examples include Google (through its search engine and YouTube) and Meta, both of which offer free services to users. As more users join a platform, the network effects increase its value to others, attracting additional users and, crucially, advertisers. This feedback loop allows dominant platforms to charge higher advertising prices and allocate more ad volumes. Although advertising is essential for product visibility, concentrating most advertising spending on a few dominant platforms strengthens their market power and makes it harder for smaller players to compete.

[1] CMA (2020)

In platform markets such as social media, search engines, or e-Commerce, multi-homing occurs when users or advertisers engage with multiple platforms simultaneously, for instance, a user accessing both Instagram and TikTok, or an advertiser placing ads on Google and Facebook. Multi-homing is often seen as pro-competitive because it reduces user lock-in. However, it can lead to trade-offs and welfare losses, depending on market structure and platform strategies. In markets dominated by a few platforms, strong network effects, integrated ecosystems, and high user lock-in can magnify the costs of multi-homing. For these users, duplicated efforts, such as interacting with the same users on different platforms, encountering similar ads, and processing redundant content, can reduce overall utility and satisfaction[2].

This chapter examines three key issues: first, the factors that shape advertising spending on ad-based platforms; second, the conditions under which multi-homing has adverse effects on user surplus; and third, how a merger of two competing platforms into a monopoly affects user surplus under multi-homing. To address the first issue, the chapter constructs an ad-based dominant platform model grounded in existing literature to explain the drivers of increasing ad expenditures on such platforms. The discussion then turns to the literature on multi-homing, which focuses on measuring user surplus while avoiding double-counting of network effects, generally highlighting that multi-homing benefits users. The analysis focuses on multi-homing by users, whereby individuals allocate their attention across multiple platforms. Advertisers are assumed to reach users through the platforms they participate in, and their behaviour is not modelled as multi-homing in this framework. However, an important and often overlooked factor is that user utility also depends on the stand-alone benefits of each platform and the associated advertising nuisance. These effects can vary significantly with the degree of substitutability between platforms when users divide their attention across multiple services. Building on these insights, this chapter incorporates platform substitutability into a formal model to assess when multi-homing enhances or undermines user welfare. The aim is to provide a more accurate measure of user welfare and to identify conditions under which users may experience reduced welfare, either when competing platforms coexist or when they merge into a single dominant platform.

[2] ACCC (2023)

Based on the above, the following research questions have been derived to guide the investigation.

(1) How is advertising spending shaped by key factors on an ad-based dominant platform?

(2) Under what conditions does multi-homing reduce user welfare relative to single-homing?

(3) How does user surplus under multi-homing differ between a duopoly and a monopoly platform market?

The literature on ad-based platforms provides a foundation for understanding non-price-based digital markets. Foundational models, such as Armstrong (2006), show how platforms subsidise users to attract advertisers when cross-group network effects are strong. This framework is extended by Reisinger (2012), which depicts how platforms optimise advertising to monetise user attention and the impact of excessive advertising on user welfare. The ad-based model proposed by Zennyo (2020) explicitly incorporates advertising disutility, providing a more realistic measure of user welfare. According to the CMA (2020) report, there is a growing dominance of digital advertising, which has become the largest and fastest-growing segment within the UK's advertising industry. Digital advertising spending reached £15.7 billion in 2019, up from £13.6 billion in 2018, representing approximately 62% of total advertising expenditure, compared to only 25% in 2010. While these studies highlight the scale of the market, they often do not explain the mechanisms driving advertiser concentration on particular platforms.

The literature on multi-homing offers additional insights and contrasts. Some models, such as Belleflamme and Peitz (2019), suggest that multi-homing improves match efficiency and benefits all market participants when platforms are differentiated. However, these models typically assume symmetric cross-side benefits and do not account for the disutility arising from repeated or redundant exposure to ads, which is particularly relevant in ad-supported platforms. Bakos and Halaburda (2020) examine multi-homing by both users and advertisers, showing that strategic interdependence weakens and platforms may charge both sides. However, their assumption that repeated exposure provides no incremental benefit limits its applicability to real ad-based platforms, where second or third impressions can significantly influence advertising outcomes. Conversely, Ambrus et al. (2016) demonstrate that multi-homing can reduce advertising levels when user preferences are highly correlated, as duplicate exposure diminishes the marginal value of shared users and can motivate platform consolidation. Post-merger, advertising levels can rise while advertiser surplus falls, producing ambiguous effects on user welfare. Similarly, Anderson et al. (2019) demonstrate that in multi-homing settings, mergers increase advertising prices without affecting consumer prices, highlighting the importance of understanding both user and advertiser behaviour. Although platform entry can increase user welfare via variety, these gains can be limited if the negative effects of advertising, or ad nuisance, are ignored.

Further work by Anderson et al. (2018) shows that when second impressions are valuable, competing platforms targeting overlapping users tend to converge in content or features. This convergence can improve the welfare for overlapping users but leaves niche users underserved. Bryan and Gans (2019), though in the context of ride-sharing, highlights that multi-homing-induced competition can reduce prices but degrade service quality, suggesting that increased competition does not always enhance user welfare.

On the one hand, the literature shows that ad-based platforms typically attract and subsidise users while generating revenue from advertisers. This sets the stage for further theoretical investigation into ad-based platform strategies and their implications for users. On the other hand, although multi-homing in ad-based platforms benefits users, such effects depend on factors including platform differentiation, the extent of repeated exposure, and the presence of advertising nuisance across the platforms. Building on these insights, this chapter develops a formal framework that incorporates direct network effects to examine how stronger network effects reinforce the market power of dominant platforms and drive heavy advertising spending. Further, it introduces a parameter to evaluate the impact of multi-homing on user surplus when platforms are substitutable and examines user surplus under duopoly and monopoly settings, providing clearer insight into when multi-homing enhances or reduces user welfare.

This chapter contributes to the literature on ad-based digital platform markets in three main ways. First, it provides a structural explanation of market outcomes driven by advertising nuisance, advertising effectiveness, and direct network benefits, thereby linking theoretical models of ad-based platforms to policy discussions such as CMA (2020) on digital advertising and spending. Second, it extends the multi-homing literature Anderson et al. (2018) by showing that the welfare effects of multi-homing are not unambiguously positive, but depend critically on platform substitutability and ad-related disutility. Third, it complements work on platform competition and mergers (e.g. Ambrus et al. (2016), Anderson et al. (2019)) by demonstrating that, in ad-based markets, consolidation can increase user welfare under a restricted set of parameter values, contrasting with standard results from price-based competition models. Together, these results clarify when consolidation and reduced platform choice can be welfare-enhancing, and when they are not, adding nuance to existing theoretical and policy debates.

The rest of this chapter is organised as follows. Section 3.2 develops the Ad-based dominant platform model, Section 3.3 discusses the determinants of advertising spending on an Ad-based dominant platform, and Section 3.4 extends the analysis to multi-homing and platform mergers and their implications for user welfare.

3.2 Advertisement-based Dominant Platform Model

Consider a dominant ad-based platform A that operates in the market and serves two groups: users (i) and advertisers (Ad). This setup is similar to the monopoly model discussed in Chapter 2, with one key difference: the platform provides free services to users but charges advertisers. Users are constantly exposed to advertisements (ads), and excessive ads can reduce their satisfaction. Given its dominant position, the platform A has a large user base, which enhances its attractiveness to advertisers and grants it significant pricing power. Let n_i denote the number of users and n_{Ad} denote the number of advertisers on the platform A. Examples include YouTube, Google Search, TikTok, and Meta.

The following assumptions define the model setup:

(i) The population of each group is normalised to 1.

(ii) Users and advertisers decide whether to join the platform A based on their positive individual utility, where $U_i \geq 0, U_{Ad} \geq 0$

(iii) Group-*Ad* derive no direct network benefit. This is because advertisers do not directly benefit from interacting with each other.

(iv) Each user independently draws their stand-alone value from a continuous distribution of the platform.

3.2.1 Two-stage game:

Ad-based digital platforms are modelled as a two-stage game because the platform sets its pricing policy before users and advertisers make their participation decisions. The game proceeds as follows:

1. The platform sets the price charged to advertisers for access to its user base.

2. Users and Advertisers decide whether to join the platform.

The above game can be solved using backward induction. In the second stage, given the price of the platform, users and advertisers decide whether to participate based on the positive utility they derive from joining the platform. Their participation decisions determine the equilibrium number of users and advertisers. In the first stage, anticipating these participation responses, the platform chooses the optimal advertising price, considering how the price affects both advertiser demand and user participation. The resulting outcome, comprising the optimal advertising price and the corresponding equilibrium levels of user and advertiser participation, constitutes the *Subgame-Perfect Nash Equilibrium* of the two-stage game.

3.2.2 Payoff Functions

The utility functions for both users and advertisers are adopted from the foundational framework of Zennyo (2020). The key innovation of this ad-based model is the inclusion of direct network effects on the user's side, reflecting a more realistic structure of digital platforms where a user's utility is influenced by the presence of other users. To further capture the negative effects of advertisements on users, the model incorporates an ad nuisance parameter, denoted λ_i, following the approach introduced in Reisinger (2012). The rationale here is that, in ad-based digital platforms, while ads generate revenue and subsidise free access for users, they often reduce user satisfaction when they become excessive. Including an ad nuisance parameter allows the model to capture user utility more accurately by accounting for the negative impact of advertising exposure. Since this part of the analysis considers a single-platform environment, the superscript A is redundant and is therefore omitted to simplify the notation. The payoff function for an agent joining platform A is calculated as follows:

$$U_i = s_i + \alpha_i \cdot n_i - \lambda_i \cdot n_{\text{Ad}} \qquad ; \quad U_{\text{Ad}} = s_{\text{Ad}} + \beta_{\text{Ad}} \cdot n_i - p_{\text{Ad}} \cdot n_i$$

The utility function for users consists of the stand-alone benefits (s_i) of the platform A, adjusted upward by direct network effects ($\alpha_i \cdot n_i$): α_i captures the direct network benefits due to the presence of the same group of users n_i and adjusted downward by advertisement disutility ($\lambda_i \cdot n_{\text{Ad}}$) experienced by users: λ_i captures the ad nuisance due to the presence of the advertisers' group n_{Ad}.

On the other hand, the utility function for the advertisers' group consists of stand-alone benefits (s_{Ad}) of the platform A, adjusted upward by the indirect network effects or ad reach ($\beta_{\text{Ad}} \cdot n_i$): β_{Ad} captures the advertising effectiveness that advertisers derive from the presence of users n_i on the platform. The ad reach for advertisers reflects how the platform's value to them increases as the user base grows. Specifically, as the number of users n_i increases, each ad placed on the platform reaches a wider audience, improving the expected return on investment of the advertisers. In other words, each individual user represents a potential engagement with an advertisement and thus contributes to the overall payoff received by advertisers. Lastly, adjusted downward by price ($p_{\text{Ad}} \cdot n_i$): p_{Ad} is the price paid by each advertiser per-view of their ads by users n_i. Here, advertisers are assumed to pay a per-view price (p_{Ad}) for each user impression (n_i). This specification captures the platform revenue from advertisers and the corresponding ad-related disutility for users. Alternative pricing schemes (e.g., per-click or per-conversion) or heterogeneous advertiser valuations could also be considered. These extensions would change the exact functional form of advertiser demand, but would not alter the main qualitative insights regarding the relationship between advertising prices, platform revenue, and user welfare.

To explain this model with a real-world example, on YouTube, users receive value in several ways. First, there are the *stand-alone benefits* that capture the intrinsic benefits provided by the platform itself, such as the interface, video playback features, search and recommendation functions, and access to preloaded or curated content, independent of how many other users are on the platform. Beyond this, users generate *direct network effects*: as more users join YouTube, the platform becomes more valuable to each individual. A larger audience encourages more content creation, improves recommendation quality, and enhances community engagement, so users directly benefit from the presence of others. At the same time, YouTube monetises its platform primarily through advertising. While this allows the service to remain free for users, they experience *advertisement disutility.* Users are exposed to ads, some of which can be intrusive, and the overall negative impact grows with the number of advertisers, reducing user satisfaction.

On the other side of the platform, advertisers also gain *stand-alone benefits* from the unique features and capabilities of the platform, such as advanced targeting tools that allow advertisers to reach very specific audiences based on demographics, interests, and behaviour. This precision increases the likelihood that the ads will be relevant to viewers, increasing the effectiveness of the ads. Besides, advertisers benefit from YouTube's large user base because it offers a broad and diverse audience to target. The *ad reach* increases with the number of active users, as each user represents a potential engagement. For advertisers, more users typically mean better reach and potentially higher returns on their investment. However, they must pay for this access, typically on a *price per-user view* basis. If the price they pay per-user view exceeds the value they derive from it, for example, if ads are skipped frequently or do not convert, then advertising on YouTube becomes less attractive. YouTube, as the platform operator, must balance the interests of both groups. While it seeks to attract advertisers and generate revenue, it must also avoid overwhelming users with ads, which could reduce engagement.

Returning to the model, the payoff function of the platform A captures its total revenue generated from advertisers and the fixed cost incurred in operating the platform:

$$\pi = n_{\mathrm{Ad}}(p_{\mathrm{Ad}}) \cdot p_{\mathrm{Ad}} \cdot n_i$$

The platform's profit π depends on three key factors: the number of advertisers who participate at a given price p_{Ad}, the size of the user base n_i, and the fixed costs are assumed to be sunk and therefore do not affect the platform's optimisation problem in the static setting considered here. For this reason, they are omitted from the profit function. The price paid by each advertiser is influenced by the platform's *ad reach*, which is proportional to the number of users: a larger user base increases the value of the platform for advertisers and allows the platform to charge higher prices.

Importantly, user participation is *indirectly influenced by the advertiser's price*. Lower prices can attract more advertisers, which increases the volume of ads on the platform. This can reduce users' utility due to ad nuisance, potentially discouraging some users from joining or remaining on the platform. Thus, there is a two-sided interdependence: the platform's pricing affects advertiser participation, which in turn affects user participation, and user participation then feeds back into advertiser value and platform revenue.

3.2.3 Joining Behaviour

A user decides to join the platform A only if his utility is positive and is given as follows:

$$\begin{gathered} U_i > 0 \\ s_i + \alpha_i \cdot n_i - \lambda_i \cdot n_{\text{Ad}} > 0 \\ \text{is rearranged as} \\ s_i > \lambda_i \cdot n_{\text{Ad}} - \alpha_i \cdot n_i \end{gathered} \tag{1}$$

Similarly, an advertiser decides to join the platform A if:

$$\begin{gathered} U_{\text{Ad}} > 0 \\ s_{\text{Ad}} + \beta_{\text{Ad}} \cdot n_i - p_{\text{Ad}} \cdot n_i > 0 \\ \text{is rearranged as} \\ s_{\text{Ad}} > p_{\text{Ad}} \cdot n_i - \beta_{\text{Ad}} \cdot n_i \end{gathered} \tag{2}$$

Inequalities (1) and (2) define threshold valuations. Users whose s_i exceeds the threshold value (ad disutility minus direct network effects) will decide to join platform A. Advertisers whose s_{Ad} exceed the threshold value (price minus ad reach) will decide to join the platform A.

From the above, the proportion of agents in each group who join the platform A is determined by the cumulative distribution functions (CDFs) of the agent's valuations, denoted by $G_i(.)$ for group-i and $G_{Ad}(.)$ for group-Ad. An agent joins if his $s_{i,Ad}$ exceeds the threshold.

The proportion of group-i users with s_i above the threshold is-

$$\Pr\big(s_i > \text{threshold}\big) = 1 - G_i\left(\lambda_i \cdot n_{\text{Ad}} - \alpha_i \cdot n_i\right)$$

Similarly, the proportion of group-Ad advertisers with s_{Ad} above the threshold is-

$$\Pr\left(s_{\text{Ad}} > \text{threshold}\right) = 1 - G_{\text{Ad}}\left(p_{\text{Ad}} \cdot n_i - \beta_{\text{Ad}} \cdot n_i\right)$$

Here, $G_i(.)(G_{Ad}(.))$ gives the probability that an agent's valuation is below the threshold. Therefore, the probability that $s_i(s_{Ad})$ exceeds the threshold is its complement, that is, $1 - G_i(.)(1 - G_{Ad}(.))$.

Then the proportion of users and advertisers who join platform A is given by:

$$n_i = 1 - G_i\big(\lambda_i \cdot n_{\text{Ad}} - \alpha_i \cdot n_i\big)$$
$$n_{Ad} = 1 - G_j\big(p_{\text{Ad}} \cdot n_i - \beta_{\text{Ad}} \cdot n_i\big)$$

The above system of equations defines a second-stage equilibrium that determines outcomes for users and advertisers for any particular price that might be set in the first-stage expressions. Further, these expressions are a system of interdependent equations, where n_i and n_{Ad} depend on each other. The right-hand side of each equation contains the left-hand side as part of a function, making the system non-linear and implicit. The interdependence of n_i and n_{Ad} implies that solving for equilibrium user and advertiser allocations involves finding values of n_i and n_{Ad} that simultaneously satisfy both equations. This is a classic *fixed point problem*, where the equilibrium values are fixed points of a function that maps the user shares into themselves.

Equilibrium Analysis: From the above, the equilibrium is determined based on the independent stand-alone values s_i and s_{Ad}, which are assumed to be drawn from the uniform distributions (UD). Assume that G_i, G_{Ad} is a

UD that is defined over the interval (l_i, h_i) for the group-i and (l_{Ad}, h_{Ad}) for the group-Ad.

For a UD with $x \sim UD[l, h]$, the CDF is given by:

$$G(x) = \frac{x - l}{h - l}, \quad \text{for } x \in [l, h]$$

This CDF is used to calculate the proportion of users and advertisers whose valuations exceed the thresholds for the platform A.

For the two agent groups, the CDFs at the respective thresholds are-

$$G_i(\lambda_i \cdot n_{\text{Ad}} - \alpha_i \cdot n_i) = \frac{\lambda_i \cdot n_{\text{Ad}} - \alpha_i \cdot n_i - l_i}{h_i - l_i}$$
$$G_{\text{Ad}}(p_{\text{Ad}} \cdot n_i - \beta_{\text{Ad}} \cdot n_i) = \frac{p_{\text{Ad}} \cdot n_i - \beta_{\text{Ad}} \cdot n_i - l_{\text{Ad}}}{h_{\text{Ad}} - l_{\text{Ad}}}$$

The above will be substituted to calculate the equilibrium users and advertisers for platform A as follows:

$$n_i = 1 - \frac{\lambda_i \cdot n_{\text{Ad}} - \alpha_i \cdot n_i - l_i}{h_i - l_i}$$
$$n_{\text{Ad}} = 1 - \frac{p_{\text{Ad}} \cdot n_i - \beta_{Ad} \cdot n_i - l_{\text{Ad}}}{h_{\text{Ad}} - l_{\text{Ad}}}$$

After the simplification of the above expressions are as follows:

$$n_i = \frac{h_i + \alpha_i \cdot n_i - \lambda_i \cdot n_{\text{Ad}}}{h_i - l_i}$$
$$n_{\text{Ad}} = \frac{h_{\text{Ad}} + \beta_{Ad} \cdot n_i - p_{\text{Ad}} \cdot n_i}{h_{\text{Ad}} - l_{\text{Ad}}}$$

After substitution, the solution is as follows:

$$n_i(p_{\text{Ad}}) = \frac{h_{\text{Ad}}(\lambda_i - h_i) + h_i l_{\text{Ad}}}{\alpha_i(h_{\text{Ad}} - l_{\text{Ad}}) - \beta_{\text{Ad}}\lambda_i - h_{Ad}(h_i + l_i) + l_{Ad}(h_i - l_i) + \lambda_i p_{\text{Ad}}}$$

$$n_{Ad}(p_{\mathrm{Ad}}) = \frac{\alpha_i h_{\mathrm{Ad}} - \beta_{\mathrm{Ad}} h_i - h_{\mathrm{Ad}}(h_i + l_{\mathrm{Ad}}) + h_i p_{\mathrm{Ad}}}{\alpha_i(h_{\mathrm{Ad}} - l_{\mathrm{Ad}}) - \beta_{\mathrm{Ad}}\lambda_i - h_{Ad}(h_i + l_i) + l_{Ad}(h_i - l_i) + \lambda_i p_{\mathrm{Ad}}}$$

For any price (p_{Ad}) in the first-stage, the above expressions are the equilibrium number of users and advertisers joining each side of the platform in the second stage. These equilibrium participation levels are then substituted into the platform's profit function to determine the optimal price for the advertisers of platform A:

$$\pi = n_{\mathrm{Ad}}(p_{\mathrm{Ad}}) \cdot p_{\mathrm{Ad}} \cdot n_i(p_{\mathrm{Ad}})$$

$$\text{maximise } \pi \text{ w.r.t. } p_{\mathrm{Ad}}$$

The equilibrium price is calculated by maximising the profit function with respect to the price of advertisers (p_{Ad}). The p_{Ad} will be:

$$p_{\mathrm{Ad}} = \left(\frac{h_{\mathrm{Ad}}^2(\alpha_i - h_i + l_i)^2 + h_i^2\left(\beta_{\mathrm{Ad}}(h_{\mathrm{Ad}} - l_{\mathrm{Ad}}) - h_{\mathrm{Ad}} l_{\mathrm{Ad}}\right) - h_{\mathrm{Ad}} l_{\mathrm{Ad}}\left(\alpha_i^2 + l_i^2 + 2h_i(\alpha_i + l_i)\right)}{h_i^2(2h_{\mathrm{Ad}} - l_{\mathrm{Ad}}) + \alpha_i\left(h_{\mathrm{Ad}}(\lambda_i - 2h_i) + 2h_i l_{\mathrm{Ad}}\right) + \lambda_i h_i(\beta_{\mathrm{Ad}} - h_{\mathrm{Ad}}) + l_i\left(h_{\mathrm{Ad}}\lambda_i - 2h_i(h_{\mathrm{Ad}} - l_{\mathrm{Ad}})\right)} \right)$$
$$+ \left(\frac{\beta_{\mathrm{Ad}} h_{\mathrm{Ad}} h_i(\lambda_i - l_i) + \beta_{\mathrm{Ad}} l_i(h_i l_{\mathrm{Ad}} - h_{\mathrm{Ad}}\lambda_i) + h_i(\beta_{\mathrm{Ad}}^2\lambda_i - 2\alpha_i h_{\mathrm{Ad}} l_i)}{h_i^2(2h_{\mathrm{Ad}} - l_{\mathrm{Ad}}) + \alpha_i\left(h_{\mathrm{Ad}}(\lambda_i - 2h_i) + 2h_i l_{\mathrm{Ad}}\right) + \lambda_i h_i(\beta_{\mathrm{Ad}} - h_{\mathrm{Ad}}) + l_i\left(h_{\mathrm{Ad}}\lambda_i - 2h_i(h_{\mathrm{Ad}} - l_{\mathrm{Ad}})\right)} \right)$$

After determining the advertisers' price, it will be substituted to determine the equilibrium number of users and advertisers. Given the complexity of the expression for price above, the relatively simple expressions for the number of users and advertisers arise from the structure of the model and the cancellation of several terms when the price is substituted. The expressions are as follows:

$$n_i = \frac{2h_{\mathrm{Ad}} h_i - 2h_i^2(h_{\mathrm{Ad}} - l_{\mathrm{Ad}}) - \alpha_i - h_{\mathrm{Ad}}\lambda_i - 2h_i\lambda_i - \beta_{\mathrm{Ad}} h_i \lambda_i - h_{\mathrm{Ad}}\lambda_i(l_i - h_i)}{2(\alpha_i - h_i + l_i)(h_{\mathrm{Ad}}(h_i - l_i - \alpha_i) + l_{\mathrm{Ad}}(\alpha_i - h_i + l_i) + \beta_{\mathrm{Ad}}\lambda_i)}$$

$$n_{\mathrm{Ad}} = \frac{\beta_{\mathrm{Ad}} h_i - h_{\mathrm{Ad}}(\alpha_i - h_i + l_i)}{2(h_{\mathrm{Ad}}(h_i - l_i - \alpha_i) + l_{\mathrm{Ad}}(\alpha_i - h_i + l_i) + \beta_{\mathrm{Ad}}\lambda_i)}$$

After the computation of the equilibrium number of users, advertisers, and their prices, the surplus will be calculated for each group of users and advertisers who join platform A.

3.2.4 User Surplus Calculation

As discussed in Chapter 2 on the User Surplus calculation in the case of partial-market coverage, where not all agents join the platform A. A cut-off stand-alone value $\hat{s}_i, \hat{s}_{Ad}$ is determined for marginal agents who are indifferent between joining and not joining. This situation arises due to the relatively high price set by the dominant ad-based platform, which makes participation unattractive for agents with lower stand-alone valuations. These agents participate only if their utility for joining is positive (i.e., $U_i = 0, U_{Ad} = 0$ at the cut-off). These values are given as follows:

$$\hat{s}_i = \lambda_i \cdot n_{\text{Ad}} - \alpha_i \cdot n_i$$
$$\hat{s}_{Ad} = p_{\text{Ad}} \cdot n_i - \beta_{\text{Ad}} \cdot n_i$$

The average agent surplus for each group is obtained by integrating the utility of all agents who join the platform A, which is given by:

$$\text{Group-}i : \bar{US}_i = \int_{\hat{s}_i}^{h_i} U_i(s_i) \times \frac{1}{h_i - \hat{s}_i} \times ds_i$$
$$\text{Group-}Ad : \bar{US}_{Ad} = \int_{\hat{s}_{Ad}}^{h_j} U_{Ad}(s_{Ad}) \times \frac{1}{h_{Ad} - \hat{s}_{Ad}} \times ds_{Ad}$$

In the above, for group-i, the integration is performed on all users whose s_i is above the cut-off $\hat{s}_i$, that is, $s_i \geq \hat{s}_i$ since only those users receive a positive utility from participation. The term $U_i(s_i)$ represents the utility of an individual user of type s_i, while the factor $\frac{1}{h_i - \hat{s}_i}$ corresponds to the probability density function that assigns equal likelihood to all users joining within the interval of a uniform distribution $[\hat{s}_i, h_i]$. Integrating the individual surplus over this interval and normalising by $\frac{1}{h_i - \hat{s}_i}$ yields the *average surplus per joining user*. The same interpretation holds for users in the group-Ad who join the platform A, with its respective parameters. Thus, $\bar{US}_i$ and $\bar{US}_{Ad}$ measure the average surplus generated by the agents in each group who join the platform A.

After solving the above integrals, the average agent surplus is:

$$\bar{US}_i = \frac{h_i + \hat{s}_i}{2} + \alpha_i n_i - \lambda_i n_{Ad}$$
$$\bar{US}_{Ad} = \frac{h_{Ad} + \hat{s}_{Ad}}{2} + \beta_{Ad} n_i - p_{Ad} n_i$$

The aggregate agent surplus for each group is obtained by multiplying the average surplus by the equilibrium number of users (n_i) or number of advertisers (n_{Ad}), for each group.

$$US_i = n_i * \bar{US}_i$$
$$US_{Ad} = n_{Ad} * \bar{US}_{Ad}$$

Finally, the total agent surplus on the dominant platform A is the sum of the surpluses generated by two participating groups i and Ad:

$$US = US_i + US_{Ad}$$

This formulation is used for the surplus computation in the following analysis.

3.3 Determinants of Advertising Spending on Ad-based Dominant Platform

In an ad-based dominant platform, the price charged to advertisers is determined by the interaction between users within their group and the behaviour of advertisers. Advertising spending on the ad-based monopoly platform by an advertiser is given by-

$$\text{Advertising Spending per Advertiser} = p_{\text{Ad}} \times n_i$$

where p_{Ad} is the price charged to advertisers and n_i denotes the number of users that advertisers can reach. This formulation implies that advertising spending depends on how much an advertiser pays per unit of advertising (the price) and how many users are available to view or interact with those advertisements (the user base).

The equilibrium price charged to advertisers (p_{Ad}), derived in the previous section, depends on three key parameters: α_i, β_{Ad}, and λ_i. The first parameter, α_i captures the **direct network benefits**, where users benefit from the presence of other users within the same group- i. The stronger α_i leads to greater user participation, as users yield greater utility from the platform. A larger user base, in turn, improves the advertising reach for advertisers on the platform. Hence, **Comparative Statics Analysis 1** investigates the impact of α_i on the equilibrium advertiser price (p_{Ad}) and on the market outcomes. Secondly, the parameter β_{Ad} captures the **advertising effectiveness** for the advertiser group. The higher this parameter means that it enhances the ad reach or the visibility of ads, so the platform becomes more valuable to advertisers, attracting more of them to join. **Comparative Statics Analysis 2** examines the relationship between β_{Ad} and p_{Ad} and its implications on market outcomes. Lastly, λ_i captures the **advertisement nuisance** that users experience from exposure to advertisers. When λ_i increases, users derive less utility from the platform, making them more likely to leave. To mitigate this risk, the way the platform adjusts the ad volume is investigated in **Comparative Statics Analysis 3** along with the market outcomes. Eventually, numerical examples are considered in which the values are set for some of the variables to demonstrate the trends in each of the numerical simulations.

When the equilibrium advertisers' price is differentiated with respect to each of these parameters, it is found that the numerator of the resulting expressions involves multiple variables. Owing to the analytical intractability of the numerator, whose sign is determined by the interaction of multiple parameters, it is not possible to establish the sign of the derivative in closed form. Therefore, numerical simulations are considered to examine its behaviour. In each simulation, the equilibrium price is differentiated with respect to one parameter at a time, and the sign of the derivative value is evaluated by varying α_i, β_{Ad}, and λ_i sequentially, while holding all other variables fixed in each case. These parameters directly govern the strength and direction of user and advertiser interactions and thus constitute the primary determinants of the platform's pricing incentives. The uniform distributions are held fixed to isolate the marginal effect of each parameter on the sign of the derivative and to avoid confounding pricing responses with changes in user or advertiser heterogeneity. This approach allows to assess how variations in network benefits, advertising effectiveness, and ad nuisance affect the direction of the platform's optimal pricing response .

The numerical examples presented in the following simulation findings were solved using MATLAB, and the corresponding code is included at the end of this thesis for reference.

