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Engineering

Electromagnetic Modelling of a Tokamak Fusion Reactor Magnet System using 2G High-temperature Superconductors

Felix Huber

Department of Electronic and Electrical Engineering

University of Strathclyde

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Supervisor: Prof. Min Zhang

Declaration

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Date: June 14, 2024

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Abstract

The current challenges in reducing carbon emissions and the increasing demand for electric energy necessitate novel solutions for large-scale energy production. Nuclear fusion, a focus of research for nearly a century, has made significant breakthroughs in the past decade. Among these, the commercialization of high-temperature superconductors (HTS) stands out as a promising technology for future commercial fusion reactors. HTS materials, with their high critical current limits, enable high magnetic field operation, surpassing the capabilities of current low-temperature superconductors (LTS). Since the size and power of a fusion reactor scale with the fourth power of the magnetic field, achieving high magnetic fields is crucial for making fusion reactors economically viable.

The Tokamak fusion reactor uses magnetic field coils arranged in a toroidal configuration to confine plasma, where fusion reactions occur. Effective confinement and control of the plasma are essential for generating fusion power, making the magnet system a critical component in Tokamak design. The complex electromagnetic behaviour of HTS necessitates numerical investigations, which are conducted using finite element modelling (FEM). Numerical methods like the **T-A** formulation enable the simulation of complex geometries with numerous turns, essential for producing the required magnetic field strengths in a fusion reactor.

This thesis develops a novel design process for a large-scale fusion reactor magnet system using HTS, evaluated through the **T-A** formulation to assess the feasibility of exploiting HTS materials for large-scale fusion reactors. This evaluation considers achievable magnetic fields, required surface area, and the length of superconducting material. The initial study investigates loss behaviour in HTS, providing advanced insights into reducing hysteresis losses at magnetic fields higher than the full penetration field. Subsequently, an original method is devised to design a fully insulated HTS coil, from a single tape to a complete coil. This approach allows for the design of a coil comprising multiple pancakes and winding packs and optimizing the number of tapes needed to achieve a target magnetic field.

This method is then applied to design the poloidal field coils (PFC) and toroidal field coils (TFC) systems for an ITER-like fusion reactor using HTS tapes. Each magnet system is investigated independently concerning current and magnetic field distribution and losses,

particularly focusing on the identified reduction in losses at high magnetic fields. While the PFC system, consisting of three PFCs, is analysed in 2D, the TFCs require a 3D analysis due to the D-shape of the coil. A novel modelling strategy is developed, utilizing the **T-A** homogenisation advantages in a 3D model. This advanced numerical modelling strategy allows for the investigation of complex 3D geometries, such as a D-shaped coil.

The work presented in this thesis enables the design and electromagnetic analysis of HTS magnet systems for Tokamak fusion reactors. This is achieved through advanced insights into the loss behaviour of HTS materials in high magnetic fields. Additionally, the research introduces an original design process for high-field HTS magnets. Finally, it incorporates a comprehensive 2D and 3D modelling strategy for the electromagnetic investigation of these magnets.

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List of Symbols

Fusion Reactor Physics

n	particle density
T	temperature
τ_E	confinement time
β_N	normalized plasma pressure
q_*	safety factor
H	confinement quality
R	reactor radius
B	magnetic field
P_{fus}	power output
ρ	radius
R_m	mean radius

Superconductivity

T_c	critical temperature
$\mathbf{H}$	magnetic field
H_c	critical magnetic field
$\mathbf{B}$	magnetic flux density
B_p	full penetration magnetic field
$B_{\parallel}$	parallel magnetic field
$B_{\perp}$	perpendicular magnetic field
B_e	external magnetic field
B_i	self-induced magnetic field
B_{target}	target magnetic field
B_{max}	maximum magnetic field
B_{centre}	magnetic field at the centre of the innermost turn of the coil

$\mathbf{A}$	magnetic vector potential
θ	magnetic angle
$\mathbf{E}$	electric field
E_0	characteristic electric field
φ	electric scalar potential
I	transport current
I_{op}	operating current
I_c	critical current
$\mathbf{J}$	current density
J_e	external surface current density
J_c	critical current density
$\mathbf{T}$	current vector potential
Q	dissipated power
γ	electron mass anisotropy ratio
ω	peak sharpness parameter
ct	computational time
$\mathbf{n}$	normal vector
$\mathbf{i}$	normal vector in a cartesian coordinate system
$\boldsymbol{\rho}$	normal vector in a cylindrical coordinate system
$\vec{e}_t$	tangential vector parallel to the winding direction
n	n-value
ρ_{HTS}	resistivity of HTS material
μ_0	magnetic permeability
μ_r	relative permeability
δ	HTS thickness
Λ	unit cell thickness
S	conductor cross-section
∂S	boundary edges of the cross-section

List of Acronyms

ARC	Affordabel Robust Compact
BCS	Bardeen-Cooper-Schrieffer
BSCCO	Bismuth-Strontium-Calcium-Copper-Oxide
CC	Curvilinear Coordinates
CFS	Commonwealth Fusion Systems
CICC	Cable-In-Conduit-Conductor
CP	Cut Plane
CS	Central Solenoid
CT	Compact TokamakTokamak
DOF	Degrees of Freedom
FEM	Finite Element Method
HTS	High Temperature Superconductor
INS	Insulated coil
ITER	International Thermonuclear Experimental Reactor
JAERI	Japan Atomic Energy Research Institute
JET	Joint European Torous
LHD	Large Helical Device
Lhe ₂	Liquid Helium
LN ₂	Liquid Nitrogen
LSCO	Lanthanum-Strontium-Copper-Oxide
LTS	Low Temperature Superconductor
MEMEP	Minimum Electromagnetic Entropy Production
MHD	Magneto HydroDynamics
MIT	Massachusetts Institute of Technology
MMEV	Minimum Magnetic Energy Variation
NHMFL	National High Magnetic Field Laboratory

NI	Non-insulated coil
NIF	National Ignition Facility
PC	Pancake
PDE	Partial Differential Equation
PFC	Poloidal Field Coil
RACC	ROEBLE Assembled Coated Conductor
REBCO	Rare-Earth-Barium-Copper-Oxide
RVB	Resonating Valence Bond
ST	Spherical TokamakTokamak
TFC	Toroidal Field Coil
W7-X	Wendelstein 7-X
WP	Winding Pack
YBCO	Yttrium-Barium-Copper-Oxide

1 Introduction

Energy and electricity demand are critical factors for the economic and social development of every nation. Currently, the world heavily relies on fossil fuels such as coal, oil, and natural gas for over 80% of its total energy, while the rest comes from nuclear, hydropower, and renewable energy sources [1]. However, when looking at global electricity, fossil fuels contribute around two-thirds, while nuclear energy generates as much as all the renewables (except hydropower) combined, accounting for approximately 10% of the global electricity generation. Although this source of energy does not produce greenhouse gas emissions, has a high energy density, and can operate continuously, providing reliable baseload power to the grid, the risks of nuclear catastrophes and the challenge of nuclear waste disposal impede the expansion of nuclear fission.

An alternative to nuclear fission is nuclear fusion, a promising source of energy that has the potential to provide a virtually limitless and clean source of electricity [2]. Unlike nuclear fission, which involves splitting atoms, nuclear fusion involves combining atomic nuclei to release energy. This process, which occurs naturally in stars like the sun, is a highly desirable source of energy because it produces no greenhouse gas emissions and does not generate long-lived nuclear waste. There are several advantages to nuclear fusion as a potential energy source. First and foremost, it is an extremely efficient energy source, with the potential to produce more energy than it requires for the reaction to occur. Additionally, nuclear fusion produces no greenhouse gas emissions or long-lived nuclear waste, making it a highly desirable option for combating climate change and reducing the risks associated with nuclear fission [3]. The primary by-products of nuclear fusion reactions are helium and trace amounts of other light elements, which are not radioactive and do not pose long-term environmental hazards, further enhancing the appeal of nuclear fusion as a clean and sustainable energy source.

The occurrence of nuclear fusion is subject to certain prerequisites, including high temperature in the range of millions of degrees Celsius to overcome the electrostatic repulsion of positively charged nuclei, as well as high pressure to counteract the natural tendency of nuclei to repel each other. The density of the fuel must also be adequate to promote frequent collisions among

atomic nuclei. Additionally, the fuel must remain confined long enough for fusion reactions to occur, which is typically achieved through the application of magnetic fields or other means. Finally, appropriate isotopes, typically hydrogen isotopes like deuterium and tritium, must be available for fusion to occur. Meeting these conditions is necessary for the initiation and maintenance of fusion reactions, which in turn leads to a substantial release of energy. These conditions are expressed as the triple product, which is a measure of the occurrence of fusion reactions in a plasma. The Lawson criteria in equation (1.1) is defined by the triple product of particle density n in [ions/m³], temperature T in [keV] and confinement time τ_E in [s] and gives the minimum value required for energy gain [4].

$$nT\tau_E \sim \frac{\beta_N H}{q^2} R^{1.3} B^3 \quad (1.1)$$

Here, β_N denotes the normalized pressure of the plasma, q is the safety factor, H is the confinement quality, R is the major radius of the reactor and B the magnetic field. The influence of the cubic magnetic field is the driving force in achieving nuclear fusion.

To achieve nuclear fusion, there are four main types of confinement used to contain and control the hot plasma where fusion reactions occur. These include gravitational confinement, magnetic confinement, inertial confinement, and electrostatic confinement. Gravitational confinement is the natural confinement mechanism in stars, where the gravitational force compresses the hydrogen gas until fusion occurs. However, this approach is not feasible for controlled fusion on Earth, as we are not able to control or generate gravitational forces. Magnetic confinement is the most widely used approach for fusion research and involves using strong magnetic fields to confine the plasma in a Tokamak, a stellarator, or a magnetic mirror device. In this method, the magnetic field lines keep the hot plasma away from the walls of the confinement vessel, allowing it to reach the high temperatures and densities needed for fusion to take place [5], [6], [7]. In inertial confinement, laser beams or other high-energy particles are used to heat and compress a small target containing fusion fuel, creating a small, hot, and dense region where fusion reactions occur. This approach is often used in laboratory experiments and can also be used in a potential fusion power plant concept called a "laser fusion" or "inertial fusion" reactor [8], [9]. Electrostatic confinement uses electrostatic fields to confine and accelerate charged particles, such as ions or electrons, towards a target containing fusion fuel [10].

In 1920, Eddington proposed that nuclear fusion reactions were the main energy source powering stars, which was a ground-breaking theory that laid the foundation for the modern field of astrophysics. He demonstrated that the fusion of hydrogen atoms in the core of stars could produce helium, resulting in the release of energy [11]. The underlying physics of nuclear fusion began to be understood by the 1940s when scientists studied the nuclear reactions taking place in stars. In 1946, the Soviet scientist Sakharov proposed using magnetic fields to confine plasma in a fusion reactor, initiating the research in magnetic confinement fusion. Sakharov introduced the concept of magnetic mirrors in 1950, which involves generating a magnetic field along a cylindrical chamber that is stronger at the ends to confine the plasma and prevent its escape. In 1956, Artsimovich published a paper describing the concept of a Tokamak reactor, which uses a toroidal magnetic field to confine plasma in a donut-shaped chamber, and a poloidal magnetic field to create a current in the plasma, heating it to the required temperatures for fusion. In the same decade, the American physicist Spitzer invented the stellarator device [12], which eliminates the need for plasma current by generating a complex three-dimensional magnetic field by using a series of twisted coils. This design has the advantage of a more stable plasma, but in turn, is also more difficult to design and build. In the 1960s the construction and operation of different reactor concepts took place. While the Soviet Union built a series of Tokamak devices, the United States built the first mirror device as well as the first stellarator. In the 1970s the focus of fusion research shifted towards Tokamak devices, which showed greater promise for achieving high plasma temperatures and fusion power output. In the 1980s, the Joint European Torus (JET) Tokamak in the United Kingdom started operating and achieved significant milestones in nuclear fusion over the years, including sustained fusion reactions and plasma confinement, providing crucial data for future fusion reactors [13] [14]. The focus on Tokamak devices continued in the 1990s, with the launch of the International Thermonuclear Experimental Reactor (ITER) project aims at building a Tokamak that can demonstrate the feasibility of commercial fusion power. The Large Helical Device (LHD) stellarator was also built and operated in Japan during this time [15]. In the 2000s, construction of ITER began in France, with notable contributions from JET concerning advanced plasma confinement and control, diagnostic techniques, materials, components testing, tritium handling and fuel cycle technology. Experimental work on magnetic confinement fusion has continued in the 2010s, with new devices being developed and tested. The Wendelstein 7-X (W7-X) in Germany is a successful stellarator-type fusion reactor that has demonstrated high-density plasma confinement necessary for fusion reactions, achieved a record-breaking plasma temperature of 100 million degrees Celsius, and stable plasma operation for up to six minutes,

significantly longer than other fusion experiments [16] [17]. The National Ignition Facility (NIF) at the Lawrence Livermore National Laboratory in the United States is exploring fusion via inertial confinement using the world's largest highest-energy laser and achieved a breakthrough when the energy output of 3.15 MJ exceeded the laser energy of 2.05 MJ [18][19].

To date, the largest project in nuclear fusion research is ITER, which started in 1985 as a joint mission of seven members China, the European Union, India, Japan, Korea, Russia, and the United States of America. The project is intended to prove the viability of nuclear fusion as an energy source. Therefore, the goals of ITER as stated in [20] are:

- 1) Achieve a deuterium-tritium plasma in which the fusion conditions are sustained mostly by internal fusion heating
- 2) Generate 500 MW of fusion power in its plasma for long pulses
- 3) Contribute to the demonstration of the integrated operation of technologies for a fusion power plant
- 4) Test tritium breeding
- 5) Demonstrate the safety characteristics of a fusion device

The complexity of this project pushed a whole range of leading-edge technologies to new levels of performance. Initially, the project was estimated to cost \$6.6 billion, based on the 2001 design [21]. The modifications, such as the addition of vertical stability and Edge Localization Mode (ELM) coils, to the reactor design have been incorporated in the 2010 baseline design, due to advancements in fusion science. Furthermore, the increases in member states from four to seven, and the increase in building construction costs are the main reasons for the dramatic increase in overall project costs to approximately \$20 billion [22]. Due to setbacks in the manufacturing of the first-of-a-kind components and technological challenges, the first plasma for ITER is planned for late 2025 and deuterium-tritium operation will start in 2035 according to ITER announcements. The main goals of the ITER project are to achieve a deuterium-tritium plasma in which the reaction is sustained through internal heating and produces 500 MW of fusion power for pulses of 400 s, as well as test the technology for tritium breeding. The project will also demonstrate the integrated operation of technologies for a fusion power plant and set a baseline for the safety characteristics of a Tokamak reactor.

Fusion research has provided a fundamental understanding of plasma physics and fusion reactions, leading to the development of various fusion device concepts. The accumulated knowledge and technological advancements have laid the foundation for today's private fusion companies, enabling them to pursue innovative approaches for developing commercially viable fusion power plants.

According to the market research of [23], covering more than 15 countries and 14 market players, the global fusion energy market is estimated to be \$429.6 billion in 2030 and is projected to reach \$840.3 billion by 2040. The major companies in today's fusion industry and their approaches are listed in Table 1.1.

While many novel fusion device concepts are explored by these companies, the magnetic confinement method is expected to maintain its market dominance [23] and occupy a larger market share in the fusion energy sector, as it is backed by advancements in experimentation, theory, modelling, and simulation efforts. Furthermore, the magnetic confinement approach offers effective plasma control and confinement using magnetic forces, leading to increased energy output, and contributing to its growth. Tokamak Energy and Commonwealth Fusion Systems (CFS) are among the leading companies advancing the Tokamak approach for magnetic confinement fusion. Tokamak Energy initiated the development of their spherical

Table 1.1: Private companies and their approaches to achieve nuclear fusion

Company	Country	Year	Approach
Lockheed Martin Corporation	United States	1995	Compact Fusion Reactor
TAE Technologies	United States	1998	Advanced beam-driven Field Reversed Configuration
General Fusion	Canada	2002	Magnetized Targeted Fusion
Helion Energy	United States	2009	Magneto-Inertial Fusion
HyperJet Fusion Corporation	United States	2009	Magnetized Targeted Fusion
Kyoto Fusioneering Ltd	Japan	2009	Field Reverse Configuration
Tokamak Energy	United Kingdom	2009	Spherical Tokamak
First Light Fusion	United Kingdom	2011	Fast Ignition Inertial Confinement Fusion
Agni Fusion	India	2015	Hybrid Beam-Targeted Fusion
Zap Energy	United States	2018	Sheared Flow Stabilised Fusion
HB11 Energy Holdings Pty Ltd	Australia	2018	Aneutronic Fusion
Commonwealth Fusion Systems	United States	2018	Compact Tokamak

Tokamak (ST), which is smaller and more compact than traditional Tokamaks, but capable of achieving higher plasma temperatures and fusion power output. The company's approach to fusion involves using high-temperature superconducting (HTS) magnets to create strong magnetic fields that confine the plasma, while also developing new technologies to improve the efficiency and stability of the fusion reaction. The company has built and operated several prototype ST reactors, including the ST-25 [24] and ST-40 reactors [25], which have achieved record-breaking plasma temperatures of 100 million degrees Celsius [26]. Furthermore, Tokamak is currently holding two patents for their innovative magnet technology for the use of stacked HTS cables in TFCs for spherical Tokamaks [27] and partially insulated HTS coils [28]. Commonwealth Fusion Systems (CFS) is a private fusion energy company that was founded in 2018. CFS was spun out of the Massachusetts Institute of Technology (MIT) and is dedicated to developing commercial fusion energy systems based on the ARC (affordable, robust, compact) fusion reactor concept. The company is working on the design of a compact Tokamak (CT) fusion reactor using HTS materials [29]. The VIPER cable [30], which is at the heart of their magnet technology, is designed to enable the creation of stronger magnetic fields with less power consumption and is the enabling technology for the reactor design. It consists of stacked HTS tapes in a twisted copper former around a central cooling channel to enable high magnetic fields and withstand high mechanical, electrical and thermal conditions. Furthermore, the electrical joints with a simple, robust and demountable design as well as the integrated fibre-optic quench detection set a new milestone for HTS cable technology.

While nuclear fusion has been a subject of research for several decades, it has yet to be developed into a commercial power source. With the ITER project under construction and the development of new technologies by private companies the Tokamak concept seems to be the most promising approach, especially if high-temperature superconductors are used [4][31]. The magnet system of a conventional Tokamak consists of three main components, namely the central solenoid (CS), the toroidal field coils (TFCs), and the poloidal field coils (PFCs). The magnetic confinement of the plasma is achieved by a toroidal magnetic field created by the TFCs superimposed with the poloidal magnetic field from the PFCs and the plasma current induced by the CS. The resulting magnetic field lines form a series of nested toroidal surfaces that confine the plasma and prevent it from touching the walls of the vacuum vessel. The plasma is heated to very high temperatures, causing the ions and electrons to become ionized and creating a high-pressure plasma state that allows for fusion reactions to occur. Therefore, the magnet system plays a critical role in the design of such a reactor. With an estimated cost

of around \$18 billion for a Tokamak fusion reactor, the magnet system is estimated to be \$3.8 billion, representing about 21% of the total capital cost [32]. When including the related subsystems of the magnet system, the estimated cost can be between 30% and 60% of the fusion reactor capital cost [33]. The high costs for this type of reactor can be linked to the size which scales with the power output P_{fus} and is strongly influenced by the plasma current and magnetic field strength [34]. Therefore, the size and shape of the reactor must be carefully selected to minimize the required size of the magnet system and determine the magnetic field strength necessary to contain and control the plasma [35].

$$P_{fus} = c_{fus} \frac{\beta_N^2 B^4 R^3}{q^2 A^4} \quad (1.2)$$

Here, c_{fus} is a constant factor for the efficiency of the fusion reaction and A is the aspect ratio of the torus (i.e. ratio of major to minor radius).

The scaling of Tokamak reactors with magnetic field strength is governed by the so-called "B³ - B⁴ scaling law", where the fusion power scales with the fourth power (1.2) and the plasma pressure scales with the third power of the magnetic field (1.1). This means that increasing the magnetic field strength for a given power output can lead to a smaller reactor due to higher achievable plasma pressures. However, magnetic fields also have practical limits in terms of the materials that can be used to generate them, adding to the cost and complexity of the magnet system. Therefore, a trade-off must be made between the desired fusion power output and the cost and size of the reactor, considering the limitations of the available technology. In general, the trend in Tokamak design has been towards stronger magnetic fields, as this allows for higher fusion power output, but with the associated challenges of larger size and higher cost. An example of this trend is the European DEMO reactor, which is intended to demonstrate the commercial feasibility of nuclear fusion producing a net electrical power of 2-4 GW [36][37]. The magnet system for future fusion reactors based on the ITER design concept, such as DEMO, faces several challenges, including higher mechanical loads, long-term availability of magnets at full performance, scalability issues, and cost reduction. The mechanical loads in DEMO are much higher than in ITER, and a robust design is needed to sustain high forces under fatigue load over a long period. To demonstrate the commercial viability of fusion power, the availability of the plant is a key driver, and a robust design is needed to guarantee full performance over several decades. The scalability issue arises due to the industrial and logistic

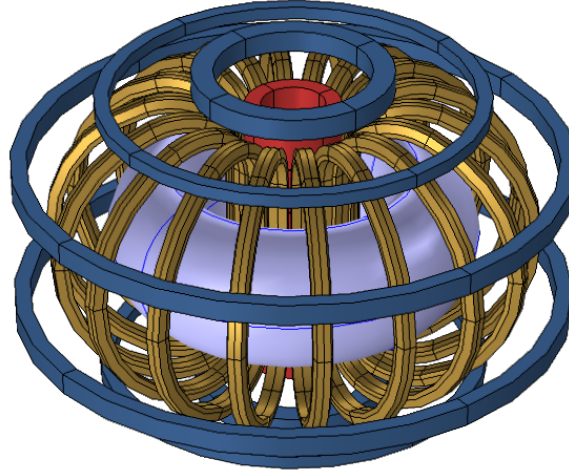
limits which are already being stretched for the ITER coils, and innovative designs and manufacturing approaches will be needed to overcome this challenge.

Reducing the cost of the magnet system is a significant factor in making fusion energy economically viable and a competitive source of energy [38], which can be achieved through improving technology and design, using advanced manufacturing techniques, and international collaboration [39]. The use of advanced high-temperature superconducting materials for the magnet system was found to significantly reduce the size and cost of the magnet system, and thus the overall size and cost of the reactor. HTS magnets can generate magnetic field strengths significantly higher than low-temperature superconducting (LTS) magnets. This can potentially improve plasma confinement and reduce the size of the Tokamak reactor. Additionally, the potential of HTS magnets to operate at higher temperatures can simplify the cooling system design and reduce the need for cryogenic cooling, further reducing the size and cost of the Tokamak reactor. Hence, HTS coils have the potential to significantly improve performance and reduce the cost of Tokamak fusion reactors [24].

1.1 Research objectives

Devices such as ST and CT can take advantage of the high-field capabilities of HTS coils and use a more compact toroidal geometry with a stronger toroidal magnetic field and a weaker poloidal magnetic field, eliminating the need for a CS. This allows for higher plasma pressure and density in ST and CT devices compared to conventional Tokamaks, which can lead to better fusion performance. However, these devices face challenges such as lower energy confinement time and increased plasma instability due to their small size and high magnetic field strength [40][41]. The conventional Tokamak design still offers advantages such as more stable plasma confinement and better fusion performance, but ST and CT have the potential as alternative options for practical fusion energy generation [42].

The design process of the overall fusion reactor geometry and physics is usually done by using system codes that combine engineering, physical, and economic aspects [43], [44], [45], [46]. The scope of these system codes is very wide and goes well beyond reactor physics, including conversion of heat to electricity, buildings, and costs, while some are specifically programmed for the evaluation of plasma physics and magnetic field calculations. Since the focus of the system codes is on the full reactor rather than a specific subsystem, additional modelling



*Figure 1.1: Magnet system geometry of an ITER-like Tokamak
The CS is shown in red, the TFCs in yellow, the PFCs in blue
and the circulating plasma in purple.*

strategies of the magnet system are essential to understand the potential benefits and limitations of using HTS materials in Tokamak reactors, as well as optimizing the design of the magnet system, predicting the performance of the reactor, and identifying any potential issues or challenges that may arise. To tackle these challenges specific to the design of a fusion reactor, numerical approaches to simulate the magnet system for HTS materials must be developed. Finite element software (FEM) is a crucial tool to simulate the electromagnetic behaviour of the superconducting materials, allowing for a more detailed design and analysis of the magnet system.

In this thesis, a modelling strategy for an ITER-like Tokamak is developed. The design is based on ITER, due to the availability of geometrical data and operating conditions. Furthermore, this reactor will be the largest fusion reactor ever built, with a major radius of 6.2 m (radius at the plasma centre) and a height and width of over 20 m. The general design of the magnet system for ITER will be adapted to substitute the LTS materials with HTS to achieve the same target magnetic field at the coils. ITER features 18 toroidal field coils, 6 poloidal field coils and 6 coils in the central solenoid which are shown in Figure 1.1.

Each field coil consists of a configuration of double pancakes, which in turn consist of multiple cable strands. While the TFC double pancakes (PCs) in ITER are made of Nb_3Sn , the PFC PCs are made of NbTi and use different configurations in each PFC due to their non-symmetrical placement and magnetic field requirements. In this study, a symmetrical distribution of the PFCs is assumed to exploit the benefits of symmetry in the numerical model. The target

Table 1.2: Main design parameters of the ITER magnet system

Parameter	Unit	TFC	PFC	CS
Number of coils		18	6	6
Conductor		Nb ₃ Sn	NbTi	Nb ₃ Sn
No. of double PC		7	8 (PF1,3,4,5) 6 (PF2) 9 (PF6)	
No. of turns/coil		134	115-459	549
Operating temperature	K	4.5	4.5	4.5
Max. operating current	kA	68	45	45
Nominal peak field	T	11.8	6	13
Total conductor length	m	82,260	63,142	36,114
Superconducting strands		900	1440, 1152 and 720	

magnetic field of the individual PFCs is taken into account as well as the difference in the available coil area. The number of turns as well as the total conductor length will be evaluated and compared with the adopted HTS design. The main relevant design parameters for the magnet system based on ITER are listed in Table 1.2.

Using HTS enables the operation of the magnet system at higher temperatures, well above the 4.5 K limit for LTS. While HTS tend to be operated at 77 K in electrical machines and power applications, their critical field and critical current density increase with decreasing temperature. This is especially desirable in high-field magnets as the fields can only be generated if the current carrying capabilities are high. In hybrid magnet concepts where LTS magnets generate a background magnetic field and HTS coils are located on the inside of the magnet, the temperature margin is narrow, since the LTS require cooling to liquid helium temperature. Although this enables high-field performance, the required magnetic fields can be achieved by HTS magnets at higher temperatures, which simplifies the needs of the cryogenic system and can reduce the cost of the reactor.

The research on fusion reactor magnets is limited to researchers directly involved in fusion projects, and private companies, which do not share all findings and technologies due to their competitive interest. The implementation of the modelling strategy in the commercial FEM software COMSOL can help to accelerate the development of high-field magnets made from HTS materials for fusion applications since it provides an easy-to-use tool for a wider range of researchers.

1.2 Research contributions

The scope of this study is to develop a modelling strategy to evaluate the performance of an HTS magnet system for a Tokamak fusion reactor and identify whether there are benefits of using HTS materials instead of LTS. The model presented in this thesis, expands the numerical capabilities of existing models used in the simulation of HTS materials. This includes a novel 3D homogenisation strategy using the **T-A** formulation for complex geometries, such as the D-shape of the TFCs. The design and electromagnetic modelling of a simplified toroidal field coil model in 2D is conducted. There, two different models are compared, evaluating the difference between an axial-symmetric approach and a cross-sectional model, which also considers the magnetic field influence of the neighbouring coils. This allows for a representation of the toroidal field distribution in the midplane of the reactor. The current and magnetic field distribution in addition to the losses during a simple operating scenario are evaluated.

Furthermore, the full-scale poloidal field coil system is modelled and the electromagnetic behaviour in terms of the current and magnetic field distribution as well as the losses during different operation scenarios such as ramp-up, AC conditions, fault scenario and ramp down are evaluated.

In addition, a fundamental study on the loss behaviour of HTS-carrying transport current in high external fields is conducted. This study is used as a simplified approach to investigate the loss behaviour that can be observed in high-field magnets with a large diameter that uses stacked HTS tape configurations.

In summary, this thesis presents an electromagnetic model for the individual magnetic field coils of a Tokamak fusion reactor. To ensure its reliability, the model is validated by comparing it with various established modelling strategies (differential and integral), all previously confirmed through experimental investigations. While direct validation of the full magnet models is not feasible within the scope of this study, small-scale experimental investigations would be required to evaluate the accuracy of the approach in real-world applications.

1.3 Structure of the thesis

In this first chapter, the background of nuclear fusion is reviewed. This includes an examination of the different approaches to fusion, a brief history of research conducted on various fusion reactor concepts, and the progress made by private companies in developing economically viable fusion reactors

The second chapter introduces the topic of superconductivity. The basics of superconductivity such as the Meissner effect, London equations, and the classifications of type-I and type-II as well as LTS and HTS conductors are discussed. Then the superconductors for fusion are detailed, including their advantages and drawbacks, currently available materials for high-field magnets and their achievements, as well as novel cable concepts that could accelerate the development of commercial fusion reactors. Then the critical state models of HTS are introduced, namely the Bean model, Kim-Anderson model, and **E-J** power law. The final section of this chapter will review numerical modelling approaches and detail the ones used in this thesis. These approaches include the integral approach based on Brandt's formulation and the **T-A** formulation which is highly advantageous when simulating large-scale applications.

In the third chapter, the modelling approach using the **T-A** formulation is outlined. The formulation is presented in 2D and 3D as well as for advanced modelling strategies such as the homogenisation method. The validation of the modelling approach is based on investigations conducted in the literature. Furthermore, the critical current data used for the numerical investigations in the following chapters are detailed.

The fourth chapter is dedicated to the investigation of the loss phenomenon that occurs when HTS tapes carry a transport current in high magnetic fields. The model is validated using an integral approach and two different numerical methods, namely, **H** and **T-A** formulations. In the first section of this chapter, a single tape is investigated in three operating conditions: 1) when carrying transport current, 2) in a perpendicular magnetic field, and 3) with transport current in a perpendicular magnetic field. In the second part of the chapter, a stack of tapes is investigated under the same conditions. The results of this chapter lay the foundation for the following case studies of the PFC and TFC systems.

In chapter five the PFC system is investigated, which is responsible for the shape and control of the plasma in a fusion reactor. The magnet for each of the symmetrically distributed poloidal

field coils is designed. Then a ramp study is conducted to identify the impact of ramp rate on the total losses of the coils. The final part of this study is to evaluate the current and magnetic field distribution as well as the losses during different operation scenarios for two different tapes and temperatures.

In chapter six the toroidal field coil system is evaluated. A novel approach to model the D-shaped coil in 3D is introduced. These results have been published as part of a research initiative of the Cooperation in Science and Technology. Additional figures showing the 3D magnetic field lines are presented. The case study of the toroidal field coil system will follow the same points as the PFC system. In addition, two modelling strategies for the 2D representation of the toroidal field coil are compared.

In the final chapter, the results and findings of this thesis are discussed and concluded. Future work will be outlined, followed by concluding remarks.

2 Superconductivity background

2.1 Introduction

To understand the impact of superconducting technology in the development of high-field magnets for nuclear fusion reactors, this chapter aims to address the basic principles of superconductivity. Furthermore, the status of superconductors in fusion is investigated, reviewing prospective design options for high-current HTS cables to enable the development of commercial fusion reactors. Due to the high aspect ratio and anisotropic properties of HTS materials, the critical state models to describe the electromagnetic behaviour of these conductors are reviewed, leading up to the different modelling strategies that can be used to investigate their performance and behaviour in various applications. Depending on the complexity and application, integral or differential approaches can be used for the macroscopic investigation of HTS.

The literature review in this chapter provides an overview of the concepts of superconductivity and the use of superconductors in fusion. Furthermore, the modelling methodologies currently available to investigate the behaviour of HTS materials are outlined.

2.2 Basics of superconductivity

In 1911, Onnes discovered the phenomenon of superconductivity while studying the electrical resistance of solid mercury at a temperature of 4.2 K [47]. He observed a sudden and complete drop in electrical resistance when cooling mercury to very low temperatures using the newly developed technique of helium liquefaction. Superconductivity refers to the complete loss of electrical resistance in certain materials when they are cooled below a specific temperature called the critical temperature (Figure 2.1). The critical temperature is unique to each material and can range from a few Kelvin to several hundred Kelvin. When a superconductor is exposed to an electric current, it can generate a persistent current that can flow indefinitely without any noticeable decay.

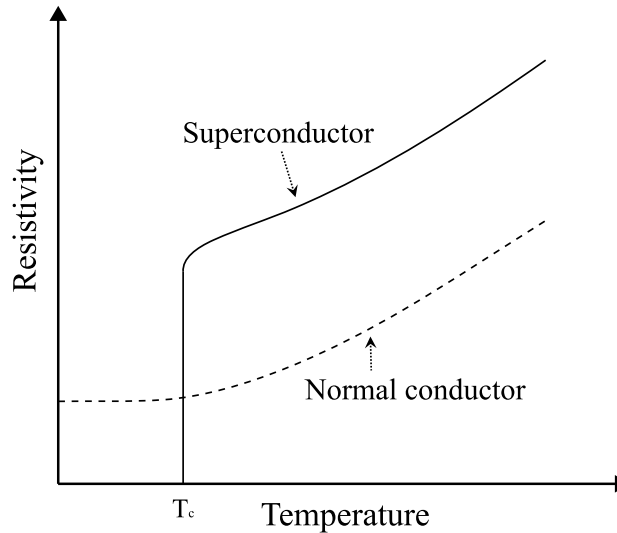


Figure 2.1: Resistivity vs. temperature curve

Shown for superconducting materials and normal conductors

The manifestation of superconductivity in a material is subject to various external physical factors. One of the crucial factors is the critical temperature (T_c), which signifies the temperature at which a superconductor undergoes a transition from a normal state to a superconducting state. Besides temperature, the superconducting state can be compromised if the material is exposed to an external magnetic field that exceeds a specific threshold value (critical magnetic field, H_c) or if the transport current density surpasses a certain threshold value (critical current density, J_c). In practical applications, the critical magnetic field is often expressed by the critical magnetic flux density B_c , which are related by the vacuum permeability $\mu_0 = 4\pi \cdot 10^{-7}$ H/m. In Figure 2.2, the relation between the three critical parameters (T_c , H_c , and J_c) is shown, which set the limits of the superconducting state. [48]

Although zero resistance is a straightforward way to describe superconductivity, it is not sufficient to describe the phenomenon. In 1933, Meissner and Ochsenfeld discovered the "Meissner effect", which describes the phenomenon where superconductors expel external magnetic fields [49]. This effect was observed by cooling pure tin exposed to an external magnetic field. Below the critical temperature, a superconductor exhibits ideal diamagnetism, which leads to the complete expulsion of the magnetic flux from its interior.

The development of quantum mechanics during the 1920s and 1930s provided a deeper understanding of superconductivity. In 1935, the London brothers developed the London equations, marking the first successful attempt to theoretically describe the electrodynamic behaviour of superconductors [50]. The London equations not only explain the Meissner effect,

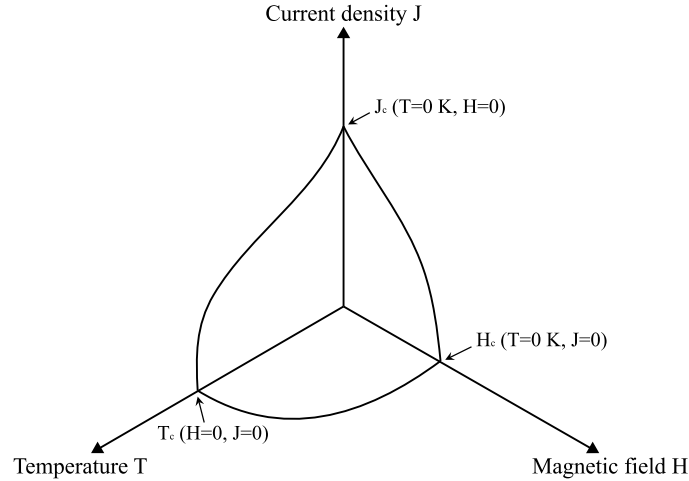


Figure 2.2: Relationship of critical parameters

but they also enable the prediction of the field penetration depth in a superconductor. The London equation demonstrates that the magnetic field and current density are related by an exponential decay from the surface of a superconductor. This relationship is only valid for a thin outer layer of the superconductor that has a depth of penetration known as the London penetration depth. Additionally, the London equation describes that the Meissner effect in superconductors arises due to the minimization of the electromagnetic free energy carried by the superconducting current.

In 1950, the Ginzburg-Landau theory was proposed by Ginzburg and Landau [51]. This theory postulates that the free energy of a superconductor in proximity to the superconducting transition can be represented through a complex order parameter field, which characterizes the behaviour of the superconducting state of a material near the critical temperature. This provides a theoretical framework for understanding the properties of superconductors. Type-I superconductors are characterized by the complete expulsion of magnetic flux within the superconductor during the superconducting state, the Meissner effect. This effect gives rise to a distinct surface boundary of energy, termed the critical magnetic field H_c , which separates the normal and Meissner regions. H_c denotes the upper limit of the magnetic field that type-I superconductors can withstand before becoming resistive. The magnitude of H_c is dependent on the temperature of the superconductor, increasing with decreasing temperature. Upon reaching the critical temperature T_c , H_c diminishes to zero and the superconductor transitions to the normal state. The critical temperature of type-I superconductors typically ranges from 0 K to 10 K.

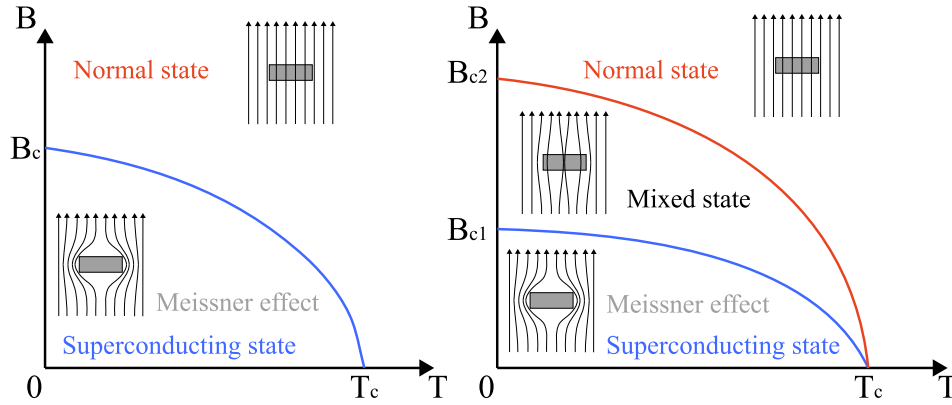


Figure 2.3: State transition of type-I and type-II superconductors

Shown for varying temperature and magnetic field

Figure adapted from [53]

A new class of type-II superconductors was discovered in 1957 by Abrikosov [52], based on the Ginzburg-Landau theory. Type-II superconductors exhibit two surface boundaries of energy, the lower critical field B_{c1} and the upper critical field B_{c2} , separate the Meissner state from the mixed and the normal state, shown in Figure 2.3 [53]. When the external magnetic field is below B_{c1} , the type-II superconductor expels the magnetic field completely from its interior. However, magnetic flux begins to penetrate the material at B_{c1} , unlike type-I superconductors. As the external magnetic field increases, the superconducting region between the cores of the flux lines decreases in size until it disappears completely at B_{c2} , causing the superconductor to transition to the normal state. However, when $B_{c1} < B < B_{c2}$, the type-II superconductor enters the mixed state, characterized by the partial penetration of flux lines into the material.

The partial penetration creates a flux vortex area consisting of one quantum of flux, which forms an hexagonal lattice known as the Abrikosov vortex lattice (Figure 2.4). The flow of current induces a magnetic field that forms a vortex lattice passing through the superconductor. The Lorentz force acting on each flux vortex can cause the vortices to move, generating an electric field and a non-zero resistivity. Therefore, for the practical use of a superconductor with current flow, the flux lattice must be pinned to impede magnetic fluxes from moving. Superconductors with high pinning forces are referred to as "hard" type-II superconductors. Flux pinning is an important determinant of the critical current in type-II superconductors and allows for higher current-carrying capacity at higher external magnetic fields than type-I superconductors because B_{c2} of type-II superconductors can be hundreds of times greater than B_c of type-I superconductors.

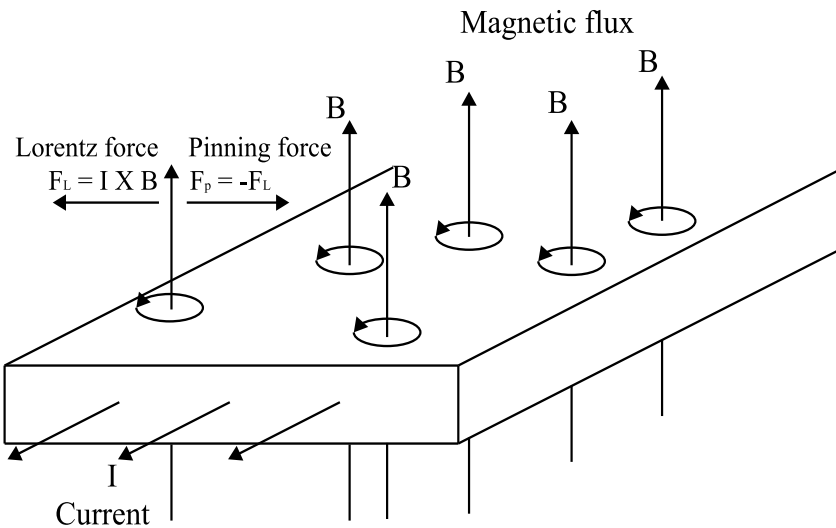


Figure 2.4: Abrikosov vortex lattice of a type-II superconductor in mixed state.

The flux lines partially penetrate the superconductor due to the vortices of persistent current which oppose the surface screening current.

Type-I superconductors are pure elemental superconductors, while type-II superconductors are predominantly metallic compounds and alloys. The critical field of type-I superconductors is below 120 mT, while type-II superconductors at a temperature of 4.2 K have reported upper critical fields of 12 T for NbTi, 27 T for Nb₃Sn, 15 T for MgB₂, and over 100 T for YBCO and BSCCO [54].

The Bardeen-Cooper-Schrieffer (BCS) theory, proposed in 1957 by Bardeen, Cooper and Schrieffer, was the first accepted quantum theory describing superconductivity as a microscopic effect [55], [56]. According to the BCS theory, the electrons in a superconductor interact with the crystal lattice and create an attractive force that binds them together in pairs, known as Cooper pairs. The formation of Cooper pairs is enabled by the exchange of lattice vibrations, or phonons, between the electrons. When the temperature of a superconductor drops below a critical temperature, the Cooper pairs condense into a single quantum mechanical state, known as the superconducting state. In this state, the Cooper pairs can move through the superconductor without experiencing any resistance or energy loss. The theory explains superconductivity of most elemental superconductors and simple alloys, such as NbTi and Nb₃Sn, at low temperatures, classifying them as low-temperature superconductors.

In 1986, the discovery of high-temperature superconductors revolutionized the field of superconductivity [57]. Copper oxides, such as lanthanum-strontium-copper-oxide (LSCO), bismuth-strontium-calcium-copper-oxide (BSCCO), and rare-earth-barium-copper-oxide

(REBCO, where rare earth materials such as yttrium, gadolinium, europium, or neodymium can be substituted), belong to this category of materials, with critical temperatures much higher than LTS materials, with values ranging up to 110 K for BSCCO. YBCO, a second-generation HTS, was discovered in 1986 and 1987 [58] and has a critical temperature above 77 K, which is the boiling point of liquid nitrogen (LN_2). This characteristic makes it attractive for practical applications, as it is less expensive to cool than LTS using liquid helium (LHe_2). In 1993, the highest T_c value observed to date was reported, with superconductivity in Hg-1223 occurring at around 133 K under ambient pressure [59]. An evolution of the discovery of superconductors is shown in Figure 2.5 [60].

It should be noted that despite significant research progress, the origin of high-temperature superconductivity remains uncertain. There are two theories that are widely accepted by physicists: the resonating valence bond (RVB) theory and the spin fluctuation theory. The RVB theory [61], which was proposed in 1987, suggests that electrons from neighbouring copper atoms in copper oxide lattices can form valence bonds that act as mobile Cooper pairs with doping. On the other hand, the spin fluctuation theory [62] was proposed in 1991 and provides a good explanation for the high critical temperatures and energy-gap behaviours of cuprate superconductors such as YBCO.

YBCO is widely utilized in commercial coated conductors because of its distinctive crystal structure that comprises yttrium atoms located between CuO_4 planes and barium atoms

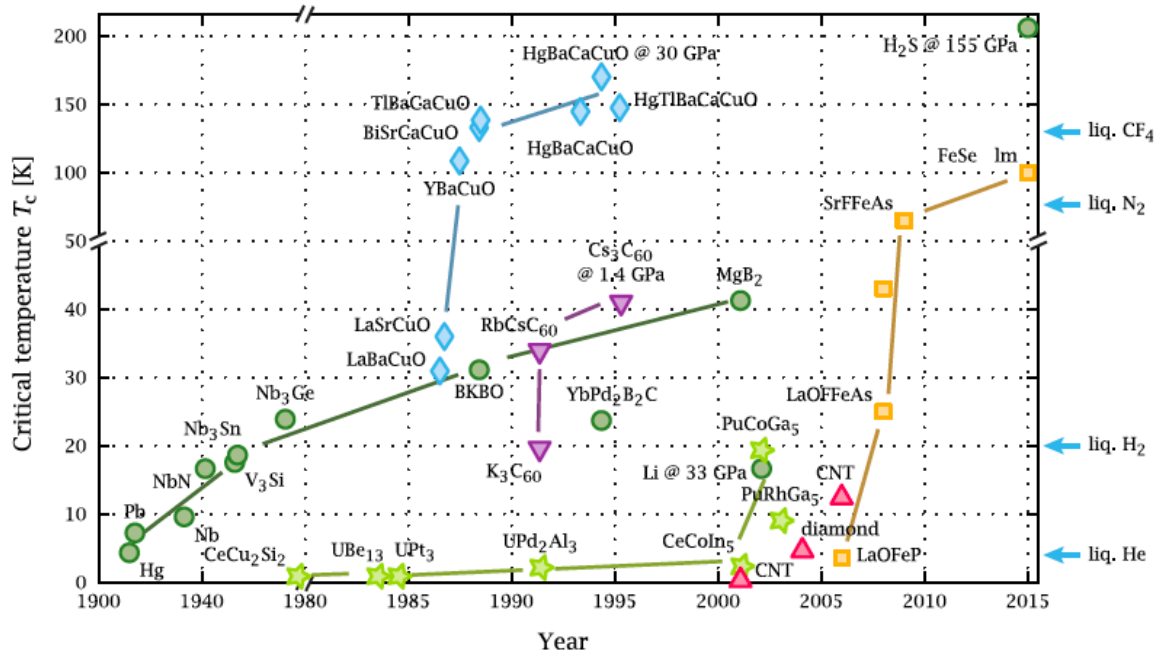


Figure 2.5: Timeline of the discovery of superconducting materials [60]

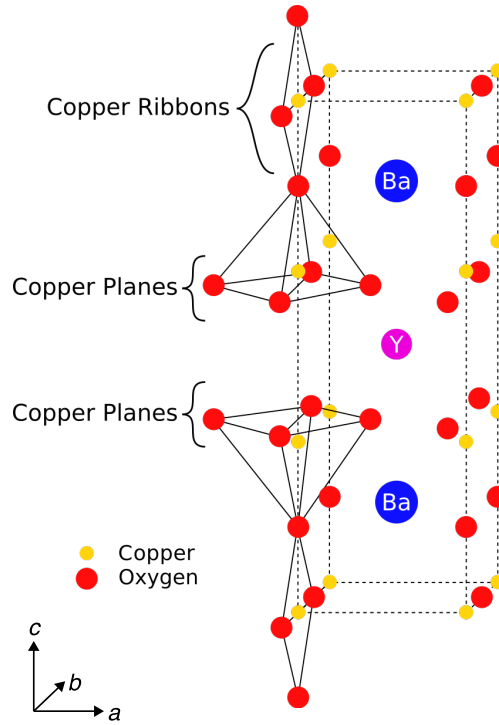


Figure 2.6: Part of the lattice structure of YBCO [63]

positioned between CuO_2 ribbons and CuO_4 planes (Figure 2.6 [63]). This structure results in a notable difference in superconducting and non-superconducting properties due to lower electrical resistivity within the CuO_4 planes in comparison to the CuO_2 ribbons. The smaller ionic radius of yttrium (Y) in contrast to other rare earth elements like gadolinium (Gd) or europium (Eu) contributes to a higher charge carrier density in the superconducting CuO_4 planes. Moreover, YBCO displays a lower electronic anisotropy and a higher irreversibility field than GdBCO and EuBCO, making it a favourable option for the production of commercial HTS-coated conductors. The increasing availability of YBCO has allowed their use in large-scale high-field magnets.