3.3.1 Comparative Statics Analysis 1: Impact of Direct Network Benefits on Advertiser's Price

The equilibrium advertiser's price (p_{Ad}) is a function of the direct network benefit parameter (α_i) and can therefore be differentiated with respect to α_i. The sign of the resulting comparative static derivative value provides intuition about the movement of the equilibrium advertiser's price under different strengths of the direct network benefits. Differentiation yields the following analytical expression:

$$\frac{\begin{array}{c}-((2h_{\text{Ad}}^2(h_i-\alpha_i-l_i)+h_{\text{Ad}}(\beta_{\text{Ad}}h_i+2\alpha_i l_{\text{Ad}}+\beta_{\text{Ad}}\lambda_i-2h_i l_{\text{Ad}}+2l_{\text{Ad}}l_i)-\beta_{\text{Ad}}h_i l_{\text{Ad}}) \\ \times(2h_i^2(h_{\text{Ad}}-l_{\text{Ad}})+h_{\text{Ad}}(\alpha_i\lambda_i-2\alpha_i h_i-h_i\lambda_i-2h_i l_i+\lambda_i l_i)+h_i(2\alpha_i l_{\text{Ad}}+\beta_{\text{Ad}}\lambda_i+2l_{\text{Ad}}l_i))) \\ +(h_{\text{Ad}}(\lambda_i-2h_i)+2h_i l_{\text{Ad}})(h_{\text{Ad}}(\alpha_i-h_i+l_i)-\beta_{\text{Ad}}h_i)(\alpha_i(l_{\text{Ad}}-h_{\text{Ad}})+\beta_{\text{Ad}}\lambda_i+h_{\text{Ad}}(h_i-l_i)+l_{\text{Ad}}(l_i-h_i))\end{array}}{(2h_i^2(h_{\text{Ad}}-l_{\text{Ad}})-\alpha_i h_{\text{Ad}}(2h_i+\lambda_i)+2h_i(\alpha_i l_{\text{Ad}}-h_{\text{Ad}}\lambda_i-2h_{\text{Ad}}l_i+2l_{\text{Ad}}l_i)+h_{\text{Ad}}\lambda_i l_i)^2}$$

The denominator of the above expression is a perfect square and therefore strictly positive. Hence, the sign of the derivative is completely determined by the numerator.

Due to the complexity of the numerator and the presence of multiple variables, it is difficult to determine its sign analytically. Therefore, numerical simulations are performed by varying parameters such as direct network benefits (α_i), advertising effectiveness (β_{Ad}), and ad nuisance (λ_i) one at a time, while keeping the upper and lower bounds of the platform's distribution for users (h_i, l_i) and advertisers (h_{Ad}, l_{Ad}) fixed. Holding these bounds fixed allows the effect of each parameter on the derivative to be isolated.

Simulation Finding 1. *Numerical simulations of the derivative* ($\frac{\partial p_{Ad}}{\partial \alpha_i}$) *indicate that, for* $\alpha_i > 0$ *and within the range of positive parameter values considered and holding all other parameters fixed, the derivative is consistently negative. This result reflects the simulated parameter space rather than a general analytical property. Similar simulations confirm that the derivative value remains negative when* $\beta_{Ad} > 0$ *and* $\lambda_i > 0$, *again within the specified parameter ranges and with all other parameters held constant.*

Numerical Example 1: Consider a uniform distribution with $h_i = 20$, $h_{Ad} = 10$, and $l_i = l_{Ad} = 2$. In the first case, the parameter α_i is varied over the range $[1, 2]$, while $\beta_{Ad} = 2$ and $\lambda_i = 6$ are held fixed. In the second case, β_{Ad} is varied over the range $[1, 2]$, with $\alpha_i = 2$ and $\lambda_i = 6$. In the third case, λ_i^A is varied over the range $[6, 7]$, while $\alpha_i = \beta_{Ad} = 2$.

The upper limits of these ranges are determined by the normalisation of the total population, as assumed in the original model, and the values are chosen to comply with this assumption, as reflected in Simulation Findings 1, 2, and 3. Further, by keeping the distributional bounds fixed, the simulations isolate the effect of each parameter on the equilibrium outcomes without confounding changes in the underlying dispersion of stand-alone values.

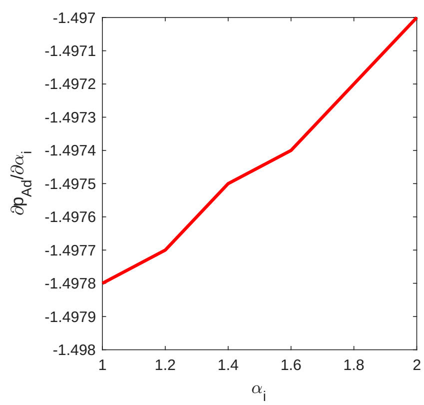
Under the parameter restrictions of the numerical example considered, increases in α_i, β_{Ad}, and λ_i result in a negative value of the derivative, while the upper and lower bounds of the uniform distributions of users and advertisers are held constant in all three cases discussed above. This confirms that:

$$\frac{\partial p_{Ad}}{\partial \alpha_i} < 0 \quad \text{for positive values of } \alpha_i, \beta_{Ad} \text{ and } \lambda_i$$

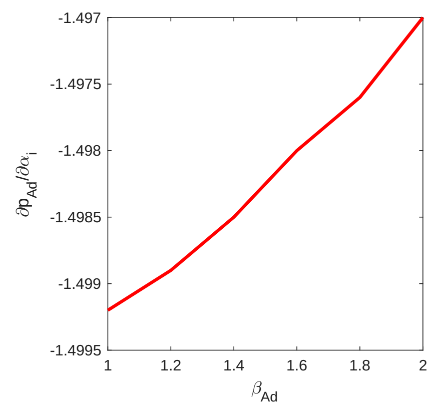
Notably, The sign of the derivative value is not determined analytically in the model, as it depends on the interaction of multiple opposing effects. On the one hand, an increase in α_i expands the user base, which raises advertisers' willingness to pay and creates upward pressure on the advertiser price. On the other hand, a larger user base increases the cost of user loss due to advertising, which induces the platform to adjust advertiser participation through pricing. Because these effects work in opposite directions, the overall impact on the advertiser price is ambiguous *a priori*. Numerical simulations indicate that, for the baseline parameter values considered, the latter effect dominates, leading to a negative derivative value. This result is robust across a range of alternative parameter values, with the derivative value remaining negative for all economically relevant configurations examined.

Regarding the intuition on demand elasticity, changes in α_i affect both the level and the responsiveness of advertiser participation through the endogenous interaction between users and advertisers. While this may be interpreted as a change in effective demand elasticity, the model does not impose this directly; instead, it arises endogenously from the two-sided structure of the platform.

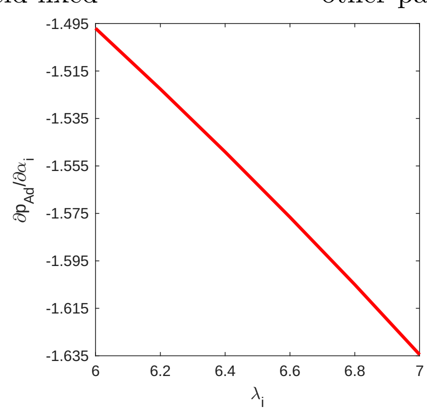
In the first case, as shown in Figure 3.1(a), when α_i increases while all other parameters are held fixed, the derivative value remains negative. The stronger the direct network benefits, the more value users derive from inter-



(a) A numerical example illustrating the sign of the derivative ($\frac{\partial p_{Ad}}{\partial \alpha_i}$) as α_i varies, while all other parameters are held fixed



(b) A numerical example illustrating the sign of the derivative ($\frac{\partial p_{Ad}}{\partial \alpha_i}$) as β_{Ad} varies, while all other parameters are held fixed



(c) A numerical example illustrating the sign of the derivative ($\frac{\partial p_{Ad}}{\partial \alpha_i}$) as λ_i varies, while all other parameters are held fixed

Figure 3.1: Graphical representation of the comparative static derivative ($\frac{\partial p_{Ad}}{\partial \alpha_i}$) for different values of α_i, β_{Ad}, and λ_i.

acting with others, which expands the user base. A larger user base makes the platform more attractive for advertisers because their ads reach more users, increasing advertisers' willingness to pay. This creates an upward pressure on the optimal advertisers' price (p_{Ad}), which weakens the downward effect of stronger network benefits on prices and therefore makes the derivative value less negative. At the same time, however, stronger network effects also make users more sensitive to ad nuisance. As the platform becomes more valuable to users, losing users due to ad overload becomes more costly for the platform. Since advertiser demand is downward sloping in price, the platform can attract more advertisers by lowering the ad price, but that increases ad volume and raises nuisance. The platform therefore chooses p_{Ad} to balance advertiser participation against the negative impact of advertising on user utility. The key point is that the platform takes into account the trade-off between advertiser participation and the negative effect of advertising on user utility. When network effects strengthen, retaining users becomes more valuable, increasing the cost of losing users due to excessive advertising. At the same time, a larger user base raises advertisers' willingness to pay. The optimal pricing decision reflects the balance between these opposing forces.

In the second case, when β_{Ad} increases while all other parameters are held fixed, the derivative value remains negative but moves upward, as shown in Figure 3.1(b). A higher value of β_{Ad} means that each user becomes more valuable to advertisers, increasing the effectiveness of advertising. This raises advertisers' willingness to pay for access to users, which creates an upward pressure on the optimal advertisers' price (p_{Ad}). Although this does not reverse the negative sign of the derivative, it weakens the downward pressure on prices, causing the derivative value to become less negative.

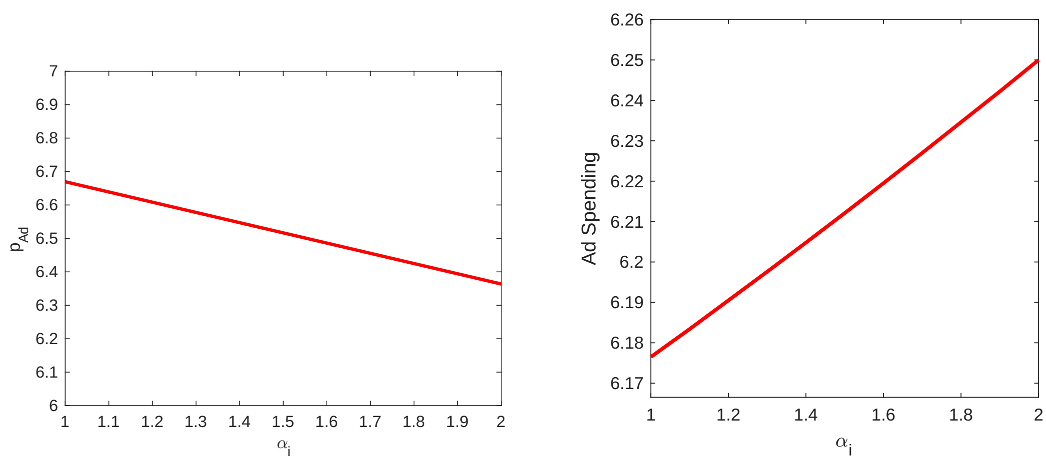
In the third case, when λ_i increases while the remaining parameters are held constant, the numerical values of the derivative decrease monotonically and become more negative, as shown in Figure 3.1(c). A higher λ_i implies that users suffer greater disutility from advertisements. When ad nuisance becomes more severe, then the platform has a stronger incentive to limit ad volume in order to retain users. This is achieved by lowering p_{Ad}, which reduces advertiser participation and ad volume, since there is a trade-off between ad revenue and user retention. As a result, increases in λ_i strengthen the downward pressure on advertising prices, making the derivative value more negative.

This numerical simulation illustrates how the derivative of the equilibrium advertising price with respect to direct network benefits responds to changes in direct network benefits, advertising effectiveness, and ad nuisance parameters. The analysis further examines how changes in α_i affect market outcomes and advertising spending.

3.3.1.1 Impact of Changes in α_i on Market Outcomes and Ad Spending

Consider a numerical example with $h_i = 20$, $h_{Ad} = 10$, and $l_i = l_{Ad} = 2$, $\beta_{\text{Ad}} = 2$ and $\lambda_i = 6$. In this setting, α_i is varied within the range $[1, 2]$.

In this numerical example, α_i is varied only within the range $[1, 2]$ to focus on the effects of changes in direct network benefits on market outcomes and advertising spending.



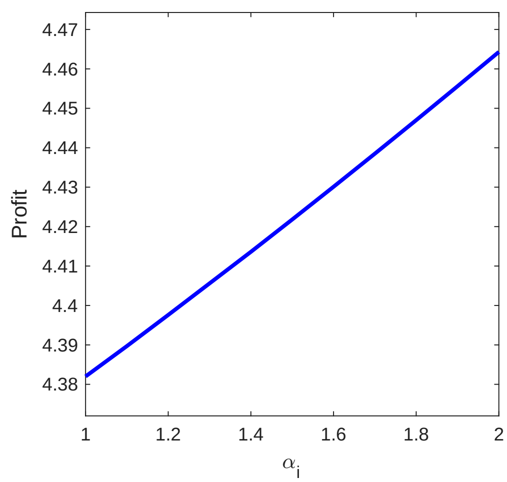
(a) illustrates the impact of stronger direct network benefits on the advertisers' price

(b) illustrates the impact of stronger direct network benefits on the advertising spending by an advertiser

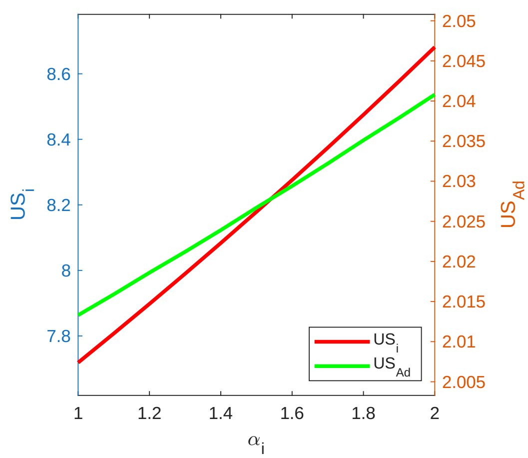
Figure 3.2: Numerical example showing the change in ad prices and spending when direct network benefits get stronger

In Figure 3.2(a) as α_i increases from 1 to 2, p_{Ad} decreases consistently. This confirms that stronger network effects among users lead to lower equilibrium advertisers' price. The optimal ad price continues to fall as network effects strengthen, but it does so at a constant rate of decrease. A higher α_i implies stronger direct network effects, where each user benefits more from the presence of others on the platform. Consequently, a change in the value of α_i leads to stronger network effects due to higher user participation, which affects the advertiser's price and their participation.

Figure 3.2(b) illustrates the impact of changes in α_i on total advertising spending by an advertiser. At first glance, it may appear counterintuitive that the platform lowers its price for advertisers when an increase in α_i makes it more attractive to advertisers. However, this outcome arises because the platform internalises the two-sided nature of the market. A higher α_i expands the user base n_i, which enhances the value of the platform for advertisers by increasing ad reach. The platform responds by reducing p_{Ad} to encourage more participation of advertisers, thereby amplifying the total advertising volume. The fall in the unit price is therefore a strategic response to the expanded user base: the platform prefers to trade off some price margin to increase the volume of ads, which becomes more valuable when reach is high. As a result, total advertising spending rises even though the per-unit price falls. This reflects the platform's strategic use of pricing to balance participation across both sides of the market and to maximise total surplus.



(a) illustrates the impact of stronger direct network benefits on the platform's profit and advertisers' ad spending



(b) illustrates the impact of stronger direct network benefits on the surpluses of both user and advertiser groups

Figure 3.3: Numerical example showing the change in profit and surpluses when direct network benefits get stronger

In an ad-based platform model, the platform sets prices to maximise profit. However, because its revenue depends on participation from both users and advertisers, the profit-maximising outcome implicitly balances the interests of the two groups through cross-side network effects. In this static setting, a larger user base is not a strategic objective in itself, but a consequence of the platform's choice to maximise profit, as greater user participation increases the value of ads and therefore the overall revenue.

Figures 3.3(a) and 3.3(b) illustrate the impact of α_i on the platform's profit, as well as on user and advertiser surplus. A higher α_i strengthens the direct network effects on the user side, leading to a larger user base and, consequently, greater value for advertisers. Stronger network effects increase each user's utility from participation due to the presence of other users, thereby raising user surplus. At the same time, the resulting reduction in the equilibrium advertising price encourages greater participation of the advertisers and increases advertising spending by each advertiser. The combined effect of a growing user base and lower entry costs for advertisers enhances the value of participation on both sides of the market, driving higher advertising demand and ultimately increasing the platform's profit.

The numerical results clearly illustrate: As direct network benefits strengthen, advertising prices decline, leading to a substantial increase in advertisers' participation and their spending. This generates a higher surplus for both users (due to enhanced user networks) and advertisers (due to greater reach at lower cost). Consequently, the platform experiences increased profitability, validating a strategy centered on user acquisition and the reinforcement of network effects as a means of value creation and monetisation.

A real-world example that illustrates this is Facebook's early growth strategy. Facebook focused on rapidly expanding its user base by offering the platform for free and enhancing user experience, thereby increasing user participation. As more users joined, the platform became more valuable to each individual, reflecting the realisation of existing direct network effects rather than a change in their underlying strength. This growing user base made the platform increasingly attractive to advertisers. Rather than raising advertising prices, Facebook kept them relatively low to encourage greater participation from advertisers. This strategy led to higher overall ad spending, increased surplus for both users (through improved content and connectivity) and advertisers (through greater reach and targeting), and ultimately higher platform profits. In the context of the model, this example illustrates how a zero-pricing strategy increases user participation (n_i), which in turn amplifies the realised value of network effects, without altering the structural network effect parameter itself.

3.3.2 Comparative Statics Analysis 2: Impact of Advertising Effectiveness on Advertiser's Price

The analysis now focuses on deriving the equilibrium advertiser's price with respect to the advertising effectiveness parameter, as this parameter influences advertisers' participation and their willingness to pay, and the participation of users. The sign of the resulting comparative static derivative determines how the equilibrium advertiser's price moves as the strength of the parameter varies. Differentiation yields the following analytical expression:

$$\frac{\begin{aligned}&(h_i^2(h_{\mathrm{Ad}} - l_{\mathrm{Ad}}) - \alpha_i h_{\mathrm{Ad}}(h_i - \lambda_i) + \alpha_i h_i l_{\mathrm{Ad}} + 2\beta_{\mathrm{Ad}} h_i \lambda_i + h_{\mathrm{Ad}} h_i \lambda_i - h_{\mathrm{Ad}} h_i l_i - h_{\mathrm{Ad}} l_{\mathrm{Ad}} l_i + h_i l_{\mathrm{Ad}} l_i)\\ &\quad \times (2h_{\mathrm{Ad}} h_i^2 - 2h_i^2 l_{\mathrm{Ad}} - 2\alpha_i h_{\mathrm{Ad}} h_i + \alpha_i h_{\mathrm{Ad}} \lambda_i + 2\alpha_i h_i l_{\mathrm{Ad}} + \beta_{\mathrm{Ad}} h_i \lambda_i - h_{\mathrm{Ad}} h_i \lambda_i - 2h_{\mathrm{Ad}} h_i l_i + h_{\mathrm{Ad}} \lambda_i l_i + 2h_i l_{\mathrm{Ad}} l_i)\\ &+ h_i \lambda_i\, (\alpha_i h_{\mathrm{Ad}} - \beta_{\mathrm{Ad}} h_i - h_{\mathrm{Ad}} h_i + h_{\mathrm{Ad}} l_i)\, (\alpha_i l_{\mathrm{Ad}} - \alpha_i h_{\mathrm{Ad}} + \beta_{\mathrm{Ad}} \lambda_i + h_{\mathrm{Ad}} h_i - h_{\mathrm{Ad}} l_{\mathrm{Ad}} - h_i l_{\mathrm{Ad}} + l_{\mathrm{Ad}} l_i)\end{aligned}}{(2h_{\mathrm{Ad}} h_i^2 - 2h_i^2 l_{\mathrm{Ad}} - 2\alpha_i h_{\mathrm{Ad}} h_i + \alpha_i h_{\mathrm{Ad}} \lambda_i + 2\alpha_i h_i l_{\mathrm{Ad}} + \beta_{\mathrm{Ad}} h_i \lambda_i - h_{\mathrm{Ad}} h_i \lambda_i - 2h_{\mathrm{Ad}} h_i l_i + h_{\mathrm{Ad}} \lambda_i l_i + 2h_i l_{\mathrm{Ad}} l_i)^2}$$

The denominator of the above expression is a perfect square and therefore strictly positive. Hence, the sign of the derivative $\frac{\partial p_{Ad}}{\partial \beta_{Ad}}$ is entirely determined by the numerator. Since the numerator is not tractable and cannot be signed analytically, numerical simulations are conducted over relevant regions of the parameter space.

The analysis focuses on variations in α_i, β_{Ad}, and λ_i, while the upper and lower bounds of the uniform distributions for users and advertisers are held constant, as they determine the stand-alone benefits drawn by each side of the platform. Varying these bounds would alter the range of stand-alone values and could move the system past critical thresholds where participation constraints become active, making it difficult to isolate the marginal effects of changes in α_i, β_{Ad}, and λ_i on the derivative. In the first case, direct network benefits are increased while the remaining parameters are held fixed. In the second case, advertising effectiveness is varied with all other parameters fixed. In the third case, the ad nuisance parameter is varied while keeping the remaining parameters constant.

Simulation Finding 2. *Numerical simulations of* $\frac{\partial p_{Ad}}{\partial \beta_{Ad}}$ *indicate that, within the range of positive parameter values considered, the derivative value remains positive when* $\alpha_i > 0$, $\beta_{Ad} > 0$, *and* $\lambda_i > 0$ *with the remaining parameters fixed in each case.*

Numerical Example 2: Consider a uniform distribution with $h_i = 20$, $h_{Ad} = 10$, and $l_i = l_{Ad} = 2$. In the first case, the parameter α_i is varied over the range $[1, 2]$, while $\beta_{Ad} = 2$ and $\lambda_i = 6$ are held fixed. In the second case, β_{Ad} is varied over the range $[1, 2]$, with $\alpha_i = 2$ and $\lambda_i = 6$. In the third case, λ_i is varied over the range $[6, 7]$, while $\alpha_i = \beta_{Ad} = 2$.

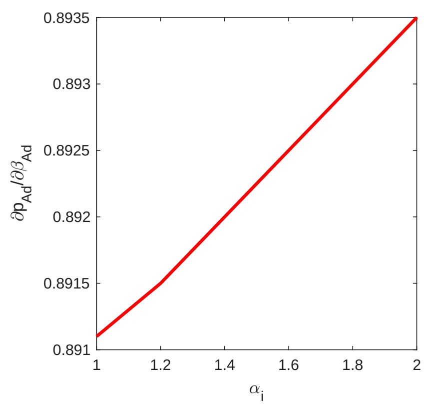
Under the parameter restrictions of the numerical example considered, increases in α_i, β_{Ad}, and λ_i result in a positive value of the derivative, while the upper and lower bounds of the uniform distributions of users and advertisers are held constant in all three cases discussed above. This confirms that:

$$\frac{\partial p_{Ad}}{\partial \beta_{Ad}} > 0 \quad \text{for positive values of } \alpha_i, \beta_{Ad} \text{ and } \lambda_i$$

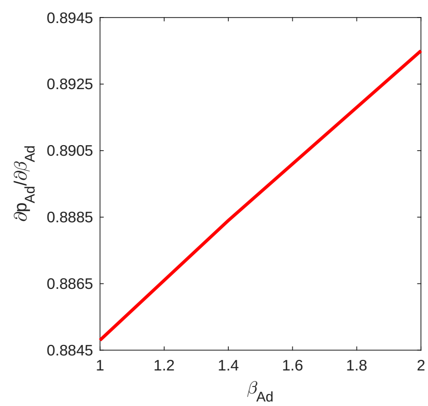
In the first case, as α_i increases while all other parameters are held constant, the derivative value remains positive, indicating that the increase in user participation enhances ad reach for the advertisers, which in turn raises the optimal advertising price. As illustrated in Figure 3.4(a), the monotonic behaviour of the equilibrium advertiser's price occurs as stronger network effects expand the user base and enhance advertisers' willingness to pay for access to the larger user base.

In the second case, as illustrated in Figure 3.4(b), when β_{Ad} increases while all other parameters are fixed, the derivative value remains positive and shows an upward trend. A higher β_{Ad} implies that each user generates more value for advertisers, through better ad targeting, higher user engagement, or greater ad effectiveness, making advertising on the platform more profitable. As a result, advertisers are willing to pay more to access the user base, which strengthens the demand for advertising. This increase in willingness to pay translates into a higher equilibrium advertiser's price (p_{Ad}). The upward movement of the derivative value indicates that the marginal effect of further increases in advertising effectiveness on the price becomes stronger: each additional unit of β_{Ad} raises the optimal advertiser's price by a slightly larger amount, reflecting the reinforcing effect of higher advertising returns on advertiser participation and platform revenue.

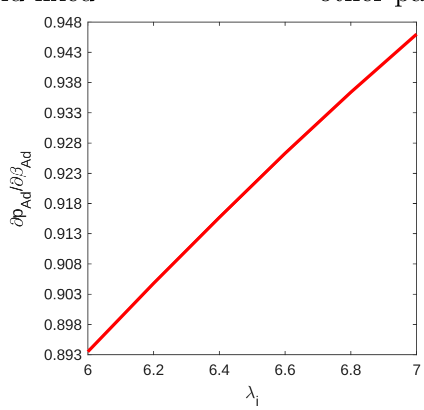
In the third case, when λ_i increases with the remaining parameters held constant, the derivative value is positive, as shown in Figure 3.4(c). The



(a) A numerical example illustrating the sign of the derivative ($\frac{\partial p_{Ad}}{\partial \beta_{Ad}}$) as α_i varies, while all other parameters are held fixed



(b) A numerical example illustrating the sign of the derivative ($\frac{\partial p_{Ad}}{\partial \beta_{Ad}}$) as β_{Ad} varies, while all other parameters are held fixed



(c) A numerical example illustrating the sign of the derivative ($\frac{\partial p_{Ad}}{\partial \beta_{Ad}}$) as λ_i varies, while all other parameters are held fixed

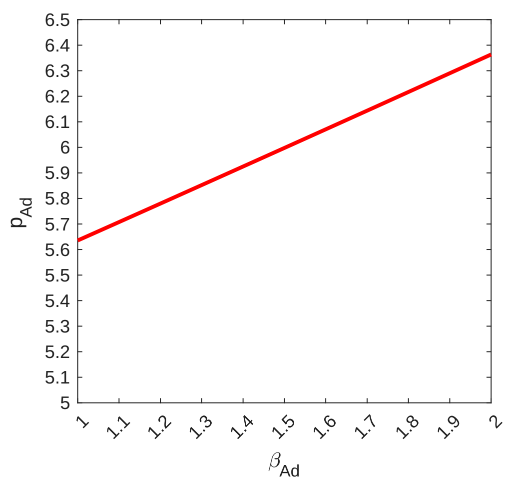
Figure 3.4: Graphical representation of the comparative static derivative ($\frac{\partial p_{Ad}}{\partial \beta_{Ad}}$) for different values of α_i, β_{Ad}, and λ_i.

derivative captures a trade-off between the direct positive effect of stronger advertising effectiveness on advertisers' willingness to pay and the indirect negative effect arising from reduced user participation due to ad disutility. When λ_i increases, the ad disutility dominates; however, stronger β_{Ad} enhances the advertisers' surplus without causing a large contraction in the user base, allowing the platform to raise the equilibrium advertiser's price.

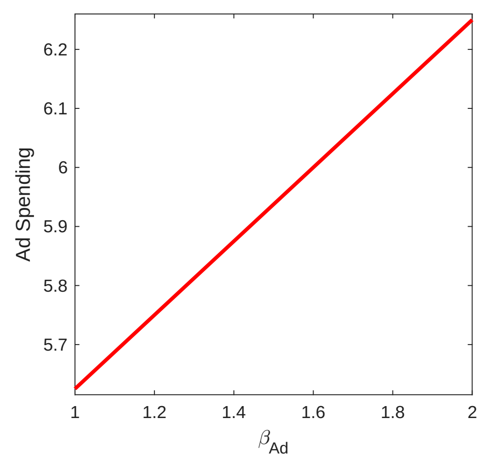
This numerical simulation illustrates how the derivative of the equilibrium advertiser's price with respect to advertising effectiveness responds to changes in the direct network benefits, advertising effectiveness, and ad nuisance parameters. The analysis further examines how changes in β_{Ad} affect market outcomes and advertising spending, without confounding effects from other variables.

3.3.2.1 Impact of Changes in β_{Ad} on Market Outcomes and Ad Spending

Consider a numerical example with $h_i = 20$, $h_{Ad} = 10$, and $l_i = l_{Ad} = 2$, $\alpha_i = 2$ and $\lambda_i = 6$. In this setting, β_{Ad} is varied within the range $[1, 2]$.



(a) illustrates the impact of stronger ad effectiveness on the advertiser's price

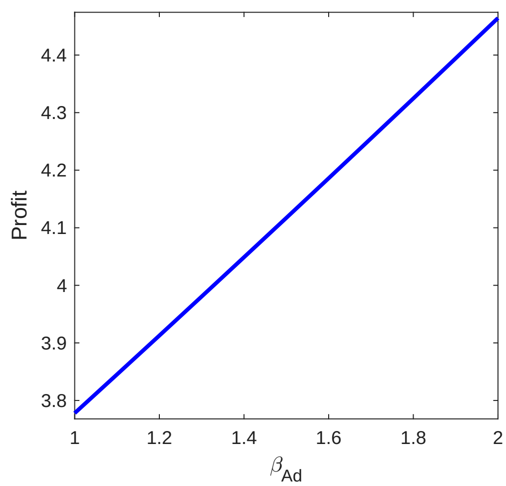


(b) illustrates the impact of stronger ad effectiveness on each advertiser's spending

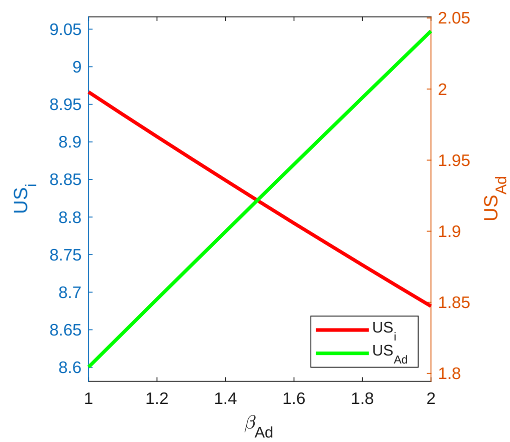
Figure 3.5: Numerical example showing the change in ad prices and spending when when ad effectiveness gets stronger

Figure 3.5(a) illustrates how changes in advertising effectiveness β_{Ad} affect the optimal advertiser's price of the platform. In the figure, as β_{Ad} increases from 1 to 2, the equilibrium advertisers' price (p_{Ad}) increases steadily. This pattern reflects the fact that greater ad effectiveness increases the value advertisers obtain from each user reached on the platform, which in turn strengthens their willingness to pay for access. By capturing part of this additional surplus, the platform optimally sets a higher price for advertisers. This numerical pattern in the figure is consistent with the derivative result, which implies that the advertising price increases as the ad effectiveness improves.