2.3 Superconductors for fusion

For a Tokamak fusion reactor, superconductors are favoured over conventional conductors, which produce considerable resistive losses, and therefore would only be suitable for short-pulsed operation and be subject to continuous maintenance and repairs [24] [64]. Maintenance and repairs of conventional conductors would primarily involve addressing issues such as overheating, material degradation, and electrical insulation breakdown, which can occur due to the high heat generated by resistive losses, necessitating frequent intervention to maintain

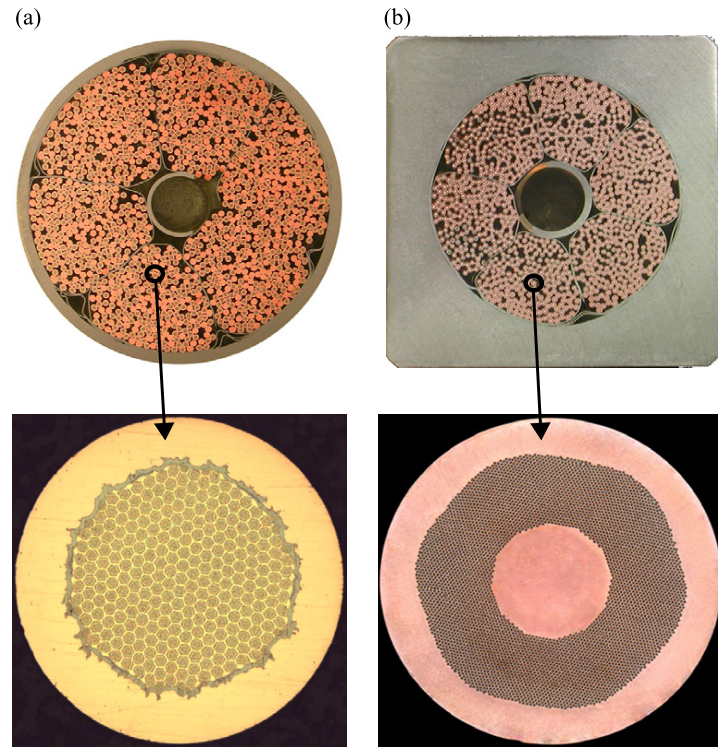


Figure 2.7: Cross section of a CICC

(a) Nb₃Sn and (b) NbTi [69]

operational efficiency and prevent system failure. The use of superconductors in Tokamak reactors, due to their capacity to carry high currents without resistance, significantly diminishes the need for maintenance and repairs when compared to conventional conductors, consequently improving operational efficiency and longevity. The desirable properties for superconductors include high critical temperature, high critical magnetic field, high current density, high mechanical strength, high thermal conductivity, high radiation resistance, low cost, and low activation properties [40].

Tokamak reactors currently under construction are conventionally designed using low-temperature superconductors for the magnet system [65]. Presently, in large experimental fusion reactors, high magnetic fields are produced for plasma control by using LTS such as Nb₃Sn [66], NbTi [67], and Nb₃Al [68]. Coils made from these materials are known to produce high magnetic fields and demonstrate high reliability, albeit with the disadvantage of being heavy and bulky. LTS materials require cooling with liquid helium to operate, which adds to the complexity and cost of the system. Over the past two decades, despite various design challenges, all fusion conductors have converged on the Cable-In-Conduit Conductor (CICC) layout [69] shown in Figure 2.7, which involves the winding and insulation of superconducting

elements within a structural conduit that also functions as the coolant channel for fusion devices [70].

The CICC design has several advantages, including flexibility in conductor design, effective cooling, large heat removal capabilities, strong mechanical function, and high voltage insulation. The flexibility of the design allows for variations in the number of strands, cabling patterns, and conductor shape. Cooling is achieved through the circulation of supercritical helium in forced flow conditions, which is particularly effective for large magnets with high stored energies. The CICC conductors have high heat exchange coefficients, allowing them to absorb large heat loads due to AC losses in varying regimes and nuclear heating. The fluid containment structure (jacket) has a strong mechanical function and can be adapted to the specific expected stress level. Electrical insulation can be applied externally, allowing for high voltage insulation levels without affecting the heat exchange efficiency between the coolant and current-carrying elements. The main drawback of the CICC design is its relatively low engineering current density, which is due to the limited fraction of the overall conductor cross-section occupied by the superconducting strands and the low critical current of LTS. The rest of the cross-section is taken up by materials such as copper, helium, and structural elements, which limits the maximum current that can be carried and hence the magnetic field strength that can be achieved [71]. Additionally, the large size of the ITER CICC is identified as the intrinsic reason for transverse load degradation and self-field-induced quenching in NbTi [70]. Traditional low-temperature superconductors such as NbTi, Nb₃Sn, and Nb₃Al encounter significant reductions in their current carrying capacity when exposed to high magnetic fields. Furthermore, these materials are unable to withstand magnetic fields exceeding 20 T, surpassing the limits of current low-temperature superconductors.

Compared to LTS, HTS offer advantages such as a high magnetic field, high cryogenic temperature, high current densities, and larger tensile stress. HTS materials have the potential to significantly relax the design restrictions placed on LTS materials for Tokamak reactor applications, such as survival of insulation, stabiliser and superconductor under irradiation, and removal of nuclear heating [54], [72]. The use of HTS allows for an increase in an on-axis magnetic field, leading to higher energy gain and power density in smaller devices, accelerating fusion development. Additionally, high-field compact devices operate far from all intrinsic disruptive limits and use normalized plasma regimes already demonstrated in present devices. These advantages result in smaller burning plasma experiments, improved performance,

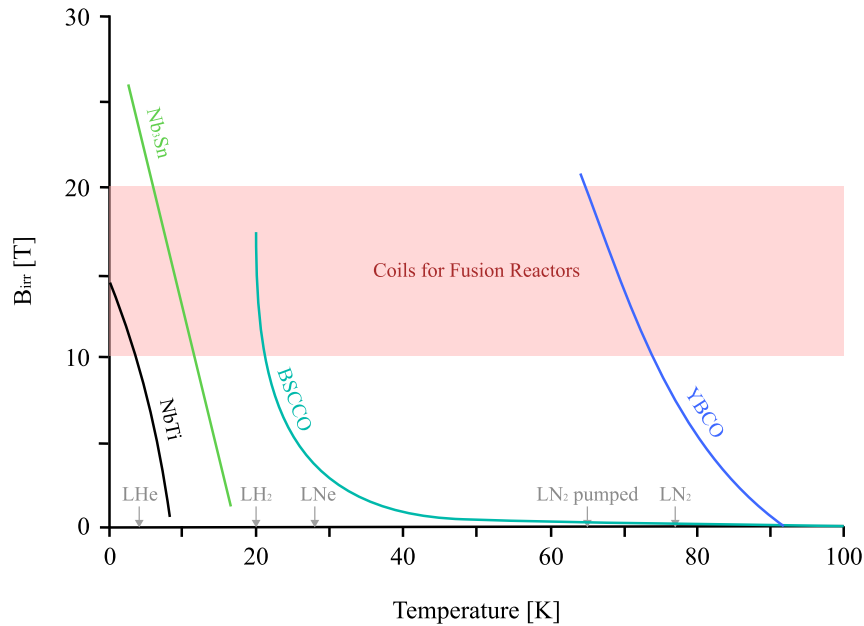


Figure 2.8: Irreversible field as a function of temperature for superconducting materials.

Figure adapted from [54], [72]

operational robustness, and steady-state physics. Analysis suggests that high-gain, robust steady-state operation will arise from the combination of high-field, small size, high safety factor, and associated improvements in the current drive enabled by high magnetic fields [73].

High-temperature superconducting coils offer several potential benefits, including the effect of screening current at a macroscopic level. However, REBCO magnets have a screening current-induced field that is five times larger than that of BSCCO magnets, requiring advancements in the REBCO conductor to suppress this effect. Reinforced Bi-2223 conductors are relatively easy to handle, and their effect on screening current is moderate, although their strengths and current densities are sometimes insufficient. Bi-2223 magnets for NMR, MRI, magnetic levitation, and other applications have been successfully developed. REBCO conductors have a high axial tensile strength of over 700 MPa, allowing for a high overall current density of over 300 A/mm². Recent advances in the heat treatment process for Bi-2212 conductors and the enhancement of mechanical strength to 500 MPa for Bi-2223 conductors will enable their use for high-field and high-current density HTS magnets. The best mix of Bi-2223, Bi-2212, and REBCO for high-field magnet applications will be pursued in the future. [74]

However, HTS coils have some disadvantages, including degradation in coil performance due to factors like mechanical stress, thermal cycling, electromagnetic forces, and material

imperfections, as well as issues related to excessive screening currents and difficulty in coil protection. In HTS coils, protection strategies are crucial to mitigate the risks of thermal runaway and quench events. Thermal runaway occurs when the coil temperature exceeds its critical point, leading to loss of superconductivity, while quenching results from factors like excessive current or magnetic field disturbances, necessitating the implementation of effective cooling systems and quench detection mechanisms to maintain coil stability and prevent damage. For the use of HTS in high-field magnets two options are available: insulated and non-insulated coils.

Non-insulated (NI) HTS coils have garnered significant interest due to their superior performance compared to traditional insulated (INS) coils [75]. The direct turn-to-turn contact in NI coils allows current to bypass local quench zones, preventing further damage and improving the overall thermal stability of the coil. Additionally, the simplified protection method of NI coils reduces the need for complex insulation systems, making them lighter and simpler in design. However, NI coils have distinct electrical characteristics, including different charging and discharging behaviours, and their magnetic field may not be directly proportional to the operating current. These differences require careful monitoring and a tailored protection algorithm to ensure safe operation.

In contrast, insulated HTS coils provide electrical isolation that can reduce nonlinearity and saturation tendencies, contributing to increased durability and longevity. However, the added insulation introduces weight and complexity, and the insulation material itself can generate heat, which may complicate cooling efforts. Thus, while NI coils offer improved thermal stability and simplified protection, they require specific management strategies to address their unique electrical and thermal behaviours. In this thesis only insulated HTS coils will be investigated as the study focuses on the electromagnetic behaviour of large-scale electromagnetic coils and does not consider thermal effects.

Despite the progress in the development of BSCCO materials, REBCO, specifically, YBCO is the only feasible HTS material that can be used in the field range of 10-20 T, required in a Tokamak fusion reactor (Figure 2.8) [76]. REBCO has several capabilities that make it a promising material for fusion applications. These include the ability to operate at higher temperatures, making it more cryo-stable and allowing for greater temperature margins, which eliminates the need for costly annealing procedures, and simplifies testing and installation. It is also easy to manufacture cables and joints with REBCO tapes, as no high-temperature

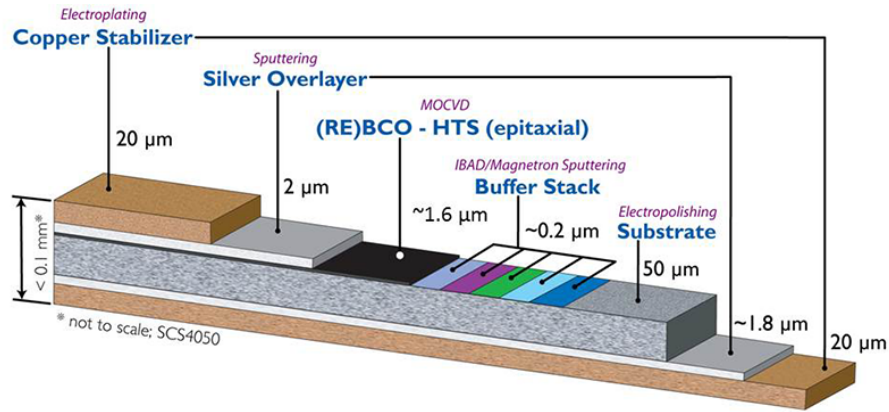


Figure 2.9: Architecture of a standard SuperPower 2G HTS tape

Figure taken from [78]

activation heat treatment is required. Furthermore, due to the well-understood electromagnetic and thermal properties of high-temperature superconductors, along with advancements in manufacturing techniques and quality control processes, HTS cable and joint performance is reliably predictable, a feature that is not always characteristic of other conductor types. REBCO also has excellent cyclic performance, particularly when designed in a monolithic conductor configuration. Finally, the use of demountable joints and magnets [77] improves the reliability, availability, maintainability, and inspectability of the magnets in Tokamak fusion devices [38].

A standard high-temperature superconducting tape (Figure 2.9) comprises multiple distinct layers with specific functionalities [78]. The central layer consists of high-quality superconducting material, such as YBCO. This layer is typically deposited on a buffered metallic substrate, which provides mechanical support and acts as a barrier against thermal and chemical reactions. Surrounding the superconducting layer are intermediate layers, which act as diffusion barriers and enhance the epitaxial growth of the superconducting layer. Finally, a protective layer, often composed of metals like silver or copper, shields the superconducting layer from external influences while maintaining good electrical conductivity. The precise composition and arrangement of these layers in an HTS tape contribute to its overall performance and functionality depending on the application.

While REBCO tapes offer high critical currents and good mechanical properties when operated below 20 K, high-field HTS magnets still require strain tolerance and management and have mechanical limitations. The use of strong substrates and situating the REBCO layer on the neutral axis or even in compression can help to minimize strain contributions. Another issue regarding HTS is quench protection and an understanding of the time evolution of voltages and

temperatures during a quench. Effective quench detection schemes and protective responses must be implemented quickly to prevent damage to the magnet. The behaviour of HTS magnets is quantitatively different from LTS magnets [79] since the detection of a quench is slow and complicated by high electromagnetic noise and complex coupled inductances. This may require new technologies such as fibre optics [80] and acoustic sensors. The radiation resistance of HTS is also noted as a potential challenge in fusion reactor magnets [73]. The effect of neutron irradiation was studied in [81] and it was found that current REBCO materials can only be used in a reactor at lower temperatures. In addition, AC losses still represent a significant challenge for REBCO coils, and optimizing conductor geometries and cable stabilisers will likely be required.

The economic challenges for the use of REBCO tapes in large-scale magnets include tape cost reduction and production. While the performance of present-day REBCO tape is sufficient for next-generation fusion devices, cost reduction is critical for widespread commercial application. Scaling up production volumes is expected to drive prices down, but this will require a shift in R&D emphasis towards novel volume production technologies. The cost of HTS materials for fusion is currently higher than conventional LTS such as Nb₃Sn, and NbTi compared to REBCO tapes as shown in Table 2.1. Although improvements in fabrication processes and increasing current carrying capabilities may bring down the cost, a market for fusion devices is needed to achieve significant cost reductions. The cost of HTS magnets is estimated to be double that of conventional magnets, making it difficult for MCF devices to compete with other energy sources. However, the use of HTS can be economically justified in specific cases such as HTS current leads, which pay back the high capital cost with large

Table 2.1: Reported commercial superconductor specifications as presented in [53]

Material	Unit cost	Critical temperature	Critical current	Manufacturer
REBCO (12 mm)	~227 \$/(kAm)	up to 119 K	400-600 A, 77 K	SuperPower
Bi-2223	17.4 \$/(kAm)	110 K	170-200 A, 77 K	SEI
NbTi	0.8 \$/(kAm)	9.5 K	Up to 3 kA, 4.2 K	SuperCon Inc.
Nb ₃ Sn	<5 \$/(kAm)	18.3 K	Up to 2 kA, 4.2 K	SuperCon Inc.
MgB ₂	20 \$/(kAm)	39 K	~157 A, 4.2 K	
Copper	11.5 \$/kg	-	-	

savings in cooling power during operation. Incorporating a grading of conductors or rotating HTS tapes can help minimize the necessary HTS volume and reduce costs.

Another drawback is the availability of HTS in long lengths. However, extremely high current density superconducting wires and large volume production is being developed [82]. Research has been conducted to optimize the processing conditions for manufacturing 2G HTS wires using hastelloy substrates, which offer improved engineering current density and strong mechanical support [83]. It has also been found that increasing the wire's absolute critical current by growing thicker HTS layers can result in an approximately 70% increase in engineering current density in commercially produced SuperOx 2G HTS wire [84]. The projected annual production capacity of pilot-scale facilities is approximately 100 km of 12 mm wide tape. Physical and chemical approaches can be combined to simplify the tape architecture, reduce the materials cost, and increase the material utilization and processing yield, which can further improve the production of HTS tapes [85].

To realize the potential impact of HTS magnets on fusion devices and their commercialization to target climate change, these innovations need to be scaled to high fields and large bores. The mass production of high-field magnets is a future challenge that will require new materials, fabrication, integration, and testing methods [38].

Despite these challenges, high-field magnets using HTS conductors have shown impressive advancements in magnetic field strength. In China, a 25 T all-superconducting magnet is being developed using YBCO inner coils and LTS outer coils [74]. In the United States, the National High Magnetic Field Laboratory (NHMFL) is engaged in several HTS high-field magnet projects, including the construction of a 7 T Bi2212 demonstration insert magnet for 25 T and an all-superconducting 32 T research magnet using 17 T REBCO insert and 15 T LTS outsert [86]. In another magnet at the NHMFL, one of the coils has repeatedly reached 35.4 T [87] using a single 100 m length of REBCO tape wet wound with epoxy and nested in a 31 T background magnet. Furthermore, a hybrid magnet, which combines the resistive insert and 14 T by the superconducting outsert, can achieve a field strength of 45 T [88]. The NOUGAT project aims to build a 10 T REBCO magnet operating inside a 20 T resistive background which has already achieved 19 T [89]. Similarly, the design and performance estimation of a 35 T, 40 mm no-insulation REBCO magnet has been proposed [90]. In addition, MIT and HTS tape manufacturer SuNAM have collaborated to design and build a GdBCO multi-width no-insulation superconducting magnet, which has achieved a field strength of 26 T [91].

These projects are supported by a broader R&D effort to address magnet technology issues and indicate that 30 T all-superconducting magnets are viable and strengthen the conviction to pursue further research in this area. The development of high-field magnets using HTS conductors is a promising avenue for creating stronger and more efficient magnetic fields, which can have a significant impact on scientific research and various industrial applications [92].

High-field solenoids typically take advantage of the thin flat geometry of HTS tapes, which enables a simple coil winding process, where the tapes are first wound into pancakes. These single pancakes can then be stacked on top of each other during the assembly process. Due to the comparably small scale of high-field solenoids where the inner diameter is one order of magnitude lower than in large-scale magnets for fusion reactors, the cooling of the superconductors is achieved by submerging the magnet in cryogenic liquid. As the size of the magnet increases, cooling via forced flow becomes more cost-effective and requires the development of novel HTS cable concepts.

Superconducting cables have become a vital component in the development of high-field magnets for fusion applications. Various cable technologies have been developed to enhance the current carrying capacity of the cables while simultaneously minimizing losses (Figure 2.10). The design and construction of a 60 kA prototype cable made of coated conductors have been developed for fusion magnets [93], and a 100 kA HTS conductor was produced for joint sections [94]. The CICC design was studied using stacks of HTS tapes [95], and a twisted stacked-tape configuration was proposed for the full industrial manufacturing of a CICC using 150 HTS tapes [96]. The HTS-coated conductor technology has been used to design HTS fusion conductors for TF and CS coils. Based on this technology, a flat Rutherford cable design has been proposed using the strands made of a twisted stack of tapes soldered into copper profiles. Design options have been developed for both TFCs operating at 63 kA and CS coils operating at 50 kA [97]. The ROEBEL assembled coated conductor (RACC) cable concept opens the possibility for low AC loss cables with a current carrying capacity of several kA, which is required for the application in devices like transformers, motors/generators, and magnets and future use in the next generation of superconducting magnets for accelerators and fusion reactors. However, a twisted filamentary structure of the YBCO layer of the strands for reduced hysteresis losses is still an unsolved challenge [98]. The VIPER cable, which is known for its high mechanical strength, cryo-stability, and rapid quench detection was developed for

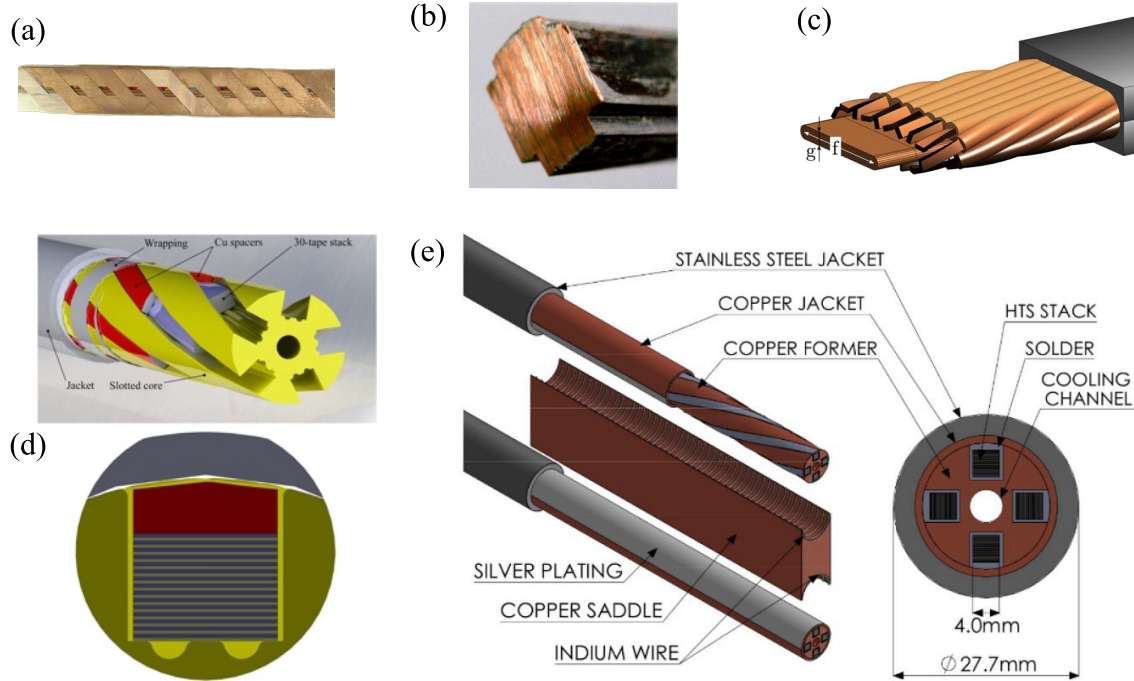


Figure 2.10: HTS cable concepts

(a) Rutherford cable [97] (b) CroCo cable [100] (c) RACC [98] (d) Twisted stacked CICC [96] (e) VIPER cable [30]

the SPARC reactor. The performance of the VIPER cable is predictable and repeatable, allowing for a high-confidence design. Its manufacturing process is simple and scalable to long lengths, and the simple, demountable joints allow for repairability, accessibility, inspectability, and maintenance required in real-world applications [30]. HTS cables for DEMO TFC magnets have been developed focusing on the novel CroCo layout [99]. The CroCo cable is a stacked HTS conductor optimized for high currents and long-length production. The HTS CroCo is a promising candidate as a strand for long-length high-current cables, for example, with a critical current of approximately 30 kA at 4.2 K and self-field [100]. With the future REBCO development, this cable layout can be optimized to higher operating temperatures, lower costs by using fewer REBCO tapes, or even achieve higher magnetic fields at the plasma centre for future fusion devices. A characterization of HTS cables for TFC magnets in the DEMO reactor [101] demonstrated the feasibility of using non-twisted REBCO-coated conductor cables as a component in ultra-high-field magnetic confinement devices. The electromechanical behaviour of REBCO-coated conductor TFCs for ultra-high-field magnetic-confinement plasma devices was studied in [102].

Since the TFCs are a crucial component in fusion power plants, researchers have explored the use of HTS in TFCs as an alternative to traditional materials like Nb_3Sn , to achieve higher

efficiency. Recent studies have shown that HTS materials are feasible for TFCs and offer several benefits over traditional materials. One study focused on the conceptual design improvement of a TFC for the next-generation fusion power plant, EU DEMO, using HTS material [103]. The study investigated the electromagnetic, structural mechanics, cooling, and thermohydraulic properties of the HTS TFC, including operation current, magnetic field, Lorentz forces, operation temperature, structural requirements, cooling, and quench performance. The study found that the HTS winding pack fits in the given winding pack area and can produce the required magnetic field at the plasma axis, with a peak magnetic field at the superconductor of 13.27 T. The use of HTS can increase the temperature margin to 11.9 K, providing enough margin for possible degradation during conductor and coil manufacturing. The temperature cannot be further increased due to LTS materials being used as well, therefore limiting the maximum temperature of the cryogenic system. Another study investigated the electromagnetic design of 12 D-shaped coils for a toroidal-type superconducting magnet. The magnet was operated at 251 A and 20 K and showed that the critical current of the magnet is determined by the upper and lower arc segments near the linear segment of the D-shaped coils. The leakage magnetic field of the toroidal-type magnet was weak and could be ignored [104]. In a third study, researchers aimed to develop a conceptual design for the TFC for future power plants using the PROCESS system code. From the input, the winding pack of the TF coil was designed, with options for pancake winding or layer winding. The magnetic field at the plasma axis was evaluated to ensure it was equal to the required magnetic field, and the peak magnetic field was also calculated to define the operating point of the conductor [105].