Figure 3.5(b) illustrates how changes in β_{Ad} affect advertising spending by each advertiser on the platform. As β_{Ad} increases, the effectiveness of advertising improves, making the platform more valuable to advertisers. This increases their willingness to participate and allows the platform to charge a higher equilibrium advertisers' price (p_{Ad}). At the same time, greater exposure to ads resulting from higher β_{Ad} can reduce the utility of users, causing a gradual decline in the user base (n_i). The combined effect of these opposing forces, a shrinking user base, and a rising advertiser's price, produces an overall increase in advertising spending. This outcome highlights that the gain from higher prices more than offsets the loss from a smaller user base, leading to higher advertising spending by an advertiser on the platform.



(a) illustrates the impact of stronger ad effectiveness on the platform's profit and advertisers' ad-spending



(b) illustrates the impact of stronger ad effectiveness on the surpluses of both user and advertiser groups

Figure 3.6: Numerical example showing the change in profit and surpluses when ad effectiveness get stronger

Figures 3.6(a) and 3.6(b) illustrate the impact of β_{Ad} on the platform's profit, as well as on user and advertiser surplus. When β_{Ad} becomes stronger, advertisers derive greater value from reaching users. It happens through enhanced targeting and better engagement tools that allow ads to be delivered more precisely to relevant users, increasing conversion rates and returns on investment. As a result, the platform becomes more attractive to advertisers, which allows it to raise the advertiser's price (p_{Ad}) while still attracting more advertisers n_{Ad}. This, in turn, leads to an increase in overall advertising revenue and platform profitability. However, stronger ad effectiveness can also have adverse effects on the user side. Higher ad effectiveness often involves showing more frequent, better-targeted, or more intrusive advertisements. While this increases the returns for advertisers, it can reduce the perceived utility of the platform for users, lowering their surplus and discouraging engagement. Thus, the platform faces a fundamental trade-off. While increasing β_{Ad} improves the advertiser's surplus and platform's revenue, it also simultaneously reduces user participation. The numerical results make this trade-off explicit: stronger ad effectiveness increases the optimal advertiser's price, boosts advertisers' participation, and raises advertising spending per advertiser and platform profit, but at the cost of diminished user surplus and potentially lower user engagement.

This outcome supports a platform strategy that leverages ad effectiveness to maximise monetisation while recognising its limits. Advertisers are willing to pay more when the ads perform better, which drives up both advertising prices and participation. At the same time, the platform must manage the negative impact on users to sustain long-term growth. Balancing these opposing effects, higher advertiser surplus and platform revenue versus potential declines in user surplus, is essential to maintaining a stable and profitable ecosystem.

3.3.3 Comparative Statics Analysis 3: Impact of Advertisement Nuisance on Advertiser's Price

This analysis focuses on deriving the equilibrium advertisers' price (p_{Ad}) with respect to the advertisement (ad) nuisance parameter (λ_i), as this parameter influences participation of users and advertisers. The sign of the resulting comparative static derivative determines how the equilibrium advertiser's price moves as the strength of the parameter varies. Differentiation yields the following analytical expression:

$$\frac{-(h_{Ad}-l_{Ad})(\alpha_i-h_i+l_i)(h_{Ad}(\alpha_i+l_i-h_i)-\beta_{Ad}h_i)^2}{\left(2h_{Ad}(h_i^2-l_{Ad})-2\alpha_i h_{Ad}h_i+\alpha_i h_{Ad}\lambda_i+2\alpha_i h_i l_{Ad}+\beta_{Ad}h_i\lambda_i-h_{Ad}h_i\lambda_i-2h_{Ad}h_i l_i+h_{Ad}\lambda_i l_i+2h_i l_{Ad}l_i\right)^2}$$

The denominator of the above expression is a perfect square and therefore strictly positive. Hence, the sign of $\frac{\partial p_{Ad}}{\partial \lambda_i}$ is entirely determined by the numerator. Since the numerator has multiple variables and cannot be signed analytically, numerical simulations are conducted over relevant regions of the parameter space.

The analysis focuses on variations in α_i and β_{Ad}, while λ_i and the upper and lower bounds of the uniform distributions for users and advertisers remain constant. In the first case, direct network benefits are strengthened while the remaining parameters are held fixed. In the second case, advertising effectiveness is varied with all other parameters fixed.

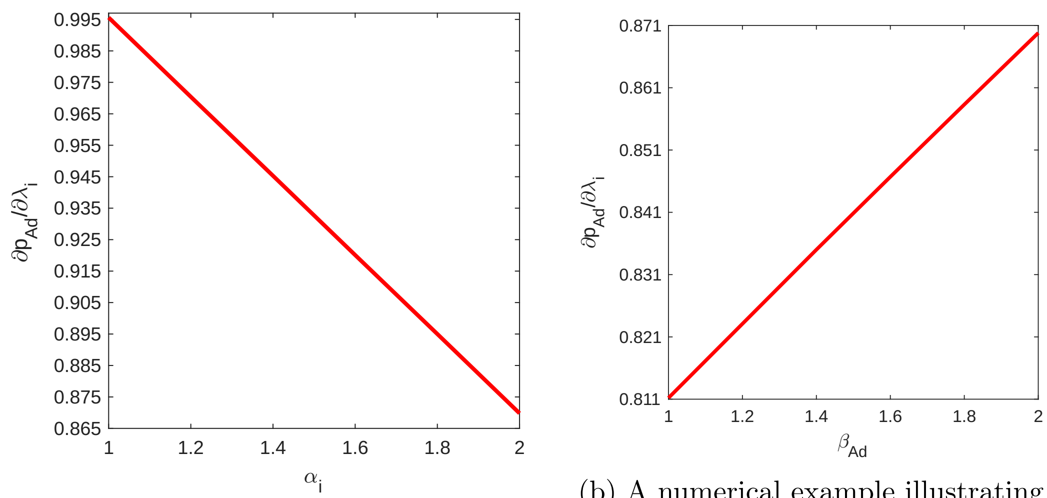
Simulation Finding 3. *Numerical simulations of $\frac{\partial p_{Ad}}{\partial \lambda_i}$ indicate that, within the range of positive parameter values considered, the derivative value remains positive when $\alpha_i > 0$ and $\beta_{Ad} > 0$, with the remaining parameters held fixed in each case.*

Numerical Example 3: Consider a uniform distribution with $h_i = 20$, $h_{Ad} = 10$, and $l_i = l_{Ad} = 2$. In the first case, the parameter α_i is varied over the range $[1, 2]$, while $\beta_{Ad} = 2$ and $\lambda_i = 6$ are held fixed. In the second case, β_{Ad} is varied over the range $[1, 2]$, with $\alpha_i = 2$ and $\lambda_i = 6$.

Under the parameter restrictions of the numerical example considered, increases in α_i and β_{Ad} result in a positive value of the derivative, while the upper and lower bounds of the uniform distributions of users and advertisers are held constant in two cases discussed above. This confirms that:

$$\frac{\partial p_{Ad}}{\partial \lambda_i} > 0 \quad \text{for positive values of } \alpha_i \text{and } \beta_{Ad}$$

In the first case, as illustrated in Figure 3.7(a), when the direct network benefits α_i are relatively weak, the expansion of the user base is limited. In this regime, each advertiser is relatively more valuable to the platform, which raises the equilibrium advertisers' price (p_{Ad}) in order to maintain revenue,



(a) A numerical example illustrating the sign of the derivative ($\frac{\partial p_{Ad}}{\partial \lambda_i}$) as α_i varies, while all other parameters are held fixed

(b) A numerical example illustrating the sign of the derivative ($\frac{\partial p_{Ad}}{\partial \lambda_i}$) as β_{Ad} varies, while all other parameters are held fixed

Figure 3.7: Graphical representation of the comparative static derivative ($\frac{\partial p_{Ad}}{\partial \lambda_i}$) for different values of α_i and β_{Ad}.

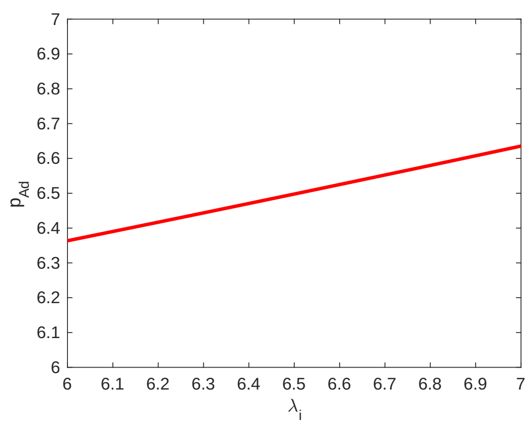
resulting in a positive derivative value. As α_i increases within this range, the derivative exhibits a downward trend. The platform faces a trade-off: higher advertisers' prices can drive users away due to increased ad nuisance (λ_i), so the optimal response is to lower the advertisers' price. The declining value of the derivative reflects the platform's strategy of reducing advertisers' prices to manage ad nuisance and retain users.

In the second case, as illustrated in Figure 3.7(b), when β_{Ad} increases while all other parameters are fixed, the derivative value remains positive and displays an upward movement, as advertisers derive greater utility from advertising on the platform. This increased value strengthens the demand for advertising, as advertisers are willing to pay higher prices to reach the platform's user base. Consequently, the equilibrium advertisers' price rises, and the derivative value remains positive. The upward movement of the derivative value indicates that the marginal impact of advertising effectiveness on the optimal ad price grows stronger: each incremental increase in β_{Ad} further boosts advertiser participation and willingness to pay, reinforcing the platform's pricing power.

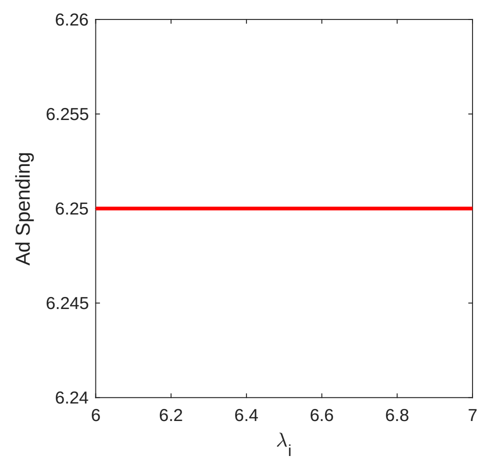
This numerical simulation illustrates how the derivative of the equilibrium advertiser's price with respect to ad nuisance responds to changes in the direct network benefits and advertising effectiveness. The analysis further examines how changes in λ_i within the limited region affect market outcomes and advertising spending.

3.3.3.1 Impact of Changes in λ_i on Market Outcomes and Ad Spending

Consider a numerical example with $h_i = 20$, $h_{Ad} = 10$, and $l_i = l_{Ad} = 2$, $\alpha_i = 2$, $\beta_{Ad} = 2$ and λ_i is examined within the range $[6, 7]$.



(a) illustrates the impact of stronger ad nuisance on the advertisers' price

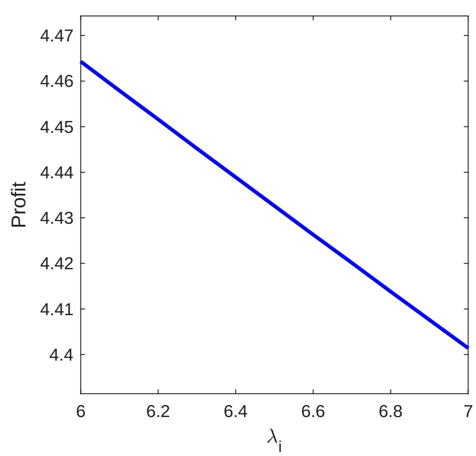


(b) illustrates the impact of stronger ad nuisance on the ad spending by each advertiser

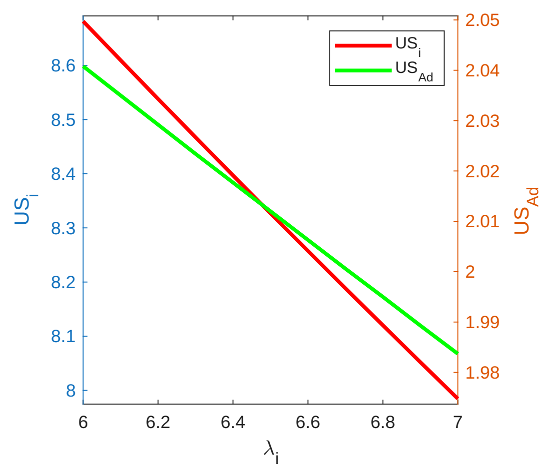
Figure 3.8: Numerical example showing the change in ad prices when ad nuisance gets stronger

Figure 3.8(a) shows the relationship between the stronger λ_i and p_{Ad}. In the simulation, as λ_i rises from 6 to 7, the optimal price per advertisement increases steadily. This behaviour is consistent with the proof $\frac{\partial p_{\text{Ad}}}{\partial \lambda_i} > 0$: when advertisements become more intrusive, the user base shrinks, and the number of participating advertisers declines. In response, the platform raises the unit price of advertising to increase revenue from a smaller pool of advertisers. Thus, a higher level of ad disutility enables the platform to extract more revenue per advertiser despite a declining number of users.

Figure 3.8(b) depicts the effect of stronger λ_i on advertising spending by each advertiser on the platform. As λ_i increases, the platform becomes less attractive to both users and advertisers: users disengage due to greater ad disutility, and advertisers' participation reduces as the effective user reach shrinks. At the same time, the platform charges a higher price per unit of advertisement. Over the numerical range considered, these opposing forces approximately offset each other: the fall in the number of advertisers and impressions is balanced by the rise in price per unit. This results in advertising spending per advertiser remaining roughly constant. In other words, although both the user base and advertiser participation shrink, the higher price per unit offsets this decline, keeping ad spending unchanged. This occurs because the rise in p_{Ad} compensates for the fall in n_i, maintaining ad spending. However, this balance is only sustained *up to a certain threshold* of ad disutility. Once λ_i crosses a critical level, the deterioration of the user experience becomes so severe that user participation collapses, eliminating the users of the platform and sharply reducing the demand for advertisers.



(a) illustrates the impact of stronger ad nuisance on the platform's profit



(b) illustrates the impact of stronger ad nuisance on the surpluses of both user and advertiser groups

Figure 3.9: Numerical example showing the change in profit and surpluses when ad nuisance gets stronger

Figures 3.9(a) and 3.9(b) illustrate the impact of stronger λ_i on the profit of the platform and on the surplus of both users and advertisers. As λ_i

increases, users experience greater disutility from advertising. This occurs because of higher ad frequency, more intrusive formats, or otherwise intensified exposure. As a result, user surplus declines, and participation shrinks, reducing the size of the audience available to advertisers. Advertisers, who value platforms with large and engaged user bases, also reduce their participation when user numbers decline. At the same time, the platform responds by increasing the advertiser's price to monetise the remaining agents more intensively. Thus, higher ad nuisance simultaneously reduces user and advertiser participation while raising the unit price of advertising. The numerical analysis makes this trade-off explicit. As ad disutility rises, the platform initially observes an increase in p_{Ad}, but the contraction in n_{Ad} and n_i eventually outweighs the price effect, causing a decline in both the advertiser and the user surplus. Consequently, platform profit also decreases because the aggregate advertising demand falls.

The above investigation provides a clear insight into the first research question: *How are advertising spendings shaped by key factors on an ad-based dominant platform?*

To explain in detail in response to the above, the total advertising spending on the platform is given by the product $p_{\text{Ad}} \times n_i \times n_{\text{Ad}}$, where p_{Ad} denotes the price per ad impression, n_i represents the number of active users, and n_{Ad} is the number of participating advertisers. Total spending, therefore, increases when more advertisers join, when the user base expands, or when the platform raises the price per impression. This relationship is reflected in the profit graphs presented earlier (Figures 3.3(a), 3.6(a), and 3.9(a)), which show that the profit of the platform (total revenue generated from advertisers, since the fixed cost is constant and eliminated after the optimisation) moves closely with changes in total advertising spending as parameters α_i, β_{Ad}, and λ_i vary.

Stronger direct network benefits reduce ad prices, thereby encouraging greater participation of advertisers and an increase in ad spending and the platform's profitability. Greater ad effectiveness enables the platform to charge higher ad prices, as improved ad reach increases advertisers' willingness to pay. Since both advertiser participation and prices increase, the total advertising spending and platform profit increase accordingly. In contrast, higher ad nuisance reduces user participation and advertiser entry, leading to a lower overall ad spending when λ_i becomes large. This reveals a critical trade-off: excessive ad nuisance degrades user experience, lowering both user surplus and participation. Although advertising spending per advertiser remains relatively stable over a moderate range of ad nuisance levels, due to the offsetting effects of higher prices and fewer advertisers,beyond a critical threshold, user attrition becomes severe enough to trigger a collapse in platform activity.

The numerical analysis demonstrates how total advertising spending, platform profit, and the surplus of both users and advertisers are jointly determined by direct network effects, advertising reach, and ad disutility. These results highlight the central role of direct network benefits, ad effectiveness, and nuisance in shaping platform outcomes and reveal the trade-offs inherent in advertising-based monetisation. It contributes to the literature by clarifying how these factors interact to determine the price for advertisers and participation outcomes on ad-based platforms. Further, it informs the broader policy discussion by providing a structural explanation for the patterns observed in the digital advertising markets. They also complement the findings of CMA (2020) on online platforms and digital advertising, illustrating how key factors jointly shape advertising spending and overall market outcomes.

3.4 Multi-homing and Platform Mergers: Implications for User Welfare

This section analyses user welfare in a scenario where *multi-homing* occurs, that is, where users simultaneously engage with more than one ad-based digital platform. Consider the case of Instagram and TikTok, two free-to-join platforms that both provide primarily entertainment content. Because neither platform charges access fees, it is reasonable to assume that a substantial share of users participate in both, dividing their attention between them. Users are assumed to allocate a fixed amount of time, say one hour, across these platforms according to their individual preferences. Since preferences are heterogeneous, the proportion of time spent on each platform will vary between users. The two platforms also perform similar functions and host users and content that overlap, making them at least partially substitutable. This overlap raises an important methodological issue: if users consume essentially the same content or interact with the same users on both platforms, the measured benefits may be duplicated, leading to an *overes-*

timation of user surplus. Any assessment of user welfare must therefore account for redundancy in consumption or social connections. At the same time, multi-homing generates incremental value by allowing users to connect with distinct networks, discover unique features, or access differentiated content available on only one of the platforms. This diversity can enhance the utility of the user beyond what could be achieved on a single platform. It is assumed in this framework that every user is active on at least one platform, though the degree of multi-homing differs across individuals.

Further, a question arises when considering the effect of a merger between the two platforms. If Instagram and TikTok were to combine into a single monopoly platform, users would face a single point of access to both networks and content. Whether this increases or decreases the user surplus depends on the balance of effects. On the one hand, consolidation could reduce duplication, improve cross-platform connectivity, and offer convenience gains. On the other hand, the loss of competition can degrade the user experience, especially if the merged entity increases the intensity of advertising, and there could be upward pressure on prices, which harms advertisers (and, ultimately, consumers). Understanding these trade-offs is essential for evaluating the welfare consequences of platform mergers in markets characterised by multi-homing. Before turning to the analysis, it is useful to note that this section builds on the framework developed in this chapter, particularly the introduction of advertising and user multi-homing, which are not present in Chapter 2. As a result, the merger analysis here differs from that in Chapter 2, which focuses on price-based platform competition with user single-homing. The aim is to highlight how merger outcomes change in this extended setting.

3.4.1 The Model

3.4.1.1 Model setup

Consider two competing ad-based digital platforms, denoted by A and B, whose objective is to attract users in group- i. Let $\tilde{n}_i^A$ be the number of users who join platform A only, $\tilde{n}_i^B$ be the number of users who join platform B only, and $n_i^{A,B}$ be the total number of users who multi-home by joining both platforms. The following assumptions define the user behaviour and market environment under the multi-homing scenario:

(i) Continuum of users: There exists a continuum of users with total measure 1, i.e., the user base is normalised to one.

(ii) Platform substitutability: Platforms A and B provide similar types of utility to users, such as entertainment and social interaction. Because of this, users can switch between them or split their attention according to individual preferences, content offerings, or perceived user experience. This behaviour is captured by the parameter δ_i, which reflects how substitutable the two platforms are for the user group-i.

(iii) User heterogeneity: Users differ in their individual stand-alone value, which represents the direct utility they derive from the platform. Each of these values is drawn from a distribution across the population. This specification captures heterogeneity among users while keeping the analysis tractable.

(iv) Free access: Access to both platforms is free, as both operate under an ad-based revenue model. Hence, every user can join at least one platform. This removes the possibility of being excluded due to cost. If the platforms together cover all relevant user needs (such as entertainment or social interaction), then every individual has an incentive to participate in at least one of them. Under these conditions, the model assumes *full market coverage*, which means that the entire population is active on at least one platform, although some users choose only one while others use both.

(v) The parameter governing direct network benefits is the same between platforms $\alpha_i^A = \alpha_i^B = \alpha_i$

These assumptions provide the foundation for analysing user allocation across platforms and evaluating the welfare implications in scenarios that allow for multi-homing. Although the payoff function builds on the advertising model presented in the previous section, this formulation introduces a key parameter: a user-specific substitutability parameter δ_i, which captures how similar the two platforms are perceived by the user group-i. A related concept appears in Anderson et al. (2018), where a uniform discount factor is used to adjust the utility from a user's second platform. In contrast, the current model allows the overlap adjustment to vary between users and across utility components (stand-alone benefits and ad disutility); asymmetry between platforms A and B is captured via the $\min(\cdot)$ terms in their payoff function.

The payoff function for users who choose to multi-home is defined as follows:

$$U_i^{A+B} = \left[(s_i^A + \alpha_i \tilde{n}_i^A - \lambda_i^A n_{\text{Ad}}^A) + (s_i^B + \alpha_i \tilde{n}_i^B - \lambda_i^B n_{\text{Ad}}^B) \right] + \alpha_i n_i^{A,B} - \delta_i \cdot \min(s_i^A, s_i^B) + \delta_i \cdot \min(\lambda_i^A n_{\text{Ad}}^A, \lambda_i^B n_{\text{Ad}}^B)$$

The utility of a multi-homing user, denoted by U_i^{A+B} comprises the stand-alone benefits of each platform (s_i^A and s_i^B), adjusted upward by the direct network effects ($\alpha_i \tilde{n}_i^A$) generated by users exclusive to platform A ($\tilde{n}_i^A$), exclusive to Platform B ($\tilde{n}_i^B$), and adjusted downward by the advertisement disutility, represented by $\lambda_i^A n_{\text{Ad}}^A$ and $\lambda_i^B n_{\text{Ad}}^B$, where λ_i^A and λ_i^B capture the ad nuisance on platforms A and B due to the presence of advertisers indicated by n_{Ad}^A and n_{Ad}^B on platforms A and B. The term $(\alpha_i n_i^{A,B})$ captures the direct network effects generated by those users who multi-home, the term $-\delta_i \cdot \min(s_i^A, s_i^B)$ removes the portion of stand-alone benefits that is duplicated when platforms provide similar content or functionality, while the term $+\delta_i \min(\lambda_i^A n_{\text{Ad}}^A, \lambda_i^B n_{\text{Ad}}^B)$ accounts for the fact that exposure to similar advertisements on both platforms does not double the perceived nuisance. In this way, δ_i captures how substitutable the platforms are for group-i users. A high δ_i indicates that the two platforms provide overlapping experiences and similar ad exposure, so the incremental utility of using both is limited. A low δ_i means that users perceive the platforms as distinct, obtaining nearly full stand-alone benefits from each while bearing a separate ad nuisance.

This formulation extends the standard single-homing utility by recognising that users gain both unique and duplicated benefits from each platform. Together, these components ensure that the payoff function reflects both the additional utility gained from access to multiple platforms and the diminishing returns that arise when platforms are perceived as close substitutes.

While the literature Anderson et al. (2018) addresses double counting of welfare in multi-homing by introducing a discount factor $\alpha \in [0, 1]$ that scales down the utility from a user's second platform, the current model generalises this approach. In this framework, multi-homing utility explicitly incorporates both stand-alone platform benefits and advertising disutility. Redundancy between platforms is adjusted through user-specific substitutability parameters (δ_i) applied to the minimum of each component, ensuring that duplicated stand-alone benefits or advertising exposures are not counted twice. This formulation captures heterogeneity in user perceptions of platform substitutability and extends the earlier framework to account for both the positive and negative dimensions of multi-homing. In other words, this model does not assume that every user experiences the same overlap discount as set out in Anderson et al. (2018). In this framework, each user has a fixed substitutability parameter δ_i. For users with high δ_i, the additional benefit of joining a second platform is small, as much of the value is already captured in the first. For users with low δ_i, platforms are perceived as distinct, so they obtain nearly the full benefit from both. While δ_i is not drawn from a distribution, this parameter captures deterministic differences in how users value multiple platforms.

Example: Consider a user who engages with both Instagram and TikTok. When $\delta_i = 0.5$, the platforms provide very similar experiences. The user perceives the entertainment and functionality as largely repeated, so the additional benefit from joining both is only partial. At the same time, exposure to advertising on one platform feels similar to the other, meaning that the annoyance of ads is not fully additive. When $\delta_i = 0.1$, the platforms are more distinct. The user derives separate enjoyment from each, gaining nearly the full stand-alone benefit of both. However, the irritation from ads on one platform is not mitigated by the other, so the total ad disutility is close to the sum across platforms. In this way, δ_i adjusts the overall user surplus to avoid redundancy of stand-alone benefits and ad disutilities when platforms are similar. It ensures that the model captures the trade-off between the added value of multi-homing and the diminishing marginal benefit when platforms are close substitutes.

3.4.1.2 Users' behaviour and Equilibrium

The equilibrium conditions describe how users choose between single-homing and multi-homing across the two platforms. Each user compares the utility obtained from joining one or both platforms and chooses the option that maximises their utility. The equilibrium conditions for the user will be as follows:

(i) A user joins the platform A if and only if $U_i^A \geq 0$.

(ii) A user joins the platform B if and only if $U_i^B \geq 0$.

(iii) A user multi-homes (joins both A and B) if and only if $U_i^{A+B} > max(U_i^A, U_i^B)$.

In equilibrium, the set of users on each platform is determined by these participation conditions. Multi-homing occurs when the combined utility from using both platforms, after accounting for redundant benefits, exceeds the maximum of the utilities from single-homing on each platform. It also allows the model to explicitly identify the share of users who overlap between the two platforms, thereby capturing realistic user allocation patterns and the extent of multi-homing across platforms.

User Draws from the Distribution: Each user draws a stand-alone value $\tilde{s}_i$ from a continuous distribution and is assumed to be a uniform distribution with lower bound l_i and upper bound h_i, i.e., $\tilde{s}_i \sim U[l_i, h_i]$.

The user's stand-alone value for platform A is given by $\hat{s}_i^A = \tilde{s}_i$, while for platform B it is defined symmetrically as $\hat{s}_i^B = h_i + l_i - \tilde{s}_i$, so that users with a low draw $\tilde{s}_i$ value platform B more highly. This symmetric construction creates a simple preference-based differentiation between the two platforms: users who place a high value on platform A naturally assign a lower value to platform B and vice versa. It also ensures that the two platforms have identical value ranges, while preserving a clean one-dimensional representation of user preferences. This setup keeps the model tractable while capturing horizontal differentiation across platforms.

The threshold value $\hat{s}_i^A$ represents the minimum stand-alone value required for a user in group-i to obtain zero net utility from joining the platform A, such that

$$U_i^A(\hat{s}_i) = 0 \quad \text{when} \quad \tilde{s}_i = \hat{s}_i^A$$

A user joins the platform A if and only if his draw $\tilde{s}_i$ exceeds the minimum stand-alone value ($\hat{s}_i^A$), i.e.,

$$\tilde{s}_i > \hat{s}_i^A$$

Hence, the set of users who join the platform A is given by

$$\tilde{s}_i \in [\hat{s}_i^A,\, h_i]$$

For platform B, the stand-alone value $\hat{s}_i^B$ is defined as the value of $\tilde{s}_i$ that gives the user zero net utility from joining the platform B:

$$U_i^B(\tilde{s}_i) = 0 \quad \text{when} \quad \tilde{s}_i = \hat{s}_i^B$$

The symmetric mapping $\hat{s}_i^B = h_i + l_i - \tilde{s}_i$ implies that a low draw of $\tilde{s}_i$ corresponds to a high valuation for platform B. Using this relation, a user joins the platform B whenever his mapped stand-alone value exceeds the threshold:

$$h_i + l_i - \tilde{s}_i > \hat{s}_i^B \quad \Longleftrightarrow \quad \tilde{s}_i < h_i + l_i - \hat{s}_i^B$$

Hence, the number of users who join the platform B is

$$\tilde{s}_i \in \left[\, l_i,\; h_i + l_i - \hat{s}_i^B \,\right]$$

This symmetric formulation reflects a Hotelling-type setting where platform A is located at one endpoint and platform B at the other, with users uniformly distributed between. It ensures that clearly defined stand-alone values separate users who prefer A or B. Further, this symmetry guarantees that both platforms face the same potential market size and variation in user allocation results from model parameters rather than differences in the underlying distribution.

User Allocation Across Platforms. Using the above thresholds, user participation can be classified as follows:

(i) **Platform A only:** If a user draws $\tilde{s}_i \in (\hat{s}_i^A, h_i)$, then he will join the platform A denoted by n_i^A. If a user draws $\tilde{s}_i \in ((h_i + l_i - \hat{s}_i^B), h_i)$, then he will join only the platform A. These users form the group $\tilde{n}_i^A$.

(ii) **Platform B only:** If a user draws $\tilde{s}_i \in (l_i, (h_i + l_i - \hat{s}_i^B))$, then he will join the platform B denoted by n_i^B. If a user draws $\tilde{s}_i \in (l_i, \hat{s}_i^A)$, then he will join only the platform B. These users form the group $\tilde{n}_i^B$.

In this way, the stand-alone values $(\hat{s}_i^A)$ of A and $(\hat{s}_i^B)$ of B jointly determine the equilibrium allocation of users between platforms A and B.

(iii) **Candidates for Multi-homing:** After determining the cut-off values above, there are some users who are located in the overlapping region of the uniform distribution, that is, between $\hat{s}_i^A$ and $(h_i + l_i - \hat{s}_i^B)$ as they draw their stand-alone values between these values:

$$\tilde{s}_i \in \left(\hat{s}_i^A, (h_i + l_i - \hat{s}_i^B)\right)$$

This interval identifies users whose characteristics place them between the two cut-off points. The measure of this set represents the users who are potential multi-homers, denoted by $n_i^{A,B}$. The number of such users is obtained as below:

$$n_i^{A,B} = \frac{(h_i + l_i - \hat{s}_i^B) - \hat{s}_i^A}{h_i - l_i}$$

Further, threshold values (t_1, t_2) are determined at the point where a user's utility from multi-homing equals the utility from single-homing. A user will prefer to multi-home only when their utility lies between these thresholds. These thresholds are given by:

$$t_1 = \lambda_i^A n_{\text{Ad}}^A - \alpha_i \tilde{n}_i^A \quad \text{such that} \quad U_i^{A,B} = U_i^B$$
$$t_2 = (h_i + l_i - \lambda_i^B n_{\text{Ad}}^B) - \alpha_i \tilde{n}_i^B \quad \text{such that} \quad U_i^{A,B} = U_i^A$$

In the above, t_1 is the point at which the user is just indifferent between multi-homing and joining only platform B. At this value, the utility from multi-homing equals the utility from single-homing on platform B. The threshold t_2 is the point at which the user is indifferent between multi-homing and joining only platform A. Here, the utility from accessing both platforms matches the utility from single-homing on platform A. Unlike the standard approach, multi-homing in this model is not determined merely by requiring $U_i^A > 0$ and $U_i^B > 0$. Instead, a user multi-homes only when the dedicated multi-homing utility $U_i^{A,B}$ exceeds the utility from single-homing on either platform. The thresholds (t_1, t_2) capture these indifference conditions.

A numerical example is presented in the next paragraph to clarify the user allocation between platforms.

Numerical Example 4. Consider a uniform distribution with $h_i = 60$, $h_{\text{Ad}} = 40$, $l_i = 6$, $l_{\text{Ad}} = 4$, $\alpha_i = 2$, $\beta_{\text{Ad}} = 2$, $\lambda_i^A = 20$, and $\lambda_i^B = 15$. Although the stand-alone values for the two platforms are drawn from the same uniform distribution, differences in the ad nuisance parameters λ_i introduce asymmetry between them. The equilibrium users and prices for the two platforms are computed independently using the equilibrium formulas from the ad-based monopoly model. The overlapping users between the platforms are then identified.

The equilibrium for the individual platform are: $p_{\text{Ad}}^A = 22.6833$, $p_{\text{Ad}}^B = 21.4321$, $n_i^A = 0.9325$, $n_i^B = 0.9870$, $n_{\text{Ad}}^A = 0.5753$, and $n_{\text{Ad}}^B = 0.5783$. From these, the stand-alone values ($\hat{s}_i^A$ and $\hat{s}_i^B$) are computed as follows:

$$U_i^A = \hat{s}_i^A + \alpha_i \cdot n_i^A - \lambda_i^A \cdot n_{\text{Ad}}^A \quad ; \quad U_i^B = (h_i^B + l_i^B - \hat{s}_i^B) + \alpha_i \cdot n_i^B - \lambda_i^B \cdot n_{\text{Ad}}^B$$
$$0 = \hat{s}_i^A + 2(0.9325) - 20(0.5753) \quad ; \quad 0 = (60 + 6 - \hat{s}_i^B) + 2(0.9870) - 15(0.5783)$$
$$\hat{s}_i^A = 9.64 \quad ; \quad \hat{s}_i^B = 59.29$$

The above can be interpreted as the stand-alone values of both platforms are drawn from the same uniform distribution. A user comes along and draws their individual stand-alone value, if:

(i) If a user draws any value of $\tilde{s}_i > 9.64$, then he will join the platform A.
If a user draws $\tilde{s}_i \in (59.29, 60)$, then he will join only the platform A.
(ii) If a user draws any value of $\tilde{s}_i < 59.29$, then he will join the platform B.
If a user draws $\tilde{s}_i \in (6, 59.29)$, then he will join only the platform B.

From the above, it is observed that some users are distributed between the region that lies between 9.64 and 59.29. This identifies the number of potential multi-homing users represented by $n_i^{A,B}$. Their measure is calculated as follows:

$$n_i^{A,B} = \frac{(h_i + l_i - \hat{s}_i^B) - \hat{s}_i^A}{h_i - l_i} = \frac{59.29 - 9.64}{60 - 6} = 0.9194$$

The user allocation between the platforms is interpreted as, out of a total population of 1, the number of users who join the platform A (n_i^A) is 0.9325.

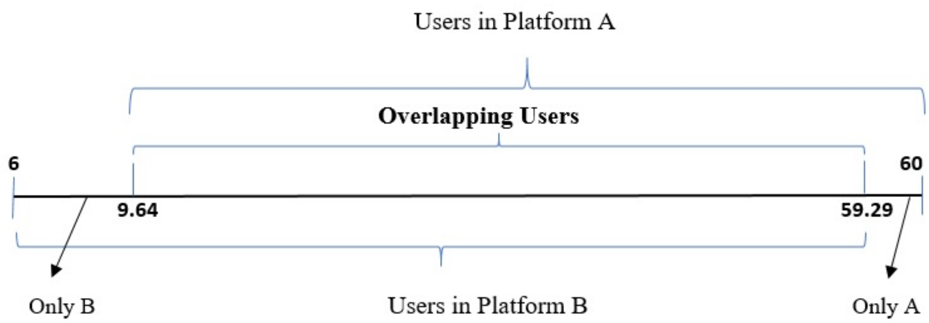


Figure 3.10: Illustrates the share of users between the platforms A and B

This implies that their stand-alone values lie between thresholds $(9.64, 60)$. Similarly, the number of users who join the platform B (n_i^B) is 0.9870. This implies that their stand-alone values lie between thresholds $(6, 59.29)$. In addition, there are 0.9194 potential multi-homing users, whose stand-alone values $(\tilde{s}_i)$ fall within the thresholds $(9.64, 59.29)$. From this, the number of users who join only Platform A $(\tilde{n}_i^A)$ can be determined, which is 0.0131 (0.9325 – 0.9194), corresponding to those whose stand-alone values lie between 59.29 and 60. Similarly, the number of users who join only Platform B $(\tilde{n}_i^B)$ is 0.0676 $(0.987 - 0.9194)$, representing those with stand-alone values between 6 and 9.64.

After determining the user who engages on platforms A and B, the following analysis examines the user surplus.

3.4.2 Users' Surplus under Multi-homing Scenario

After determining the users who are potential multi-homers on platforms A and B, a user is considered to be a candidate for multi-homing when their combined utilities of using both platforms exceed the utility from using platforms A and B after accounting for redundant benefits. The threshold values from Numerical example 4 are arrived at $t_1 = 11.4798((20 * 0.5753) - (2 * 0.0131))$ such that $U_i^{A,B} = U_i^B$ and $t_2 = 57.1903(60 + 6 - (15 * 0.5783) - (2 * 0.1352))$ such that $U_i^{A,B} = U_i^A$. Hence, a candidate chooses to multi-home when their utility falls between t_1 and t_2. If the platforms are close substitutes, meaning (δ_i) is sufficiently large, then the utility from multi-homing is lower than the utility from single-homing. In that case, a user is better off in single-homing. Based on the definition of utility for a group i user engaged in multi-homing, the average surplus denoted by $\bar{US}_i^{A,B}$ is defined as-

$$\begin{aligned}\bar{US}_i^{A,B} &= \left[\int_{\hat{s}_i^A}^{s_i^B} (s_i^A + \alpha_i \tilde{n}_i^A - \lambda_i^A n_{\text{Ad}}^A) \times \frac{1}{s_i^B - \hat{s}_i^A} \times ds_i^A\right] \\ &+ \quad \left[\int_{\hat{s}_i^A}^{s_i^B} (s_i^B + \alpha_i \tilde{n}_i^B - \lambda_i^B n_{\text{Ad}}^B) \times \frac{1}{s_i^B - \hat{s}_i^A} \times ds_i^B\right] \\ &\qquad + \alpha_i (n_i^{A,B}) - \delta_i^A \cdot \min(s_i^A, s_i^B) + \delta_i \cdot \min(\lambda_i^A n_{\text{Ad}}^A, \lambda_i^B n_{\text{Ad}}^B)\end{aligned}$$

The above expression captures the average surplus for multi-homing users. The integration limits, $\hat{s}_i^A$ and s_i^B, represent the range of stand-alone valuations for the user type i, where $\hat{s}_i^A$ is the threshold at which a user begins to find it worthwhile to join the platform A and also join the platform B until s_i^B. Within the integrand, the first terms s_i^A and s_i^B reflect the stand-alone benefits of accessing both platforms. The following terms, $\alpha_i \tilde{n}_i^A$, $\alpha_i \tilde{n}_i^B + n_i^{A,B}$), account for the direct network effects created by users exclusive to platform A ($\tilde{n}_i^A$), exclusive to platform B ($\tilde{n}_i^B$). The negative components, $-\lambda_i^A n_{\text{Ad}}^A$ and $-\lambda_i^B n_{\text{Ad}}^B$, represent the ad disutility on each platform. The factor $\frac{1}{s_i^B - \hat{s}_i^A}$ represents the uniform probability density of users whose stand-alone valuations are distributed between the bounds $\hat{s}_i^A$ and s_i^B, ensuring that the integral averages surplus across the full range of multi-homing user preferences to yield the *average surplus per multi-homing user*. Adjusted upward by direct network effects from the multi-homing users ($n_i^{A,B}$) and the final adjustment term that involves δ_i captures the degree of substitutability between the two platforms. Specifically, δ_i reduces the duplicated stand-alone benefits and scales the minimum of the two ad disutilities, $\min(\lambda_i^A n_{\text{Ad}}^A, \lambda_i^B n_{\text{Ad}}^B)$, recognising that exposure to similar advertisements on both platforms does not double the perceived nuisance.

After solving the above integral, the aggregate user surplus for multi-homing users is obtained by multiplying it by the number of multi-homing users, which is as follows:

$$US_i^{A,B} = n_i^{A,B}\Bigg[\left(\frac{(s_i^B + \hat{s}_i^A)}{2} + \alpha_i\tilde{n}_i^A - \lambda_i^A n_{\text{Ad}}^A\right) + \left(\frac{(s_i^B + \hat{s}_i^A)}{2} + \alpha_i\tilde{n}_i^B - \lambda_i^B n_{\text{Ad}}^B\right)$$
$$+ \alpha_i(n_i^{A,B}) - \delta_i^A \cdot \min\left(\frac{(s_i^B + \hat{s}_i^A)}{2}, \frac{(s_i^B + \hat{s}_i^A)}{2}\right) + \delta_i \cdot \min(\lambda_i^A n_{\text{Ad}}^A, \lambda_i^B n_{\text{Ad}}^B)\Bigg]$$

To illustrate this, **Numerical Example 4** is considered to analyse the welfare implications for multi-homing users. Given that $\delta_i = 0.5$, the surplus can be computed as follows:

$$\text{US}_i^{A,B} = 0.9194\Bigg[\left(\frac{(59.29 + 9.64)}{2} + (2 \times 0.0131) - (20 \times 0.5753)\right)$$
$$+ \left(\frac{(59.29 + 9.64)}{2} + (2 \times 0.0676) - (15 \times 0.5783)\right)$$
$$+ (2 \times 0.9194) - 0.5(34.465) + 0.5(8.6745)\Bigg]$$

$$US_i^{A,B} = 34.8032$$

The above computation shows that when platforms are moderately substitutable, specifically when the substitutability parameter is set at $\delta_i = 0.5$, the surplus for users who multi-home is 34.8032, which falls between the threshold values ($t_1 = 11.4798, t_2 = 57.1903$). This surplus is also substantially higher than the surplus the same user would obtain from single-homing on platform A (22.8232) or on Platform B (25.5267). Under this degree of substitutability, multi-homing users clearly gain more from accessing both platforms than from joining just one. For instance, consider Instagram and TikTok, if a user encounters similar content on both platforms (e.g. 50% overlap), the additional utility gained from accessing both diminishes, leading to a lower overall surplus. This highlights the importance of identifying the threshold of substitutability beyond which multi-homing becomes suboptimal.

This finding naturally raises a key research question: *Under what conditions does multi-homing reduce user welfare relative to single-homing?* To examine

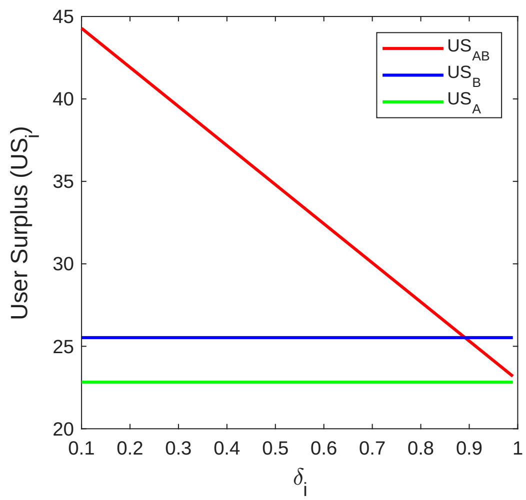


Figure 3.11: Impact of change in δ_i on multi-homing users' surplus in comparison with the user surplus for the same users on single-homing

this, the substitutability parameter δ_i is varied over a range (e.g., from 0.1 to 0.99, approaching perfect substitutability), and the resulting user surplus from multi-homing is compared with the surplus from single-platform participation for the same group of users who engage in multi-homing. This comparison focuses on how the welfare of existing multi-homing users changes as the two platforms become increasingly similar, rather than comparing two distinct user populations. When δ_i reaches 0.9, the analysis shows that the aggregate user surplus from multi-homing stood at 25.3185, while the corresponding surplus from single-homing on platform B is slightly higher at 25.5267. Further, as a response to the key research question, this result indicates that, under conditions of high substitutability, that is, when the value of δ_i crosses 0.9 from the numerical example considered, users are marginally better off remaining on a single platform rather than using both. Figure 3.11 illustrates that as δ_i increases, the surplus for multi-homing users decreases. This decline in surplus occurs because, as the two platforms become closer substitutes, the additional benefit from joining the second platform becomes increasingly small. When most of the value provided by one platform is already replicated on the other, the marginal utility from multi-homing drops, which reduces the overall surplus from accessing both. In other words, once the platforms offer nearly the same experience, the second platform contributes little extra value, so the advantage of multi-homing disappears.

In other words, as δ_i increases, the incremental welfare benefit of multi-homing decreases and can even reverse when the platforms become too similar in content and ad exposure. Although multi-homing generally offers access to a larger network and broader content, its relative advantage diminishes sharply when the overlap between platforms grows, suggesting that excessive similarity between competing platforms can erode user welfare.

3.4.3 User Surplus under Platform Consolidation

Further, the above analysis also raises a key policy-relevant question: *How does user surplus under multi-homing differ between a duopoly and a monopoly platform market?* To address this, user surplus in a merged (monopoly) scenario is compared with the surplus obtained under duopoly multi-homing. This comparison allows for a direct assessment of whether consolidation improves or diminishes user welfare.

The analysis considers the impact of mergers between platforms. When two platforms have identical ad nuisance parameters, they attract the same number of users and advertisers and charge identical advertising prices. In such a case, a merger between the two platforms does not change equilibrium outcomes, as users continue to face the same level of advertising nuisance and platform behaviour remains unchanged. As a result, the fully symmetric case offers limited insight into the effects of consolidation. Allowing for a small difference in ad nuisance keeps the setting realistic and shows how even minor quality differences influence user allocation and competitive pressure. The symmetric case remains a useful benchmark, but the asymmetric version offers a more informative and economically meaningful comparison. Building on this distinction, two merger scenarios are considered. In both cases, platforms A and B are initially symmetric in terms of uniform distribution, and the only difference lies in the degree of advertising nuisance each generates. Specifically, the ad nuisance parameter is set at $\lambda_i^A = 15$ for platform A and $\lambda_i^B = 20$ for platform B, introducing a mild asymmetry in user experience while maintaining a symmetric underlying UD. A mild asymmetry in ad nuisance is introduced to avoid the knife-edge outcomes that arise in a fully symmetric setup.

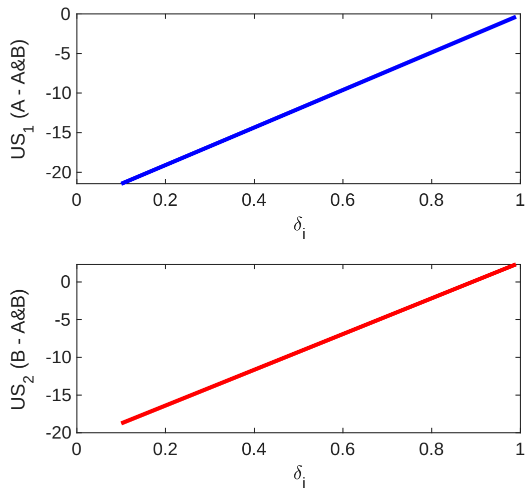


Figure 3.12: Change in the surplus of multi-homing users:(1) when platform A merges with B and A remains in the market ($US_1 : A - A\&B$) and (2) when platform B acquires A and B and B remains in the market ($US_2 : B - A\&B$). The negative values indicate that user surplus is lower under a monopoly compared to a duopoly with multi-homing. When $\delta_i = 0.9$ the surplus is positive in $US_2 : B - A\&B$.

(i) **Platforms A and B merge and A remains in the market.** Platforms A and B merge into a single unified platform A, and users interact within this integrated environment. This scenario eliminates inter-platform competition and allows the merged platform to internalise all network effects, but also increases ad exposure since all advertisers now concentrate on a single platform A.

(ii) **Platforms A and B merge and B remains in the market.** Platform B acquires A but continues to operate the merged entity as an integrated service under platform B's structure. In this case, users effectively access a single system that combines the content and user bases of both platforms, but with potentially lower overall ad nuisance and less duplicated content.

The user surplus is then evaluated for different degrees of substitutability, δ_i, ranging from weak to strong. When the two platforms are close substitutes ($\delta_i = 0.9$), the numerical results in Figure 3.12 show a distinct difference between the two merger scenarios. In scenario (i), the merged firm operates solely as platform A. Due to the concentration of users and advertisers on a monopoly platform, the total volume of ads shown to users increases. Under the duopoly, users effectively spread their activity across two platforms, which limits the ad disutility they experience on each. After the merger, this dispersion disappears, and the merged platform has no competitive incentive to restrain ad volumes. As a result, user surplus falls relative to the multi-homing benchmark.

In scenario (ii), after the merger, platform B operates as a monopoly platform. In this setup, the combined user and advertiser base is integrated in a way that removes duplicated material and overlapping ads that multi-homing users previously encountered on both platforms. Users are therefore experiencing fewer repeated ads and less repetition in content. This cuts down the total ad disutility they experience compared with multi-homing across two separate platforms. The drop in repeated ads improves the user experience enough to outweigh the loss of having two distinct platforms. As a result, when $\delta_i = 0.9$, the user surplus in scenario (ii) ends up higher than in the multi-homing duopoly.

This comparison also connects back to the earlier discussion on consideration of different levels of ad disutilities. Varying the ad-nuisance parameter shows how the merger reshapes the user surplus by either concentrating or reducing total exposure. In scenario (i), the higher ad nuisance raises the effective disutility faced by the users, which reduces the surplus. In scenario (ii), the lower ad nuisance and repeated ads reduce the overall disutility, so users benefit despite the loss of platform variety.

This investigation yields two key findings on user welfare in ad-based digital platform markets.

First, multi-homing does not always enhance user welfare. Although existing literature, such as Anderson et al. (2018) suggests that multi-homing can generate full network effects by allowing users to interact with all other users across the different platforms, the current analysis shows that this benefit is sensitive to the degree of substitutability between platforms. Specifically, at lower levels of substitutability, users derive distinct stand-alone benefits from each platform, making multi-homing attractive and welfare-enhancing.

However, as the substitutability parameter δ_i increases and goes beyond a threshold, users experience a reduced surplus from multi-homing due to redundant content and increased ad exposure, which ultimately lowers their surplus. This result diverges from prior studies that suggest consistent welfare gains from multi-homing and suggests that in some cases, users are better off just engaging in a single platform because of the lower ad disutility. This result clarifies that the relative advantage of one platform over the other arises not from intrinsic asymmetry in user valuations, but from differences in ad-related disutility.

Second, the analysis shows that user welfare can, under specific conditions, be higher in a monopoly (merged) environment than in a duopoly with multi-homing. When two highly substitutable ad-based platforms consolidate, the reduction in duplicated content and ads can outweigh the loss of inter-platform choice, leading to a net welfare gain for users. This outcome contrasts with findings in price-based digital platform markets, such as those discussed in Ambrus et al. (2016) and Anderson et al. (2019), where mergers generally lead to a reduction in user welfare due to higher prices and reduced competition. In ad-based markets, the main welfare channel operates through the balance between network effects and ad nuisance, rather than price. Hence, under high substitutability and strong advertisement disutilities, consolidation improves user surplus by eliminating redundant exposure and enhancing efficiency in content delivery.

In sum, this analysis challenges prevailing assumptions in the platform economics literature by demonstrating that (i) multi-homing is not inherently welfare-improving, and (ii) platform consolidation does not necessarily harm users. When platforms are highly substitutable and advertising disutility is significant, users are better off either single-homing on the less ad-intensive platform or benefiting from a consolidated service that reduces duplication. However, when platforms remain differentiated, competition between them continues to yield higher welfare through greater variety and lower aggregate ad exposure.

3.5 Conclusion

This chapter has examined how direct network benefits, advertising effectiveness, and advertising disutility jointly determine advertising spending, user participation, and platform profitability in ad-based digital platforms. By extending the analysis to environments with multi-homing and potential platform mergers, the chapter provides a richer and more detailed understanding of user welfare under different market structures.

The model shows that a stronger direct network benefits lower advertiser prices, increases advertiser participation, and raises both advertiser and user surplus. This combination improves the profitability of the platform and supports strategies focused on user acquisition and network growth. In contrast, improvements in advertising effectiveness allow platforms to charge higher prices without deterring advertisers' participation, further boosting revenues. However, the results also reveal a key trade-off: excessive ad nuisance undermines user experience, reduces surplus, and discourages participation.

Advertising spending remains roughly stable at moderate levels of ad disutility because higher prices offset the decline in advertiser numbers. Yet, beyond a critical threshold, user attrition becomes so severe that advertiser demand and platform profit collapse. This dynamic clarifies both the gains from expanding user networks and improving ad performance and the limits imposed by user tolerance of advertising.

Turning to multi-homing, the chapter challenges the widely held assumption that accessing multiple platforms is always beneficial to users. When platforms provide sufficiently distinct stand-alone value, multi-homing increases user welfare because it allows users to connect with different networks, discover unique features, or access differentiated content available on only one platform.

However, as the substitutability parameter δ_i increases, reflecting greater duplication in content, user interactions, or experience, the marginal benefits of multi-homing diminish. Beyond a certain threshold, duplicated content and higher cumulative ad exposure reduce user surplus, making single-homing preferable. This finding diverges from earlier studies that suggested consistent welfare gains from multi-homing, showing instead that its benefits are conditional on platform differentiation.

The analysis also offers a fresh perspective on mergers. In price-based digital markets, a merger to a monopoly platform typically reduces user surplus through higher prices and reduced competition. However, in ad-based platforms, allowing mergers between highly substitutable platforms can, under specific conditions, increase user surplus. This occurs through the elimination of duplicate content and advertising, lower cumulative ad nuisance, and stronger consolidated network effects. In such cases, a monopoly platform may deliver higher welfare than a competitive multi-homing environment.

This result highlights the need to evaluate mergers not only through the lens of competition but also by considering the structure of user benefits and costs in ad-based markets. When platforms are close substitutes, consolidation can reduce redundancy and improve efficiency; when they are complementary, mergers may instead harm users by removing variety and choice.

The findings carry several important implications for policy and regulatory oversight of digital advertising markets:

(i) Encouraging network effects: Policies that support open access, interoperability, or data portability can strengthen direct network effects. In the model, stronger direct network effects are associated with greater user participation and lower equilibrium prices for advertisers, which can improve user surplus and overall market outcomes.

(ii) Monitoring ad effectiveness and targeting: Since improvements in ad performance allow platforms to raise prices without losing advertisers, regulators should focus on transparency of targeting practices and their effects on both advertisers and users. This ensures that efficiency gains do not come at the cost of reduced consumer welfare.

(iii) Limit advertising nuisance: The model identifies a natural limit to monetisation through more frequent or intrusive ads. Guidelines on acceptable ad load or ad quality standards could protect user experience, benefiting users, advertisers, and the long-term sustainability of the platform ecosystem.

(iv) Assessment of mergers in ad-based markets: Merger reviews should weigh not only the risks of market concentration, but also the potential welfare gains from eliminating duplicate content and advertising when platforms are close substitutes. This is particularly relevant for competition authorities to evaluate large-scale platform consolidations.

This chapter advances the understanding of ad-based digital platforms by:

(i) Demonstrating how network effects and advertising effectiveness interact to influence advertisers' participation, advertising expenditure, and platform profitability.

(ii) Establishing that multi-homing does not universally enhance user welfare, and can lead to welfare losses when platforms are close substitutes with high ad nuisance.

(iii) Providing a structural rationale for how platform mergers can, under a restricted set of parameter values, improve user welfare in ad-based digital markets, contrasting with standard results in price-based competition settings.

Taken together, these insights show that platform policies and regulatory interventions must balance the interests of users, advertisers, and platforms by promoting healthy network growth, fair advertising practices, and safeguards against excessive ad nuisance. They also suggest that merger reviews in ad-based platform markets should be more nuanced than traditional competition analysis, accounting for both the risks and potential welfare gains of consolidation. This framework provides a solid foundation for future research and policy evaluation in the rapidly evolving landscape of digital platforms and online advertising.

Thesis Conclusion

This thesis set out to deepen the understanding of how digital platforms function, compete, and affect user surplus by developing and applying unified theoretical and simulation-based frameworks. Traditional models of industrial organisation provide powerful insights into market competition and efficiency. Through three interconnected chapters, this research contributes to bridging that gap by integrating price-based and ad-based platforms within a single analytical framework and by examining how network effects, mergers, and advertising strategies jointly shape welfare outcomes.

Chapter 1 established the conceptual and theoretical foundation by developing a unified and flexible model for analysing digital platform markets. This framework integrates both price-based and non-price-based platforms into a single structure, enabling consistent analysis of market strategies, user participation, and welfare outcomes. By allowing smooth transitions between different monetisation approaches, the model provides a comprehensive analytical base for studying digital ecosystems that range from subscription-based to ad-supported and hybrid configurations. This unified framework not only strengthens the theoretical foundations of platform economics but also enhances the ability to evaluate regulatory and policy interventions across diverse digital markets.

Chapter 2 applied this framework to examine how competition and market consolidation influence user surplus in price-based digital markets. Using a simulation-based approach, the chapter showed that mergers can yield welfare gains when strong network effects outweigh the loss of competition, particularly in the transition from triopoly to duopoly. However, further consolidation from duopoly to monopoly consistently led to welfare losses, even under strong network effects, due to higher prices and reduced competitive pressure. By incorporating both symmetric and asymmetric market structures, this analysis demonstrated that the welfare implications of mergers are context-dependent and shaped by network intensity and platform heterogeneity. These findings provide important insights for policymakers, suggesting that merger assessments in digital markets should account for the quantitative strength of network effects and the degree of asymmetry among competing platforms.

Chapter 3 extended the analysis to ad-based platforms, focusing on how network effects, advertising effectiveness, and ad disutility jointly determine advertising expenditure, user participation, and welfare. The chapter also introduced multi-homing users engaging with multiple platforms, and showed that its welfare effects depend critically on platform differentiation. When platforms are distinct, multi-homing enhances user welfare; when they are close substitutes, duplicated content and higher ad exposure reduce it. Unlike price-based markets, mergers among ad-based platforms can, under specific conditions, improve user welfare by reducing redundancy and ad nuisance. These results call for a more nuanced approach to merger evaluation in digital advertising markets, recognising that consolidation can sometimes enhance efficiency and welfare if platforms are highly substitutable.

Together, the three chapters provide an integrated view of how digital platforms operate under different monetisation models and market structures. The thesis advances both theory and policy by:

(i) Developing a unified, switchable framework that captures price- and ad-based platform behaviour within one structure.

(ii) Demonstrating how network effects, asymmetry, and market concentration interact to shape welfare outcomes.

(iii) Challenging conventional wisdom by showing that merger impacts depend on platform characteristics and the nature of user interactions.

(iv) Highlighting the conditional nature of multi-homing benefits and the welfare trade-offs between advertising intensity, user engagement, and market power.

While this thesis advances the understanding of digital platform markets, a few limitations remain and open opportunities for future work. The analysis is largely static, capturing equilibrium outcomes but not dynamic processes such as innovation or technological change. Although user heterogeneity is considered, behavioural aspects could be explored further. Some results rely on numerical simulations due to analytical intractability, suggesting scope for empirical validation with real-world data. The study extends from monopoly to duopoly and triopoly, implicitly addressing market entry, but future research could examine multi-market or vertically integrated platforms. Finally, as most platforms use hybrid monetisation models, further work could explore pricing–advertising trade-offs and their effects on user welfare.