For ST power plants the TFC system is also crucial to the plant's design and performance due to its high electrical power requirements. This is mainly due to the centre rod conductor, which dominates the power consumption. The low aspect ratio and high TF coil current, combined with the effects of neutron irradiation on the structures, create challenges in choosing materials and dimensions for the centre column. To reduce resistive power dissipation in the coils, the electrical conductivity of metals is lowered by decreasing temperatures. The optimum temperature for a water-cooled copper centre conductor appears to be around room temperature, while the optimum temperature for a helium-cooled aluminium conductor is around 30 K. Two different TFC designs are being developed for use in the centre column of ST power plants [106]. The first uses a dispersion-strengthened copper alloy for the centre rod and minimal shielding, allowing for a simple design. The second option uses pure aluminium

and requires significant shielding and more complex structural supports, which offers advantages in terms of power consumption.

In the case of a low aspect ratio Tokamak fusion power reactor, such as the VECTOR designed by the Japan Atomic Energy Research Institute (JAERI), HTS YBCO tape is considered for use in the TFC conductor. YBCO tape has a higher critical current density but also has magnetic instabilities due to flux jumps. To prevent flux jumps, the winding is divided into three winding packs, with the maximum self-magnetic field produced by each pack being less than 6.8 T. The winding packs are cooled in a specific order to maintain the magnetic stability of the system, allowing for the generation of 20 T without a flux jump. The nominal current for every winding pack is 40 kA. By using YBCO tape, it is possible to realize low construction and operation costs and high-performance plasma in Tokamak fusion power reactors [107]

Private fusion companies have successfully secured substantial private capital investments, empowering them to pursue ambitious timelines in response to the pressing climate crisis. This substantial financial backing enables them to accelerate the advancement of cutting-edge fusion technologies. As a result, a novel dynamic is emerging in the fusion landscape, where governmental organizations concentrate on fundamental scientific research, while private companies embrace the responsibility of tackling commercial fusion technology obstacles. Notably, this relationship is further strengthened by the introduction of new government initiatives. These initiatives either offer direct support to private fusion companies or promote innovative fusion concepts that are backed by private firms, fostering collaboration between the public and private sectors in this critical field [38]

2.4 Critical state models

To design superconducting devices, various models have been developed, including analytical and numerical methods. Analytical methods are typically used for models with simple geometries and provide solutions for basic electromagnetic behaviour. To study the performance of superconducting devices, it is crucial to know the electromagnetic characteristics of the adopted superconductors. The most classical and basic model to describe the current density and magnetic flux density in HTS was introduced by Bean, but many researchers have extended it to a more precise level, such as the model proposed by Rhyner and Kim. These critical state models are used to simulate the macroscopic behaviour of superconducting materials and simplify the distribution of magnetic field and current density.

They are based on quasi-static conditions, and most of the critical state models separate the model into two parts: the critical state region and the virgin region. The critical state always appears at the outer layer of the material when an applied field exceeds the lower critical magnetic field B_{c1} of a type-II superconductor. At this point, magnetic flux vortices with circulating superconducting shielding currents penetrate the material to shield the interior from the applied current or field. The following sections aim to illustrate these fundamental models.

2.4.1 Bean model

Bean first introduced Bean's critical-state model in 1962, which provides a macroscopic explanation of the hysteretic magnetization phenomenon in type-II superconductors [108], [109]. The model simplifies the distribution of the magnetic field and current density in the superconducting strip, making it easier to analyse the magnetic field, current density, penetrated magnetic flux, and losses. According to the Bean model, the HTS carrying transport current or under the influence of an external magnetic field can be divided into the current carrying and current free region. While the current density in the current carrying region is assumed to be in the critical state, where the critical current density takes on the values of $\pm J_c$. In the region free of current, the current density is considered to be zero [110].

This can be illustrated when considering an infinite slab with a thickness of $2a$ in a uniform magnetic field in a parallel direction. Taking a cross-section of the slab, the magnetic flux density and current density distributions along the thickness are demonstrated in Figure 2.11 for a slab in an external magnetic field. In Figure 2.12 the behaviour for a slab carrying

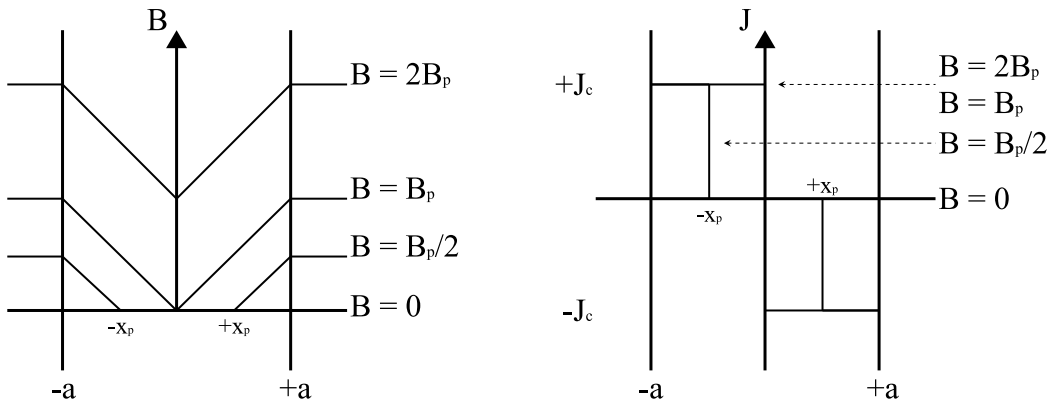


Figure 2.11: Magnetic field (left) and current (right) distribution for an HTS slab in an external magnetic field

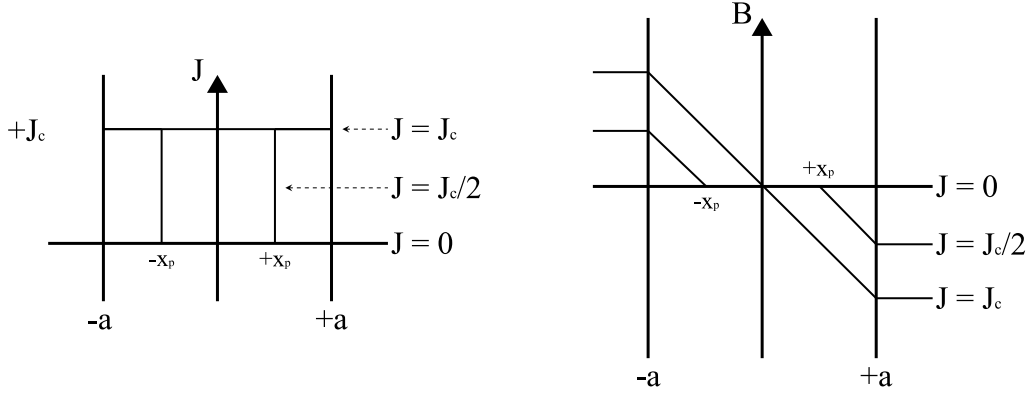


Figure 2.12: Current distribution (left) and magnetic field (right) distribution for an HTS slab carrying transport current

transport current in the absence of an external magnetic field is shown. This model assumes that the variation of electric field $\mathbf{E}$ induces a current density $\mathbf{J}$, with a value assumed to be constant, J_c . The magnetic field distribution inside the superconductor can be obtained by solving Ampere's Law, where the magnetic field inside the slab declines with the penetration depth x_p . The external magnetic field penetrates the material from the edge of the material, and the penetration depth is dependent on the magnitude of the magnetic field and the critical current density of the HTS. The full penetration magnetic field B_p , is determined by $B_p = \mu_0 J_c d$. When the external field exceeds B_p , the current density reaches its maximum threshold and remains constant at $\pm J_c$.

If instead of an external magnetic field, the HTS is carrying transport current, the respective current and magnetic field distribution is shown in Figure 2.12.

The Bean model is a useful tool to qualitatively analyse the electromagnetic properties of HTS due to its ability to simplify loss calculations by introducing a domain-like structure into the current density. However, this model has limitations since it assumes that the current density is limited by the constant critical current density throughout the superconductor. Despite this, the simplified domain-like structure introduced by the Bean model seems to be retained in modified models, which allow for J_c to depend on the magnetic field. However, it is difficult to define the boundary between the critical and subcritical currents, which is challenging to apply the unknown boundary condition of Bean's critical-state model to complex systems [111].

2.4.2 Kim-Anderson model

The Kim-Anderson model has been proposed to describe the electromagnetic distributions in type-II superconductors more accurately. The Kim-Anderson critical-state model was proposed in 1962 by Kim, Hempstead, and Strnad to study the critical phenomenon of type-II superconductors [112], [113]. This modified version of Bean's critical-state model incorporates the concept of a local variable critical current density J_c that is dependent on both the external magnetic field and surrounding temperature. The critical current density represents the maximum current carrying capacity of a superconductor and is influenced by the inherent properties of the superconducting material, such as the microstructure properties. In type-II superconductors, the critical currents can vary with the local magnetic field and ambient temperature. The Kim-Anderson model [114] characterizes the field and temperature dependency of the critical current density, making it a more realistic representation of superconducting behaviour.

$$J_c(\mathbf{B}, T) = \frac{J_{c0}(T)}{1 + \frac{|\mathbf{B}|}{B_0}} \quad (2.1)$$

Here, $J_{c0}(T)$ represents the critical current density in self-field at a temperature T . The local magnetic field is represented by $\mathbf{B}$, and B_0 denotes the characteristic field that is determined by the properties of the material. The relationship between $J_{c0}(T)$ and superconductors was first discovered by Onnes in 1913, who used an expression to describe how the critical current density of mercury changes with temperature.

$$J_{c0}(T) = J_{c0}(B = 0) \left(1 - \frac{T}{T_c}\right) \quad (2.2)$$

By assuming that the cooling power of the cryogenic system is sufficient to remove the heat generated by the superconductor, the dependence of $J_{c0}(T)$ can be treated as a constant. This assumption is used in electromagnetic modelling of HTS to simplify the analysis, but it may not always hold true in practical scenarios and should be validated experimentally or through detailed coupled electromagnetic-thermal modelling. Previous studies have highlighted that the magnetic field perpendicular to the a-b plane of type-II superconductors has a greater impact on the losses than transverse or parallel fields and therefore, a simplified expression of equation (2.2) was proposed in [115]. A semi-empirical Kim model with an orthonormal field

dependence to examine the relationship between critical current and field orientation was then presented in [116].

$$J_c(\mathbf{B}) = \frac{J_{c0}}{\left(1 + \sqrt{\frac{k^2 |B_{\parallel}|^2 + |B_{\perp}|^2}{B_0}}\right)^{\alpha}} \quad (2.3)$$

The variable $B_{\parallel}$ refers to the magnetic field component parallel to the surface of the HTS, while $B_{\perp}$ denotes the magnetic field component perpendicular to the surface. The anisotropy factor is represented by k , which is always less than one. The exponent of the $J_c(\mathbf{B})$ dependence is denoted by α .

The equation was further refined following an investigation of the magneto-angular relationship of critical current density and the more comprehensive $J_c(\mathbf{B}, \theta)$ dependence [117], [118] can be written as:

$$J_c(\mathbf{B}, \theta) = \frac{J_{c0}}{\left[1 + \varepsilon_{\theta} \left(\frac{|B|}{B_0}\right)^{\beta}\right]^{\alpha}} \quad (2.4)$$

where
$$\varepsilon_{\theta} = \sqrt{\gamma^{-1} \sin^2(\theta) + \cos^2(\theta)} \quad (2.5)$$

The angle θ indicates the deviation between the crystallographic a-b plane and the field vector of the HTS. When the magnetic field is perpendicular to the wide surface of the conductor, θ is equal to $\pi/2$. The electron mass anisotropy ratio is denoted by γ , while α and β are constants. The critical current density diminishes with the magnetic field at a rate specified by the parameters B_0 and β .

Precise estimations of the critical currents of superconductors are critical for practical applications of superconductivity. Equations that consider both the transport current and external magnetic field can produce more accurate outcomes. Additionally, considering the orientation of the magnetic field relative to the superconductor can also increase the precision of these predictions.

2.4.3 E-J power law

Bean's model has highlighted the requirement for additional exploration of the interdependence between current density and electric field. As a reaction, Rhyner presented the **E-J** power law [119], a significant model that captures the fundamental electromagnetic features of high-temperature superconductors. Additionally, the **E-J** power law effectively describes the theory of flux creep, proposed by Anderson [120]. Flux creep occurs in a superconductor when a transport current flows through it, resulting in the generation of an electric field and losses due to thermal fluctuations. The power law is defined as,

$$\mathbf{E}(\mathbf{J}) = E_0 \left(\frac{\mathbf{J}}{J_c} \right)^n, \quad (2.6)$$

where $E_0 = 10^{-4}$ V/m denotes the characteristic electrical field, J_c is the critical current density and n is the power exponent which depends upon the intrinsic properties of the HTS material. The **E-J** power law accurately represents the non-linear relationship between the electric field and current density within HTS. As a result, it can effectively fit the non-linear experimental I-V curves of numerous high-temperature superconductors. Equation (2.6) enables the derivation of a non-linear resistivity for high-temperature superconductors, expressed as:

$$\rho = \frac{\mathbf{E}}{\mathbf{J}} = \frac{E_0}{J_c} \left(\frac{|\mathbf{J}|}{J_c} \right)^n \quad (2.7)$$

This formulation displays a continuously changing resistivity within the superconductor, in contrast with the Bean model's assumption of zero resistivity. In cases where $n = 1$, the conductor maintains a consistent resistivity (under fixed temperature and magnetic field) and follows a linear Ohm's law, similar to normal conductors such as copper. As the n -value increases beyond 1, the model approaches the Bean model, with the Bean model corresponding to $n = \infty$. To obtain a good approximation of the Bean model, the value of n should typically exceed 20.

2.5 Numerical modelling methods

2.5.1 Integral models

Integral equation methods are advantageous in that they eliminate the need for boundary conditions. The method has also been used in superconductor modelling, where the current distribution within the superconducting area is defined and integrated parameters such as magnetic flux density and magnetic vector potential are used to calculate the total energy within the domain. Mathematical optimization algorithms can be combined with the integral method to determine the optimal current distributions of superconductors.

One of the early formulations was developed by Carr who investigated the AC loss from the combined action of transport current and external magnetic fields [121]. Further investigations into the critical state of HTS using integral equations were carried out by Mawatari in [122]. The Brandt model introduced by Brandt is recognized as a powerful tool to describe the penetration of a type-II superconductor in an applied field, also when a transport current is applied [123], [124], [125]. This model, which solves the integral equation for the time derivative of the current density in simple geometries, was later adapted by Otten and Grilli for thin films, rectangular bars, and cylindrical bulks [126]. In contrast to the initially proposed formulation, a discrete geometry is considered to avoid dealing with integral equations. By splitting the conductor into N elements, each element i (where $i = 1, 2, \dots, N$) can be assigned a uniform current density J_i . At the centre of each element, the magnetic vector potential is expressed as

$$\mathbf{A}_i = \mathbf{A}_{ext,i} + \frac{\mu_0}{2\pi} \sum_{j=1}^N K_{ij} J_j, \quad (2.8)$$

which combines the current J_i and external field $\mathbf{A}_{ext}$ contribution. Here, K is a $N \times N$ matrix and depends on the geometry. The field contribution from the external magnetic field is selected to satisfy,

$$\nabla \times \mathbf{A}_{ext} = \mathbf{B}_{ext}. \quad (2.9)$$

The Maxwell equations state that

$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla \phi. \quad (2.10)$$

Where the electric scalar potential is denoted by ϕ . Differentiating equation (2.9) with respect to time, the vector potential $\mathbf{A}$ can be removed.

$$\frac{\partial \mathbf{A}_{ext,i}}{\partial t} + \frac{\mu_0}{2\pi} \sum_{j=1}^N K_{ij} \frac{\partial \mathbf{J}_j}{\partial t} = -\mathbf{E}_i - (\nabla \phi)_i \quad (2.11)$$

Rearranging the terms of equation (2.11) gives,

$$\sum_{j=1}^N K_{ij} \frac{\partial \mathbf{J}_j}{\partial t} = -\frac{2\pi}{\mu_0} \left(\mathbf{E}_i + \frac{\partial \mathbf{A}_{ext,i}}{\partial t} + (\nabla \phi)_i \right), \quad (2.12)$$

where the current density and the magnetic vector potential are related through the matrix K . Before equation (2.12) can be numerically integrated, the values of K need to be identified by using Poisson's equation to relate the vector potential and the current density in the Coulomb gauge (2.13).

$$\nabla \cdot \mathbf{A} = 0 \quad (2.13)$$

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J} \quad (2.14)$$

In the case of a narrow strip with a width w , located in the x-y plane that has been partitioned into N elements, each element carries a sheet current density $\mathbf{J}_i$ in the x-direction. The elements are defined by $a_i < y < b_i$ and $z = c_i$ and the vector potential of a single element in terms of the current distribution can be obtained by integrating equation (2.14) over space.

$$A_x(y, z) = -\frac{\mu_0}{2\pi} J \int_a^b \ln \left(\sqrt{(y - y')^2 + (z - c)^2} \right) dy' \quad (2.15)$$

This expression can be evaluated by parts,

$$\begin{aligned} A_x(y, z) &= \frac{\mu_0 J}{4\pi} \int_{y-a}^{y-b} \ln(u^2 + (z - c)^2) du \\ &= \frac{\mu_0 J}{4\pi} \left[u \ln(u^2 + (z - c)^2) - 2u \right. \\ &\quad \left. + 2(z - c) \tan^{-1} \left(\frac{u}{z - c} \right) \right]_{u=y-a}^{y-b}. \end{aligned} \quad (2.16)$$

Since K_{ij} relates the current density of element j with the vector potential at the position (y_i, z_i) , this proportionality can be expressed by:

$$K_{ij} = \begin{cases} \left[u \left(\frac{1}{2} \ln(u^2 + (z_i - c_j)^2) - 1 \right) + (z_i - c_j) \tan^{-1} \left(\frac{u}{z_i - c_j} \right) \right]_{u=y_i-a_j}^{y_i-b_j} \\ [u(\ln|u| - 1)]_{u=y_i-a_j}^{y_i-b_j} \end{cases} \quad (2.17)$$

$$\begin{aligned} z_i &\neq c_j \\ z_i &\neq c_j \end{aligned}$$

The current distribution can be calculated by numerically integrating equation (2.12) over time, while the vector potential is obtained at the midpoint of each element at the coordinates $(y_i = a_i + b_i/2, z_i = c_i)$. In equation (2.12), the electric field E can be calculated by using the power law expressed previously in equation (2.6). Since the external potentials A_{ext} and φ depend on the condition of the simulation, A_{ext} is chosen in such a way that the condition of zero transport current is satisfied. Therefore, the gradient of the electric potential can be set to zero.

By calculating the current distribution for a specified amount of time, the dissipated energy can be computed by integrating the dissipated power over time.

$$Q = \int_0^t \sum_{i=1}^N w_i J_i(t) E(J_i(t)) dt \quad (2.18)$$

In equation (2.18), the width of element i is expressed as $w_i = b_i - a_i$. The outlined approach is presented in more detail in [126] and used in this study to investigate the loss behaviour of HTS in high fields in section 4.2. By using a constant critical current density, the model resembles the Bean model and is used to validate the modelling strategy used in this thesis.

Although integral formulations can solve the critical state in type-II superconductors more accurately than the fundamental modelling strategies such as the Bean model and are computationally less demanding, they are difficult to apply to complex 3D problems and therefore limited to simple geometries such as single tapes, rectangular or circular shapes. Using numerical solvers to efficiently solve these integral problems for larger HTS configurations led to the development of the Minimum Magnetic Energy Variation (MMEV) method [127] by Pardo, which was introduced as a numerical technique for modelling pancake

coils made of YBCO with an aspect ratio of up to 10000 and up to 200 turns. The YBCO superconducting layer is usually less than 1 μm thick, making it a thin sheet. The integral method is promising for simulating future power applications that utilize REBCO tapes. The MMEV as well as the minimum electromagnetic entropy production (MEMEP) [128], [129] methods are computationally efficient for demanding 3D problems, but less available in comparison to FEM numerical models that can be implemented in commercial software.

2.5.2 Differential methods

Analytical equations can help understand the loss behaviour of HTS theoretically, but they are limited to simple structures and have fundamental assumptions. Therefore, numerical models using finite element methods (FEM) are an indispensable tool for the design of superconducting applications. To calculate the losses in HTS, FEMs are commonly used to solve partial differential equations (PDEs) based on Ampere's law, which relates the magnetic field and current source. The FEM can transform Maxwell's equations into different formats for various variables, including magnetic fields, current distribution, and losses. Different FEM models have been proposed, including the $\mathbf{T}$ - ϕ formulation, $\mathbf{A}$ - $\mathbf{V}$ formulation, $\mathbf{E}$ formulation, and $\mathbf{H}$ formulation, among others, named according to the solved state variables [130]. Before applying these formulations, boundary conditions, initial values, and the nonlinear $\mathbf{E}$ - $\mathbf{J}$ power law of HTS must be predefined. The main step to calculate the losses is to solve the electromagnetic state variables, such as current density $\mathbf{J}$, magnetic field $\mathbf{B}$, electric field $\mathbf{E}$, current vector potential $\mathbf{T}$, magnetic vector potential $\mathbf{A}$, and magnetic scalar potential ϕ . Once these variables are obtained, the losses can be calculated according to the methods presented in the literature.

The $\mathbf{T}$ - ϕ formulation [131] was proposed to simulate 2D superconducting wires by defining the current vector potential $\mathbf{T}$ on each node to describe the current density $\mathbf{J}$, with $\mathbf{J} = \nabla \times \mathbf{T}$. Afterwards, the thin film approximation is applied to the HTS, resulting in a 1D problem by neglecting the current component perpendicular to the wide surface of the conductor [132]. This approach allows for a high calculation efficiency for simulating the HTS layer with the $\mathbf{T}$ - ϕ formulation. However, errors may be introduced to the simulation of HTS coils as the magnetic field components parallel to the wide surface of the HTS cannot be considered with the thin film approximation. On the other hand, the $\mathbf{T}$ - ϕ formulation, similar to the $\mathbf{A}$ - $\mathbf{V}$ formulation, uses the current vector potential $\mathbf{T}$ and the scalar magnetic potential ϕ as state

Table 2.2: Differential models used for the simulation of the electromagnetic behaviour of HTS [130]

Formulation	Equations	Definitions
$\mathbf{T} - \phi$	$\nabla \times \rho \nabla \times \mathbf{T} = -\mu \frac{\partial(\mathbf{T} - \nabla \phi)}{\partial t}$ $\nabla^2 \phi = 0$	$\mathbf{J} = \nabla \times \mathbf{T}$ $\mathbf{H} = \mathbf{T} - \nabla \phi$ $\rho = \rho(\mathbf{J})$
$\mathbf{A} - \mathbf{V}$	$\nabla^2 \mathbf{A} = \mu \sigma \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V \right)$ $\nabla \left(\sigma \frac{\partial \mathbf{A}}{\partial t} + \sigma \nabla V \right) = 0$	$\mathbf{B} = \nabla \times \mathbf{A}$ $\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla V$ $\sigma = \sigma(\mathbf{E})$
$\mathbf{E}$	$\nabla \times \nabla \times \mathbf{E} = -\mu \frac{\partial(\sigma \mathbf{E})}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ $\sigma = \sigma(\mathbf{E})$
$\mathbf{H}$	$\nabla \times \rho \nabla \times \mathbf{H} = -\mu \frac{\partial \mathbf{H}}{\partial t}$	$\mathbf{J} = \nabla \times \mathbf{H}$ $\rho = \rho(\mathbf{J})$

variables to provide current streamlines in a 2D view, with an advantage in reducing computation time compared to the $\mathbf{A}$ - $\mathbf{V}$ formulation [133].

The $\mathbf{A}$ - $\mathbf{V}$ formulation is a well-established method for simulating conventional materials and some superconductors [134]. It involves using the magnetic vector potential $\mathbf{A}$ and electrostatic potential $\mathbf{V}$ as state variables, where $\mathbf{V}$ is often disregarded in many cases [135]. In Flux2D an $\mathbf{A}$ - $\mathbf{V}$ formulation-based simulation module was developed [136], while Flux3D is used for solving 3D problems [137]. In [138] the mathematical background of a co-tree gauged $\mathbf{T}$ - ϕ FEM solver is presented and the $\mathbf{A}$ - $\mathbf{V}$ - $\mathbf{J}$ formulation in [139] for computing the hysteresis losses of superconductors, which were compared with $\mathbf{H}$ formulation in terms of degrees of freedom (DOF), computation time, and accuracy [140]. It was found that the $\mathbf{A}$ - $\mathbf{V}$ - $\mathbf{J}$ formulation needs a denser mesh to get solid outcomes compared to the $\mathbf{H}$ and $\mathbf{T}$ - ϕ formulations, but the $\mathbf{A}$ - $\mathbf{V}$ - $\mathbf{J}$ formulation-based solver can be less time-consuming than the other solvers in computation speed with the same mesh. The magnetic vector potential of $\mathbf{B}$ is defined by $\mathbf{A}$ in the $\mathbf{A}$ - $\mathbf{V}$ formulation, despite generally having three components. Recently, the $\mathbf{A}$ - $\mathbf{V}$ formulation in 3D was used to simulate the magnetic field of a toroidal field coil system of a Tokamak fusion reactor [141].