Overall, this thesis develops a unified framework for analysing digital platform markets that bridges gaps between existing theories of industrial organisation and the evolving realities of digital markets. By integrating price-based and ad-based competition within a single structure, it advances a more coherent understanding of how network effects, platform strategies, and market structure shape welfare. While the analysis remains theoretical, it offers a foundation for both empirical validation and policy application in areas such as merger regulation, advertising standards, and platform governance.

As digital markets continue to expand and diversify, understanding their economic mechanisms becomes increasingly vital. Future research building on this framework can contribute to more effective policies that balance innovation, competition, and welfare in the digital age.

Appendix A: User Surplus Calculation

Chapter 2: User Surplus Calculation

Integration step by step in the case of Monopoly Full Market Coverage: The utilities are specified as

$$U_i^A(s_i^A) = s_i^A + \alpha n_i^A + \beta n_j^A - p_i^A$$

$$U_j^A(s_j^A) = s_j^A + \beta n_i^A - p_j^A$$

Group-i average surplus on A. By definition

$$\bar{US}_i^A = \int_{l_i^A}^{h_i^A} U_i^A(s_i^A) \times \frac{1}{h_i^A - l_i^A} \times ds_i^A = \int_{l_i^A}^{h_i^A} \left(s_i^A + \alpha n_i^A + \beta n_j^A - p_i^A\right) \times \frac{1}{h_i^A - l_i^A} \times ds_i^A$$

Split the integral into the s_i^A-term and the constant terms:

$$\begin{aligned}\bar{US}_i^A &= \frac{1}{h_i^A - l_i^A} \int_{l_i^A}^{h_i^A} s_i^A \, ds_i^A + \frac{\alpha n_i^A + \beta n_j^A - p_i^A}{h_i^A - l_i^A} \int_{l_i^A}^{h_i^A} 1 \, ds_i^A \\ &= \frac{1}{h_i^A - l_i^A} \left[\frac{(s_i^A)^2}{2} \Bigg|_{l_i^A}^{h_i^A} \right] + \alpha n_i^A + \beta n_j^A - p_i^A\end{aligned}$$

Evaluate the quadratic term:

$$\frac{(h_i^A)^2 - (l_i^A)^2}{2(h_i^A - l_i^A)} = \frac{(h_i^A - l_i^A)(h_i^A + l_i^A)}{2(h_i^A - l_i^A)} = \frac{h_i^A + l_i^A}{2}$$

Hence

$$\boxed{\bar{US}_i^A \; = \; \frac{h_i^A + l_i^A}{2} \; + \; \alpha n_i^A \; + \; \beta n_j^A \; - \; p_i^A}$$

(Interpretation: the average of the s_i^A-term is the midpoint $(h_i^A + l_i^A)/2$ and the other terms are constants that simply add.)

Group–j average surplus on A. By definition

$$\bar{US}_j^A = \int_{l_j^A}^{h_i^A} U_j^A(s_j^A) \times \frac{1}{h_j^A - l_j^A} \times ds_j^A = \int_{l_j^A}^{h_j^A} \left(s_j^A + \beta n_i^A - p_j^A\right) \times \frac{1}{h_j^A - l_j^A} \times ds_j^A$$

Split the integral into the s_j^A-term and the constant terms:

$$\bar{US}_j^A = \frac{1}{h_j^A - l_j^A} \int_{l_j^A}^{h_j^A} s_j^A \, ds_j^A + \frac{\beta n_i^A - p_j^A}{h_j^A - l_j^A} \int_{l_j^A}^{h_j^A} 1 \, ds_j^A$$

$$= \frac{1}{h_j^A - l_j^A} \left[\frac{(s_j^A)^2}{2} \Bigg|_{l_j^A}^{h_j^A} \right] + \beta n_j^A - p_j^A$$

Evaluate the quadratic term:

$$\frac{(h_j^A)^2 - (l_j^A)^2}{2(h_j^A - l_j^A)} = \frac{(h_j^A - l_j^A)(h_j^A + l_j^A)}{2(h_j^A - l_j^A)} = \frac{h_j^A + l_j^A}{2}$$

Hence

$$\boxed{\bar{US}_j^A \;=\; \frac{h_j^A + l_j^A}{2} \;+\; \beta n_i^A \;-\; p_j^A}$$

(Interpretation: the average of the s_j^A-term is the midpoint $(h_j^A + l_j^A)/2$ and the other terms are constants that simply add.)

Integration step by step in the case of Monopoly Partial Market Coverage:

Group–i average surplus on A.

$$
\begin{aligned}
\bar{US}_i^A &= \frac{1}{h_i^A - \hat{s}^A} \int_{\hat{s}_i^A}^{h_i^A} s_i^A + \alpha n_i^A + \beta n_j^A - p_i^A \, ds_i^A \\
&= \frac{1}{h_i^A - \hat{s}^A} \left[\int_{\hat{s}_i^A}^{h_i^A} s_i^A \, ds_i^A + (\alpha n_i^A + \beta n_j^A - p_i^A) \int_{\hat{s}_i^A}^{h_i^A} 1 \, ds_i^A \right] \\
&= \frac{1}{h_i^A - \hat{s}^A} \left[\frac{(s_i^A)^2}{2} \Big|_{\hat{s}_i^A}^{h_i^A} + (\alpha n_i^A + \beta n_j^A - p_i^A)(h_i^A - \hat{s}_i^A) \right] \\
&= \frac{1}{h_i^A - \hat{s}^A} \left(\frac{(h_i^A)^2 - (\hat{s}_i^A)^2}{2} + (\alpha n_i^A + \beta n_j^A - p_i^A)(h_i^A - \hat{s}_i^A) \right) \\
&= \frac{(h_i^A + \hat{s}_i^A)}{2} + \alpha n_i^A + \beta n_j^A - p_i^A
\end{aligned}
$$

Group–j average surplus on A.

$$
\begin{aligned}
\bar{US}_j^A &= \frac{1}{h_j^A + \hat{s}_j^A} \int_{\hat{s}_j^A}^{h_j^A} s_j^A + \beta n_i^A - p_j^A \, ds_j^A \\
&= \frac{1}{h_j^A - \hat{s}_j^A} \left[\int_{\hat{s}_j^A}^{h_j^A} s_j^A \, ds_j^A + \beta n_i^A - p_j^A \int_{\hat{s}_j^A}^{h_j^A} 1 \, ds_j^A \right] \\
&= \frac{1}{h_j^A - \hat{s}_j^A} \left[\frac{(s_j^A)^2}{2} \Big|_{\hat{s}_j^A}^{h_j^A} + \beta n_i^A - p_j^A (h_j^A - \hat{s}_j^A) \right] \\
&= \frac{1}{h_j^A - \hat{s}_j^A} \left(\frac{(h_j^A)^2 - (\hat{s}_j^A)^2}{2} + \beta n_i^A - p_j^A (h_j^A - \hat{s}_j^A) \right) \\
&= \frac{h_j^A + \hat{s}_j^A}{2} + \beta n_i^A - p_j^A
\end{aligned}
$$

Thus, the average surplus for a user in group-i and j is:

$$\boxed{\bar{US}_i^A = \frac{(h_i^A + \hat{s}_i^A)}{2} + \alpha n_i^A + \beta n_j^A - p_i^A \quad ; \quad \bar{US}_j^A = \frac{h_j^A + \hat{s}_j^A}{2} + \beta n_i^A - p_j^A}$$

Proof for Simulation Finding 1 (Figure 2.1 (a))						
Monopoly Platform A: h = 60, l = 20 (Partial Market Coverage)						
	α = β = 0	α = β = 1	α = β = 2	α = β = 3	α = β = 4	α = β = 5
P_i	30	30	30	30	30	30
P_j	30	30	30	30	30	30
n_i	0.75	0.78897	0.83113	0.87695	0.92697	0.98182
n_j	0.75	0.76972	0.79156	0.81577	0.8427	0.87273
Min. s_i value	30	28.4413	26.7546	24.9218	22.9213	20.7273
Min. s_j value	30	29.211	28.3377	27.3691	26.2921	25.0909
Aggregate US_i	11.25	12.4494	13.8157	15.381	17.1854	19.2793
Aggregate US_j	11.25	11.8495	12.5313	13.3097	14.2028	15.2331
Total US (i+j)	22.5	24.2989	26.347	28.6907	31.3882	34.5124
Profit	45	46.7607	48.6807	50.7818	53.0899	55.6364
Duopoly_Sym: A = B: 60, 20, s = 40 , Duopoly_Asym: A: 60, 20; B: 50, 30						
	α = β = 0	α = β = 1	α = β = 2	α = β = 3	α = β = 4	α = β = 5
US_SYM_2	40	44.5	49	53.5	58	62.5
US_ASYM_2	60	64.5	69	73.5	78	82.5
US_SYM (1-2)	-17.5	-20.2011	-22.653	-24.8093	-26.6118	-27.9876
US_ASYM (1-2)	-37.5	-40.2011	-42.653	-44.8093	-46.6118	-47.9876

Proof for Simulation Result 1 (Figure 2.1 (b))						
Monopoly Platform A: h = 60, l = 20 (Full Market Coverage)						
	α = β = 0	α = β = 1	α = β = 2	α = β = 3	α = β = 4	α = β = 5
n_i	1	1	1	1	1	1
n_j	1	1	1	1	1	1
P_i (l + α*n_i + β*n_j)	20	22	24	26	28	30
P_j (l + β*n_i)	20	21	22	23	24	25
Average s_i	40	40	40	40	40	40
Average s_j	40	40	40	40	40	40
Aggregate US_i	20	20	20	20	20	20
Aggregate US_j	20	20	20	20	20	20
Total US (i+j)	40	40	40	40	40	40
Profit	40	43	46	49	52	55
Duopoly_Sym: A = B: 60, 20, s = 40 , Duopoly_Asym: A: 60, 20; B: 50, 30						
	α = β = 0	α = β = 1	α = β = 2	α = β = 3	α = β = 4	α = β = 5
US_SYM_2	40	44.5	49	53.5	58	62.5
US_ASYM_2	60	64.5	69	73.5	78	82.5
US_SYM (1-2)	0	-4.5	-9	-13.5	-18	-22.5
US_ASYM (1-2)	-20	-24.5	-29	-33.5	-38	-42.5

Proof for Simulation Finding 1 (Figure 2.1 (c))						
Monopoly Platform A: h = 60, l = 20, β = 5 (Partial Market Coverage)						
	α = 0	α = 1	α = 2	α = 3	α = 4	α = 5
P_i	30	30	30	30	30	30
P_j	30	30	30	30	30	30
n_i	0.85714	0.87948	0.90301	0.92784	0.95406	0.98182
n_j	0.85714	0.85993	0.86288	0.86598	0.86926	0.87273
Min. s_i value	25.7143	24.8208	23.8796	22.8866	21.8375	20.7273
Min. s_j value	25.7143	25.6026	25.4849	25.3608	25.2297	25.0909
Aggregate US_i	14.6939	15.4697	16.3085	17.2176	18.2047	19.2793
Aggregate US_j	14.6939	14.7898	14.8911	14.9984	15.1122	15.2331
Total US (i+j)	29.3878	30.2594	31.1997	32.216	33.3169	34.5124
Profit	51.4286	52.1824	52.9766	53.8144	54.6996	55.6364
Duopoly_Sym: A = B: 60, 20, s = 40 , Duopoly_Asym: A: 60, 20; B: 50, 30						
	α = 0	α = 1	α = 2	α = 3	α = 4	α = 5
US_SYM_2	55	56.5	58	59.5	61	62.5
US_ASYM_2	75	76.5	78	79.5	81	82.5
US_SYM (1-2)	-25.6122	-26.2406	-26.8003	-27.284	-27.6831	-27.9876
US_ASYM (1-2)	-45.6122	-46.2406	-46.8003	-47.284	-47.6831	-47.9876

Proof for Simulation Finding 1 (Figure 2.1 (d))						
Monopoly Platform A: h = 60, l = 20, α = 5 (Partial Market Coverage)						
	β = 0	β = 1	β = 2	β = 3	β = 4	β = 5
P_i	30	30	30	30	30	30
P_j	30	30	30	30	30	30
n_i	0.85714	0.8792	0.90258	0.92739	0.95376	0.98182
n_j	0.75	0.77198	0.79513	0.81955	0.84538	0.87273
Min. s_i value	25.7143	24.832	23.8968	22.9044	21.8497	20.7273
Min. s_j value	30	29.1208	28.1948	27.2178	26.185	25.0909
Aggregate US_i	14.6939	15.4598	16.293	17.2011	18.1931	19.2793
Aggregate US_j	11.25	11.9191	12.6446	13.4334	14.2932	15.2331
Total US (i+j)	25.9439	27.3789	28.9376	30.6344	32.4863	34.5124
Profit	48.2143	49.5354	50.9312	52.4083	53.974	55.6364
Duopoly_Sym: A = B: 60, 20, s = 40 , Duopoly_Asym: A: 60, 20; B: 50, 30						
	β = 0	β = 1	β = 2	β = 3	β = 4	β = 5
US_SYM_2	47.5	50.5	53.5	56.5	59.5	62.5
US_ASYM_2	67.5	70.5	73.5	76.5	79.5	82.5
US_SYM (1-2)	-21.5561	-23.1211	-24.5624	-25.8656	-27.0137	-27.9876
US_ASYM (1-2)	-41.5561	-43.1211	-44.5624	-45.8656	-47.0137	-47.9876

3

Proof for Simulation Result 1 (Figure 2.1 (e))						
Monopoly Platform A: h = 60, l = 20, β = 5 (Full Market Coverage)						
	α = 0	α = 1	α = 2	α = 3	α = 4	α = 5
n_i	1	1	1	1	1	1
n_j	1	1	1	1	1	1
P_i (l + α*n_i + β*n_j)	25	26	27	28	29	30
P_j (l + β*n_i)	25	25	25	25	25	25
Average s_i	40	40	40	40	40	40
Average s_j	40	40	40	40	40	40
Aggregate US_i	20	20	20	20	20	20
Aggregate US_j	20	20	20	20	20	20
Total US (i+j)	40	40	40	40	40	40
Profit	50	51	52	53	54	55
Duopoly_Sym: A = B: 60, 20, s = 40 , Duopoly_Asym: A: 60, 20; B: 50, 30						
	α = β = 0	α = β = 1	α = β = 2	α = β = 3	α = β = 4	α = β = 5
US_SYM_2	55	56.5	58	59.5	61	62.5
US_ASYM_2	75	76.5	78	79.5	81	82.5
US_SYM (1-2)	-15	-16.5	-18	-19.5	-21	-22.5
US_ASYM (1-2)	-35	-36.5	-38	-39.5	-41	-42.5

Proof for Simulation Result 1 (Figure 2.1 (f))						
Monopoly Platform A: h = 60, l = 20 α = 5 (Full Market Coverage)						
	β = 0	β = 1	β = 2	β = 3	β = 4	β = 5
n_i	1	1	1	1	1	1
n_j	1	1	1	1	1	1
P_i (l + α*n_i + β*n_j)	25	26	27	28	29	30
P_j (l + β*n_i)	20	21	22	23	24	25
Average s_i	40	40	40	40	40	40
Average s_j	40	40	40	40	40	40
Aggregate US_i	20	20	20	20	20	20
Aggregate US_j	20	20	20	20	20	20
Total US (i+j)	40	40	40	40	40	40
Profit	45	47	49	51	53	55
Duopoly_Sym: A = B: 60, 20, s = 40 , Duopoly_Asym: A: 60, 20; B: 50, 30						
	α = β = 0	α = β = 1	α = β = 2	α = β = 3	α = β = 4	α = β = 5
US_SYM_2	47.5	50.5	53.5	56.5	59.5	62.5
US_ASYM_2	67.5	70.5	73.5	76.5	79.5	82.5
US_SYM (1-2)	-7.5	-10.5	-13.5	-16.5	-19.5	-22.5
US_ASYM (1-2)	-27.5	-30.5	-33.5	-36.5	-39.5	-42.5

Proof for Simulation Finding 2

SYMMETRIC PLATFORMS

Figure 2.2 (a): Numerical Example (A = B = C: h = 14, l = 2; β = 2)

	US/ α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
Three Platforms	US_i	5.1667	5.2213	5.2749	5.3273	5.3784	5.4282	5.4765	5.5234	5.5686	5.6122	5.6538	5.6936	5.7312	5.7665	5.7995	5.8297	5.8571	5.8815	5.9025	5.9199	5.9333
	US_j	5.1667	5.1571	5.1472	5.1369	5.1261	5.1149	5.1033	5.0911	5.0784	5.0652	5.0513	5.0367	5.0215	5.0055	4.9887	4.971	4.9524	4.9327	4.9119	4.89	4.8667
	US_3	10.3333	10.3785	10.4221	10.4641	10.5045	10.5431	10.5798	10.6145	10.6471	10.6773	10.7051	10.7303	10.7527	10.772	10.7882	10.8007	10.8095	10.8142	10.8145	10.8099	10.8
Duopoly	US_i	5	5.15	5.3	5.45	5.6	5.75	5.9	6.05	6.2	6.35	6.5	6.65	6.8	6.95	7.1	7.25	7.4	7.55	7.7	7.85	8
	US_j	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
	US_2	10	10.15	10.3	10.45	10.6	10.75	10.9	11.05	11.2	11.35	11.5	11.65	11.8	11.95	12.1	12.25	12.4	12.55	12.7	12.85	13
US	US(2-3)	-0.3333	-0.2285	-0.1221	-0.0141	0.0955	0.2069	0.3202	0.4355	0.5529	0.6727	0.7949	0.9197	1.0473	1.178	1.3118	1.4493	1.5905	1.7358	1.8855	2.0401	2.2

Figure 2.2 (b): Numerical Example (A = B = C: h = 14, l = 2; α = 2)

	US/ β	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
Three Platforms	US_i	5.1667	5.2244	5.2807	5.3357	5.3893	5.4412	5.4915	5.5401	5.5868	5.6314	5.6739	5.7141	5.7518	5.7868	5.8188	5.8477	5.8731	5.8948	5.9123	5.9253	5.9333
	US_j	4	4.0579	4.1149	4.171	4.226	4.2798	4.3325	4.3838	4.4338	4.4822	4.529	4.574	4.617	4.658	4.6966	4.7328	4.7662	4.7966	4.8237	4.8472	4.8667
	US_3	9.1667	9.2823	9.3957	9.5067	9.6154	9.721	9.824	9.9239	10.0206	10.1136	10.2029	10.2881	10.3688	10.4447	10.5154	10.5805	10.6393	10.6914	10.736	10.7725	10.8
Duopoly	US_i	5	5.15	5.3	5.45	5.6	5.75	5.9	6.05	6.2	6.35	6.5	6.65	6.8	6.95	7.1	7.25	7.4	7.55	7.7	7.85	8
	US_j	2	2.15	2.3	2.45	2.6	2.75	2.9	3.05	3.2	3.35	3.5	3.65	3.8	3.95	4.1	4.25	4.4	4.55	4.7	4.85	5
	US_2	7	7.3	7.6	7.9	8.2	8.5	8.8	9.1	9.4	9.7	10	10.3	10.6	10.9	11.2	11.5	11.8	12.1	12.4	12.7	13
US	US(2-3)	-2.1667	-1.9823	-1.7957	-1.6067	-1.4154	-1.221	-1.024	-0.8239	-0.6206	-0.4136	-0.2029	0.0119	0.2312	0.4553	0.6846	0.9195	1.1607	1.4086	1.664	1.9275	2.2

Proof for Simulation Finding 2: Symmetric Scenario: A = B = C: 14,2, β = 2

α = 0					
	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	3.5	3.5	3.5	4	4
P_j	3.5	3.5	3.5	4	4
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Profit	2.3333	2.3333	2.3333	4	4
Total Profit	7			8	

α = 0.5					
	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	3.4052	3.4052	3.4052	3.5	3.5
P_j	3.5517	3.5517	3.5517	4	4
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Profit	2.319	2.319	2.319	3.75	3.75
Total Profit	6.9569			7.5	

α = 1					
	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	3.3462	3.3462	3.3462	3	3
P_j	3.6154	3.6154	3.6154	4	4
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Profit	2.3205	2.3205	2.3205	3.5	3.5
Total Profit	6.9615			7	

α = 1.5					
	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	3.337	3.337	3.337	2.5	2.5
P_j	3.6957	3.6957	3.6957	4	4
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Profit	2.3442	2.3442	2.3442	3.25	3.25
Total Profit	7.0326			6.5	

α = 2					
	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	3.4	3.4	3.4	2	2
P_j	3.8	3.8	3.8	4	4
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Profit	2.4	2.4	2.4	3	3
Total Profit	7.2			6	

Proof for Simulation Finding 2: Symmetric Scenario: A = B = C: 14,2, α = 2

β = 0					
	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	3.5	3.5	3.5	4	4
P_j	4	4	4	6	6
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Profit	2.5	2.5	2.5	5	5
Total Profit	7.5			10	

β = 0.5					
	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	3.3921	3.3921	3.3921	3.5	3.5
P_j	3.8868	3.8868	3.8868	5.5	5.5
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Profit	2.4263	2.4263	2.4263	4.5	4.5
Total Profit	7.2789			9	

β = 1					
	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	3.3261	3.3261	3.3261	3	3
P_j	3.8043	3.8043	3.8043	5	5
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Profit	2.3768	2.3768	2.3768	4	4
Total Profit	7.1304			8	

β = 1.5					
	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	3.319	3.319	3.319	2.5	2.5
P_j	3.7672	3.7672	3.7672	4.5	4.5
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Profit	2.3621	2.3621	2.3621	3.5	3.5
Total Profit	7.0862			7	

β = 2					
	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	3.4	3.4	3.4	2	2
P_j	3.8	3.8	3.8	4	4
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Profit	2.4	2.4	2.4	3	3
Total Profit	7.2			6	

Proof for Simulation Finding 3

ASYMMETRIC PLATFORMS

Figure 2.3 (a): Numerical Example (A: h = 60, l = 20: h = 50, l = 16; C: h = 40, l = 14; β = 5)

	US/ α	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
Three Platforms	US_i	28.7478	29.0836	29.4081	29.7197	30.0166	30.2968	30.5579	30.7972	31.0116	31.1972	31.3498	31.4641	31.5334
	US_j	28.7478	28.7029	28.6542	28.6012	28.5435	28.4805	28.4115	28.336	28.2531	28.1616	28.0604	27.9476	27.8205
	US_3	57.4955	57.7865	58.0623	58.3209	58.5601	58.7773	58.9695	59.1332	59.2647	59.3588	59.4102	59.4116	59.3538
Duopoly (A & B)	US_i	27.3214	28.0931	28.8671	29.6438	30.4234	31.2065	31.9937	32.7856	33.5831	34.3871	35.1991	36.0205	36.8535
	US_j	27.3214	27.3277	27.3345	27.3419	27.3499	27.3587	27.3682	27.3787	27.3903	27.4131	27.4174	27.4334	27.4514
	US_2	54.6427	55.4208	56.2016	56.9857	57.7733	58.5652	59.3619	60.1643	60.9734	61.7902	62.6165	63.4539	64.3049
US	US(2-3)	-2.8528	-2.3657	-1.8607	-1.3352	-0.7868	-0.2121	0.3924	1.0311	1.7087	2.4314	3.2063	4.0423	4.9511
Duopoly (A & C)	US_i	29.9071	30.8417	31.8044	32.8013	33.8401	34.9309	36.0869	37.326	38.6273	40.1597	41.8365	43.7736	46.0785
	US_j	29.9071	29.9771	30.0555	30.1438	30.2441	30.3587	30.4909	30.645	30.8264	31.0429	31.3051	31.6283	32.0349
	US_2	59.8142	60.8188	61.8599	62.9451	64.0842	65.2896	66.5779	67.9709	69.4978	71.2026	73.1416	75.4019	78.1134
US	US(2-3)	2.3187	3.0323	3.7976	4.6242	5.5241	6.5123	7.6084	8.8377	10.2331	11.8438	13.7314	15.9903	18.7596
Duopoly (B & C)	US_i	24.9062	25.6956	26.4909	27.2935	28.105	28.9276	29.7641	30.6183	31.4953	32.4024	33.3498	34.3527	35.4339
	US_j	24.9063	24.9212	24.9379	24.9567	24.978	25.0024	25.0306	25.0634	25.1021	25.1482	25.2041	25.2729	25.3595
	US_2	49.8125	50.6167	51.4287	52.2501	53.083	53.93	54.7947	55.6817	56.5974	57.5506	58.5538	59.6256	60.7934
US	US(2-3)	-7.683	-7.1698	-6.6336	-6.0708	-5.4771	-4.8473	-4.1748	-3.4515	-2.6673	-1.8082	-0.8564	0.214	1.4396

Figure 2.3 (b): Numerical Example (A: h = 60, l = 20; B: h = 50, l = 16; C: h = 40, l = 14; α = 5)

	US/ β	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
Three Platforms	US_i	28.7478	29.0984	29.4355	29.7573	30.0618	30.3465	30.6083	30.844	31.0494	31.22	31.3498	31.4318	31.4563
	US_j	25.1777	25.5308	25.8752	26.2094	26.5315	26.8395	27.1306	27.4021	27.6505	27.8716	28.0604	28.2104	28.3124
	US_3	53.9255	54.6291	55.3107	55.9667	56.5934	57.1859	57.7389	58.2461	58.6999	59.0915	59.4102	59.6421	59.7687
Duopoly (A & B)	US_i	27.3214	28.0914	28.8639	29.6391	30.4177	31.2	31.9868	32.7789	33.5774	34.3836	35.1991	36.026	36.8674
	US_j	19.6601	20.4242	21.1898	21.9572	22.7268	23.4988	24.2738	25.0524	25.8352	26.6232	27.4174	28.2195	29.0313
	US_2	46.9815	48.5156	50.0536	51.5963	53.1444	54.6988	56.2606	57.8313	59.4126	61.0067	62.6165	64.2455	65.8986
US	US(2-3)	-6.944	-6.1135	-5.2571	-4.3704	-3.449	-2.4871	-1.4783	-0.4148	0.7127	1.9152	3.2063	4.6034	6.1299
Duopoly (A & C)	US_i	29.9071	30.8166	31.755	32.7287	33.7463	34.8189	35.9622	37.1974	38.555	40.0792	41.8365	43.9303	46.529
	US_j	21.2222	22.0693	22.9319	23.814	24.7206	25.6583	26.6364	27.6675	28.7696	29.969	31.3051	32.8398	34.6736
	US_2	51.1293	52.8858	54.6869	56.5427	58.4668	60.4773	62.5986	64.8649	67.3246	70.0482	73.1416	76.7701	81.2026
US	US(2-3)	-2.7962	-1.7433	-0.6238	0.576	1.8734	3.2914	4.8597	6.6188	8.6247	10.9567	13.7314	17.128	21.4339
Duopoly (B & C)	US_i	24.9062	25.6902	26.4803	27.278	28.085	28.9037	29.7375	30.5909	31.4703	32.3852	33.3498	34.386	35.5298
	US_j	17.1538	17.9245	18.6985	19.4766	20.26	21.05	21.8486	22.6585	23.4835	24.3292	25.2041	26.1212	27.1021
	US_2	42.0601	43.6147	45.1789	46.7547	48.345	49.9537	51.5861	53.2493	54.9538	56.7144	58.5538	60.5072	62.6319
US	US(2-3)	-11.8654	-11.0144	-10.1318	-9.212	-8.2484	-7.2322	-6.1528	-4.9968	-3.7461	-2.3771	-0.8564	0.8651	2.8632

Proof for Simulation Finding 3: Asymmetric Scenario: Numerical Example (A: h = 60, l = 20; B: h = 50, l = 16; C: h = 40, l = 14)

	α = 0; β = 5			α = 5; β = 0		
	A	B	C	A	B	C
P_i	11.0257	9.087	3.5562	11.0256	9.0871	3.5562
P_j	11.0257	9.087	3.5562	12.9526	10.3809	3.9684
n_i	0.55793	0.28428	0.15779	0.55793	0.28428	0.15779
n_j	0.55793	0.28428	0.15779	0.56329	0.2811	0.15561
Profit	12.303	5.1665	2.3333	13.4477	5.5013	1.1787
Total Profit	18.5919			20.1277		

	α = 1; β = 5			α = 5; β = 1		
	A	B	C	A	B	C
P_i	10.6703	8.8649	3.5221	10.6536	8.8436	3.4987
P_j	11.0897	9.1714	3.6432	12.582	10.1428	3.9002
n_i	0.55553	0.28508	0.15939	0.55619	0.28493	0.15888
n_j	0.55575	0.28479	0.15946	0.56185	0.28192	0.15623
Profit	12.0908	5.1391	1.1423	12.9946	5.3793	1.1652
Total Profit	18.3723			19.5391		

	α = 2; β = 5			α = 5; β = 2		
	A	B	C	A	B	C
P_i	10.3458	8.6842	3.5354	10.3185	8.6491	3.4962
P_j	11.1647	9.2702	3.7453	12.2393	9.9408	3.8711
n_i	0.55206	0.28604	0.1619	0.55309	0.28581	0.1611
n_j	0.55322	0.28538	0.1614	0.5594	0.28294	0.15766
Profit	11.8882	5.1295	1.1768	12.5538	5.2846	1.1736
Total Profit	18.1945			19.0119		

	α = 3; β = 5			α = 5; β = 3		
	A	B	C	A	B	
P_i	10.0607	8.5546	3.6107	10.0302	8.5152	3.5658
P_j	11.2538	9.3863	3.8675	11.9337	9.7859	3.8955
n_i	0.5473	0.28725	0.16545	0.54836	0.28701	0.16463
n_j	0.5503	0.2861	0.1636	0.55566	0.28424	0.16011
Profit	11.6991	5.1428	1.2301	12.1311	5.2255	1.2108
Total Profit	18.072			18.5674		

	α = 4; β = 5			α = 5; β = 4		
	A	B	C	A	B	C
P_i	9.827	8.4917	3.7717	9.8038	8.4619	3.7358
P_j	11.3611	9.5255	4.018	11.6781	9.6961	3.9954
n_i	0.54108	0.2888	0.17011	0.54179	0.28861	0.1696
n_j	0.547	0.28698	0.16602	0.55037	0.28589	0.16374
Profit	11.5317	5.1861	1.3087	11.7389	5.2142	1.2878
Total Profit	18.0265			18.2409		

	α = 5; β = 5			α = 5; β = 5		
	A	B	C	A	B	C
P_i	9.6641	8.5198	4.0598	9.6641	8.5198	4.0598
P_j	11.4935	9.6985	4.2115	11.4935	9.6985	4.2115
n_i	0.53345	0.29079	0.17576	0.53345	0.29079	0.17576
n_j	0.54349	0.28805	0.16847	0.54349	0.28805	0.16847
Profit	11.4019	5.2711	1.4231	11.4019	5.2711	1.4231
Total Profit	18.0961			18.0961		

	α = 6; β = 5			α = 5; β = 6		
	A	B	C	A	B	C
P_i	9.6081	8.6752	4.575	9.6629	8.7434	4.6811
P_j	11.6651	9.9211	4.4855	11.421	9.8421	4.645
n_i	0.52509	0.29376	0.18114	0.52454	0.29423	0.18123
n_j	0.54027	0.28957	0.17015	0.53583	0.29124	0.17293
Profit	11.3475	5.4213	1.5919	11.1883	5.439	1.6516
Total Profit	18.3608			18.2789		

Asymmetric Scenario: Numerical Example (A: h = 60, l = 20B: h = 50, l = 16; C: h = 40, l = 14)

	α = 0; β = 5					
	Duopoly		Duopoly		Duopoly	
	A	B	A	C	B	C
P_i	14.3333	9.6667	12.3333	3.6667	10	6
P_j	14.3333	9.6667	12.3333	3.6667	10	6
n_i	0.59722	0.40278	0.77083	0.22917	0.625	0.375
n_j	0.59722	0.40278	0.77083	0.22917	0.625	0.375
Profit	17.1204	7.787	19.0139	1.6806	12.5	4.5
Total Profit	24.9074		20.6944		17	

	α = 1; β = 5					
	Duopoly		Duopoly		Duopoly	
	A	B	A	C	B	C
P_i	13.3333	8.6667	11.3333	2.6667	9	5
P_j	14.3333	9.6667	12.3333	3.6667	10	6
n_i	0.60391	0.39609	0.79771	0.20229	0.6374	0.3626
n_j	0.59919	0.40081	0.78117	0.21883	0.62977	0.37023
Profit	16.6406	7.3072	18.6751	1.3418	12.0344	4.0344
Total Profit	23.9478		20.017		16.0687	

	α = 2; β = 5					
	Duopoly		Duopoly		Duopoly	
	A	B	A	C	B	C
P_i	12.3333	7.6667	10.3333	1.6667	8	4
P_j	14.3333	9.6667	12.3333	3.6667	10	6
n_i	0.61159	0.38841	0.83051	0.16949	0.65254	0.34746
n_j	0.60145	0.39855	0.79379	0.20621	0.63559	0.36441
Profit	16.1638	6.8304	18.3719	1.0386	11.5763	3.5763
Total Profit	22.9942		19.4105		15.1525	

	α = 3; β = 5					
	Duopoly		Duopoly			
	A	B	A	C	B	C
P_i	11.3333	6.6667	9.3333	0.66667	7	3
P_j	14.3333	9.6667	12.3333	3.6667	10	6
n_i	0.6205	0.3795	0.87143	0.12857	0.67143	0.32857
n_j	0.60407	0.39593	0.80952	0.19048	0.64286	0.35714
Profit	15.6907	6.3573	18.1175	0.78413	11.1286	3.1286
Total Profit	22.048		18.9016		14.2571	

	α = 4; β = 5					
	Duopoly		Duopoly		Duopoly	
	A	B	A	C	B	C
P_i	10.3333	5.6667	8.3333	0	6	2
P_j	14.3333	9.6667	12.3333	3.6667	10	6
n_i	0.63905	0.36905	0.92391	0.076087	0.69565	0.30435
n_j	0.60714	0.39286	0.82971	0.17029	0.65217	0.34783
Profit	15.2222	5.8889	17.9324	0.6244	10.6957	2.6957
Total Profit	21.1111		18.5314		13.3914	

	α = 5; β = 5					
	Duopoly		Duopoly		Duopoly	
	A	B	A	C	B	C
P_i	9.3333	4.6667	7.3333	0	5	1
P_j	14.3333	9.6667	12.3333	3.6667	10	6
n_i	0.64339	0.35661	0.99367	0.00633	0.72785	0.27215
n_j	0.6108	0.3892	0.85654	0.14346	0.66456	0.33544
Profit	14.7598	5.4264	17.8509	0.526	10.2848	2.2848
Total Profit	20.1862		18.3769		12.5696	

	α = 6; β = 5					
	Duopoly		Duopoly		Duopoly	
	A	B	A	C	B	C
P_i	8.3333	3.6667	6.3333	0	4	0
P_j	14.3333	9.6667	12.3333	3.6667	10	6
n_i	0.65844	0.34156	1	0	0.77273	0.22727
n_j	0.61523	0.38477	0.89394	0.10606	0.68182	0.31818
Profit	14.3052	4.9719	17.3585	0.38889	9.9091	1.90908
Total Profit	19.2771		17.7474		11.8182	

Proof for Simulation Finding 3: Asymmetric Scenario: Numerical Example (A: h = 60, l = 20; B: h = 50, l = 16; C: h = 40, l = 14)

	α = 5; β = 0					
	Duopoly		Duopoly		Duopoly	
	A	B	A	C	B	C
P_i	14.3333	9.6667	12.3333	3.66667	10	6
P_j	19.3333	14.6667	17.3333	8.6667	15	11
n_i	0.59722	0.40278	0.77083	0.22917	0.625	0.375
n_j	0.56863	0.43137	0.66667	0.33333	0.57692	0.42308
Profit	19.5536	10.2203	21.0625	3.7292	14.9038	6.9038
Total Profit	29.774		24.7917		21.8077	

	α = 5; β = 1					
	Duopoly		Duopoly		Duopoly	
	A	B	A	C	B	C
P_i	13.3333	8.6667	11.3333	2.6667	9	5
P_j	18.3333	13.6667	16.3333	7.6667	14	10
n_i	0.60345	0.39655	0.7945	0.2055	0.63592	0.36408
n_j	0.57471	0.42529	0.68932	0.31068	0.58738	0.41262
Profit	18.5824	9.249	20.2632	2.9299	13.9466	5.9466
Total Profit	27.8314		23.1931		19.8932	

	α = 5; β = 2					
	Duopoly		Duopoly		Duopoly	
	A	B	A	C	B	C
P_i	12.3333	7.6667	10.3333	1.6667	8	4
P_j	17.3333	12.6667	15.3333	6.6667	13	9
n_i	0.61083	0.38917	0.825	0.175	0.65	0.35
n_j	0.58167	0.41833	0.71667	0.28333	0.6	0.4
Profit	17.6158	8.2825	19.5139	2.1806	13	5
Total Profit	25.8983		21.6944		18	

	α = 5; β = 3					
	Duopoly		Duopoly			
	A	B	A	C	B	C
P_i	11.3333	6.6667	9.3333	0.66667	7	3
P_j	16.3333	11.6667	14.3333	5.6667	12	8
n_i	0.61966	0.38034	0.86491	0.13509	0.66842	0.33158
n_j	0.58974	0.41026	0.75088	0.24912	0.61579	0.38421
Profit	16.6553	7.3219	18.8351	1.5018	12.0684	4.0684
Total Profit	23.9772		20.3368		16.1368	

	α = 5; β = 4					
	Duopoly		Duopoly		Duopoly	
	A	B	A	C	B	C
P_i	10.3333	5.6667	8.3333	0	6	2
P_j	15.3333	10.6667	13.3333	4.6667	11	7
n_i	0.63032	0.36968	0.91856	0.081439	0.69318	0.30682
n_j	0.59929	0.40071	0.79545	0.20455	0.63636	0.36364
Profit	15.7024	6.3691	18.2607	0.954573	11.1591	3.1591
Total Profit	22.0715		19.2153		14.3182	

	α = 5; β = 5					
	Duopoly		Duopoly		Duopoly	
	A	B	A	C	B	C
P_i	9.3333	4.6667	7.3333	0	5	1
P_j	14.3333	9.6667	12.3333	3.6667	10	6
n_i	0.64339	0.35661	0.99367	0.00633	0.72785	0.27215
n_j	0.6108	0.3892	0.85654	0.14346	0.66456	0.33544
Profit	14.7598	5.4264	17.8509	0.526	10.2848	2.2848
Total Profit	20.1862		18.3769		12.5696	

	α = 5; β = 6					
	Duopoly		Duopoly		Duopoly	
	A	B	A	C	B	C
P_i	8.3333	3.6667	6.3333	0	4	0
P_j	13.3333	8.6667	11.3333	2.6667	9	5
n_i	0.65972	0.34028	1	0	0.77941	0.22059
n_j	0.625	0.375	0.94608	0.05392	0.70588	0.29412
Profit	13.831	4.4977	17.0555	0.14379	9.4706	1.4706
Total Profit	18.3287		17.1993		10.9412	

Proof for Simulation Finding 4 (Figure 2.4 (a)) Symmtric Platforms: A = B = C = 60,20; Asymmtric Platforms: A = 60,20; B = C = 50,30

α = β = 0

	Symmetric			Asymmetric		
	A	B	C	A	B	C
P_i	13.3333	13.3333	13.3333	12.3617	8.0566	8.0566
P_j	13.3333	13.3333	13.3333	12.3617	8.0566	8.0566
n_i	0.33333	0.33333	0.33333	0.30904	0.34548	0.34548
n_j	0.33333	0.33333	0.33333	0.30904	0.34548	0.34548
Average US_i	26.6667	26.6667	26.6667	27.6383	31.9434	31.9434
Aggregate US_i	26.6667			30.6129		
Average US_j	26.6667	26.6667	26.6667	27.6383	31.9434	31.9434
Aggregate US_j	26.6667			30.6129		
Total US (i+j)	53.3334			61.2258		
Profit	8.8889	8.8889	8.8889	7.6405	5.5668	5.5668
Total Profit	26.6667			18.7742		

α = β = 1

	Symmetric			Asymmetric		
	A	B	C	A	B	C
P_i	12.6939	12.6939	12.6939	12.1133	7.3416	7.3416
P_j	13.018	13.018	13.018	12.2633	7.7001	7.7001
n_i	0.33333	0.33333	0.33333	0.29624	0.35188	0.35188
n_j	0.33333	0.33333	0.33333	0.3019	0.34905	0.34905
Average US_i	27.9728	27.9728	27.9728	28.4848	33.3593	33.3593
Aggregate US_i	27.9728			31.9153		
Average US_j	27.3154	27.3154	27.3154	28.033	32.6518	32.6518
Aggregate US_j	27.3154			31.2574		
Total US (i+j)	55.2881			63.1727		
Profit	8.5706	8.5706	8.5706	7.2907	5.2711	5.2711
Total Profit	25.7119			17.8329		

α = β = 2

	Symmetric			Asymmetric		
	A	B	C	A	B	C
P_i	12.1198	12.1198	12.1198	12.9643	6.6301	6.6301
P_j	12.7453	12.7453	12.7453	12.7985	7.3413	7.3413
n_i	0.33333	0.33333	0.33333	0.26809	0.36595	0.36595
n_j	0.33333	0.33333	0.33333	0.28393	0.35804	0.35804
Average US_i	29.2135	29.2135	29.2135	28.1397	34.8179	34.8179
Aggregate US_i	29.2135			33.0275		
Average US_j	27.9214	27.9214	27.9214	27.7377	33.3906	33.3906
Aggregate US_j	27.9214			31.7856		
Total US (i+j)	57.1349			64.8131		
Profit	8.2884	8.2884	8.2884	7.1095	5.0548	5.0548
Total Profit	24.8651			17.2191		

α = β = 3

	Symmetric			Asymmetric		
	A	B	C	A	B	C
P_i	11.6324	11.6324	11.6324	13.4002	6.0188	6.0188
P_j	12.5282	12.5282	12.5282	13.2684	7.1464	7.1464
n_i	0.33333	0.33333	0.33333	0.28224	0.35888	0.35888
n_j	0.33333	0.33333	0.33333	0.28538	0.35731	0.35731
Average US_i	30.3676	30.3676	30.3676	28.3025	36.1297	36.1297
Aggregate US_i	30.3676			33.9207		
Average US_j	28.4718	28.4718	28.4718	27.5783	33.9302	33.9302
Aggregate US_j	28.4718			32.1176		
Total US (i+j)	58.8394			66.0383		
Profit	8.0535	8.0535	8.0535	7.5683	4.7136	4.7136
Total Profit	24.1606			16.9955		

α = β = 4

	Symmetric			Asymmetric		
	A	B	C	A	B	C
P_i	11.2632	11.2632	11.2632	8.3134	7.2314	7.2314
P_j	12.3859	12.3859	12.3859	10.2757	7.9873	7.9873
n_i	0.33333	0.33333	0.33333	0.39058	0.30471	0.30471
n_j	0.33333	0.33333	0.33333	0.36096	0.31952	0.31952
Average US_i	31.4035	31.4035	31.4035	34.6927	35.2655	35.2655
Aggregate US_i	31.4035			35.0418		
Average US_j	28.9474	28.9474	28.9474	31.2866	33.2316	33.2316
Aggregate US_j	28.9474			32.5295		
Total US (i+j)	60.3509			67.5713		
Profit	7.883	7.883	7.883	6.9561	4.7556	4.7556
Total Profit	23.6491			16.4673		

α = β = 5

	Symmetric			Asymmetric		
	A	B	C	A	B	C
P_i	11.0606	11.0606	11.0606	8.4751	12.2468	12.2468
P_j	12.3485	12.3485	12.3485	9.242	10.4127	10.4127
n_i	0.33333	0.33333	0.33333	0.49204	0.25398	0.25398
n_j	0.33333	0.33333	0.33333	0.42392	0.28804	0.28804
Average US_i	32.2727	32.2727	32.2727	36.1048	30.4632	30.4632
Aggregate US_i	32.2727			33.2392		
Average US_j	29.3182	29.3182	29.3182	33.2182	30.8572	30.8572
Aggregate US_j	29.3182			31.8581		
Total US (i+j)	61.5909			65.0973		
Profit	7.803	7.803	7.803	8.0881	6.1096	6.1096
Total Profit	23.4091			20.3073		

Proof for Simulation Finding 4 (Figure 2.4 (b) & (c))

Asymmetric: A: 60,20; B: 52,28; C: 52, 28; α = β = 5
Symmetric: A = B = C = 60,20; B: 60,20; α = β = 5

	Asymmtric					Symmetric				
	A	B	C	A	B	A	B	C	A	B
P_i	9.2459	9.605	9.605	2	2	11.0606	11.0606	11.0606	10	10
P_j	10.8392	9.8947	9.8947	7	7	12.3485	12.3485	12.3485	15	15
n_i	0.39841	0.3008	0.3008	0.5	0.5	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.3707	0.31465	0.31465	0.5	0.5	0.33333	0.33333	0.33333	0.5	0.5
Average US_i	34.5997	33.4722	33.4722	43	43	32.2727	32.2727	32.2727	35	35
Aggregate US_i	33.9214			43		32.2727			35	
Average US_j	31.1529	31.6093	31.6093	35.5	35.5	29.3182	29.3182	29.3182	27.5	27.5
Aggregate US_j	31.4401			35.5		29.3182			27.5	
Total US (i+j)	65.3615			78.5		61.5909			62.5	
US (2-3)	13.1385					0.9091				
Profit	7.7017	6.0025	6.0025	4.5	4.5	7.803	7.803	7.803	12.5	12.5
Total Profit	19.7067			9		23.4091			25	

Asymmetric: A: 60,20; B: 54,26; C: 54, 26; α = β = 5
Symmetric: A = B = C = 60,20; B: 60,20; α = β = 5

	Asymmtric					Symmetric				
	A	B	C	A	B	A	B	C	A	B
P_i	11.5606	9.3355	9.3355	4	4	11.0606	11.0606	11.0606	10	10
P_j	12.5879	10.2594	10.2594	9	9	12.3485	12.3485	12.3485	15	15
n_i	0.33027	0.33487	0.33487	0.5	0.5	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.3263	0.33685	0.33685	0.5	0.5	0.33333	0.33333	0.33333	0.5	0.5
Average US_i	31.7223	34.0231	34.0231	41	41	32.2727	32.2727	32.2727	35	35
Aggregate US_i	33.2632			41		32.2727			35	
Average US_j	29.0634	31.415	31.415	33.5	33.5	29.3182	29.3182	29.3182	27.5	27.5
Aggregate US_j	30.6476			33.5		29.3182			27.5	
Total US (i+j)	63.9108			74.5		61.5909			62.5	
US (2-3)	10.5892					0.9091				
Profit	7.9256	6.582	6.582	6.5	6.5	7.803	7.803	7.803	12.5	12.5
Total Profit	21.0896			13		23.4091			25	

Asymmetric: A: 60,20; B: 56,24; C: 56, 24; α = β = 5
Symmetric: A = B = C = 60,20; B: 60,20; α = β = 5

	Asymmtric					Symmetric				
	A	B	C	A	B	A	B	C	A	B
P_i	14.4033	9.577	9.577	6	6	11.0606	11.0606	11.0606	10	10
P_j	14.2293	10.7513	10.7513	11	11	12.3485	12.3485	12.3485	15	15
n_i	0.26038	0.36981	0.36981	0.5	0.5	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.2832	0.3584	0.3584	0.5	0.5	0.33333	0.33333	0.33333	0.5	0.5
Average US_i	28.3146	34.064	34.064	39	39	32.2727	32.2727	32.2727	35	35
Aggregate US_i	32.5672			39		32.2727			35	
Average US_j	27.0725	31.0978	31.0978	31.5	31.5	29.3182	29.3182	29.3182	27.5	27.5
Aggregate US_j	29.9579			31.5		29.3182			27.5	
Total US (i+j)	62.525			70.5		61.5909			62.5	
US (2-3)	7.975					0.9091				
Profit	7.7799	7.3949	7.3949	8.5	8.5	7.803	7.803	7.803	12.5	12.5
Total Profit	22.5698			17		23.4091			25	

Asymmetric: A: 60,20; B: 58,22; C: 58, 22; α = β = 5
Symmetric: A = B = C = 60,20; B: 60,20; α = β = 5

	Asymmtric					Symmetric				
	A	B	C	A	B	A	B	C	A	B
P_i	11.8127	10.4292	10.4292	8	8	11.0606	11.0606	11.0606	10	10
P_j	12.7601	11.6009	11.6009	13	13	12.3485	12.3485	12.3485	15	15
n_i	0.31086	0.34457	0.34457	0.5	0.5	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.31832	0.34084	0.34084	0.5	0.5	0.33333	0.33333	0.33333	0.5	0.5
Average US_i	31.3331	32.9978	32.9978	37	37	32.2727	32.2727	32.2727	35	35
Aggregate US_i	32.4804			37		32.2727			35	
Average US_j	28.7942	30.122	30.122	29.5	29.5	29.3182	29.3182	29.3182	27.5	27.5
Aggregate US_j	29.6993			29.5		29.3182			27.5	
Total US (i+j)	62.1797			66.5		61.5909			62.5	
US (2-3)	4.3203					0.9091				
Profit	7.7337	7.5477	7.5477	10.5	10.5	7.803	7.803	7.803	12.5	12.5
Total Profit	22.8292			21		23.4091			25	

Symmetric: A = B = C = 60,20; B: 60,20; α = β = 5
Symmetric: A = B = C = 60,20; B: 60,20; α = β = 5

	Symmetric					Symmetric				
	A	B	C			A	B	C	A	B
P_i	11.0606	11.0606	11.0606	10	10	11.0606	11.0606	11.0606	10	10
P_j	12.3485	12.3485	12.3485	15	15	12.3485	12.3485	12.3485	15	15
n_i	0.33333	0.33333	0.33333	0.5	0.5	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5	0.33333	0.33333	0.33333	0.5	0.5
Average US_i	32.2727	32.2727	32.2727	35	35	32.2727	32.2727	32.2727	35	35
Aggregate US_i	32.2727			35		32.2727			35	
Average US_j	29.3182	29.3182	29.3182	27.5	27.5	29.3182	29.3182	29.3182	27.5	27.5
Aggregate US_j	29.3182			27.5		29.3182			27.5	
Total US (i+j)	61.5909			62.5		61.5909			62.5	
US (2-3)	0.9091					0.9091				
Profit	7.803	7.803	7.803	12.5	12.5	7.803	7.803	7.803	12.5	12.5
Total Profit	23.4091			25		23.4091			25	

Proof for Simulation Finding 4 (Figure 2.4 (d))

Symmetric

A: 60,20; B: 60,20; C: 60, 20; $\alpha = \beta = 0$

	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	13.3333	13.3333	13.3333	20	20
P_j	13.3333	13.3333	13.3333	20	20
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Average US_i	26.6667	26.6667	26.6667	20	20
Aggregate US_i	26.6667			20	
Average US_j	26.6667	26.6667	26.6667	20	20
Aggregate US_j	26.6667			20	
Total US (i+j)	53.3334			40	
US (2-3)	-13.3334				
Profit	8.8889	8.8889	8.8889	20	20
Total Profit	26.6667			40	

Symmetric

A: 60,20; B: 60,20; C: 60, 20; $\alpha = \beta = 1$

	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	12.6939	12.6939	12.6939	18	18
P_j	13.018	13.018	13.018	19	19
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Average US_i	27.9728	27.9728	27.9728	23	23
Aggregate US_i	27.9728			23	
Average US_j	27.3154	27.3154	27.3154	21.5	21.5
Aggregate US_j	27.3154			21.5	
Total US (i+j)	55.2881			44.5	
US (2-3)	-10.7881				
Profit	8.5706	8.5706	8.5706	18.5	18.5
Total Profit	25.7119			37	

Symmetric

A: 60,20; B: 60,20; C: 60, 20; $\alpha = \beta = 2$

	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	12.1198	12.1198	12.1198	16	16
P_j	12.7453	12.7453	12.7453	18	18
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Average US_i	29.2135	29.2135	29.2135	26	26
Aggregate US_i	29.2135			26	
Average US_j	27.9214	27.9214	27.9214	23	23
Aggregate US_j	27.9214			23	
Total US (i+j)	57.1349			49	
US (2-3)	-8.1349				
Profit	8.2884	8.2884	8.2884	17	17
Total Profit	24.8651			34	

Symmetric

A: 60,20; B: 60,20; C: 60, 20; $\alpha = \beta = 3$

	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	11.6324	11.6324	11.6324	14	14
P_j	12.5282	12.5282	12.5282	17	17
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Average US_i	30.3676	30.3676	30.3676	29	29
Aggregate US_i	30.3676			29	
Average US_j	28.4718	28.4718	28.4718	24.5	24.5
Aggregate US_j	28.4718			24.5	
Total US (i+j)	58.8394			53.5	
US (2-3)	-5.3394				
Profit	8.0535	8.0535	8.0535	15.5	15.5
Total Profit	24.1606			31	

Symmetric

A: 60,20; B: 60,20; C: 60, 20; $\alpha = \beta = 4$

	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	11.2632	11.2632	11.2632	12	12
P_j	12.3859	12.3859	12.3859	16	16
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Average US_i	31.4035	31.4035	31.4035	32	32
Aggregate US_i	31.4035			32	
Average US_j	28.9474	28.9474	28.9474	26	26
Aggregate US_j	28.9474			26	
Total US (i+j)	60.3509			58	
US (2-3)	-2.3509				
Profit	7.883	7.883	7.883	14	14
Total Profit	23.6491			28	

Symmetric

A: 60,20; B: 60,20; C: 60, 20; $\alpha = \beta = 5$

	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	11.0606	11.0606	11.0606	10	10
P_j	12.3485	12.3485	12.3485	15	15
n_i	0.33333	0.33333	0.33333	0.5	0.5
n_j	0.33333	0.33333	0.33333	0.5	0.5
Average US_i	32.2727	32.2727	32.2727	35	35
Aggregate US_i	32.2727			35	
Average US_j	29.3182	29.3182	29.3182	27.5	27.5
Aggregate US_j	29.3182			27.5	
Total US (i+j)	61.5909			62.5	
US (2-3)	0.9091				
Profit	7.803	7.803	7.803	12.5	12.5
Total Profit	23.4091			25	

Asymmetric

A: 60,20; B: 50,30; C: 50, 30; $\alpha = \beta = 0$

	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	12.3617	8.0566	8.0566	10	10
P_j	12.3617	8.0566	8.0566	10	10
n_i	0.30904	0.34548	0.34548	0.5	0.5
n_j	0.30904	0.34548	0.34548	0.5	0.5
Average US_i	27.6383	31.9434	31.9434	30	30
Aggregate US_i	30.6129			30	
Average US_j	27.6383	31.9434	31.9434	30	30
Aggregate US_j	30.6129			30	
Total US (i+j)	61.2258			60	
US (2-3)	-1.2258				
Profit	7.6405	5.5668	5.5668	10	10
Total Profit	18.7742			20	

Asymmetric

A: 60,20; B: 50,30; C: 50, 30; $\alpha = \beta = 1$

	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	12.1133	7.3416	7.3416	8	8
P_j	12.2633	7.7001	7.7001	9	9
n_i	0.29624	0.35188	0.35188	0.5	0.5
n_j	0.3019	0.34905	0.34905	0.5	0.5
Average US_i	28.4848	33.3593	33.3593	33	33
Aggregate US_i	31.9153			33	
Average US_j	28.033	32.6518	32.6518	31.5	31.5
Aggregate US_j	31.2574			31.5	
Total US (i+j)	63.1727			64.5	
US (2-3)	1.3273				
Profit	7.2907	5.2711	5.2711	8.5	8.5
Total Profit	17.8329			17	

Asymmetric

A: 60,20; B: 50,30; C: 50, 30; $\alpha = \beta = 2$

	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	12.9643	6.6301	6.6301	6	6
P_j	12.7985	7.3413	7.3413	8	8
n_i	0.26809	0.36595	0.36595	0.5	0.5
n_j	0.28393	0.35804	0.35804	0.5	0.5
Average US_i	28.1397	34.8179	34.8179	36	36
Aggregate US_i	33.0275			36	
Average US_j	27.7377	33.3906	33.3906	33	33
Aggregate US_j	31.7856			33	
Total US (i+j)	64.8131			69	
US (2-3)	4.1869				
Profit	7.1095	5.0548	5.0548	7	7
Total Profit	17.2191			14	

Asymmetric

A: 60,20; B: 50,30; C: 50, 30; $\alpha = \beta = 3$

	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	13.4002	6.0188	6.0188	4	4
P_j	13.2684	7.1464	7.1464	7	7
n_i	0.28224	0.35888	0.35888	0.5	0.5
n_j	0.28538	0.35731	0.35731	0.5	0.5
Average US_i	28.3025	36.1297	36.1297	39	39
Aggregate US_i	33.9207			39	
Average US_j	27.5783	33.9302	33.9302	34.5	34.5
Aggregate US_j	32.1176			34.5	
Total US (i+j)	66.0383			73.5	
US (2-3)	7.4617				
Profit	7.5683	4.7136	4.7136	5.5	5.5
Total Profit	16.9955			11	

Asymmetric

A: 60,20; B: 50,30; C: 50, 30; $\alpha = \beta = 4$

	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	8.3134	7.2314	7.2314	2	2
P_j	10.2757	7.9873	7.9873	6	6
n_i	0.39058	0.30471	0.30471	0.5	0.5
n_j	0.36096	0.31952	0.31952	0.5	0.5
Average US_i	34.6927	35.2655	35.2655	42	42
Aggregate US_i	35.0418			42	
Average US_j	31.2866	33.2316	33.2316	36	36
Aggregate US_j	32.5295			36	
Total US (i+j)	67.5713			78	
US (2-3)	10.4287				
Profit	6.9561	4.7556	4.7556	4	4
Total Profit	16.4673			8	

Asymmetric

A: 60,20; B: 50,30; C: 50, 30; $\alpha = \beta = 5$

	Three Platforms			Duopoly	
	A	B	C	A	B
P_i	8.4751	12.2468	12.2468	0.00	0.00
P_j	9.242	10.4127	10.4127	5	5
n_i	0.49204	0.25398	0.25398	0.5	0.5
n_j	0.42392	0.28804	0.28804	0.5	0.5
Average US_i	36.1048	30.4632	30.4632	45	45
Aggregate US_i	33.2392			45	
Average US_j	33.2182	30.8572	30.8572	37.5	37.5
Aggregate US_j	31.8581			37.5	
Total US (i+j)	65.0973			82.5	
US (2-3)	17.4027				
Profit	8.0881	6.1096	6.1096	2.5	2.5
Total Profit	20.3073			5	

Appendix B: Users and Advertisers Surplus Calculation

Chapter 3: Users and Advertisers Surplus Calculation

Integration step by step in the case of Monopoly Partial Market Coverage:

Group–i average surplus on A.

$$\bar{US}_i = \int_{\hat{s}_i}^{h_i} U_i(s_i) \times \frac{1}{h_i - \hat{s}_i} \times ds_i = \int_{\hat{s}_i}^{h_i} \big(s_i + \alpha_i n_i - \lambda_i n_{Ad}\big) \times \frac{1}{h_i - \hat{s}_i} \times ds_i$$

$$\bar{US}_i = \frac{1}{h_i - \hat{s}_i} \left[\int_{\hat{s}_i}^{h_i} s_i \, ds_i + (\alpha_i n_i - \lambda_i n_{Ad}) \int_{\hat{s}_i}^{h_i} 1 \, ds_i \right]$$

$$= \frac{1}{h_i - \hat{s}_i} \left[\frac{(s_i)^2}{2} \bigg|_{\hat{s}_i}^{h_i} + (\alpha_i n_i - \lambda_i n_{Ad})(h_i - \hat{s}_i) \right]$$

Evaluate the quadratic term:

$$\frac{(h_i)^2 - (\hat{s}_i)^2}{2(h_i - \hat{s}_i)} = \frac{(h_i - \hat{s}_i)(h_i + \hat{s}_i)}{2(h_i - \hat{s}_i)}$$

Hence

$$\boxed{\bar{US}_i \; = \; \frac{h_i + \hat{s}_i}{2} \; + \; \alpha_i n_i - \lambda_i n_{Ad}}$$

Group–Ad average surplus on A. By definition

$$\bar{US}_{Ad} = \int_{\hat{s}_{Ad}}^{h_{Ad}} U_{Ad}(s_{Ad}) \times \frac{1}{h_{Ad} - \hat{s}_{Ad}} \times ds_{Ad} = \int_{\hat{s}_{Ad}}^{h_{Ad}} \big(s_{Ad} + \beta_{Ad} n_i - p_{Ad} n_i\big) \times \frac{1}{h_{Ad} - \hat{s}_{Ad}} \times ds_j$$