The **E** formulation has been proposed as an alternative to avoid the calculation of derivatives. However, as reported in [142], it may result in convergence issues when applied to finite geometries with highly nonlinear **E-J** power laws, particularly for n values exceeding 20.

The **H** formulation is currently the most widely adopted formulation due to its quick convergence, and ease of implementation within COMSOL Multiphysics and only one state variable **H** [143], [144]. However, it still has its drawbacks, such as the need for a solution of a vector field in the non-conducting regions, resulting in increased complexity and degraded matrix conditioning due to the application of a dummy resistivity in the air region. Despite these limitations, the implementation of the **H** formulation in COMSOL Multiphysics has provided a novel approach to the numerical simulation of HTS, validated by experimental data [145], [146] and subsequently improved to accommodate complex geometries and materials [147], [148]. Nevertheless, these differential methods have natural limitations, making them unsuitable for simulations involving YBCO conductors, which require extremely fine meshes to accurately represent the dramatic variation of current density, and increase the computation time due to the highly nonlinear characteristic of superconductors.

When dealing with large-scale magnets, a significant number of HTS is required. Regardless of the chosen formulation, modelling HTS coils becomes complex and time-consuming. Therefore, most researchers concentrate on the superconducting parts and use 2D models to examine the cross-section of HTS coils, stacks of tapes, and bulks to reduce simulation complexity. It is more reasonable to model both superconducting and non-superconducting sections together to accurately predict the loss distribution. Different combinations of formulations have been developed to meet this requirement.

A few researchers have used the **H- ϕ** formulation to simulate superconductors in GetDP, but only recently has its implementation into COMSOL Multiphysics been detailed in [149], [150]. The **H** formulation has been used for superconductors, and the ϕ physics has been introduced for current-free areas. The **H- ϕ** formulation can significantly reduce the size of the linear matrix to be solved and the number of DOF, resulting in almost a 50% reduction in computational time for a given relative error [151]. As a result, the **H- ϕ** formulation is thought to be a beneficial alternative for modelling superconducting machines that consider both the superconducting and non-superconducting components.

According to [152], a FEM framework based on the **H-A** formulation has been used to model rotating machines with HTS windings. It has been noted that the **H** formulation is more reliable than the **A** formulation for simulating flux dynamics in superconductors using the **E-J** power law. As a result, the **H** formulation has been used for the superconducting parts, while the **A** formulation has been used for the outer iron stator poles.

In recent years the **T-A** formulation gained popularity due to its simple implementation and efficient computation speed. A detailed review of the macroscopic electromagnetic behaviour of HTS-coated conductor applications about the **T-A** formulation was published as part of this thesis [153]. By adopting advanced modelling strategies such as the multiscale and homogenisation method to the **T-A** formulation, it is now possible to model large-scale applications with large numbers of turns over the capabilities of other formulations, such as the **H** formulation. This makes the **T-A** formulation even more promising for future modelling of HTS. With these improvements, the **T-A** formulation was used to simulate the superconducting behaviour in a multitude of applications: basic tapes and coils [154], [155], [156], [157], [158]; high-field cables such as Roebel cables [159], [160], CORC[®] cables [161], [162], [163], [164], [165], [166], [167], stacked cables [168], [169] and cables with a twisted structure [170], [171]; fault current limiters [172], electrical machines [173], [174], [175], [176], [177], [178], and high-field magnets [179], [180], [181], [182], [183], [184]. Especially for the modelling of high-field magnets, where a large number of turns in a pancake wound solenoid is simulated, the homogenisation technique reduces the numerical complexity and enables fast simulation times [181], [184]. The **T-A** formulation has shown to be a reliable alternative to the **H** formulation and is especially desired in large-scale applications since the computational speed can be increased tremendously. In chapter 3, the basic concept and implementation of the **T-A** formulation are described, and the numerical efficiency is evaluated based on the available literature.

2.6 Discussion

This chapter provided a background on the topic of superconductivity (section 2.2), detailing the discovery as well as the unique properties of superconductors, such as the Meissner effect. Superconductors can be categorized into type-I and type-II, according to their transition from superconducting to the normal state. While type-I superconductor transition from the Meissner state to the normal state, type-II superconductors enter a transition state before becoming

resistive. In this transition state magnetic flux partially penetrates the superconductor allowing it to achieve higher magnetic fields. The categorization of superconductors into LTS and HTS materials is by their adherence to the BCS theory. The behaviour of LTS materials can be explained by the BCS theory, while this is not completely true for HTS as their origin is still unknown.

The use of superconductors for fusion reactor magnets was also reviewed in section 2.3. While LTS have been used for the latest large-scale Tokamak reactor projects, HTS, specifically REBCO, are the most promising candidate for future commercial fusion reactors. REBCO-coated conductors have the desired properties for the application in a fusion reactor and have already shown their high-field capabilities in high-field solenoids, generating fields of up to 45 T. Novel cable concepts and the development of improved production and manufacturing capabilities for REBCO tapes are expected to drive down the material cost.

To understand the fundamental behaviour of HTS, critical state models such as the Bean model, Kim model or the **E-J** power law have been developed. These models (section 2.4) aim to describe the current and magnetic field distribution within a superconductor. While the Bean model assumes a constant critical current density, the Kim model also considers the contribution from the magnetic field, which will influence the local critical current density. The **E-J** power law is most used in numerical models to describe the resistivity of HTS and for large values of the superconducting n -value, the solution of the Bean model can be obtained.

The numerical modelling strategies developed to simulate the behaviour of HTS for practical applications have been reviewed in terms of integral and differential methods in section 2.5. The integral method by Brandt has been modified to be solved in MATLAB for simple geometries such as rectangular, spherical, or cylindrical conductors. Novel approaches such as the MMEV or MEMEP method can be used to efficiently simulate the macroscopic behaviour of large numbers of turns. Since these methods are not easily accessible, differential methods are commonly used for practical applications due to their simple implementation within commercially available FEM software such as COMSOL Multiphysics.

There is a variety of differential formulations, which all have their advantages and drawbacks. The **H** formulation is the most versatile tool to simulate almost any application of HTS. For large-scale applications such as high-field magnets, the formulation has the drawback of long simulation times. This is due to the required implementation of a dummy resistivity in the air

domain to prevent current sharing with the neighbouring tapes. Also, when using the homogenisation method, the added resistive air gap in the bulk will require either a high number of elements in the mesh or result in elements with bad quality. In both cases, the computational time increases significantly with an increasing number of HTS tapes.

To improve the numerical modelling efficiency, the **T-A** formulation was introduced, where the current vector potential is used to model the current distribution within the superconductor and the **A** formulation to simulate the magnetic field distribution in the whole domain. The formulation makes use of the thin strip approximation, reducing the cross-section of the HTS to a line element. Coupled with advanced modelling strategies such as the homogenisation technique, this modelling strategy is suitable for large-scale applications such as the modelling of a fusion reactor magnet system and will therefore be used in this thesis. In chapter 3, the methodology of the **T-A** formulation is described in detail for 2D and 3D problems as well as the advanced modelling techniques such as homogenisation or multi-scale. Furthermore, the validity of the **T-A** formulation is investigated in detail.

Although the **T-A** formulation can simulate large-scale applications, the size of a Tokamak fusion reactor magnet is one order of magnitude larger than high-field solenoids. High-field solenoids aim to achieve the highest field at the centre of the coil, and due to the compact size can be wound from HTS tapes. In fusion reactor magnets, on the other hand, novel HTS cable concepts will most likely be required to address the issue of the large size. Modelling HTS cables is complicated, especially if the structure is twisted since this requires a 3D modelling approach. Due to this complexity, the investigation of HTS cables is limited to sections of the cable and a model for a coil exceeds the capabilities of any numerical approach. Therefore, the modelling of large-scale magnets can only be done by assuming a simple pancake wound coil. Nevertheless, it is reasonable to investigate the electromagnetic behaviour of pancake wound large-scale high-field magnets due to the simplicity of the winding procedure and high performance compared to LTS magnets.

3 Modelling Approach

3.1 Introduction

The **T-A** formulation was identified as a powerful method to simulate the high aspect ratio of HTS tapes for large-scale applications, in the previous chapter. Here the methodology of the formulation is outlined, followed by a review of the validation reported in the literature. The subsequent Sections 3.2 and 3.3 have been published as part of this thesis in [153], which provides a detailed review of the **T-A** formulation.

In the subsequent chapters, the **T-A** formulation will be the main numerical tool used for the studies. As established in section 2.4 the critical current is dependent on the magnetic field. In section 3.4 of this chapter, the critical current characteristics used in this thesis are outlined. While the study in chapter 4, will use a mathematical formulation, the critical current for specific tapes in the PFC and TFC study will be based on experimentally validated datasets (chapters 5 and 6). This chapter is intended to provide an overview of the numerical modelling approach used throughout the thesis and outline the benefits of the formulation.

3.2 The **T-A** formulation

The **T-A** formulation was first introduced by Zhang et al. [185] and Liang et al. [186] in 2017. The concept of this numerical model is to solve the current vector potential $\mathbf{T}$ only in the superconducting domain, while the magnetic vector potential A is solved for the whole domain. The two state variables $\mathbf{T}$ and $\mathbf{A}$ are used to calculate the current density $\mathbf{J}$ and the magnetic flux density $\mathbf{B}$, respectively. The numerical model is fully coupled by solving for $\mathbf{J}$ and $\mathbf{B}$ at the same time and continuously exchanging these variables.

3.2.1 General formulation

The governing equations of the general form are,

$$\mathbf{J} = \nabla \times \mathbf{T} \quad (3.1)$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (3.2)$$

The resistivity in type-II superconductors is modelled by the **E-J** power law, coupling the local electric field with the local current density within the superconducting domain only.

$$\mathbf{E} = \rho_{HTS} \cdot \mathbf{J} = E_c \frac{\mathbf{J}}{J_c(\mathbf{B})} \left| \frac{\mathbf{J}}{J_c(\mathbf{B})} \right|^{n-1} \quad (3.3)$$

Here, ρ_{HTS} denotes the resistivity of the HTS material, E_c is the characteristic electric field at 10^{-4} V/m, J_c is the critical current density dependent on the magnetic flux density $\mathbf{B}$ and n is the superconducting n -value.

Applying Maxwell-Faraday's law to the HTS domain gives

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \quad (3.4)$$

In the whole space Maxwell-Ampere's law is applied to solve the traditional **A** formulation

$$\nabla \times \nabla \times \mathbf{A} = \mu_0 \mu_r \mathbf{J}, \quad (3.5)$$

where μ_0 and μ_r are the magnetic and relative permeability of the free space, respectively.

The current density is coupled with the **A** formulation by an applied external surface current density $\mathbf{J}_e$.

$$\mathbf{J}_e = \mathbf{J} \cdot \delta, \quad (3.6)$$

where δ denotes the thickness of the HTS tape. $\mathbf{J}_e$ is imposed in the tape in A/m to accommodate for the thin strip approximation, which is neglecting the thickness of the HTS layer.

The transport current flowing in the superconductor is applied at the tape terminals. It can be represented by an integral of the current density over the cross-section of the conductor.

$$I = \iint_S \mathbf{J} \cdot d\mathbf{S} = \iint_S \nabla \times \mathbf{T} \cdot d\mathbf{S} = \oint_{\partial S} \mathbf{T} \cdot d\mathbf{s} \quad (3.7)$$

Here, S is the cross-section of the conductor and ∂S represents the boundary edges of the cross-section.

The losses using the **T-A** formulation can be calculated by:

$$Q = \delta \int_0^t \iint_S |\mathbf{J}| \cdot |\mathbf{E}| dS dt \quad (3.8)$$

In HTS applications the losses can be categorized into magnetization losses, transport losses, and coupling losses. Magnetization losses, primarily hysteresis losses, arise from magnetic flux movement within the superconductor under time varying magnetic fields. Transport losses include hysteresis losses from time varying currents within the superconductor and eddy current losses from induced currents in surrounding conductive materials. Coupling losses occur in multi-filamentary wires due to magnetic flux interactions between filaments. It should be noted that equation (3.8) can only be used to calculate the hysteresis losses. Although the losses of HTS applications will depend on coupling as well as eddy current loss in addition to hysteresis loss, it is reasonable to approximate the losses with hysteresis loss. Therefore, the loss in subsequent chapters only consider the hysteresis loss during magnetization, transport current or transport current during magnetization.

3.2.2 Formulation in 2D (infinitely long strip, axial-symmetric)

Due to the high aspect ratio of HTS-coated conductors, modelling the whole surface requires long simulation times. The **T-A** formulation makes unique use of this issue, by applying the thin strip approximation to the tape. This geometrical simplification reduces the surface area of the HTS tape to a thin sheet (Figure 3.1).

Therefore, the current is restricted to flow within the superconducting sheet, while $\mathbf{T}$ is reduced to the component perpendicular to the wide surface of the tape. The current vector potential can then be defined as $\mathbf{T} \cdot \mathbf{n}$, where $\mathbf{n} = [n_x \ n_y \ n_z]^T$ is the normal vector perpendicular to the superconducting tape in cartesian coordinates and $\mathbf{n} = [n_r \ n_\phi \ n_z]^T$ in cylinder coordinates. In 2D the surface element of the HTS tape is reduced to a 1D line element and equation (3.1) can be written as

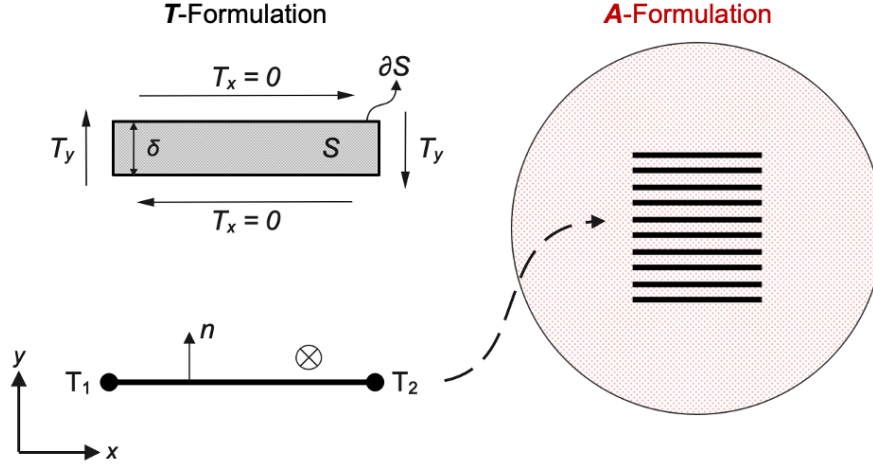


Figure 3.1: **T-A** thin strip approximation in 2D

$\mathbf{T}$ is computed only in the superconducting domain (black lines), while $\mathbf{A}$ is computed everywhere. The thickness of the HTS tape is collapsed into a superconducting layer, represented by a 1D line element. The current is enforced by boundary conditions at the edges T_1 and T_2 of the tape.

$$J_z = \frac{\partial(\mathbf{T} \cdot \mathbf{n}_y)}{\partial x} - \frac{\partial(\mathbf{T} \cdot \mathbf{n}_x)}{\partial y} \quad (3.9)$$

in cartesian coordinates, or for axial-symmetric problems in cylindrical coordinates as

$$J_\varphi = \frac{\partial(\mathbf{T} \cdot \mathbf{n}_r)}{\partial z} \quad (3.10)$$

Depending on the orientation of the tape, $\mathbf{T} \cdot \mathbf{n}_y$ or $\mathbf{T} \cdot \mathbf{n}_x$ in equation (3.9) can further be neglected. Equation (3.10) expresses a general case for a wound tape with an easy bend where the current is flowing in φ -direction and therefore, reducing the current vector potential to $\mathbf{T} \cdot \mathbf{n}_r$. Furthermore, when implementing Faraday's law, equation (3.4) can be reduced to

$$\begin{bmatrix} \frac{\partial E_z}{\partial y} \\ -\frac{\partial E_z}{\partial x} \end{bmatrix} \cdot \mathbf{n} + \begin{bmatrix} \frac{\partial B_x}{\partial t} \\ \frac{\partial B_y}{\partial t} \end{bmatrix} \cdot \mathbf{n} = 0 \quad (3.11)$$

$$\begin{bmatrix} -\frac{\partial E_\varphi}{\partial z} \end{bmatrix} \cdot \mathbf{n} + \begin{bmatrix} \frac{\partial B_r}{\partial t} \end{bmatrix} \cdot \mathbf{n} = 0 \quad (3.12)$$

for cartesian and cylindrical coordinates respectively. Again, only the x or y component of the magnetic flux density in cartesian coordinates needs to be considered, depending on the orientation of the tape.

A transport current, as defined in equation (3.7), can then be applied to the edges of the tape using

$$I = (T_1 - T_2)\delta \quad (3.13)$$

where T_1 and T_2 are the scalar values of the current vector potential $\mathbf{T}$ at the respective edge points and δ is the thickness of the superconducting tape.

3.2.3 Formulation in 3D

In a 3D geometry, the superconductor is no longer represented as a line, instead, the conductor will be modelled as a thin sheet [185]. This surface element also does not consider the thickness of the HTS. Using the same set of equations for a 3D geometry, the current density vector is defined as,

$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} \frac{\partial(\mathbf{T} \cdot \mathbf{n}_z)}{\partial y} - \frac{\partial(\mathbf{T} \cdot \mathbf{n}_y)}{\partial z} \\ \frac{\partial(\mathbf{T} \cdot \mathbf{n}_x)}{\partial z} - \frac{\partial(\mathbf{T} \cdot \mathbf{n}_z)}{\partial x} \\ \frac{\partial(\mathbf{T} \cdot \mathbf{n}_y)}{\partial x} - \frac{\partial(\mathbf{T} \cdot \mathbf{n}_x)}{\partial y} \end{bmatrix} \quad (3.14)$$

The implementation of Faraday's law in the superconducting domain can now be written as

$$\begin{bmatrix} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \end{bmatrix} \cdot \mathbf{n} + \begin{bmatrix} \frac{\partial B_x}{\partial t} \\ \frac{\partial B_y}{\partial t} \\ \frac{\partial B_z}{\partial t} \end{bmatrix} \cdot \mathbf{n} = 0. \quad (3.15)$$

The boundary condition for the transport current is applied in the same way as for a 2D problem in equation (3.13), only instead of defining it at the terminal points, it is applied to the edge of the superconducting sheet. In 3D there is an additional boundary condition to consider. When symmetry is exploited the conductor cross-section is defined with a zero-flux condition in the $\mathbf{T}$ formulation to contribute to the magnetic insulation boundary condition of the $\mathbf{A}$ formulation. If in a 3D model, symmetry is exploited to reduce the complexity of the model, then for the boundaries of the $\mathbf{A}$ formulation, a perfect magnetic conductor boundary condition needs to be defined.

3.2.4 Multi-scale and homogenisation method

Based on the general form of the **T-A** formulation, two simplification methods can be implemented to reduce the amount of computational resources needed to model large numbers of tapes in 2D and 3D. Although simplification assumptions are made, the accuracy for hysteresis losses, current distribution and magnetic field distribution is maintained [187]. Both these methods have been validated in 2D by [187],[188] and in 3D by [164], [189].

In the multi-scale method, the current vector potential is defined for each analysed tape individually.

The current density distribution is obtained by computing **T** in the analysed tapes, while the non-analysed tapes in between are approximated by linear interpolation, shown in Figure 3.2. The magnetic flux density is computed via the **A** formulation, which is defined in the whole space. To couple the **T** and **A** formulation, the current density is imposed into the **A** formulation using equation (3.6), where **J** is multiplied by the thickness of the tape, to obtain the surface current density.

The homogenisation method assumes that a stack of tapes can be considered as an anisotropic bulk, which will not compromise the electrodynamic behaviour of the coil [187]. In the **T-A** formulation, a tape is considered as a thin sheet with the thickness δ , where we now define a unit cell around it with the thickness Λ . The unit cell assumes the thickness of the additional

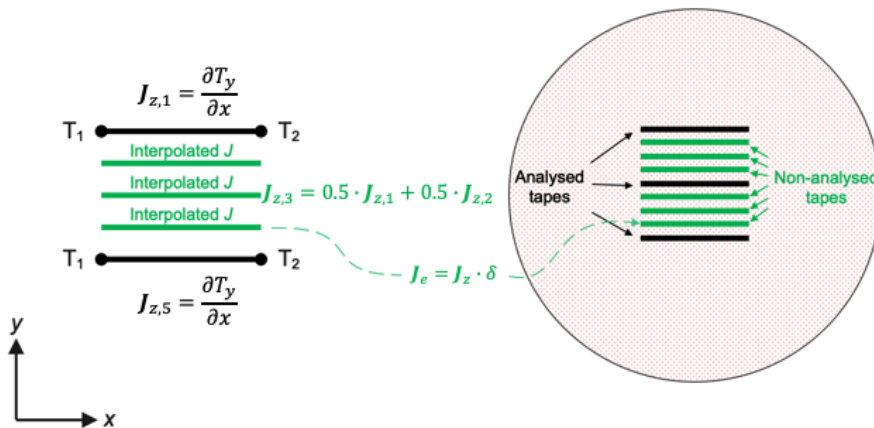


Figure 3.2: **T-A** multi-scale approach in 2D

*In this example, there are three analysed tapes (black) whereby **J** is obtained by computing **T**. **J** in the remaining non-analysed tapes (green) is approximated by linear interpolation. The surface current **J_e** is imposed in the **A** formulation by means of a boundary condition. See also [187]*

layers in an HTS tape such as copper, hastelloy, buffer layer, and insulation material, but the materials themselves are not modelled separately. Therefore, the actual size of the bulk is given by the stack of tapes with the unit cells surrounding them. The tapes are then transformed into a single block, which is shown in Figure 3.3.

The scaled external current density J_e in the homogenised bulk is imposed in the **A** formulation using

$$J_e = J \cdot \frac{\delta}{\Lambda} \quad (3.16)$$

When expanding the homogenisation method to 3D geometries, software like COMSOL only features the normal vector on boundary elements. This means that for a 2D element such as a rectangle, the normal vectors are only defined at the edges of the geometry and not for the enclosed surface. Therefore, it is necessary to define the normal vector $\mathbf{n}$ inside the bulk.

Considering a simple 3D geometry that is commonly studied like in the case of a racetrack coil, $\mathbf{n}$ will be defined for the straight and circular parts respectively.

$$\mathbf{n} = \mathbf{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (3.17)$$

$$\mathbf{n} = \boldsymbol{\rho} = \begin{bmatrix} \frac{x}{\sqrt{x^2 + y^2}} \\ y \\ \frac{y}{\sqrt{x^2 + y^2}} \\ 0 \end{bmatrix} \quad (3.18)$$

Here, $\mathbf{i}$ defines the normal vector in a cartesian coordinate system and $\boldsymbol{\rho}$ is defined in a cylindrical frame.

The boundary conditions for the transport current are applied in the same way as in the 3D tape model, with Dirichlet boundary conditions at the top and bottom of the homogenised bulk. For the additional boundaries that come from the homogenisation technique, a Neumann boundary condition is defined in the general form as,

$$\frac{\partial(T_x n_x + T_y n_y + T_z n_z)}{\partial n} = 0 \quad (3.19)$$

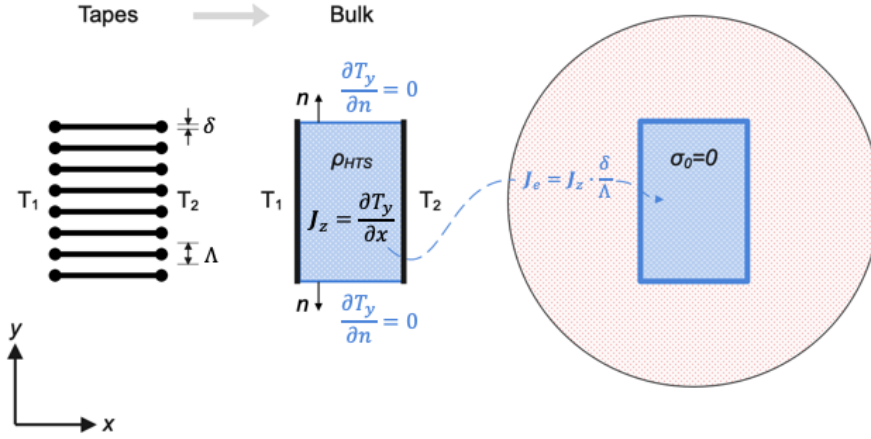


Figure 3.3: *T-A homogenisation in 2D*

The superconducting sheets are replaced by a homogeneous bulk. The current density inside the bulk is computed by boundary conditions applied to the edges T_1 and T_2 of the bulk and the scaled current density J_e is imposed in the A formulation.

In a 2D homogenisation problem, equation (3.19) can be reduced to only the scalar component of $\mathbf{T}$.

In the case of a closed coil, symmetry conditions can be useful to simplify the problem. When considering a quarter model, the symmetry planes use a zero-flux condition for the superconducting domain. In the $\mathbf{A}$ formulation, the symmetry plane is considered a perfect magnetic conductor, mirroring the current of the superconductor.