Split the integral into the s_{Ad}-term and the constant terms:

$$\bar{US}_{Ad} = \frac{1}{h_{Ad} - \hat{s}_{Ad}} \left[\int_{\hat{s}_{Ad}}^{h_{Ad}} s_{Ad} \, ds_{Ad} + (\beta_{Ad} n_i - p_{Ad} n_i) \int_{\hat{s}_{Ad}}^{h_{Ad}} 1 \, ds_{Ad} \right]$$

$$= \frac{1}{h_{Ad} - \hat{s}_{Ad}} \left[\frac{(s_{Ad})^2}{2} \bigg|_{\hat{s}_{Ad}}^{h_{Ad}} + (\beta_{Ad} n_i - p_{Ad} n_i)(h_{Ad} - \hat{s}_{Ad}) \right]$$

Evaluate the quadratic term:

$$\frac{(h_{Ad})^2 - (\hat{s}_{Ad})^2}{2(h_{Ad} - \hat{s}_{Ad})} = \frac{(h_{Ad} - \hat{s}_{Ad})(h_{Ad} + \hat{s}_{Ad})}{2h_{Ad} - \hat{s}_{Ad}}$$

Hence

$$\boxed{\bar{US}_{Ad} \ = \ \frac{h_{Ad} + \hat{s}_{Ad}}{2} \ + \ \beta_{Ad} n_i - p_{Ad} n_i}$$

Step-by-Step Integration of Multi-Homing User Surplus:

The average surplus for a multi-homing user of type i is given by:

$$\begin{aligned}
\bar{US}_i^{A,B} = & \left(\int_{\hat{s}_i^A}^{s_i^B} (s_i^A + \alpha_i \tilde{n}_i^A - \lambda_i^A n_{\text{Ad}}^A) \times \frac{1}{s_i^B - \hat{s}_i^A} \times ds_i^A \right) \\
& + \left(\int_{\hat{s}_i^A}^{s_i^B} s_i^B + \alpha_i \tilde{n}_i^B - \lambda_i^B n_{\text{Ad}}^B \times \frac{1}{s_i^B - \hat{s}_i^A} \times ds_i^B \right) \\
& + \alpha_i (n_i^{A,B}) - \delta_i^A \cdot \min(s_i^A, s_i^B) + \delta_i \cdot \min(\lambda_i^A n_{\text{Ad}}^A, \lambda_i^B n_{\text{Ad}}^B)
\end{aligned}$$

Step 1 — evaluate the first integral:

$$\begin{aligned}
& \int_{\hat{s}_i^A}^{s_i^B} \left(s_i^A + \alpha_i \tilde{n}_i^A - \lambda_i^A n_{\text{Ad}}^A\right) \times \frac{1}{s_i^B - \hat{s}_i^A} \times ds_i^A \\
& = \frac{1}{s_i^B - \hat{s}_i^A} \int_{\hat{s}_i^A}^{s_i^B} s_i^A \times ds_i^A \ + \ \frac{\alpha_i \tilde{n}_i^A - \lambda_i^A n_{\text{Ad}}^A}{s_i^B - \hat{s}_i^A} \int_{\hat{s}_i^A}^{s_i^B} ds_i^A \\
& = \frac{1}{s_i^B - \hat{s}_i^A} \left[\frac{(s_i^B)^2 - (\hat{s}_i^A)^2}{2} \right] \ + \ \frac{\alpha_i \tilde{n}_i^A - \lambda_i^A n_{\text{Ad}}^A}{s_i^B - \hat{s}_i^A} (s_i^B - \hat{s}_i^A) \\
& = \frac{1}{2}(s_i^B + \hat{s}_i^A) \ + \ \alpha_i \tilde{n}_i^A \ - \ \lambda_i^A n_{\text{Ad}}^A
\end{aligned}$$

Step 2 — evaluate the second integral:

$$\int_{\hat{s}_i^A}^{s_i^B} \left(s_i^B + \alpha_i \tilde{n}_i^B - \lambda_i^B n_{\text{Ad}}^B\right) \times \frac{1}{s_i^B - \hat{s}_i^A} \times ds_i^B$$

$$= \frac{1}{s_i^B - \hat{s}_i^A} \int_{\hat{s}_i^A}^{s_i^B} s_i^B \times ds_i^B \; + \; \frac{\alpha_i \tilde{n}_i^B - \lambda_i^B n_{\text{Ad}}^B}{s_i^B - \hat{s}_i^A} \int_{\hat{s}_i^A}^{s_i^B} ds_i^B$$

$$= \frac{1}{s_i^B - \hat{s}_i^A} \left[\frac{(s_i^B)^2 - (\hat{s}_i^A)^2}{2}\right] \; + \; \frac{\alpha_i \tilde{n}_i^B - \lambda_i^B n_{\text{Ad}}^B}{s_i^B - \hat{s}_i^A} \left(s_i^B - \hat{s}_i^A\right)$$

$$= \frac{1}{2}(s_i^B + \hat{s}_i^A) \; + \; \alpha_i \tilde{n}_i^B \; - \; \lambda_i^B n_{\text{Ad}}^B$$

Step 3 — substitute back the evaluated integrals and the final expression is as follows:

$$\bar{US}_i^{A,B} = \left[\frac{(s_i^B + \hat{s}_i^A)}{2} + \alpha_i \tilde{n}_i^A - \lambda_i^A n_{\text{Ad}}^A + \frac{(s_i^B + \hat{s}_i^A)}{2} + \alpha_i \tilde{n}_i^B - \lambda_i^B n_{\text{Ad}}^B\right] + \alpha_i (n_i^{A,B}) - \delta_i^A \cdot \min\left(\frac{(s_i^B + \hat{s}_i^A)}{2}, \frac{(s_i^B + \hat{s}_i^A)}{2}\right) + \delta_i \cdot \min(\lambda_i^A n_{\text{Ad}}^A, \lambda_i^B n_{\text{Ad}}^B)$$

Proof for Simulation Result 1											
Group-i: h_i = 20; l_i = 2; Group-Ad: h_Ad = 10; l_Ad = 2; β_Ad = 2; λ_i = 6											
	α_i = 1	α_i = 1.1	α_i = 1.2	α_i = 1.3	α_i = 1.4	α_i = 1.5	α_i = 1.6	α_i = 1.7	α_i = 1.8	α_i = 1.9	α_i = 2
P_Ad	6.6695	6.6389	6.6083	6.5777	6.5471	6.5166	6.486	6.4554	6.4248	6.3942	6.3636
n_i	0.92607	0.93139	0.93677	0.94221	0.94771	0.95328	0.95892	0.96462	0.97039	0.97623	0.98214
n_Ad	0.70946	0.70992	0.71038	0.71085	0.71133	0.71181	0.71229	0.71278	0.71328	0.71378	0.71429
s^_i	3.3307	3.235	3.1382	3.0402	2.9412	2.8409	2.7395	2.6368	2.533	2.4278	2.3214
s^_Ad	4.3243	4.3207	4.3169	4.3132	4.3094	4.3056	4.3017	4.2978	4.2938	4.2898	4.2857
s_i	11.6653	11.6175	11.5691	11.5201	11.4706	11.4205	11.3697	11.3184	11.2665	11.2139	11.1607
s_Ad	7.1622	7.1603	7.1585	7.1566	7.1547	7.1528	7.1508	7.1489	7.1469	7.1449	7.1429
Individual Utility_i	8.3347	8.3825	8.4309	8.4799	8.5294	8.5795	8.6303	8.6816	8.7335	8.7861	8.8393
User Surplus_i	7.7185	7.8074	7.8978	7.9898	8.0834	8.1787	8.2757	8.3744	8.4749	8.5773	8.6814
Individual Utility_Ad	2.8378	2.8397	2.8415	2.8434	2.8453	2.8472	2.8492	2.8511	2.8531	2.8551	2.8571
User Surplus_Ad	2.0133	2.0159	2.0186	2.0212	2.0239	2.0267	2.0294	2.0322	2.0351	2.0379	2.0408
User Surplus (I+II)	9.7318	9.8233	9.9164	10.0111	10.1074	10.2054	10.3051	10.4067	10.51	10.6152	10.7223
Profit/ Ad Spending	4.382	4.3897	4.3976	4.4056	4.4136	4.4218	4.4301	4.4385	4.447	4.4556	4.4643
Ad Spending per Advertiser	6.1765	6.1834	6.1905	6.1976	6.2048	6.2121	6.2195	6.227	6.2346	6.2422	6.25

Proof for Simulation Result 2											
Group-i: h_i = 20; l_i = 2; Group-Ad: h_Ad = 10; l_Ad = 2; α_i = 2; λ_i = 6											
	β_Ad = 1	β_Ad = 1.1	β_Ad = 1.2	β_Ad = 1.3	β_Ad = 1.4	β_Ad = 1.5	β_Ad = 1.6	β_Ad = 1.7	β_Ad = 1.8	β_Ad = 1.9	β_Ad = 2
P_Ad	5.6355	5.7076	5.7799	5.8524	5.925	5.9977	6.0706	6.1436	6.2168	6.2902	6.3636
n_i	0.99813	0.99647	0.99482	0.99319	0.99157	0.98996	0.98837	0.98679	0.98523	0.98368	0.98214
n_Ad	0.67164	0.67608	0.68047	0.68483	0.68915	0.69343	0.69767	0.70188	0.70605	0.71019	0.71429
s^_i	2.0336	2.0635	2.0932	2.1226	2.1518	2.1807	2.2093	2.2377	2.2659	2.2938	2.3214
s^_Ad	4.6269	4.5914	4.5562	4.5214	4.4868	4.4526	4.4186	4.3849	4.3516	4.3185	4.2857
s_i	11.0168	11.0318	11.0466	11.0613	11.0759	11.0903	11.1047	11.1188	11.1329	11.1469	11.1607
s_Ad	7.3134	7.2957	7.2781	7.2607	7.2434	7.2263	7.2093	7.1925	7.1758	7.1593	7.1429
Individual Utility_i	8.9832	8.9682	8.9534	8.9387	8.9241	8.9097	8.8953	8.8812	8.8671	8.8531	8.8393
User Surplus_i	8.9664	8.9366	8.907	8.8778	8.8489	8.8202	8.7919	8.7639	8.7361	8.7086	8.6814
Individual Utility_Ad	2.6866	2.7043	2.7219	2.7393	2.7566	2.7737	2.7907	2.8075	2.8242	2.8407	2.8571
User Surplus_Ad	1.8044	1.8283	1.8522	1.876	1.8997	1.9234	1.947	1.9705	1.994	2.0175	2.0408
User Surplus (I+II)	10.7709	10.7649	10.7592	10.7538	10.7486	10.7436	10.7389	10.7344	10.7302	10.7261	10.7223
Profit/ Ad Spending	3.778	3.8452	3.9127	3.9806	4.0488	4.1172	4.186	4.2552	4.3246	4.3943	4.4643
Ad Spending per Advertiser	5.625	5.6875	5.75	5.8125	5.875	5.9375	6	6.0625	6.125	6.1875	6.25

Proof for Simulation Result 3											
Group-i: h_i = 20; l_i = 2; Group-Ad: h_Ad = 10; l_Ad = 2; α_i = 2; β_Ad = 2											
	λ_i = 6	λ_i = 6.1	λ_i = 6.2	λ_i = 6.3	λ_i = 6.4	λ_i = 6.5	λ_i = 6.6	λ_i = 6.7	λ_i = 6.8	λ_i = 6.9	λ_i = 7
P_Ad	6.3636	6.3902	6.4168	6.4436	6.4706	6.4977	6.525	6.5524	6.5799	6.6076	6.6355
n_i	0.98214	0.97807	0.974	0.96995	0.96591	0.96188	0.95786	0.95385	0.94986	0.94587	0.9419
n_Ad	0.71429	0.71327	0.71225	0.71124	0.71023	0.70922	0.70822	0.70721	0.70621	0.70522	0.70423
s^_i	2.3214	2.3948	2.4679	2.5409	2.6136	2.6862	2.7585	2.8306	2.9025	2.9743	3.0458
s^_Ad	4.2857	4.2939	4.302	4.3101	4.3182	4.3262	4.3343	4.3423	4.3503	4.3583	4.3662
s_i	11.1607	11.1974	11.234	11.2704	11.3068	11.3431	11.3792	11.4153	11.4513	11.4871	11.5229
s_Ad	7.1429	7.1469	7.151	7.155	7.1591	7.1631	7.1671	7.1711	7.1751	7.1791	7.1831
Individual Utility_i	8.8393	8.8026	8.766	8.7296	8.6932	8.6569	8.6208	8.5847	8.5487	8.5129	8.4771
User Surplus_i	8.6814	8.6095	8.5381	8.4672	8.3968	8.3269	8.2575	8.1885	8.1201	8.0521	7.9846
Individual Utility_Ad	2.8571	2.8531	2.849	2.845	2.8409	2.8369	2.8329	2.8289	2.8249	2.8209	2.8169
User Surplus_Ad	2.0408	2.035	2.0292	2.0234	2.0177	2.012	2.0063	2.0006	1.995	1.9893	1.9837
User Surplus (I+II)	10.7223	10.6445	10.5673	10.4907	10.4145	10.3389	10.2638	10.1891	10.115	10.0414	9.9683
Profit/ Ad Spending	4.4643	4.4579	4.4516	4.4452	4.4389	4.4326	4.4263	4.4201	4.4138	4.4076	4.4014
Ad Spending per Advertiser	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25

Figure 3.1: Numerical Simulation 1: h_i = 20, l_i = 2; h_Ad = 10, l_Ad = 2, α_i = β_Ad= 2, λ_i=6						
α_i/β_Ad	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
∂p_Ad/α_i, when α_i vaires (3.1 (a))	-1.4978	-1.4977	-1.4975	-1.4974	-1.4972	-1.497
∂p_Ad/α_i, when β_Ad varies (3.1 (b))	-1.4992	-1.4989	-1.4985	-1.498	-1.4976	-1.497
λ_i	**6**	**6.2**	**6.4**	**6.6**	**6.8**	**7**
∂p_Ad/α_i, when λ_i varies (3.1 (c))	-1.497	-1.5226	-1.5491	-1.5766	-1.605	-1.6345

Figure 3.4: Numerical Simulation 2: h_i = 20, l_i = 2; h_Ad = 10, l_Ad = 2, α_i = β_Ad= 2, λ_i=6						
α_i/β_Ad	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
∂p_Ad/β_Ad, when α_i vaires (3.4(a))	0.8911	0.8915	0.892	0.8925	0.893	0.8935
∂p_Ad/β_Ad, when β_Ad varies (3.4(b))	0.8848	0.8866	0.8884	0.8901	0.8918	0.8935
λ_i	**6**	**6.2**	**6.4**	**6.6**	**6.8**	**7**
∂p_Ad/β_Ad, when λ_i varies (3.4(c))	0.8935	0.9048	0.9157	0.9263	0.9364	0.946

Figure 3.7: Numerical Simulation 3: h_i = 20, l_i = 2; h_Ad = 10, l_Ad = 2, α_i = β_Ad= 2, λ_i=6						
α_i/β_Ad	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
∂p_Ad/λ_i, when α_i vaires (3.7(a))	0.9956	0.9704	0.9453	0.9201	0.895	0.8698
∂p_Ad/λ_i, when β_Ad varies (3.4(b))	0.8112	0.8231	0.835	0.8467	0.8583	0.8698

Numerical example 4 A = B: (i: 60,6; Ad: 40,4); α = 2, β = 2, λ^A = 20, λ^B = 15)		
	A	**B**
P_Ad	22.6833	21.4321
n_i	0.9325	0.987
n_Ad	0.5753	0.5783
sˆ_i	9.64	6
s_i	34.82	32.645
Individual Utility_i	25.179	25.9445
User Surplus_i	23.4818	25.6034
s_i for multi-homing users	34.465	34.465
Individual Utility_Multi-hoimng User	24.824	27.7645
User Surplus_Multi-homing Users	22.8232	25.5267

Numerical example 4 on Multi-homing										
For Multi-homing Users: US_i (A): 22.8232; US_i (B): 25.5267										
Δ_i	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
US_i (Multi-homing)	44.2879	41.9167	39.5455	37.1744	34.8032	32.4320	30.0608	27.6896	25.3185	23.1844
US_i (A - A&B)	-21.4647	-19.0935	-16.7223	-14.3512	-11.9800	-9.6088	-7.2376	-4.8664	-2.4953	-0.3612
US_i (B - A&B)	-18.7612	-16.3900	-14.0188	-11.6477	-9.2765	-6.9053	-4.5341	-2.1629	0.2082	2.3423

Chapter 2: Analytical solution to Monopoly model

```
syms h_i_A h_j_A l_i_A l_j_A a b p_1 p_2 n_1 n_2;

eq1 = n_1 == ((h_i_A - l_i_A) + a * n_1 + b * n_2
+ l_i_A - p_1) / (h_i_A - l_i_A);
eq2 = n_2 == ((h_j_A - l_j_A) + b * n_1
+ l_j_A - p_2) / (h_j_A - l_j_A);

sol = solve([eq1, eq2], [n_1, n_2]);

disp(['n_1 = ', char(sol.n_1)]);
disp(['n_2 = ', char(sol.n_2)]);

expr_1 = sol.n_1 * p_1 + sol.n_2 * p_2;
result_1 = diff(expr_1, p_1);
result_2 = diff(expr_1, p_2);

expr1 = result_1;
expr2 = result_2;

solutions = solve([expr1, expr2], [p_1, p_2]);

disp(['p_1 = ', char(solutions.p_1)]);
disp(['p_2 = ', char(solutions.p_2)]);

n_1_expr = subs(sol.n_1, {p_1, p_2}, {solutions.p_1, solutions.p_2});
n_2_expr = subs(sol.n_2, {p_1, p_2}, {solutions.p_1, solutions.p_2});

n_1_expr = simplify(n_1_expr);
n_2_expr = simplify(n_2_expr);

disp(['n_1 = ', char(n_1_expr)]);
disp(['n_2 = ', char(n_2_expr)]);
```

Analytical solution to duopoly model

```
syms n_1 n_2 n_3 n_4 x_1 x_2 p_1 p_2 h_i_A l_i_A h_i_B
l_i_B h_j_A l_j_A h_j_B l_j_B a b;

eq1 = n_1 == ((h_i_A + l_i_A)/2 + a * (n_1 - n_3)
+ b * (n_2 - n_4) - x_1 + x_2 - l_i_B) / (h_i_B - l_i_B);
eq2 = n_2 == ((h_j_A + l_j_A)/2 + b * (n_1 - n_3)
- p_1 + p_2 - l_j_B) / (h_j_B - l_j_B);
eq3 = n_3 == 1 - n_1;
eq4 = n_4 == 1 - n_2;

solution = solve([eq1, eq2, eq3, eq4], [n_1, n_2, n_3, n_4]);

disp(['n_1 = ', char(solution.n_1)]);
disp(['n_2 = ', char(solution.n_2)]);
disp(['n_3 = ', char(solution.n_3)]);
disp(['n_4 = ', char(solution.n_4)]);

expr_1 = solution.n_1 * x_1 + solution.n_2 * p_1;
result_1 = simplify(diff(expr_1, x_1));
disp(result_1);

expr_2 = solution.n_1 * x_1 + solution.n_2 * p_1;
result_2 = simplify(diff(expr_2, p_1));
disp(result_2);

expr_3 = solution.n_3 * x_2 + solution.n_4 * p_2;
result_3 = simplify(diff(expr_3, x_2));
disp(result_3);

expr_4 = solution.n_3 * x_2 + solution.n_4 * p_2;
result_4 = simplify(diff(expr_4, p_2));
disp(result_4);

expr1 = result_1;
expr2 = result_2;
```

```
expr3 = result_3;
expr4 = result_4;

solutions = solve([expr1, expr2, expr3, expr4], [x_1, p_1, x_2, p_2]);

disp(['x_1 = ', char(solutions.x_1)]);
disp(['x_2 = ', char(solutions.x_2)]);
disp(['p_1 = ', char(solutions.p_1)]);
disp(['p_2 = ', char(solutions.p_2)]);

n_1_expr = subs(solution.n_1, {x_1, x_2, p_1, p_2},
{solutions.x_1, solutions.x_2, solutions.p_1, solutions.p_2});
n_2_expr = subs(solution.n_2, {x_1, x_2, p_1, p_2},
{solutions.x_1, solutions.x_2, solutions.p_1, solutions.p_2});
n_3_expr = subs(solution.n_3, {x_1, x_2, p_1, p_2},
{solutions.x_1, solutions.x_2, solutions.p_1, solutions.p_2});
n_4_expr = subs(solution.n_4, {x_1, x_2, p_1, p_2},
{solutions.x_1, solutions.x_2, solutions.p_1, solutions.p_2});

n_1_expr = simplify(n_1_expr);
n_2_expr = simplify(n_2_expr);
n_3_expr = simplify(n_3_expr);
n_4_expr = simplify(n_4_expr);

disp(['n_1 = ', char(n_1_expr)]);
disp(['n_2 = ', char(n_2_expr)]);
disp(['n_3 = ', char(n_3_expr)]);
disp(['n_4 = ', char(n_4_expr)]);
```

Three Platform Model

```
syms n1 n2 n3 n4 n5 n6 s q_1 q_2 q_3 q_4 q_5 q_6 r_1 r_2 r_3
r_4 r_5 r_6 x_1 p_1 n_1 n_2 x_2 x_3 p_2 p_3 n_3 n_4;

h_i_A = input("input the value for h_i_A: ");
l_i_A = input("input the value for l_i_A: ");
h_j_A = input("input the value for h_j_A: ");
```

```
l_j_A = input("input the value for l_j_A: ");
h_i_B = input("input the value for h_i_B: ");
l_i_B = input("input the value for l_i_B: ");
h_j_B = input("input the value for h_j_B: ");
l_j_B = input("input the value for l_j_B: ");
h_i_C = input("input the value for h_i_C: ");
l_i_C = input("input the value for l_i_C: ");
h_j_C = input("input the value for h_j_C: ");
l_j_C = input("input the value for l_j_C: ");
a = input("enter a: ");
b = input("enter b: ");

eq1 = q_1 == (s + a * (q_1 - q_3) +
b * (q_2 - q_4) - x_1 + x_2 - l_i_B) / (h_i_B - l_i_B);
eq2 = q_2 == (s + b * (q_1 - q_3) - p_1 + p_2 - l_j_B) / (h_j_B - l_j_B);
eq3 = q_3 == (s + a * (q_3 - q_5) +
b * (q_4 - q_6) - x_2 + x_3 - l_i_C) / (h_i_C - l_i_C);
eq4 = q_4 == (s + b * (q_3 - q_5) - p_2 + p_3 - l_j_C) / (h_j_C - l_j_C);
eq5 = q_5 == (s + a * (q_5 - q_1) +
b * (q_6 - q_2) - x_3 + x_1 - l_i_A) / (h_i_A - l_i_A);
eq6 = q_6 == (s + b * (q_5 - q_1) - p_3 + p_1 - l_j_A) / (h_j_A - l_j_A);

eq7 = r_1 == (s + a * (r_1 - r_5) +
b * (r_2 - r_6) - x_1 + x_3 - l_i_C) / (h_i_C - l_i_C);
eq8 = r_2 == (s + b * (r_1 - r_5) - p_1 + p_3 - l_j_C) / (h_j_C - l_j_C);
eq9 = r_3 == (s + a * (r_3 - r_1) +
b * (r_4 - r_2) - x_2 + x_1 - l_i_A) / (h_i_A - l_i_A);
eq10 = r_4 == (s + b * (r_3 - r_1) - p_2 + p_1 - l_j_A) / (h_j_A - l_j_A);
eq11 = r_5 == (s + a * (r_5 - r_3) +
b * (r_6 - r_4) - x_3 + x_2 - l_i_B) / (h_i_B - l_i_B);
eq12 = r_6 == (s + b * (r_5 - r_3) - p_3 + p_2 - l_j_B) / (h_j_B - l_j_B);

% Solve equations separately
q_solutions = solve([eq1, eq2, eq3, eq4, eq5, eq6],
[q_1, q_2, q_3, q_4, q_5, q_6]);
r_solutions = solve([eq7, eq8, eq9, eq10, eq11, eq12],
[r_1, r_2, r_3, r_4, r_5, r_6]);
```

```
% Display solutions
disp(q_solutions.q_1);
disp(q_solutions.q_2);
disp(q_solutions.q_3);
disp(q_solutions.q_4);
disp(q_solutions.q_5);
disp(q_solutions.q_6);
disp(r_solutions.r_1);
disp(r_solutions.r_2);
disp(r_solutions.r_3);
disp(r_solutions.r_4);
disp(r_solutions.r_5);
disp(r_solutions.r_6);

eq13 = n1 == int((q_solutions.q_1 * r_solutions.r_1
* 1 / (h_i_A - l_i_A)), s, l_i_A, h_i_A);
eq14 = n2 == int((q_solutions.q_2 * r_solutions.r_2
* 1 / (h_j_A - l_j_A)), s, l_j_A, h_j_A);
eq15 = n3 == int((q_solutions.q_3 * r_solutions.r_3
* 1 / (h_i_B - l_i_B)), s, l_i_B, h_i_B);
eq16 = n4 == int((q_solutions.q_4 * r_solutions.r_4
* 1 / (h_j_B - l_j_B)), s, l_j_B, h_j_B);
eq17 = n5 == 1 - n1 - n3;
eq18 = n6 == 1 - n2 - n4;

sol = solve([eq13, eq14, eq15, eq16, eq17, eq18], [n1, n2, n3, n4, n5, n6]);

disp(['n_1 = ', char(sol.n1)]);
disp(['n_2 = ', char(sol.n2)]);
disp(['n_3 = ', char(sol.n3)]);
disp(['n_4 = ', char(sol.n4)]);
disp(['n_5 = ', char(sol.n5)]);
disp(['n_6 = ', char(sol.n6)]);
```