3.3 Validation of modelling strategies

The validation of the $\mathbf{T-A}$ formulation, especially of the advanced modelling strategies (multi-scale, homogenisation), was carried out by comparisons with experiments and other numerical models, mainly the $\mathbf{H}$ formulation. This section is dedicated to showing a quantitative evaluation of the performance of the $\mathbf{T-A}$ formulation compared to the $\mathbf{H}$ formulation presented in the literature.

In [187], the authors proposed the $\mathbf{T-A}$ multi-scale and homogeneous approach as an efficient alternative for large-scale HTS applications.

The simulations compared the accuracy and computational time (ct) of the different models for a coil configuration of 200 tapes for each of the 10 pancakes. Making use of symmetry conditions reduced the total number of tapes to 500. The results of this comparison are shown

Table 3.1: *T-A full, multi-scale and homogenous model comparison.* [187]

T-A model	Losses (J/m)	Error (%)	ct (h)
full	0.8832	-	9 h 04 min
multi-scale	0.8807	0.28	1 h 13 min
homogeneous	0.9025	2.18	0 h 37 min

in Table 3.1. Furthermore, an extensive comparison with the **T** formulation was conducted to validate these approaches with different orders of elements for the **T** and **A** formulation and the results are shown in Table 3.2.

Further investigations about the validity of the **T-A** formulation in 2D were conducted in [190], where different orders for the dependent variables were evaluated and compared with the **H** formulation. The authors concluded that the **T-A** formulation is the preferred choice for large-scale applications, due to the faster computation time and high accuracy.

Another study regarding the element order of the dependent variables of the **T-A** formulation was carried out by the authors in [160]. Simulations using $T\ 1^\circ$, $A\ 1^\circ$ and $T\ 1^\circ$, $A\ 2^\circ$ were compared with the **H** formulation using first-order curl elements. Although all models showed good results, the **T-A** formulation was advantageous for complex problems like the simulated 14-strand Roebel cable.

In their latest publication to date [188], the authors expanded on their previously conducted study in [187] and provided an even more detailed comparison of the different **T-A** and **H** formulation models, including densified modelling strategies as well as iterative and simultaneous approaches. In the study, models were compared regarding the degrees of freedom. It was found that the DOF have a direct impact on the simulation time.

A compact version of these results is presented in Table 3.3 where $\bar{c}t$ denotes the normalized computation time. Especially the homogenous approach for both **H** and **T-A** formulation greatly reduced the number of DOF and subsequently reduced the computational time tremendously. In Figure 3.4, the authors compared the normalized computation time with the absolute value of the loss error shown in Table 3.3. Again, the results confirmed the improved computational speed with similar accuracy for the **T-A** formulation.

Table 3.2: **H** and **T-A** full model comparison. [187]

Model	Order	Losses (W/m)	Error (%)	ct (h)
H full		127.23	-	31 h 32 min
	T 1°, A 1°	126.02	0.96	02 h 46 min
T-A full	T 2°, A 2°	126.38	0.67	09 h 01 min
	T 1°, A 2°	128.05	0.64	03 h 14 min

It was shown that for the **T-A** formulation (striped bubbles), the homogeneous and the simultaneous multi-scale were within the first square of the grid. It was concluded that these methods have an error of less than 1% and less than 10% of the normalized computation time.

In [189], the implementation of the **T-A** homogenisation method was achieved for the first time in 3D. The validation of the method was done by the experimental analysis of an 86-turn double pancake racetrack coil. For applications such as a racetrack coil where a straight as well as a curved segment of the coil needs to be modelled, 2D simplifications are often not sufficient to accurately simulate the electromagnetic behaviour. The authors concluded that the AC loss of 2D models could show a larger discrepancy if the number of turns was increased. Furthermore, the 3D **T-A** homogenised model improves in its efficiency as the number of turns increases, compared to other FEM models such as the **H** formulation. Therefore, this method could be valuable for the numerical analysis of large-scale HTS problems, which cannot be represented accurately in 2D.

In [191], the authors continued their investigation of the homogenisation method in 3D using the **T-A** formulation. The 3D approach was compared with 2D infinitely long and axial-symmetric models to show the difference in the current distribution as well as the losses. Although a 2D estimation of a racetrack coil is often sufficient, it was shown that there are differences in the penetration of the HTS coil, due to the anisotropy of the critical current. In comparison with the homogenised **H** formulation, the higher penetration in the arc segment of the racetrack coil was validated since the influence of the self-field in this region is higher than in the straight part. The authors had to concede that the **H** formulation was faster than the **T-A**

Table 3.3: Comparison of different modelling approaches with the **H** and **T-A** formulation. [188]

	Losses (W m^{-1})	ct (h)	DOF
Reference	127.24	31 h 32 min	563'893
Model	Error (%)	$\bar{c}t$ (%)	DOF
H full	1.62	55.81	359'408
H homogenous	1.28	1.94	11'838
H densified	6.67	29.03	120'424
T-A full	0.64	10.25	548'624
T-A homogenous	0.71	0.78	20'612
T-A densified	-2.62	3.22	103'638

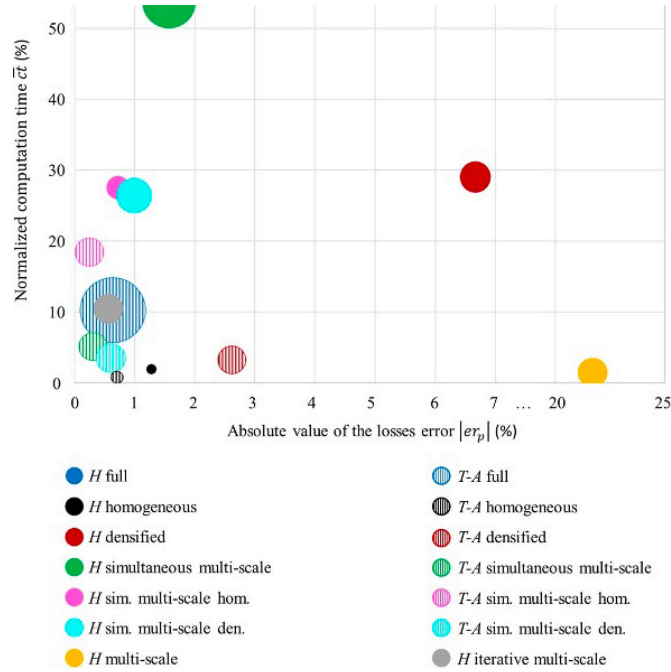


Figure 3.4: Trade-off between accuracy and computation time

The bubbles closer to the origin are those of the models that better fulfil the compromise between accuracy and computation time. The areas of the bubbles are proportional to the number of DOF.

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formulation, unlike in 2D. Nevertheless, they state that the mode is extremely necessary and will enable the design and optimisation of HTS applications in 3D in the future.

3.4 Critical current

High-temperature superconductors can operate at temperatures of up to 80 K, which is desirable for a lot of applications, especially in electrical machines. However, in nuclear fusion reactors and high-field magnets lower temperatures are preferred since the performance of HTS conductors greatly improves as the operating temperature decreases. High-field solenoids are often even operated at a liquid helium temperature of 4.2 K to achieve the highest field performance. In DEMO fusion reactor concepts, HTS insert coils are considered for the magnet system which are surrounded by LTS magnets. This results in a mandatory operating temperature of 4.2 K since the LTS coils would otherwise be in a resistive state. If a coil is solely made from HTS conductors, the operating temperature can be increased to temperatures above 4.2 K which provides a greater temperature margin and enables other cooling options, i.e., helium gas, or even liquid nitrogen. For the use of HTS in a fusion reactor magnet, two operating temperatures at 25 K and 35 K are selected for the following case studies.

To model the HTS coil at a specific temperature, the field-dependent critical current data at the selected temperature can be used. The field-dependent critical current is obtained experimentally, and a lot of data for different tapes at various temperatures is publicly available online [192]. If there is no data available for the desired temperature, a widely used approach is to use lifting factors to scale the critical current. One issue regarding the availability of critical current data is concerning high magnetic fields, where data points are often only available for magnetic fields of up to 10 T, which is below the requirements for fusion reactor magnets. Field data for higher fields are only available for specific tapes at specific temperatures (mainly 4.2 K). Another option is to use an analytical approach to calculate the critical current density, which has been achieved for a SuperPower tape in fields up to 35 T, based on experimental data [193]. For the case studies two different tapes from the manufacturers SuperPower and SuperOx are investigated.

3.4.1 Critical current dependency

For the investigation of the loss behaviour of HTS conductors (chapter 4) and the 3D study of a D-shaped coil (section 6.2.1), the critical current dependence of the tape reported in [148] is used. Using the same coil configuration allows for direct comparison and validation of the modelling approach with the well-established 3D H homogenisation used in the publication.

The critical current density dependence on the magnetic field amplitude and direction of the tape is described by the following equation:

$$J_c(B) = \frac{J_{c0}}{\left(1 + \sqrt{(kB_{\parallel})^2 + \frac{B_{\perp}^2}{B_c^2}}\right)^b} \quad (3.20)$$

where $B_{\parallel}$ and $B_{\perp}$ are the parallel and perpendicular components of the magnetic flux density, and the parameters J_{c0} , k , B_c and b have the following values 49 GAm⁻², 0.275, 32.5 mT and 0.6 [148]. The tape has a critical current I_c of 160 A at 77 K self-field. The behaviour of the critical current density described in equation ((3.20)) can be better appreciated in Figure 3.5, where the critical current density $J_c(B_{\parallel}, B_{\perp})$ was plotted by considering four different magnetic field magnitudes with different directions.

3.4.2 SuperPower tape

The critical current characteristics of tapes from SuperPower are publicly available [194] at a variety of different temperatures and fields of up to 10 T. Beyond this field strength, a

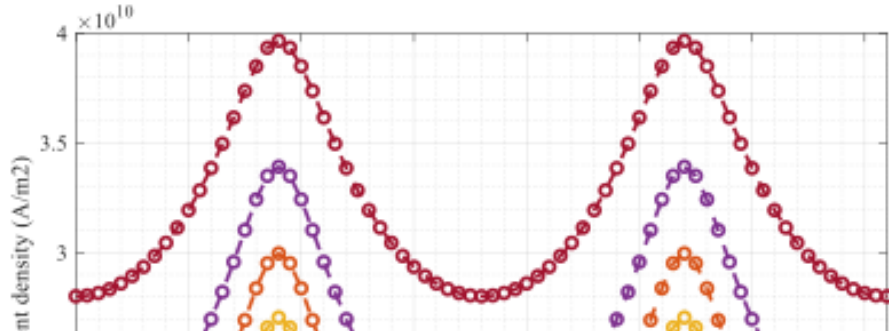


Table 3.4: Nonlinear fit parameters for SuperPower tape [183]

Parameter	Value
k_0	8870
k_1	18500
a_0	1.30
a_1	0.809
β_0	13.8
β_1	13.8
φ_1	-0.180°
c	2.15 (4.2 K), 4.7 (25 K), 7.4 (35 K)

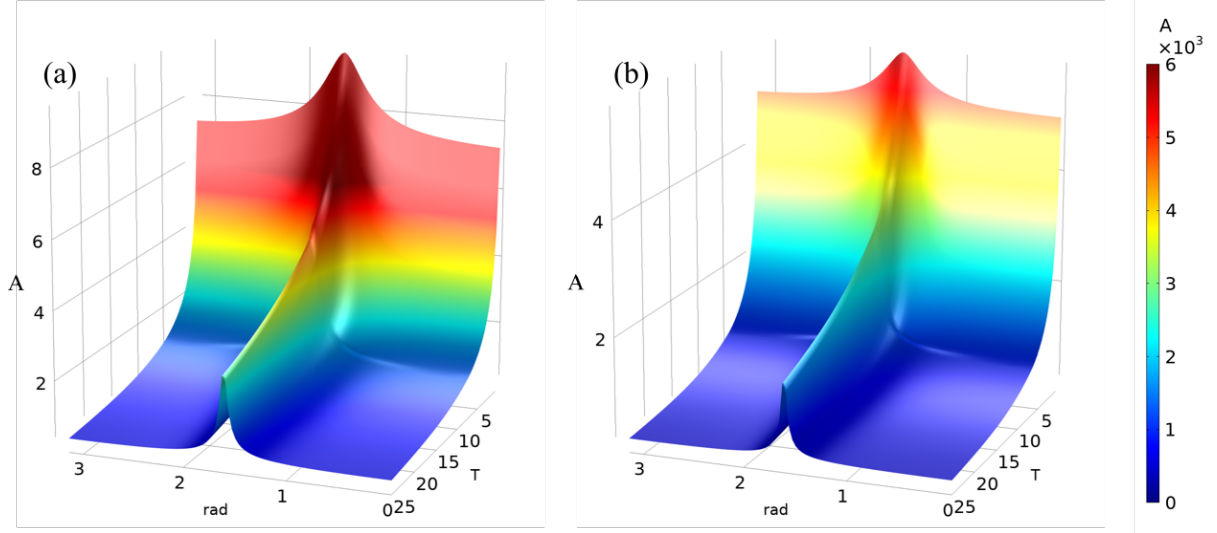


Figure 3.6: Critical current characteristic of SuperPower tape
(a) 25 K and (b) 35 K

mathematical model that systematically and comprehensively characterizes the transport critical current angular dependence was used. At fixed temperatures and high magnetic fields, this model shows an uncertainty of 10% or better. The model was developed in [193] and is based on the analysis of experimental data sets from various published sources and coated conductors with different microstructures. This approach was used and modified to fit the dataset for a SuperPower tape at 25 K and 35 K.

Due to the anisotropic nature of HTS tapes, the critical current is influenced by the direction of the magnetic flux density. As a result, a coil constructed with HTS conductors exhibits specific locations with minimal transport critical current, which are influenced by the strength and orientation of the magnetic flux density. The spatial distribution of these points within the coil is partly governed by the conductor's anisotropy. Therefore, the coil's transport critical current and overall stability are primarily dictated by these points, as they are assumed to be the starting points of coil quenching.

To describe this behaviour mathematically, a fit parameter ω is defined, which appears to obey a power law. Taking into account the magnetic field dependence, the peak sharpness parameter $\omega(B)$ can be defined as

$$\omega(B) = c \left[B + \left(\frac{1}{c} \right)^{\frac{5}{3}} \right]^{\frac{3}{5}}. \quad (3.21)$$

To extrapolate the critical current data of an existing dataset for a SuperPower tape at 25 K and 35 K for fields above 7 T, the parameter c was modified to match the available data. Then, the

general expression of a fit function was used to extrapolate the critical current profile for up to 25 T and consider the parallel and perpendicular magnetic field components.

$$I_c(B, \theta) = \frac{k_0}{(B + \beta_0)^{a_0}} + \frac{k_1}{(B + \beta_1)^{a_1}} \times [\omega_1^2(B) \cos^2(\theta - \varphi_1) + \sin^2(\theta - \varphi_1)]^{-\frac{1}{2}} \quad (3.22)$$

The parameters used in equation (3.22) are listed in Table 3.4. The resulting critical current profile is shown in Figure 3.6.

3.4.3 SuperOx tape

For the SuperOx tape characteristic, the available data from [195] was used for fields up to 10 T. The critical current measurements for temperatures from 15 K to 85 K in parallel and perpendicular fields are shown in Figure 3.7.

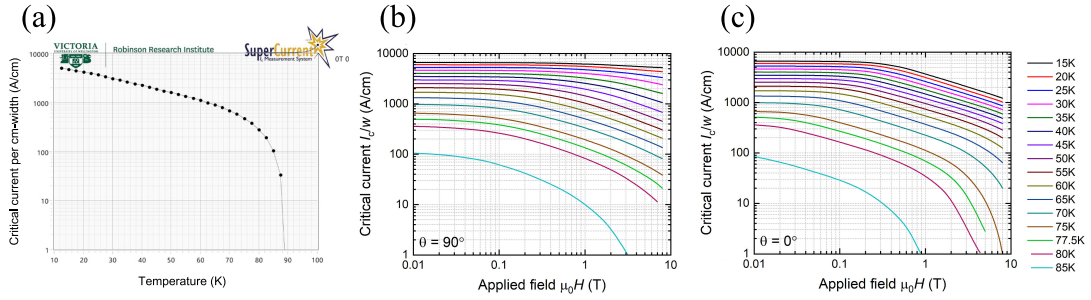


Figure 3.7: Temperature and magnetic field dependence of critical current for SuperOx tape
(a) temperature (b) parallel field (c) perpendicular field [195]

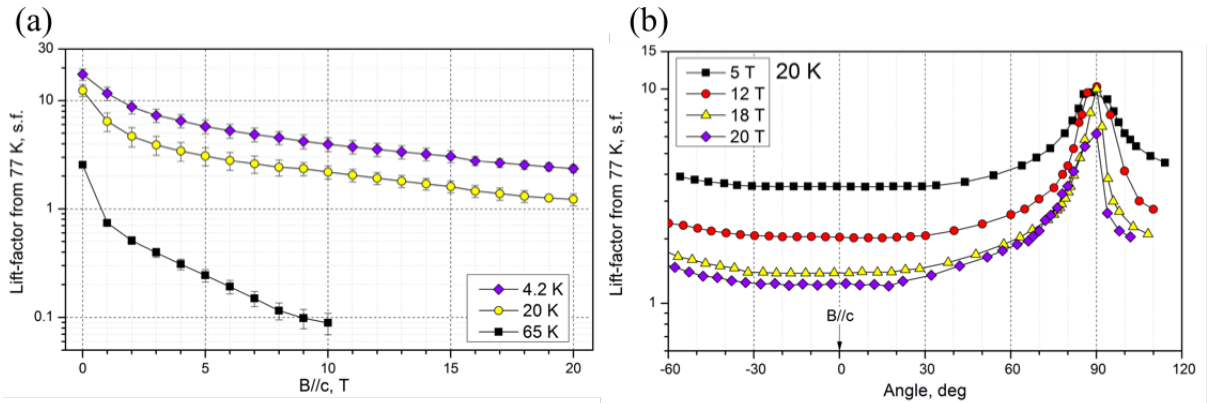


Figure 3.8: Critical current data for fields up to 20 T for SuperOx tape

(a) perpendicular field (b) magnetic angle

Measurements performed at NHMFL, University of Geneva, Tohoku

University, RRI, Lebedev Physics Institute

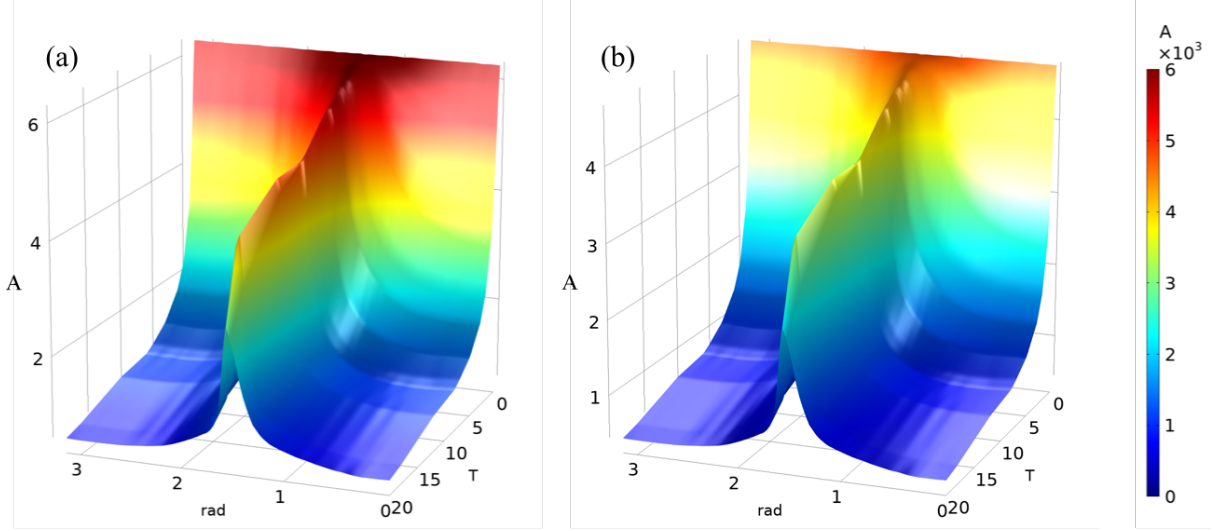


Figure 3.9: Critical current characteristic of SuperOx tape
(a) 25 K and (b) 35 K

For fields above 10 T, data points and corresponding lifting factors were provided by SuperOx for up to 20 T as shown in Figure 3.8. The surface profiles of the critical current at different temperatures were calculated using lifting factors that can be obtained from the measured temperature dependence Figure 3.8.

By employing interpolation techniques, it is possible to generate the critical current characteristic depicted in Figure 3.9 using the available data points. However, due to the limited availability of data points above 10 T, the interpolated dataset does not exhibit the same level of smoothness as the SuperPower data.

3.5 Discussion

In this chapter, the methodology of the **T-A** formulation in 2D and 3D was presented. In addition to the thin strip approximation, the homogenisation or multi-scale approach can be used to further improve the efficiency of the formulation as presented in section 3.2. This is especially advantageous when modelling large numbers of turns. The validity of the numerical model has been shown in a variety of different studies (section 3.3), by comparison with the widely established **H** formulation. Different orders of the dependent state variables as well as all the advanced modelling approaches using the **T-A** formulation have been compared and evaluated in terms of their accuracy and speed. The homogenised **T-A** formulation proves to be the best approach to achieve a high level of accuracy while at the same time reducing the simulation time.

To improve the accuracy of the simulation, field-dependent critical current characteristics need to be considered. For the investigation of the loss behaviour of HTS in chapter 4, the mathematical formulation presented in section 3.4.1 will be used. This expression is supported by experimental data and has been used to validate the 3D homogenisation method using the $\mathbf{H}$ formulation. The studies of the PFC and TFC systems (chapters 5 and 6) will compare the critical current of the tape manufacturers SuperPower and SuperOx. Due to the missing critical current data at high fields for the SuperPower tape, an extrapolation based on the formulation presented in [193] is used. In the case of the SuperOx tape, the tape manufacturer provided additional data points for the critical current at high fields.

The simulations in this thesis have been performed with the AC/DC module of the commercially available FEM software COMSOL Multiphysics 6.5. Furthermore, MATLAB R2017b was used for the 1D calculations of a single HTS tape using the integral approach outlined in [126], to validate the numerical approach of the FEM software.

All the calculations were performed with a 3.20 GHz Intel Core i7-8700 computer with 12 logical processors and 32 GB of RAM.

4 Loss investigation

4.1 Introduction

In this chapter the loss behaviour of a simplified HTS coil is investigated numerically using the **T-A** formulation and comparing the results with other formulation such as the **H** formulation and an analytical solution of the Brand model. As previously discussed in section 2.4.1, the Bean model is described for the case of either transport current or magnetic field. Although the Bean model provides a basis for the analysis of current and magnetic field distribution within the superconductor, it operates under the assumption that the critical current density is independent of the magnetic field at a constant temperature, for single tapes and simple configurations it can predict the behaviour sufficiently. In high-field coils, a transport current is applied, which will produce a magnetic field. Therefore, the HTS tape experiences the external magnetic field B_e , coming from the surrounding tapes and pancakes of the coil configuration and in addition creates a self-field B_i . Both, the induced and external magnetic fields influence the current and magnetic field distribution within the superconductor, which in turn will influence the loss behaviour and is investigated in this chapter.

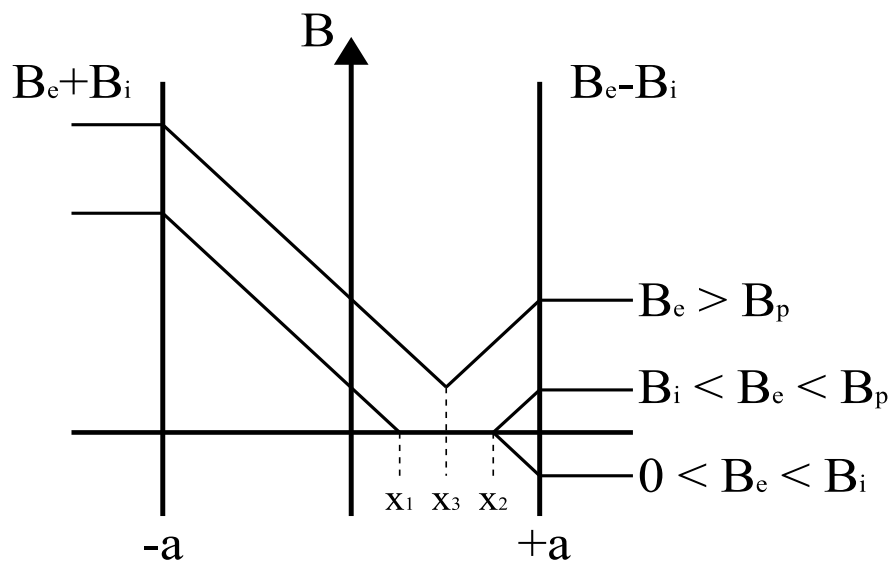


Figure 4.1: Magnetic field distribution inside an infinite slab with transport current in external magnetic field [196]

Considering the infinite slab from the Bean model with a width of a , three distinct cases can be derived (Figure 4.1). In the case, where $0 < B_e < B_i$ the flux front is shifted from the centre of the tape, but the magnetic field distribution resembles the case of a transport current. If $B_i < B_e < B_p$, the magnetization from the external field is larger than the induced field, resembling the is larger than the full penetration field $B_e > B_p$, the magnetic field will be non-zero across the tape. The points x_1 and x_2 mark the positions, where the magnetic field is zero, while x_3 denotes the lowest point of the magnetic field distribution when the external field exceeds the penetration field. These points can be calculated by [196]:

$$x_1 = (b + I - 1)a \quad (4.1)$$

$$x_2 = (1 - |b - I|)a \quad (4.2)$$

$$x_3 = Ia \quad (4.3)$$

Here b is defined as $b = B_e/B_p$ and $I = B_i/B_p = I_t/I_c$, and the critical current is assumed to be constant. It is well known that the critical current is not independent of the magnetic field, as discussed in section 2.4. Especially the influence of the perpendicular magnetic field will limit the current carrying capabilities of the coil significantly.