One dominant and two duopoly model:

```
eq13 = n5 == int((q_solutions.q_5 * r_solutions.r_5
* 1 / (h_i_C - l_i_C)), s, l_i_C, h_i_C);
eq14 = n6 == int((q_solutions.q_6 * r_solutions.r_6
```

```
*  1 / (h_j_C - l_j_C)), s, l_j_C, h_j_C);
eq15 = n3 == int((q_solutions.q_3 * r_solutions.r_3
* 1 / (h_i_B - l_i_B)), s, l_i_B, h_i_B);
eq16 = n4 == int((q_solutions.q_4 * r_solutions.r_4
* 1 / (h_j_B - l_j_B)), s, l_j_B, h_j_B);
eq17 = n1 == 1 - n3 - n5;
eq18 = n2 == 1 - n4 - n6;

sol = solve([eq13, eq14, eq15, eq16, eq17, eq18], [n1, n2, n3, n4, n5, n6]);

disp(['n_1 = ', char(sol.n1)]);
disp(['n_2 = ', char(sol.n2)]);
disp(['n_3 = ', char(sol.n3)]);
disp(['n_4 = ', char(sol.n4)]);
disp(['n_5 = ', char(sol.n5)]);
disp(['n_6 = ', char(sol.n6)]);

eq13 = n1 == int((q_solutions.q_1 * r_solutions.r_1 *
1 / (h_i_A - l_i_A)), s, l_i_A, h_i_A);
eq14 = n2 == int((q_solutions.q_2 * r_solutions.r_2 *
1 / (h_j_A - l_j_A)), s, l_j_A, h_j_A);
eq15 = n3 == int((q_solutions.q_3 * r_solutions.r_3 *
1 / (h_i_B - l_i_B)), s, l_i_B, h_i_B);
eq16 = n4 == int((q_solutions.q_4 * r_solutions.r_4 *
1 / (h_j_B - l_j_B)), s, l_j_B, h_j_B);
eq17 = n5 == int((q_solutions.q_5 * r_solutions.r_5 *
1 / (h_i_C - l_i_C)), s, l_i_C, h_i_C);
eq18 = n6 == int((q_solutions.q_6 * r_solutions.r_6 *
1 / (h_j_C - l_j_C)), s, l_j_C, h_j_C);

sol = solve([eq13, eq14, eq15, eq16, eq17, eq18], [n1, n2, n3, n4, n5, n6]);

disp(['n_1 = ', char(sol.n1)]);
disp(['n_2 = ', char(sol.n2)]);
disp(['n_3 = ', char(sol.n3)]);
disp(['n_4 = ', char(sol.n4)]);
disp(['n_5 = ', char(sol.n5)]);
disp(['n_6 = ', char(sol.n6)]);
```

```
expr_1 = sol.n1 * x_1 + sol.n2 * p_1;
result_1 = diff(expr_1, x_1);
disp(result_1);

expr_2 = sol.n1 * x_1 + sol.n2 * p_1;
result_2 = diff(expr_2, p_1);
disp(result_2);

expr_3 = sol.n3 * x_2 + sol.n4 * p_2;
result_3 = diff(expr_3, x_2);
disp(result_3);

expr_4 = sol.n3 * x_2 + sol.n4 * p_2;
result_4 = diff(expr_4, p_2);
disp(result_4);

expr_5 = sol.n5 * x_3 + sol.n6 * p_3;
result_5 = diff(expr_5, x_3);
disp(result_5);

expr_6 = sol.n5 * x_3 + sol.n6 * p_3;
result_6 = diff(expr_6, p_3);
disp(result_6);

x_1expr = matlabFunction(result_1, 'Vars', {x_1, p_1, x_2, p_2, x_3, p_3});
p_1expr = matlabFunction(result_2, 'Vars', {x_1, p_1, x_2, p_2, x_3, p_3});
x_2expr = matlabFunction(result_3, 'Vars', {x_1, p_1, x_2, p_2, x_3, p_3});
p_2expr = matlabFunction(result_4, 'Vars', {x_1, p_1, x_2, p_2, x_3, p_3});
x_3expr = matlabFunction(result_5, 'Vars', {x_1, p_1, x_2, p_2, x_3, p_3});
p_3expr = matlabFunction(result_6, 'Vars', {x_1, p_1, x_2, p_2, x_3, p_3});

initial_guess = [0; 0; 0; 0; 0; 0];

equations = @(vars) [x_1expr(vars(1), vars(2), vars(3), vars(4),
vars(5), vars(6));
p_1expr(vars(1), vars(2), vars(3), vars(4), vars(5), vars(6));
x_2expr(vars(1), vars(2), vars(3), vars(4), vars(5), vars(6));
```

```
p_2expr(vars(1), vars(2), vars(3), vars(4), vars(5), vars(6));
x_3expr(vars(1), vars(2), vars(3), vars(4), vars(5), vars(6));
p_3expr(vars(1), vars(2), vars(3), vars(4), vars(5), vars(6))];

options = optimoptions('fsolve', 'Display', 'off');

solutions = fsolve(equations, initial_guess, options);

x_1val = solutions(1);
p_1val = solutions(2);
x_2val = solutions(3);
p_2val = solutions(4);
x_3val = solutions(5);
p_3val = solutions(6);

disp(['x_1 = ', num2str(x_1val)]);
disp(['p_1 = ', num2str(p_1val)]);
disp(['x_2 = ', num2str(x_2val)]);
disp(['p_2 = ', num2str(p_2val)]);
disp(['x_3 = ', num2str(x_3val)]);
disp(['p_3 = ', num2str(p_3val)]);

n_1_val = double(subs(sol.n1, [x_1, p_1, x_2, p_2, x_3, p_3],
[x_1val, p_1val, x_2val, p_2val, x_3val, p_3val]));
n_2_val = double(subs(sol.n2, [x_1, p_1, x_2, p_2, x_3, p_3],
[x_1val, p_1val, x_2val, p_2val, x_3val, p_3val]));
n_3_val = double(subs(sol.n3, [x_1, p_1, x_2, p_2, x_3, p_3],
[x_1val, p_1val, x_2val, p_2val, x_3val, p_3val]));
n_4_val = double(subs(sol.n4, [x_1, p_1, x_2, p_2, x_3, p_3],
[x_1val, p_1val, x_2val, p_2val, x_3val, p_3val]));
n_5_val = double(subs(sol.n5, [x_1, p_1, x_2, p_2, x_3, p_3],
[x_1val, p_1val, x_2val, p_2val, x_3val, p_3val]));
n_6_val = double(subs(sol.n6, [x_1, p_1, x_2, p_2, x_3, p_3],
[x_1val, p_1val, x_2val, p_2val, x_3val, p_3val]));

disp(['n_1 = ', num2str(n_1_val)]);
disp(['n_2 = ', num2str(n_2_val)]);
disp(['n_3 = ', num2str(n_3_val)]);
```

```
disp(['n_4 = ', num2str(n_4_val)]);
disp(['n_5 = ', num2str(n_5_val)]);
disp(['n_6 = ', num2str(n_6_val)]);

U_1 = (h_i_A + l_i_A)/2 + (a * n_1_val) + (b * n_2_val) - x_1val;
U_2 = (h_j_A + l_j_A)/2 + (b * n_1_val) - p_1val;
U_A = (U_1 * n_1_val) + (U_2 * n_2_val);
U_3 = (h_i_B + l_i_B)/2 + (a * n_3_val) + (b * n_4_val) - x_2val;
U_4 = (h_j_B + l_j_B)/2 + (b * n_3_val) - p_2val;
U_B = (U_3 * n_3_val) + (U_4 * n_4_val);
U_5 = (h_i_C + l_i_C)/2 + (a * n_5_val) + (b * n_6_val) - x_3val;
U_6 = (h_j_C + l_j_C)/2 + (b * n_5_val) - p_3val;
U_C = (U_5 * n_5_val) + (U_6 * n_6_val);
U_i = (U_1 * n_1_val) + (U_3 * n_3_val) + (U_5 * n_5_val);
U_j = (U_2 * n_2_val) + (U_4 * n_4_val) + (U_6 * n_6_val);
U = U_A + U_B + U_C;

disp(['Value of U_1: ' num2str(U_1)]);
disp(['Value of U_2: ' num2str(U_2)]);
disp(['Value of U_A: ' num2str(U_A)]);
disp(['Value of U_3: ' num2str(U_3)]);
disp(['Value of U_4: ' num2str(U_4)]);
disp(['Value of U_B: ' num2str(U_B)]);
disp(['Value of U_5: ' num2str(U_5)]);
disp(['Value of U_6: ' num2str(U_6)]);
disp(['Value of U_C: ' num2str(U_C)]);
disp(['Value of U_i: ' num2str(U_i)]);
disp(['Value of U_j: ' num2str(U_j)]);
disp(['Value of U: ' num2str(U)]);

z_A = n_1_val * x_1val + n_2_val * p_1val;
z_B = n_3_val * x_2val + n_4_val * p_2val;
z_C = n_5_val * x_3val + n_6_val * p_3val;
Z = z_A + z_B + z_C;

disp(['Value of z_A: ' num2str(z_A)]);
disp(['Value of z_B: ' num2str(z_B)]);
disp(['Value of z_C: ' num2str(z_C)]);
```

```
disp(['Value of Z: ' num2str(Z)]);
```

Two Platform Model

```
syms s_i s_j a b h_i_A h_i_B h_j_A h_j_B l_i_A l_i_B l_j_A l_j_B
x_1 x_2 p_1 p_2 n_1 n_2 n_3 n_4;

expr1 = (s_i + a*(n_1 - n_3) + b*(n_2 - n_4) - (x_1 - x_2)
- l_i_B)/((h_i_B - l_i_B)*(h_i_A - l_i_A));
expr2 = (s_j + b*(n_1 - n_3) - (p_1 - p_2)
- l_j_B)/((h_j_B - l_j_B)*(h_j_A - l_j_A));
I_1 = int(expr1, s_i, l_i_A, h_i_A);
I_2 = int(expr2, s_j, l_j_A, h_j_A);
disp(I_1)
disp(I_2)

syms x_1 x_2 p_1 p_2 n_1 n_2 n_3 n_4;

h_i_A = input("input the value for h_i_A: ");
l_i_A = input("input the value for l_i_A: ");
h_j_A = input("input the value for h_j_A: ");
l_j_A = input("input the value for l_j_A: ");
h_i_B = input("input the value for h_i_B: ");
l_i_B = input("input the value for l_i_B: ");
h_j_B = input("input the value for h_j_B: ");
l_j_B = input("input the value for l_j_B: ");
a = input("enter a: ");
b = input("enter b: ");

eq1 = n_1 == ((h_i_A + l_i_A)/2 + a * (n_1 - n_3) +
b * (n_2 - n_4) - x_1 + x_2 - l_i_B) / (h_i_B - l_i_B);
eq2 = n_2 == ((h_j_A + l_j_A)/2 + b * (n_1 - n_3) -
p_1 + p_2 - l_j_B) / (h_j_B - l_j_B);
eq3 = n_3 == 1 - n_1;
eq4 = n_4 == 1 - n_2;

sol = solve([eq1, eq2, eq3, eq4], [n_1, n_2, n_3, n_4]);
```

```
disp(sol.n_1);
disp(sol.n_2);
disp(sol.n_3);
disp(sol.n_4);

expr_1 = sol.n_1 * x_1 + sol.n_2 * p_1;
result_1 = diff(expr_1, x_1);
disp(result_1);

expr_2 = sol.n_1 * x_1 + sol.n_2 * p_1;
result_2 = diff(expr_2, p_1);
disp(result_2);

expr_3 = sol.n_3 * x_2 + sol.n_4 * p_2;
result_3 = diff(expr_3, x_2);
disp(result_3);

expr_4 = sol.n_3 * x_2 + sol.n_4 * p_2;
result_4 = diff(expr_4, p_2);
disp(result_4);

p_1value = solve(result_2 == p_1, p_1);
p_2value = solve(result_4 == p_2, p_2);

disp(['p_1 = ', char(p_1value)]);
disp(['p_2 = ', char(p_2value)]);

x_1expr = matlabFunction(result_1, 'Vars', [x_1, p_1, x_2, p_2]);
p_1expr = matlabFunction(result_2, 'Vars', [x_1, p_1, x_2, p_2]);
x_2expr = matlabFunction(result_3, 'Vars', [x_1, p_1, x_2, p_2]);
p_2expr = matlabFunction(result_4, 'Vars', [x_1, p_1, x_2, p_2]);

initial_guess = [0; 0; 0; 0];

equations = @(vars) [x_1expr(vars(1), vars(2), vars(3), vars(4));
                     p_1expr(vars(1), vars(2), vars(3), vars(4));
                     x_2expr(vars(1), vars(2), vars(3), vars(4));
```

```
                   p_2expr(vars(1), vars(2), vars(3), vars(4))];

options = optimoptions('fsolve', 'Display', 'off');

solutions = fsolve(equations, initial_guess, options);

x_1val = solutions(1);
p_1val = solutions(2);
x_2val = solutions(3);
p_2val = solutions(4);

disp(['x_1 = ', num2str(x_1val)]);
disp(['p_1 = ', num2str(p_1val)]);
disp(['x_2 = ', num2str(x_2val)]);
disp(['p_2 = ', num2str(p_2val)]);

n_1_val = double(subs(sol.n_1, [x_1, p_1, x_2, p_2],
[x_1val, p_1val, x_2val, p_2val]));
n_2_val = double(subs(sol.n_2, [x_1, p_1, x_2, p_2],
[x_1val, p_1val, x_2val, p_2val]));
n_3_val = double(subs(sol.n_3, [x_1, p_1, x_2, p_2],
[x_1val, p_1val, x_2val, p_2val]));
n_4_val = double(subs(sol.n_4, [x_1, p_1, x_2, p_2],
[x_1val, p_1val, x_2val, p_2val]));

disp(['n_1 = ', num2str(n_1_val)]);
disp(['n_2 = ', num2str(n_2_val)]);
disp(['n_3 = ', num2str(n_3_val)]);
disp(['n_4 = ', num2str(n_4_val)]);

U_1 = (h_i_A + l_i_A)/2 + a * n_1_val + b * n_2_val - x_1val;
U_2 = (h_j_A + l_j_A)/2 + b_A * n_1_val - p_1val;
U_A = U_1 * n_1_val + U_2 * n_2_val;
U_3 = (h_i_B + l_i_B)/2 + a_B * n_3_val + b_B * n_4_val - x_2val;
U_4 = (h_j_B + l_j_B)/2 + b_B * n_3_val - p_2val;
U_B = U_3 * n_3_val + U_4 * n_4_val;
U_i = U_1 * n_1_val + U_3 * n_3_val;
U_j = U_2 * n_2_val + U_4 * n_4_val;
```

```
U = U_A + U_B;

disp(['Value of U_1: ' num2str(U_1)]);
disp(['Value of U_2: ' num2str(U_2)]);
disp(['Value of U_A: ' num2str(U_A)]);
disp(['Value of U_3: ' num2str(U_3)]);
disp(['Value of U_4: ' num2str(U_4)]);
disp(['Value of U_B: ' num2str(U_B)]);
disp(['Value of U_i: ' num2str(U_i)]);
disp(['Value of U_j: ' num2str(U_j)]);
disp(['Value of U: ' num2str(U)]);

z_A = n_1_val * x_1val + n_2_val * p_1val;
z_B = n_3_val * x_2val + n_4_val * p_2val;
Z = z_A + z_B;

disp(['Value of z_A: ' num2str(z_A)]);
disp(['Value of z_B: ' num2str(z_B)]);
disp(['Value of Z: ' num2str(Z)]);
```

Monopoly Platform Model: Partial-Market Coverage

```
syms p_1 p_2 n_1 n_2;

h_1 = input("Input the value for h_1: ");
h_2 = input("Input the value for h_2: ");
l_1 = input("Input the value for l_1: ");
l_2 = input("Input the value for l_2: ");
a = input("Enter a: ");
b = input("Enter b: ");

eq1 = n_1 == ((h_1 - l_1) + a * n_1 + b * n_2 + l_1 - p_1) / (h_1 - l_1);
eq2 = n_2 == ((h_2 - l_2) + b * n_1 + l_2 - p_2) / (h_2 - l_2);

sol = solve([eq1, eq2], [n_1, n_2]);
```

```
disp(sol.n_1);
disp(sol.n_2);

expr_1 = sol.n_1 * p_1 + sol.n_2 * p_2;
result_1 = diff(expr_1, p_1);
result_2 = diff(expr_1, p_2);

p_1value = solve(result_1 == p_1, p_1);
p_2value = solve(result_2 == p_2, p_2);

disp(['p_1 = ', char(p_1value)]);
disp(['p_2 = ', char(p_2value)]);

p_1expr = matlabFunction(result_1, 'Vars', [p_1, p_2]);
p_2expr = matlabFunction(result_2, 'Vars', [p_1, p_2]);

initial_guess = [0; 0];

equations = @(vars) [p_1expr(vars(1), vars(2));
                     p_2expr(vars(1), vars(2))];

options = optimoptions('fsolve', 'Display', 'off');

solutions = fsolve(equations, initial_guess, options);

p_1val = solutions(1);
p_2val = solutions(2);

disp(['p_1 = ', num2str(p_1val)]);
disp(['p_2 = ', num2str(p_2val)]);

n_1_val = double(subs(sol.n_1, [p_1, p_2], [p_1val, p_2val]));
n_2_val = double(subs(sol.n_2, [p_1, p_2], [p_1val, p_2val]));

disp(['n_1 = ', num2str(n_1_val)]);
disp(['n_2 = ', num2str(n_2_val)]);
```

```
s_1 = p_1val -  a_A * n_1_val - b_A * n_2_val;
s_2 = p_2val - b_A * n_1_val;

disp(['Value of s_1: ', num2str(s_1)]);
disp(['Value of s_2: ', num2str(s_2)]);

s_i = (h_1 + s_1)/2;
s_j = (h_2 + s_2)/2;

s_i_val = double(s_i);
s_j_val = double(s_j);

disp(['Value of s_i: ', num2str(s_i_val)]);
disp(['Value of s_j: ', num2str(s_j_val)]);

U_1 = s_i + a * n_1_val + b * n_2_val - x_1val;
U_2 = s_j + b_A * n_1_val - p_1val;
U_i = n_1_val * U_1;
U_j = n_2_val * U_2;
U = U_i + U_j;
z = n_1_val * p_1val + n_2_val * p_2val;

U_1_val = double(U_1);
U_2_val = double(U_2);
U_i_val = double(U_i);
U_j_val = double(U_j);
U_val = double(U);
z_val = double(z);
```

Chapter 3: Ad-based Monopoly Platform

```
syms p_A n_i n_A;
h_i = input("Input the value for h_i: ");
h_A = input("Input the value for h_A: ");
l_i = input("Input the value for l_i: ");
l_A = input("Input the value for l_A: ");
a_i = input("Enter a_i: ");
b_A = input("Enter b_A: ");
l = input("Enter l: ");

eq1 = n_i == (h_i + (a_i * n_i) - (l * n_A)) / (h_i - l_i);
eq2 = n_A == (h_A + (b_A * n_i) - (p_A * n_i)) / (h_A - l_A);

sol = solve([eq1, eq2], [n_i, n_A]);

disp(sol.n_i);
disp(sol.n_A);

expr_1 = sol.n_A * p_A * sol.n_i;
result_1 = diff(expr_1, p_A);

p_Avalue = solve(result_1 == p_A, p_A);

disp(['p_A (symbolic) = ', char(p_Avalue)]);

% Define p_Aexpr as a MATLAB function
p_Aexpr = matlabFunction(result_1, 'Vars', [p_A]);

% Set up fsolve with proper initialization
initial_guess = 0;  % Single initial guess for p_A

equations = @(p_A) p_Aexpr(p_A);

options = optimoptions('fsolve', 'Display', 'off');

solutions = fsolve(equations, initial_guess, options);
```

```
p_Aval = solutions;

disp(['p_A (numerical) = ', num2str(p_Aval)]);

% Substitute back to find n_i and n_A values
n_i_val = double(subs(sol.n_i, [p_A], [p_Aval]));
n_A_val = double(subs(sol.n_A, [p_A], [p_Aval]));

disp(['n_i = ', num2str(n_i_val)]);
disp(['n_A = ', num2str(n_A_val)]);

s_1 = l * n_A_val -  a_i * n_i_val;
s_2 = p_Aval * n_i_val - b_A * n_i_val;

s_1_val = double(s_1);
s_2_val = double(s_2);

disp(['Value of s_i: ', num2str(s_1_val)]);
disp(['Value of s_Ad: ', num2str(s_2_val)]);

s_i = (h_i + s_1)/2;
s_Ad = (h_A + s_2)/2;

s_i_val = double(s_i);
s_Ad_val = double(s_A);

disp(['Value of s_i: ', num2str(s_i_val)]);
disp(['Value of s_A: ', num2str(s_Ad_val)]);

U_1 = s_i_val + a_i * n_i_val - l * n_A_val;
U_2 = s_A_val + b_A * n_i_val - p_Aval * n_i_val;
U_i = n_i_val * U_1;
U_Ad = n_A_val * U_2;
U = U_i + U_Ad;
z = n_i_val * n_A_val * p_Aval;
A = n_i_val * p_Aval;
```

```
U_1_val = double(U_1);
U_2_val = double(U_2);
U_i_val = double(U_i);
U_Ad_val = double(U_Ad);
U_val = double(U);
z_val = double(z);
A_val = double(A);

disp(['Value of U_1: ', num2str(U_1_val)]);
disp(['Value of U_2: ', num2str(U_2_val)]);
disp(['Value of U_i: ', num2str(U_i_val)]);
disp(['Value of U_Ad: ', num2str(U_Ad_val)]);
disp(['Value of U: ', num2str(U_val)]);
disp(['Value of z: ', num2str(z_val)]);
disp(['Value of A: ', num2str(A_val)]);
```

Ad-based Monopoly Analytical Solution

```
syms h_i h_A l_i l_A a_i b_A p_A n_i n_A l;

% Define the equations
eq1 = n_i == (h_i + (a_i * n_i) - (l * n_A)) / (h_i - l_i);
eq2 = n_A == (h_A + (b_A * n_i) - (p_A * n_i)) / (h_A - l_A);

% Solve the equations for n_i and n_A
sol = solve([eq1, eq2], [n_i, n_A]);

disp(['n_i = ', char(sol.n_i)]);
disp(['n_A = ', char(sol.n_A)]);

% Define the expression and differentiate
expr_1 = sol.n_A * p_A * sol.n_i;
result_1 = diff(expr_1, p_A);

expr1 = result_1;

% Solve for p_A
```

```
solutions = solve([expr1], [p_A]);

% Check if 'solutions' is a symbolic array or structure
if isstruct(solutions)
    p_A_sol = solutions.p_A; % Access as a field
else
    p_A_sol = solutions; % Single symbolic solution
end

disp(['p_A = ', char(p_A_sol)]);

% Substitute p_A in n_i and n_A
n_i_expr = simplify(subs(sol.n_i, {p_A}, {p_A_sol}));
n_A_expr = simplify(subs(sol.n_A, {p_A}, {p_A_sol}));

disp(['n_i = ', char(n_i_expr)]);
disp(['n_A = ', char(n_A_expr)]);

p_A = (a_i^2*h_A^2 + h_A^2*h_i^2 + h_A^2*l_i^2 - 2*a_i*h_A^2*h_i
+ b_A*h_A*h_i^2 - a_i^2*h_A*l_A + 2*a_i*h_A^2*l_i + b_A^2*h_i*l
- b_A*h_i^2*l_A - h_A*h_i^2*l_A - 2*h_A^2*h_i*l_i - h_A*l_A*l_i^2
- 2*a_i*h_A*l_A*l_i - b_A*h_A*l*l_i + b_A*h_i*l_A*l_i
+ 2*h_A*h_i*l_A*l_i - a_i*b_A*h_A*h_i - a_i*b_A*h_A*l
+ a_i*b_A*h_i*l_A + 2*a_i*h_A*h_i*l_A + b_A*h_A*h_i*l - b_A*h_A*h_i*l_i)/
(2*h_A*h_i^2 - 2*h_i^2*l_A - 2*a_i*h_A*h_i + a_i*h_A*l + 2*a_i*h_i*l_A
+ b_A*h_i*l - h_A*h_i*l - 2*h_A*h_i*l_i + h_A*l*l_i + 2*h_i*l_A*l_i)

n_i = -(2*h_A*h_i^2 - 2*h_i^2*l_A - 2*a_i*h_A*h_i + a_i*h_A*l
+ 2*a_i*h_i*l_A + b_A*h_i*l - h_A*h_i*l - 2*h_A*h_i*l_i
+ h_A*l*l_i + 2*h_i*l_A*l_i)/(2*(a_i - h_i + l_i)
*(a_i*l_A - a_i*h_A + b_A*l + h_A*h_i - h_A*l_i - h_i*l_A + l_A*l_i))

n_A = -(a_i*h_A - b_A*h_i - h_A*h_i + h_A*l_i)/
(2*(a_i*l_A - a_i*h_A + b_A*l + h_A*h_i - h_A*l_i - h_i*l_A + l_A*l_i))
```

Ad-based Monopoly Model Differentiation Results

```
syms h_A h_i l_i l_A a_i b_A l p_A

f_a = (a_i^2*h_A^2 + h_A^2*h_i^2 + h_A^2*l_i^2 - 2*a_i*h_A^2*h_i
+ b_A*h_A*h_i^2 - a_i^2*h_A*l_A + 2*a_i*h_A^2*l_i + b_A^2*h_i*l
- b_A*h_i^2*l_A - h_A*h_i^2*l_A - 2*h_A^2*h_i*l_i - h_A*l_A*l_i^2
- 2*a_i*h_A*l_A*l_i - b_A*h_A*l*l_i + b_A*h_i*l_A*l_i
+ 2*h_A*h_i*l_A*l_i - a_i*b_A*h_A*h_i - a_i*b_A*h_A*l + a_i*b_A*h_i*l_A
+ 2*a_i*h_A*h_i*l_A + b_A*h_A*h_i*l - b_A*h_A*h_i*l_i);

g_a = (2*h_A*h_i^2 - 2*h_i^2*l_A - 2*a_i*h_A*h_i + a_i*h_A*l
+ 2*a_i*h_i*l_A + b_A*h_i*l - h_A*h_i*l - 2*h_A*h_i*l_i
+ h_A*l*l_i + 2*h_i*l_A*l_i);

% Differentiate numerator and denominator with respect to a_i
df_da = diff(f_a, a_i);
dg_da = diff(g_a, a_i);

% Differentiate numerator and denominator with respect to b_A
df_db = diff(f_a, b_A);
dg_db = diff(g_a, b_A);

% Differentiate numerator and denominator with respect to l
df_dl = diff(f_a, l);
dg_dl = diff(g_a, l);

% Apply the quotient rule for differentiation
dp_da = (df_da * g_a - f_a * dg_da) / g_a^2;
dp_db = (df_db * g_a - f_a * dg_db) / g_a^2;
dp_dl = (df_dl * g_a - f_a * dg_dl) / g_a^2;

% Simplify the resulting expressions
dp_da_simplified = simplify(dp_da);
dp_db_simplified = simplify(dp_db);
dp_dl_simplified = simplify(dp_dl);
```

```
disp('The derivative of p_A with respect to a_i is:');
disp(dp_da_simplified);

disp('The derivative of p_A with respect to b_A is:');
disp(dp_db_simplified);

disp('The derivative of p_A with respect to l is:');
disp(dp_dl_simplified);

The derivative of p_A with respect to a_i is:
-((2*h_A^2*h_i - 2*a_i*h_A^2 - 2*h_A^2*l_i + b_A*h_A*h_i + 2*a_i*h_A*l_A
+ b_A*h_A*l - b_A*h_i*l_A - 2*h_A*h_i*l_A + 2*h_A*l_A*l_i)*(2*h_A*h_i^2
- 2*h_i^2*l_A - 2*a_i*h_A*h_i + a_i*h_A*l + 2*a_i*h_i*l_A + b_A*h_i*l
- h_A*h_i*l - 2*h_A*h_i*l_i + h_A*l*l_i + 2*h_i*l_A*l_i)
- (h_A*l - 2*h_A*h_i + 2*h_i*l_A)*
(a_i*h_A - b_A*h_i - h_A*h_i + h_A*l_i)*
(a_i*l_A - a_i*h_A + b_A*l + h_A*h_i - h_A*l_i - h_i*l_A + l_A*l_i))/
(2*h_A*h_i^2 - 2*h_i^2*l_A - 2*a_i*h_A*h_i + a_i*h_A*l
+ 2*a_i*h_i*l_A + b_A*h_i*l - h_A*h_i*l - 2*h_A*h_i*l_i
+ h_A*l*l_i + 2*h_i*l_A*l_i)^2

The derivative of p_A with respect to b_A is:
((h_A*h_i^2 - h_i^2*l_A - a_i*h_A*h_i - a_i*h_A*l + a_i*h_i*l_A
+ 2*b_A*h_i*l + h_A*h_i*l - h_A*h_i*l_i - h_A*l*l_i + h_i*l_A*l_i)*
(2*h_A*h_i^2 - 2*h_i^2*l_A - 2*a_i*h_A*h_i + a_i*h_A*l + 2*a_i*h_i*l_A
+ b_A*h_i*l - h_A*h_i*l - 2*h_A*h_i*l_i + h_A*l*l_i + 2*h_i*l_A*l_i)
+ h_i*l*(a_i*h_A - b_A*h_i - h_A*h_i + h_A*l_i)*(a_i*l_A - a_i*h_A
+ b_A*l + h_A*h_i - h_A*l_i - h_i*l_A + l_A*l_i))/
(2*h_A*h_i^2 - 2*h_i^2*l_A - 2*a_i*h_A*h_i + a_i*h_A*l
+ 2*a_i*h_i*l_A + b_A*h_i*l - h_A*h_i*l - 2*h_A*h_i*l_i + h_A*l*l_i
+ 2*h_i*l_A*l_i)^2

The derivative of p_A with respect to l is:
-((h_A - l_A)*(a_i - h_i + l_i)*(a_i*h_A - b_A*h_i - h_A*h_i
+ h_A*l_i)^2)/(2*h_A*h_i^2 - 2*h_i^2*l_A
- 2*a_i*h_A*h_i + a_i*h_A*l + 2*a_i*h_i*l_A + b_A*h_i*l
- h_A*h_i*l - 2*h_A*h_i*l_i + h_A*l*l_i + 2*h_i*l_A*l_i)^2
```

Numerical Simulation Results

The derivative of p_A with respect to a_i is:

```
% Assign values to variables
h_A = input("Input the value for h_A: ");
h_i = input("Input the value for h_i: ");
l_i = input("Input the value for l_i: ");
l_A = input("Input the value for l_A: ");
l = input("Input the value for l: ");
b_A = input("Input the value for b_A: ");
a_i = input("Input the value for a_i: ");

eq1 = -((2*h_A^2*h_i - 2*a_i*h_A^2 - 2*h_A^2*l_i + b_A*h_A*h_i + 2*a_i*h_A*l_A
+ b_A*h_A*l - b_A*h_i*l_A - 2*h_A*h_i*l_A + 2*h_A*l_A*l_i)*(2*h_A*h_i^2
- 2*h_i^2*l_A - 2*a_i*h_A*h_i + a_i*h_A*l + 2*a_i*h_i*l_A + b_A*h_i*l
- h_A*h_i*l - 2*h_A*h_i*l_i + h_A*l*l_i + 2*h_i*l_A*l_i)
- (h_A*l - 2*h_A*h_i + 2*h_i*l_A)*
(a_i*h_A - b_A*h_i - h_A*h_i + h_A*l_i)*
(a_i*l_A - a_i*h_A + b_A*l + h_A*h_i - h_A*l_i - h_i*l_A + l_A*l_i))/
(2*h_A*h_i^2 - 2*h_i^2*l_A - 2*a_i*h_A*h_i + a_i*h_A*l
+ 2*a_i*h_i*l_A + b_A*h_i*l - h_A*h_i*l - 2*h_A*h_i*l_i
+ h_A*l*l_i + 2*h_i*l_A*l_i)^2;

disp('eq1 =');
disp(eq1);
```

The derivative of p_A with respect to b_A is:

```
% Assign values to variables
h_A = input("Input the value for h_A: ");
h_i = input("Input the value for h_i: ");
l_i = input("Input the value for l_i: ");
l_A = input("Input the value for l_A: ");
l = input("Input the value for l: ");
b_A = input("Input the value for b_A: ");
a_i = input("Input the value for a_i: ");

eq2 = ((h_A*h_i^2 - h_i^2*l_A - a_i*h_A*h_i - a_i*h_A*l + a_i*h_i*l_A
```

```
+ 2*b_A*h_i*l + h_A*h_i*l - h_A*h_i*l_i - h_A*l*l_i + h_i*l_A*l_i)*
(2*h_A*h_i^2 - 2*h_i^2*l_A - 2*a_i*h_A*h_i + a_i*h_A*l + 2*a_i*h_i*l_A
+ b_A*h_i*l - h_A*h_i*l - 2*h_A*h_i*l_i + h_A*l*l_i + 2*h_i*l_A*l_i)
+ h_i*l*(a_i*h_A - b_A*h_i - h_A*h_i + h_A*l_i)*(a_i*l_A - a_i*h_A
+ b_A*l + h_A*h_i - h_A*l_i - h_i*l_A + l_A*l_i))/
(2*h_A*h_i^2 - 2*h_i^2*l_A - 2*a_i*h_A*h_i + a_i*h_A*l
+ 2*a_i*h_i*l_A + b_A*h_i*l - h_A*h_i*l - 2*h_A*h_i*l_i + h_A*l*l_i
+ 2*h_i*l_A*l_i)^2;

disp('eq2 =');
disp(eq2);

The derivative of p_A with respect to l is:
% Assign values to variables
h_A = input("Input the value for h_A: ");
h_i = input("Input the value for h_i: ");
l_i = input("Input the value for l_i: ");
l_A = input("Input the value for l_A: ");
l = input("Input the value for l: ");
b_A = input("Input the value for b_A: ");
a_i = input("Input the value for a_i: ");

eq3 = -((h_A - l_A)*(a_i - h_i + l_i)*(a_i*h_A - b_A*h_i - h_A*h_i
+ h_A*l_i)^2)/(2*h_A*h_i^2 - 2*h_i^2*l_A
- 2*a_i*h_A*h_i + a_i*h_A*l + 2*a_i*h_i*l_A + b_A*h_i*l
- h_A*h_i*l - 2*h_A*h_i*l_i + h_A*l*l_i + 2*h_i*l_A*l_i)^2;

disp('eq3 =');
disp(eq3);
```

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