The losses for a single tape and simple HTS configurations can be mathematically derived and calculated. These calculations consider three cases as shown with the Bean model, 1) a tape carrying only transport current, 2) a tape in a perpendicular magnetic field, and 3) a tape carrying transport current in a perpendicular magnetic field. These three cases are investigated below to evaluate the influence of high magnetic fields on the current and magnetic field distribution as well as the loss behaviour of HTS tapes and pancakes. The case of a tape or pancake carrying transport current in a perpendicular field is intended to resemble the behaviour of a tape within a coil configuration, which will carry transport current and experience the magnetic field from the surrounding tapes and pancakes.

Comparable studies have been conducted for two-ply-bundle conductors using a constant critical current [197] and a field-dependent critical current [198]. Here, the investigated HTS tape has a width of 4 mm and a thickness of 1 μm and the field-dependent critical current from equation (3.20) is used. The resistivity of the tape is modelled using the **E-J** power law with an n -value of 30. The transport current can be applied by the boundary conditions and the magnetic field for a single tape can be directly implemented on the outer boundary of the free

space. However, in the case of a single pancake, defining the perpendicular field on the outer boundary leads to instabilities in the calculation and therefore, a source is defined far away from the superconducting region. The source generates a magnetic field with the desired strength at the conductor, ensuring that the degradation of the magnetic field remains within an error margin of less than 1%, relative to the distance from the source.

4.2 Single tape

Here, the case of a single HTS tape is investigated under three different conditions trying to mimic the conditions in a larger coil assembly. First, the transport current in a single HTS tape is investigated when increased up to the critical current. Then, the tape is exposed to an external perpendicular field much larger than the full penetration field. Finally, the transport current is ramped up to the critical current while simultaneously exposing the tape to a magnetic field in perpendicular direction.

All cases are simulated with the **T-A** and **H** formulation using a constant critical current as well as a field-dependent critical current expression and are compared to the theoretical solution calculated with the method presented [126] in section 2.5.1. This method was adapted to simulate the ramping processes for the different cases and evaluate the losses.

In the evaluation, the current distribution is shown as a percentage of the critical current for all methods. In the case of a single tape, the solid line represents the theoretical approach and is shown in all figures as a reference.

4.2.1 Transport current

The critical current of the tape under self-field conditions is 160 A, with a full penetration field of 0.05 T. In this study, a ramp current up to the critical current is applied linearly within 1 s. The figure below presents the distribution of the normalized current density J/J_c and the perpendicular magnetic field across the superconductor at different percentages of the critical current. The left side compares the theoretical approach with the FEM results using a constant critical current, while the right-hand side presents the results for a field-dependent critical current. In all figures, the solid lines represent the results from the theoretical formulation. As the current is increased, the magnetic self-field starts to penetrate the tape from the edges. The flux fronts move towards the centre of the tape and upon reaching the critical current, the

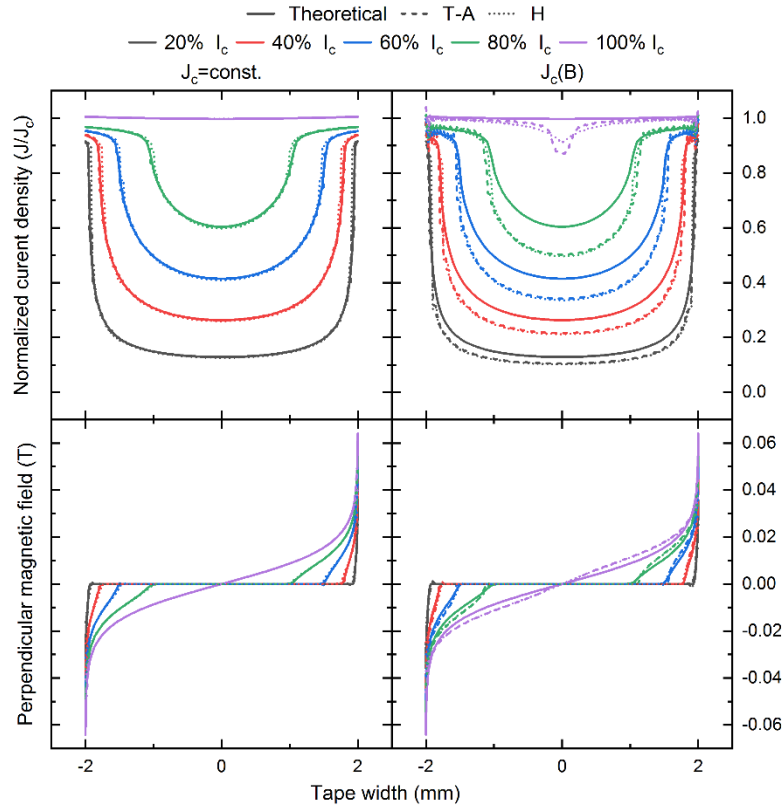


Figure 4.2: Normalized current density distribution and perpendicular magnetic field distribution in a single HTS tape carrying transport current. The left side shows the results for constant critical current, and the right side uses a field dependent critical current.

current is distributed throughout the superconductor. The magnetic field at lower currents is still expelled from the interior of the tape, but as the flux fronts move closer towards the centre, the field is fully penetrating the HTS.

Figure 4.2 demonstrates that the current and magnetic field distribution along the tape is identical for all approaches when using a constant critical current. However, a more dynamic distribution of the current along the tape can be observed when field dependency is considered. This indicates that the central regions of the tape possess a higher critical current than the edges of the tape, which is a more realistic representation.

From the current and electric field within the superconductor, the instantaneous loss can be integrated. This loss distribution over the normalized current is shown in Figure 4.3. At currents below 60 % of the critical current, the penetration is slow since the Lorentz force is not strong enough to move the flux vortices. As the current increases closer to the critical current, the

Table 4.1: Dissipated energy and power of a single HTS tape during transport current ramp to $I=I_c$

Method	Dissipated energy (J)	Dissipated power (W)
Theoretical	1.18e-3	16.05e-3
T-A formulation $J_c=\text{const.}$	1.25e-3	16.04e-3
T-A formulation $J_c(\mathbf{B})$	1.21e-3	10.6e-3
H formulation $J_c=\text{const.}$	1.26e-3	16.02e-3
H formulation $J_c(\mathbf{B})$	1.31e-3	13.87e-3

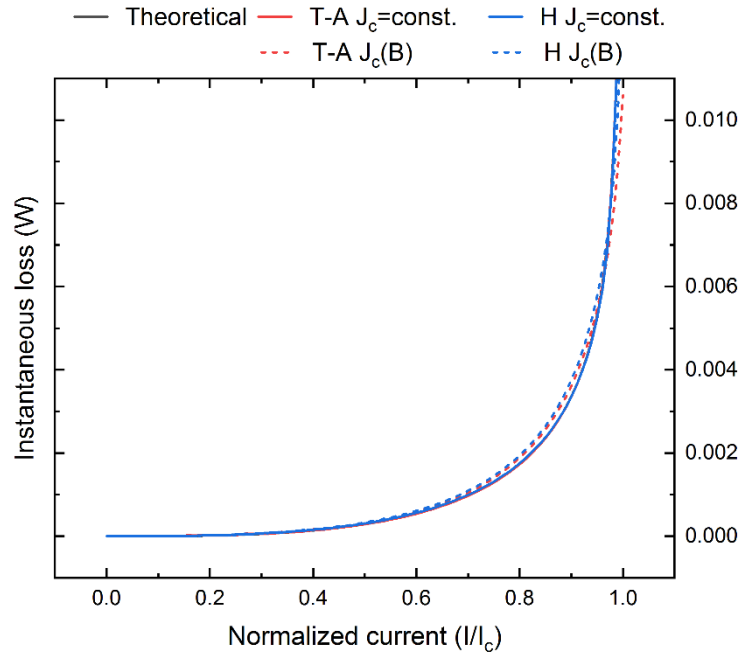


Figure 4.3: Instantaneous loss for a single HTS tape carrying transport current shown over the normalized current

losses rise rapidly because the pinning force can no longer prevent the flux from moving through the lattice.

There is no notable difference in the instantaneous loss, when comparing the different formulations. The values for the dissipated energy during ramping of the transport current up to the critical current can be taken from Table 4.1 below. The **T-A** and **H** formulation shows a very good agreement when using a constant critical current and when field dependency is considered. A difference can be observed in the dissipated power at the end of the ramp phase where the **T-A** formulation for variable critical current density has the lowest losses at the end.

4.2.2 Perpendicular magnetic field

Here, the magnetization in a perpendicular field is investigated for a single tape. Using the full penetration field as a reference, the perpendicular field is increased linearly over 1 s up to $0.5 B_p$, B_p , $2 B_p$ and $4 B_p$. The normalized current density and perpendicular magnetic field along the tape are evaluated in Figure 4.4, for the theoretical approach, **T-A** and **H** formulation. An external magnetic field will induce a persistent current in the superconductor, which is limited by $\pm J_c$. With an increase in the external magnetic field, the flux fronts will move towards the centre of the tape. The induced current penetrates the tape from the edges although in opposing directions, until the full penetration field is reached, and the current is uniformly distributed in each direction.

As the magnetic field is increased the induced current will penetrate towards the centre of the tape allowing the magnetic field to penetrate the superconductor. Once the full penetration field

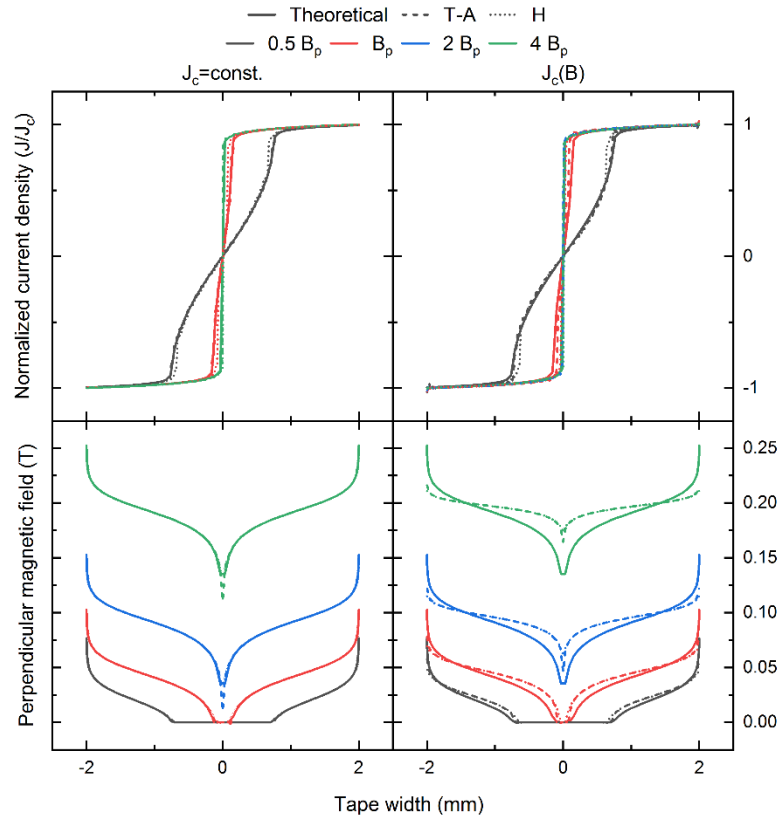


Figure 4.4: Normalized current density distribution and perpendicular magnetic field distribution in a single HTS tape in an external magnetic field in perpendicular direction. The left side shows the results for constant critical current, and the right side uses a field dependent critical current.

Table 4.2: Dissipated energy and power of a single HTS tape during magnetization up to $4 B_p$

Method	Dissipated energy (J)	Dissipated power (W)
Theoretical	28.10e-3	7.87e-3
T-A formulation $J_c=\text{const.}$	28.11e-3	7.87e-3
T-A formulation $J_c(\mathbf{B})$	16.69e-3	3.03e-3
H formulation $J_c=\text{const.}$	28.20e-3	7.87e-3
H formulation $J_c(\mathbf{B})$	16.76e-3	3.03e-3

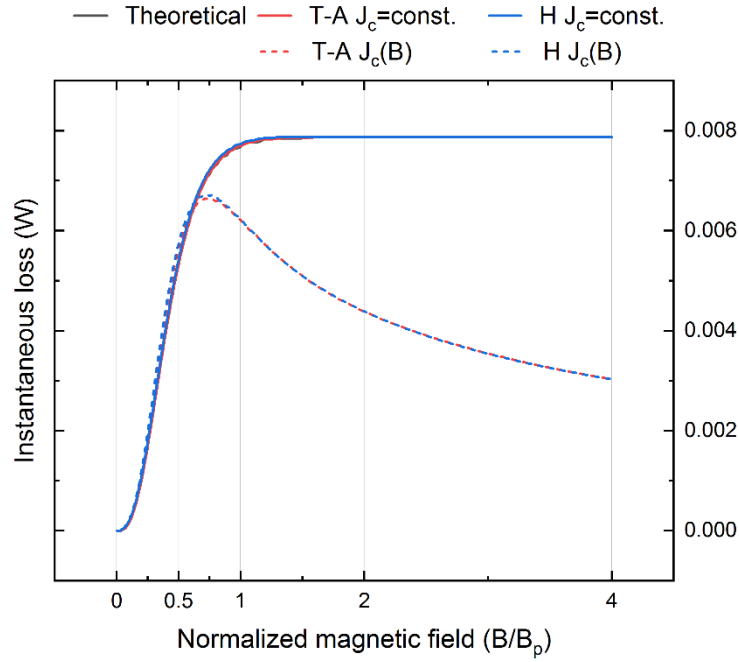


Figure 4.5: Instantaneous loss for a single HTS tape in an external magnetic field in perpendicular direction over the normalized magnetic field

is reached, the magnetic field is now going through the superconductor and the current fronts are fixed at the centre of the tape. When the external magnetic field is increased further up to $4 B_p$, the only change is the increase in magnetic field along the tape. Notably, the field dependence in the case of an external magnetic field in perpendicular direction has an impact on the distribution of the magnetic field, especially when the field is higher than $2 B_p$. While the flux fronts are fixed at the centre the increased magnetic field at $4 B_p$ shows that the field in the centre of the tape is only marginally lower than at the edges.

Below, the losses for the magnetization up to $4 B_p$ are shown for all simulation cases (Figure 4.5). The numerical approaches using a constant critical current are identical to the theoretical

formulation. The losses will increase for $B < B_p$ and after the tape is fully magnetised, the losses stay constant. Since the losses in a superconductor are generated by time-varying changes in the flux flow, a maximum should be reached when the tape is fully penetrated. Conversely, when field dependency is considered, the instantaneous losses will reach a peak at the full penetration field (indicated by the vertical lines) and then start to decrease even if the field is further increased.

When a high-temperature superconducting (HTS) tape is subjected to an external magnetic field, the hysteresis loss exhibits a distinct behaviour characterized by an initial increase, peaking at the full penetration field, followed by a subsequent decrease as the field continues to rise. Initially, as the magnetic field increases, vortices penetrate the superconductor and interact with pinning centres, resulting in increased hysteresis loss due to the energy dissipation associated with vortex motion. This phase continues until the full penetration field is reached, at which point the superconductor is fully magnetized, and vortices are uniformly distributed throughout the material. Beyond this point, further increases in the external magnetic field lead to a saturation of the pinning centres, resulting in reduced effectiveness of vortex pinning. Consequently, the critical current density decreases, and the additional vortices experience diminished resistance to their motion, leading to a reduction in incremental energy dissipation.

The dissipated energy and power during the increase of the perpendicular magnetic field up to $4 B_p$ is shown in Table 4.2. The total power loss, when considering a field-dependent critical current, is 40% lower than the results using a constant critical current indicate. When the external field reaches $4 B_p$, the dissipated power is reduced by 60% if field dependency is used.

4.2.3 Transport current and perpendicular magnetic field

When simulating a tape carrying a transport current in an external magnetic field one must consider the reduction of the critical current due to the magnetic field. Therefore, the critical current of the tape at the selected magnetic fields is evaluated below. With increasing external magnetic field, the critical current of the tape decreases as shown in Table 4.3.

The same evaluation of the normalized current density and a perpendicular magnetic field is compared below for all formulations when the transport current is ramped up to the critical current at different external magnetic fields. In the previous cases, when only the transport current or magnetic field was considered, the current as well as the magnetic field penetrated

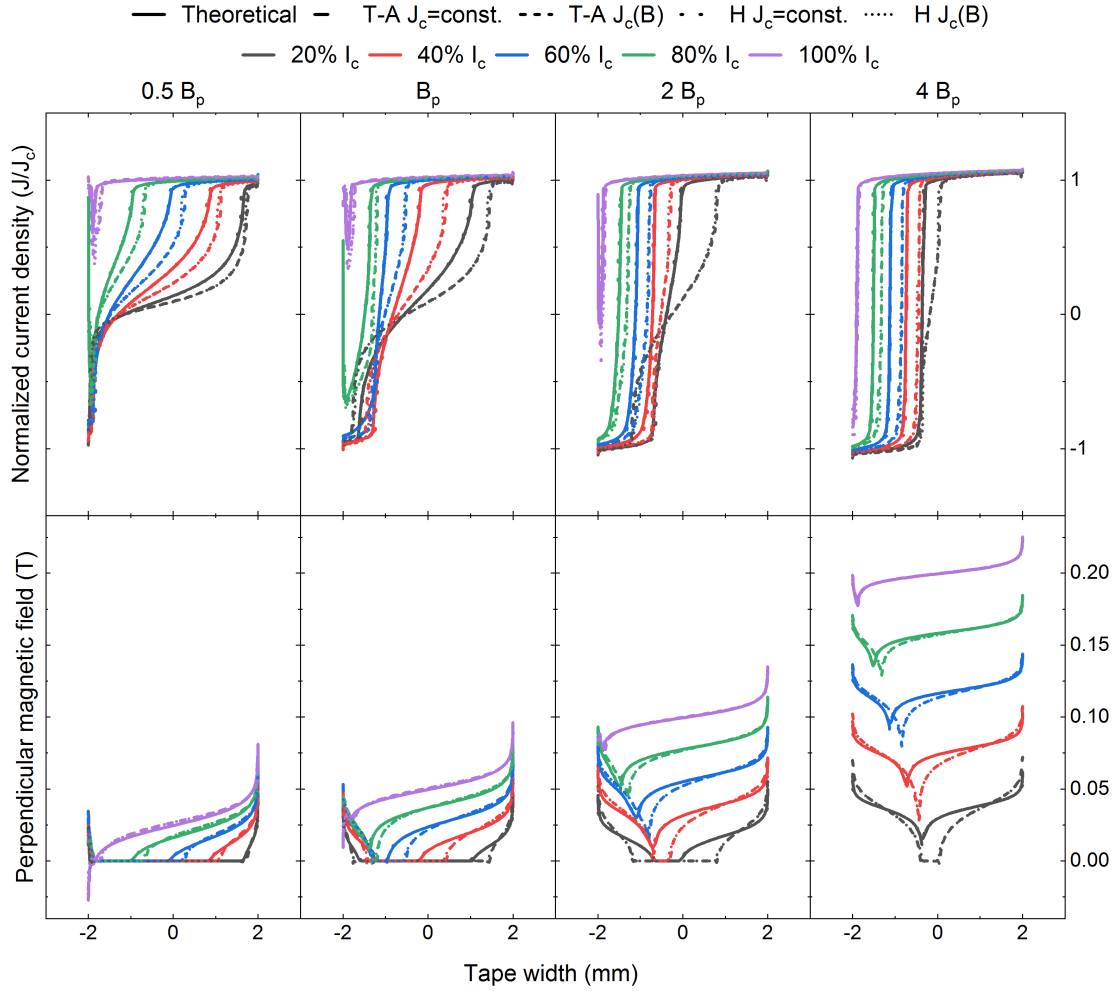


Figure 4.6: Normalized current density distribution and perpendicular magnetic field distribution in a single HTS tape carrying transport current in an external magnetic field in perpendicular direction

towards the centre of the tape. If both are applied simultaneously, the magnetic field induces a persistent current within the superconductor, while the transport current flows in only one direction. This results in a shift of the current penetration towards the edge of the tape. At the same time, the flux fronts of the magnetic field are also distributed closer to the edge of the tape. This current profile and magnetic field distribution for external magnetic field up to $4 B_p$ and transport currents up to the critical current is shown in Figure 4.6.

Table 4.3: Critical current of a single HTS tape for different external magnetic fields

	$0.5 B_p$	B_p	$2 B_p$	$4 B_p$
I_c (A)	138	112	84	60

Table 4.4: Dissipated power of a single HTS tape at the end of transport current ramp in perpendicular field

Method	Dissipated power (W)			
	0.5 B_p	B_p	2 B_p	4 B_p
Theoretical	17.95e-3	18.86e-3	21.94e-3	27.34e-3
T-A formulation $J_c=\text{const.}$	18.04e-3	18.88e-3	21.96e-3	27.39e-3
T-A formulation $J_c(\mathbf{B})$	21.02e-3	21.39e-3	23.88e-3	28.61e-3
H formulation $J_c=\text{const.}$	17.74e-3	18.51e-3	21.63e-3	26.94e-3
H formulation $J_c(\mathbf{B})$	20.72e-3	20.99e-3	23.60e-3	28.47e-3

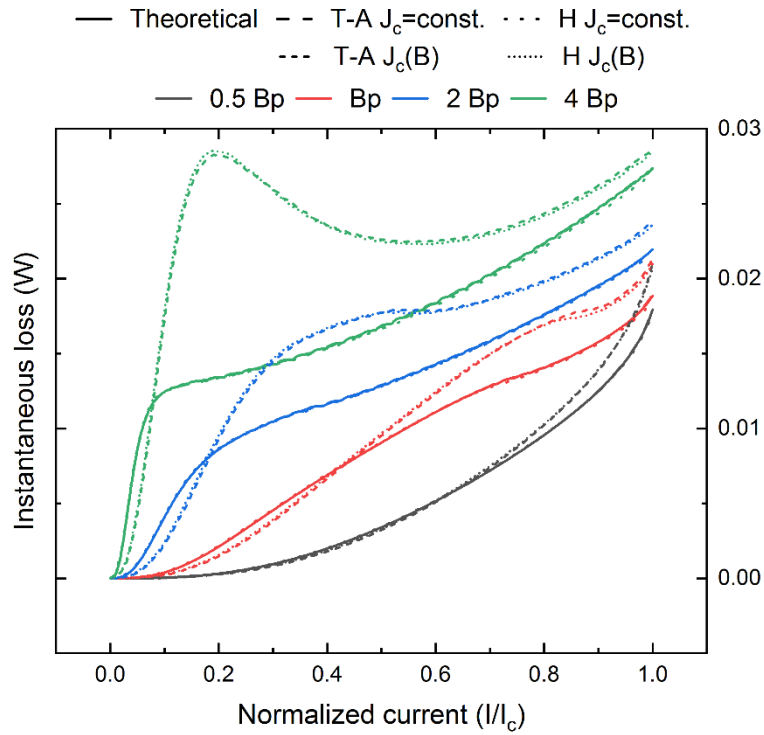


Figure 4.7: Instantaneous loss for a single HTS tape carrying transport current in an external magnetic field in perpendicular direction over the normalized current

If the HTS is subjected to a perpendicular field of $0.5 B_p$ with an increase in transport current, the induced current by the magnetic field is much lower than the applied current. Therefore, the flux fronts gradually move towards the edge of the tape. As the magnetic field is increased, less current can be applied and the magnetic field induces a much larger persistent current. This results in a current penetration at the centre of the tape when only 20% of the critical current is applied. After this point, the increasing transport current generates a high Lorentz force, moving the flux fronts towards the edge of the tape.

Table 4.5: Dissipated energy of a single HTS tape during transport current ramp in perpendicular field

Method	Dissipated energy (J)			
	0.5 B_p	B_p	2 B_p	4 B_p
Theoretical	4.93e-3	8.58e-3	12.56e-3	17.29e-3
T-A formulation $J_c=\text{const.}$	5.02e-3	8.60e-3	12.66e-3	17.41e-3
T-A formulation $J_c(\mathbf{B})$	5.32e-3	9.52e-3	15.10e-3	22.89e-3
H formulation $J_c=\text{const.}$	5.03e-3	8.66e-3	12.62e-3	17.27e-3
H formulation $J_c(\mathbf{B})$	5.37e-3	9.54e-3	15.12e-3	22.87e-3

The corresponding losses are depicted in Figure 4.7. Based on the previous results, if a constant critical current is considered, the losses under the influence of the transport current will start to rise exponentially at high currents. If only an external magnetic field is applied, the losses will stagnate at the point where the magnetic field fully penetrates the superconductor. If the external magnetic field is lower than the full penetration field, the contribution of the magnetic field does not influence the losses much. As the field increases, the losses will increase rapidly, until the full penetration field is reached. After this point, the main contribution to the losses is the increasing transport current.

If a field-dependent critical current is considered, the losses show a good agreement for external magnetic fields below the full penetration field. However, at high perpendicular fields, the losses increase up to the point where the magnetic field reaches B_p , after which the losses decrease. As the transport current is increased further, closer to the critical current, the losses will start to increase again.

The results show good agreement between the different formulations. The main difference is whether field dependency is considered for the critical current. The dissipated energy of each approach is shown in Table 4.5. While the results using constant critical current match the theoretical expression, the field dependency increases the total loss by up to 30% if the external field reaches 4 B_p .

In terms of the dissipated power (Table 4.4) when the current approaches the critical current, the results using a variable critical current show an increase of $\sim 8\%$. Again, the **T-A** and **H** formulation match well with the theoretical results when using a constant critical current, and with each other if field dependency is considered.

4.3 Single pancake

While the case for a single tape is a very simple depiction of the actual behaviour of a coil, the following study will investigate the behaviour of the losses in a pancake made from 50 tapes in an infinitely long domain with an air gap of 0.04 mm between the tapes. The same scenarios for transport current, an external field in perpendicular direction and transport current in a perpendicular magnetic field are investigated. It should be mentioned that the theoretical formulation will no longer be used as a reference as this method can only be used for the evaluation of a single tape. By establishing that the results of the **T-A** and **H** formulation show a good agreement, only the **T-A** formulation using constant critical current, and field-dependent critical current profile are investigated. Two different approaches, the tape model and the homogenised model are used to compare the accuracy of the results.

4.3.1 Transport current

Although the magnetic field can be increased by stacking HTS tapes into a single pancake, this also reduces the critical current. The critical current of 100 A of the pancake is defined by the weakest turn, which is the outermost turn. In Figure 4.8 the current profiles and magnetic field distribution at 30 %, 60 % and 90 % of the critical current are shown in the weakest turn.

In a pancake configuration, the current penetration starts at the corners of the outermost tapes and gradually penetrates towards the inside. The pancake is considered to be fully penetrated after the weakest turn reaches the critical current, at which point the tapes in the centre of the pancake are still superconducting, well below their critical current. Once the weakest turn loses its superconducting state, the pancake coil could be damaged.

As the transport current increases, the current gradually penetrates towards the centre. Like in the case of a single tape, the field dependency of the critical current influences the current profile, indicating a lower critical current at the centre of the tape. The magnetic field distribution along the tape shows a good agreement for both approaches. When the weakest turn, and therefore the pancake, is fully penetrated, the full penetration field can be evaluated at the edges of the centre of the configuration. Here, the full penetration field corresponds to 0.13 T.

Table 4.6: Dissipated energy and power of a single HTS pancake during transport current ramp to $I=I_c$

Method	Dissipated energy (J)	Dissipated power (W)
T-A tape $J_c=\text{const.}$	203.98e-3	708.26e-3
T-A tape $J_c(\mathbf{B})$	242.96e-3	983.93e-3
T-A homogenised $J_c=\text{const.}$	203.92e-3	675.32e-3
T-A homogenised $J_c(\mathbf{B})$	240.90e-3	934.62e-3

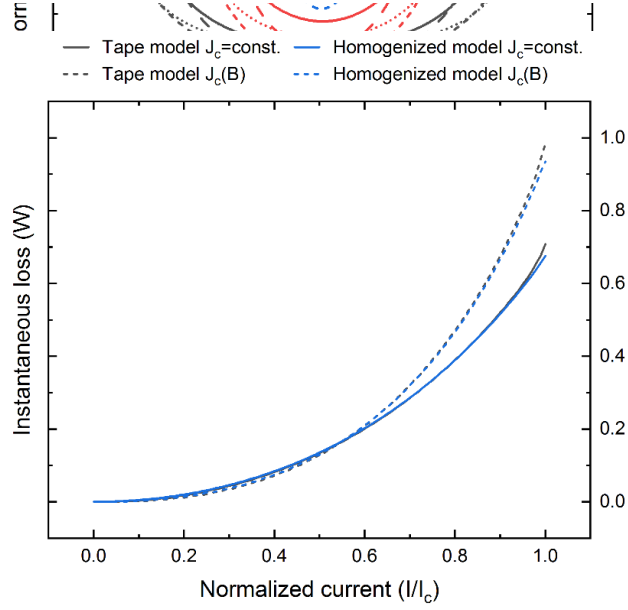


Figure 4.9: Instantaneous loss for a single HTS pancake carrying transport current over the normalized current

Figure 4.9 shows the instantaneous loss for the whole pancake using the tape and homogenised model. While for a single tape, the field dependency did not have a major impact on the behaviour of the losses, the intricate tape-tape interactions of a pancake resulted in an earlier increase in the losses. This behaviour is also shown when comparing the dissipated energy during the ramp phase (Table 4.6). Here, the field dependency accounts for an increase of approximately 20% of the total loss. When looking at the dissipated power, the formulations using a constant critical current have a $\sim 30\%$ lower power loss when the current reaches I_c in the weakest turn.

4.3.2 Perpendicular magnetic field

The evaluated full penetration field of 0.13 T is used as a reference to apply an external field of up to $4 B_p$. As the flux fronts are moved towards the centre, the induced current fronts are

established at the centre with current moving in a persistent loop. This process is shown in Figure 4.10, where the formulations are in good agreement. At fields lower than the full penetration field, the current fronts are moving faster towards the centre if field dependency is considered. As for the magnetic field distribution, the results with a constant critical current show a lower magnetic field strength across the weakest turn, especially if the external field is larger than B_p .

Similar to the case of a single tape, the evaluation of the instantaneous loss for simulations conducted with the constant critical current shows that the losses stay constant after reaching the full penetration field. On the other hand, if a field-dependent critical current is used, then the losses increase rapidly until the full penetration field is reached, and afterwards decline. This is shown in Figure 4.11 below. Notably, the results obtained with the homogenised and tape model match very well. Table 4.7 presents the dissipated energy as well as the dissipated power when the external field is increased up to $4 B_p$. In the case of a field-dependent critical current, the error between the tape and homogenised model in terms of both, total loss and

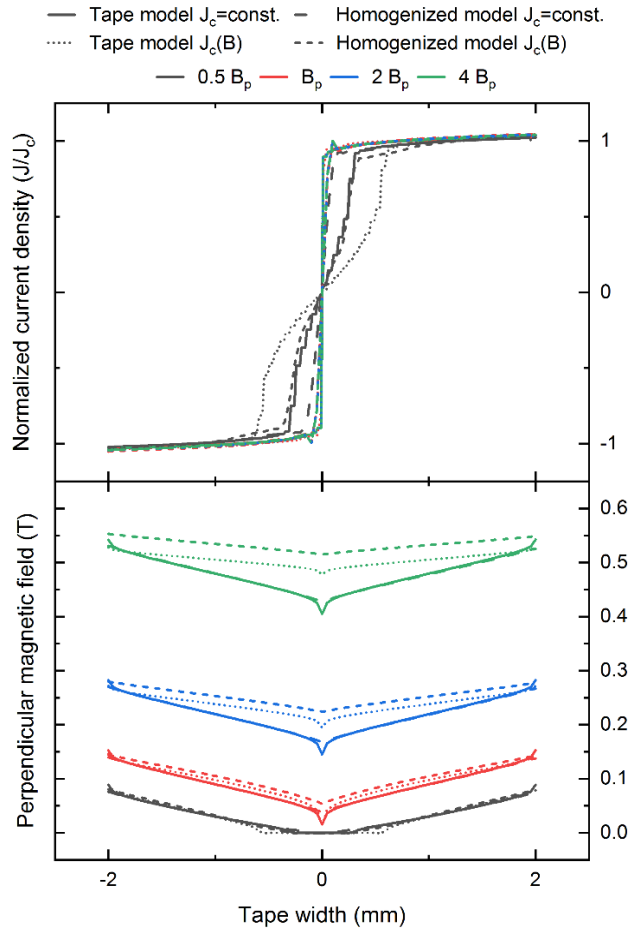


Figure 4.10: Normalized current density distribution and perpendicular magnetic field distribution in a single HTS pancake in an external magnetic field in perpendicular direction

Table 4.7: Dissipated energy and power of a single HTS pancake during magnetization up to $4 B_p$

Method	Dissipated energy (J)	Dissipated power (W)
T-A tape $J_c = \text{const.}$	2.18556	0.65970
T-A tape $J_c(B)$	1.42607	0.24911
T-A homogenised $J_c = \text{const.}$	2.19614	0.66186
T-A homogenised $J_c(B)$	1.48395	0.25518

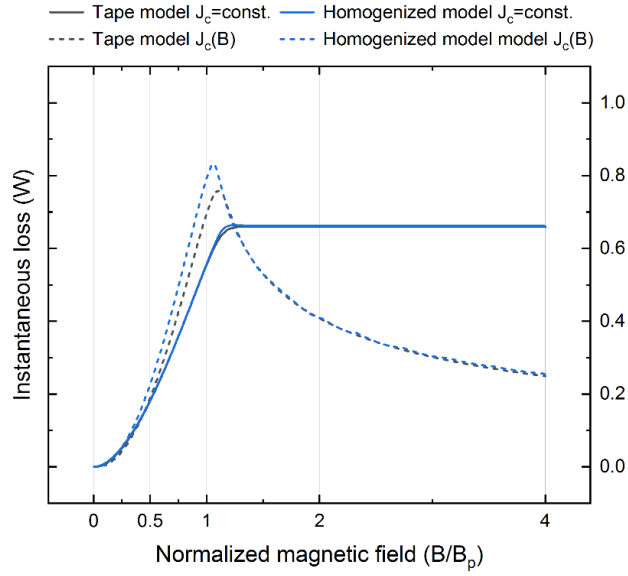


Figure 4.11: Instantaneous loss for a single HTS pancake in an external magnetic field in perpendicular direction over the normalized magnetic field

dissipated power at the end is $\sim 3\%$. When comparing the influence of a variable critical current to a constant assumption, there is a discrepancy of 35%.

4.3.3 Transport current and perpendicular magnetic field

The final case of this study is aimed to imitate the electromagnetic behaviour of a large-scale, high-field coil made from stacked HTS pancakes. The coil generates a self-field, which can be much higher than the full penetration field of the single pancake. Therefore, an external magnetic field is applied to simulate the influence of high fields on the current profile and magnetic field distribution in the weakest turn.

At first, the critical current of the pancake needs to be evaluated at the selected external magnetic fields. The critical current for each field strength is shown in Table 4.8. It can be seen

that increasing the magnetic field from $0.5 B_p$ to $4 B_p$ reduces the current carrying capabilities by 61%.

Using the evaluated critical current of the weakest turn as the maximum transport current for the respective magnetic field strength, the current profiles and magnetic field distribution are evaluated and shown in Figure 4.12. Similar to the case of a single tape, the current gradually penetrates towards the edge of the tape when the external field is below the full penetration field. At magnetic fields higher than B_p , the current fronts are established earlier, with a persistent current running in opposing directions. As the transport current increases, the flux fronts are moving towards the edge of the tape.

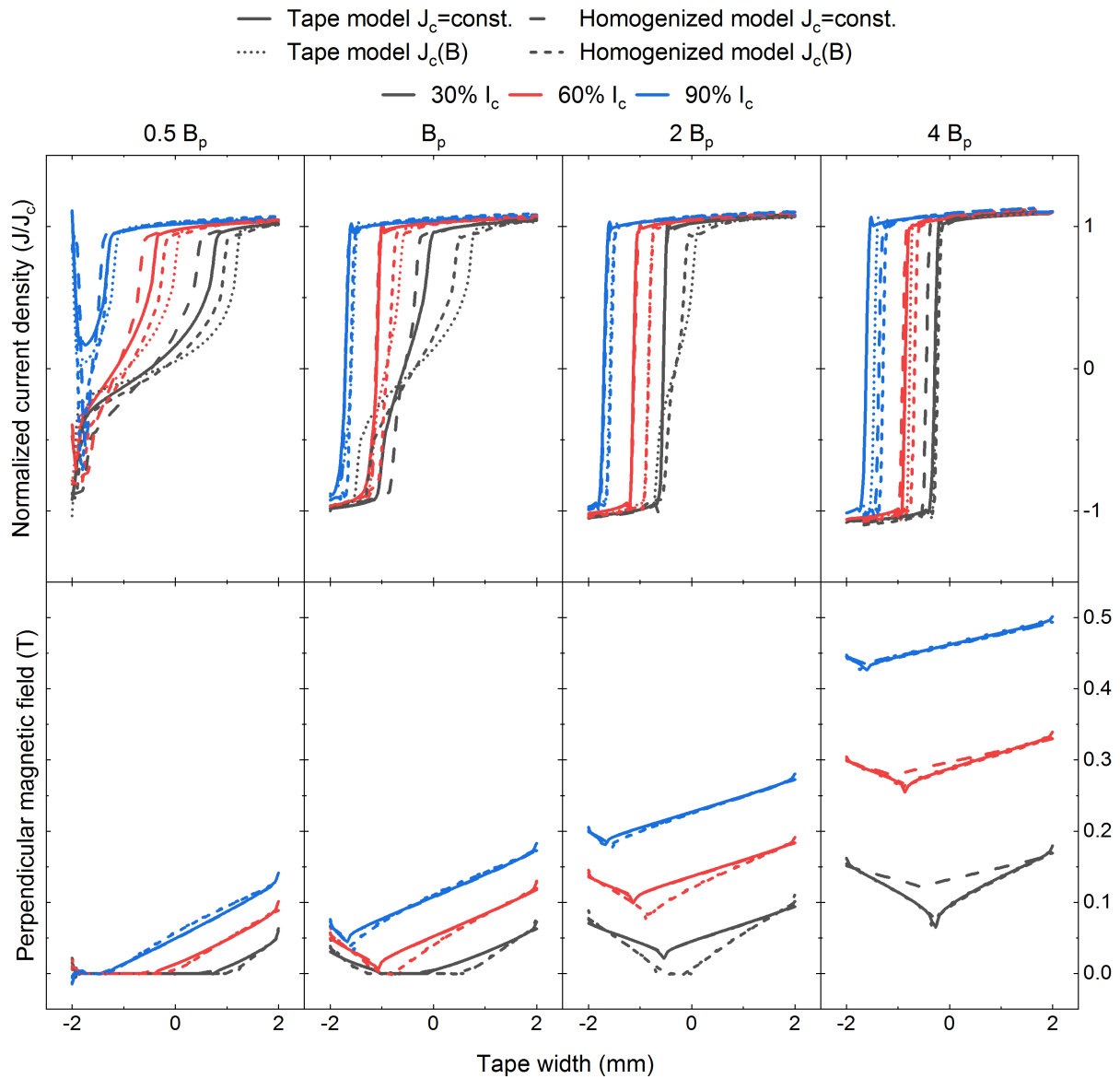


Figure 4.12: Normalized current density distribution and perpendicular magnetic field distribution in a single HTS pancake carrying transport current in an external magnetic field in perpendicular direction

Table 4.8: Critical current of a single HTS pancake for different external magnetic fields

	0.5 B_p	B_p	2 B_p	4 B_p
I_c (A)	93	74	52	36

The resulting instantaneous loss of the whole pancake is shown in Figure 4.13. At low fields, the transport current is the driving force in the penetration of the pancake. With the flux fronts gradually moving towards the edge of the tape, the losses increase slowly at first and will then rise exponentially as the transport current approaches the critical current. At high external fields, the vortices are moved rapidly as the field increases until the full penetration field is reached. After this point, the losses decline, as the vortices are pinned down in the defects of the material, and only when the transport current is close to the critical current, the losses start to increase again. If field dependency is not considered, the transport current causes a steady increase in the losses after the full penetration field is established.

While the error in terms of the dissipated energy in the tape model compared to the homogenised model is negligible, the use of a field-dependent critical current has a significant influence. The dissipated energy is shown in Table 4.10, for all approaches and scenarios.

It can be stated that there is no discrepancy in terms of the accuracy between the approaches. The most significant difference can be seen in the variation of the critical current density at high external magnetic fields. If a field-dependent critical current is considered at 4 B_p , the dissipated energy is 30% higher.

Table 4.9: Dissipated power of a single HTS pancake during transport current ramp in perpendicular field

Method	Dissipated power (W)			
	0.5 B_p	B_p	2 B_p	4 B_p
T-A tape $J_c = \text{const.}$	1.0711	1.7003	1.7285	2.0636
T-A tape $J_c(\mathbf{B})$	1.5489	2.4050	2.0295	2.1787
T-A homogenised $J_c = \text{const.}$	1.0415	1.6716	1.7230	2.0732
T-A homogenised $J_c(\mathbf{B})$	1.5052	2.4035	1.9786	2.1852

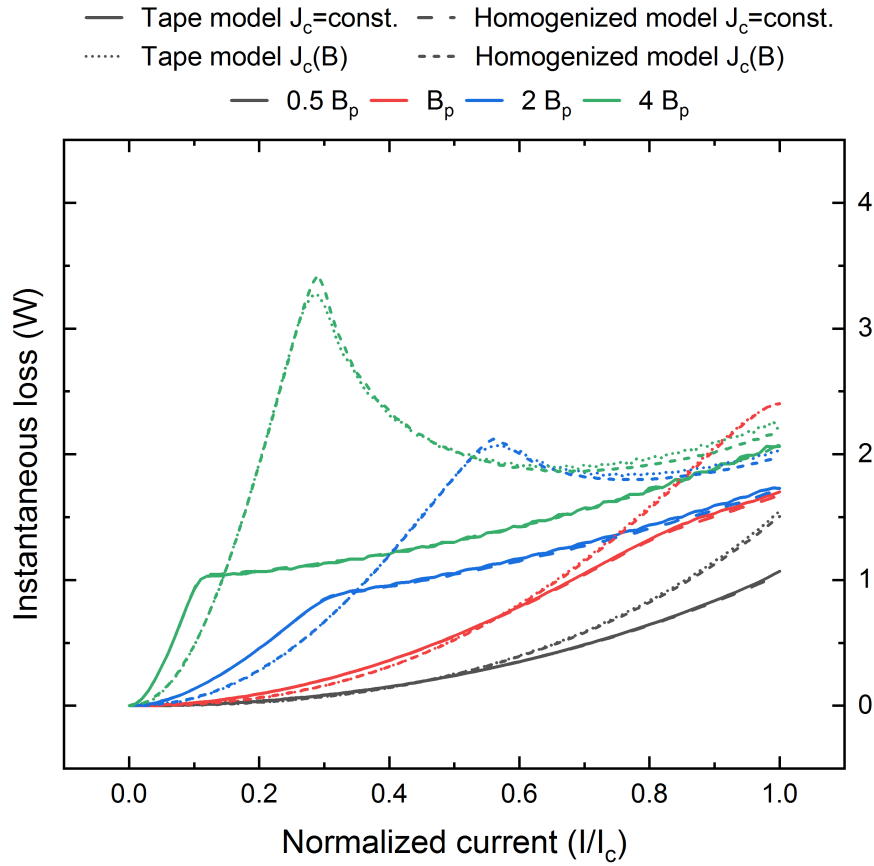


Figure 4.13: Instantaneous loss for a single HTS pancake carrying transport current in an external magnetic field in perpendicular direction over the normalized current

In Table 4.9, the dissipated power at the end of the ramping of the transport current and magnetic field is listed. Although the dissipated energy shows a large discrepancy at higher fields, with regards to the critical current, the difference in dissipated power is more significant if the external field is below the full penetration field. At $0.5 B_p$, the dissipated power at the end is 30% higher if a field-dependent critical current is considered.

Table 4.10: Dissipated energy of a single HTS pancake during transport current ramp in perpendicular field

Method	Dissipated energy (J)			
	$0.5 B_p$	B_p	$2 B_p$	$4 B_p$
T-A tape $J_c=\text{const.}$	0.34006	0.68579	0.98730	1.33070
T-A tape $J_c(\mathbf{B})$	0.42869	0.79941	1.27626	1.89695
T-A homogenised $J_c=\text{const.}$	0.33755	0.67842	0.97409	1.33087
T-A homogenised $J_c(\mathbf{B})$	0.42126	0.79310	1.26239	1.88453

4.4 Discussion

In this chapter an investigation of the behaviour of a simplified large electromagnetic coil wound from HTS tapes was conducted. HTS coils carry a transport current and are simultaneously exposed to the self-field created by the magnet. As the whole coil usually consists of multiple stacked pancakes, every single pancake will experience a magnetic field from the neighbouring pancakes. This behaviour is replicated when simulating a single pancake carrying transport current in an external field. To further break down the investigation, the single tape with transport current will experience the magnetic field generated by the tapes within the pancake. The investigation showed that the full penetration field of a single tape is significantly lower than for a pancake since the current can be distributed among the tapes. Similarly, the full penetration field of a coil will be higher than for a single pancake due to the screening current shielding the inner turns of the magnet from the magnetic field. While the full penetration field increases in stacked coil configurations, the critical current will decrease as more tapes and pancakes are used. This is due to the higher self-field produced by the magnet. The magnetic field produced by high-field magnets will be higher than the full penetration field and therefore, the investigation was conducted for magnetic field up to $4 B_p$.

Three different scenarios (transport current, external magnetic field, and transport current in an external magnetic field) have been used to simulate the electromagnetic behaviour of a single HTS tape and a single HTS PC. For the single tape, the **T-A** and **H** formulation are compared to the Brandt model to validate the accuracy of the modelling approach. Furthermore, a constant, as well as a field-dependent critical current density, are employed in the FEM models. For all scenarios, the **T-A** and **H** formulations show a good agreement with the Brandt model if a constant critical current is used. The influence of the field-dependent critical current can be observed if an external magnetic field is applied, which is strong enough to exceed the full penetration field of the conductor. While the current and magnetic field distribution across the tape is similar, the resulting losses show a significant difference. After the initial increase of the losses up to the full penetration field, it can be observed that the losses decrease at external fields higher than B_p . If in addition to the external field, a transport current is applied, the losses will only start to increase again when the current approaches the critical current.

For the simulations of a single PC, only the **T-A** formulation is used, but two different modelling approaches are compared: the tape model and the homogenisation method. Again,

two critical current densities are used in the simulation (constant and field-dependent). For all scenarios, the different approaches and critical current densities show a good agreement. The most significant difference can again be observed in the losses when an external field is applied.

As the magnetic field at the HTS approaches the full penetration field, the magnetic flux fronts are moving from the edges towards the centre. The induced current is moving in a loop along the length of the tape resulting in zero net currents. If transport current is applied to the HTS in addition to the external magnetic field, the current moving along the tape will be increased in one direction. Therefore, the current penetration will be higher on one side which results in a shift of the magnetic flux fronts from the centre of the tape towards the edge of the conductor. Especially at fields higher than the full penetration field, the established magnetic and current penetration in the conductor leads to a decrease in the losses until the transport current approaches the critical current. The results of this study can be compared to the results obtained in [197] and [198], which show the same loss behaviour in a two-ply bundle conductor.

The observations of this study will be further examined in the subsequent chapters 5 and 6 by investigating the full-scale PFC and TFC magnet system of an ITER-like Tokamak fusion reactor.

5 Poloidal Field Coils

5.1 Introduction

After investigating the loss behaviour in a simplified coil model in chapter 4, the poloidal field coil system for a large-scale fusion reactor is investigated here. The PFC system is one of the vital components of a Tokamak fusion reactor needed to confine and shape the plasma during different operational scenarios. While the central solenoid and the toroidal field coils provide a static magnetic field, the poloidal field is dynamic and is responsible for shaping and confining the plasma in different plasma modes during the fusion process. The PFCs are located outside the vacuum vessel and provide a magnetic field that is orthogonal to the toroidal magnetic field produced by the toroidal field coils.

Because the primary purpose of the PFCs is to provide a stable and precise magnetic field that shapes the plasma and maintains its position, the design of the PFCs must consider factors such as magnetic field strength, stability, and control, as well as mechanical and thermal stress. This means that the PFCs must be energized with an electrical current to produce a magnetic field of sufficient strength to confine the plasma. The current flowing through the PFCs must be precisely controlled to ensure stable plasma confinement. The magnetic field must also be stable, without any fluctuations or perturbations, to maintain the position of the plasma. The

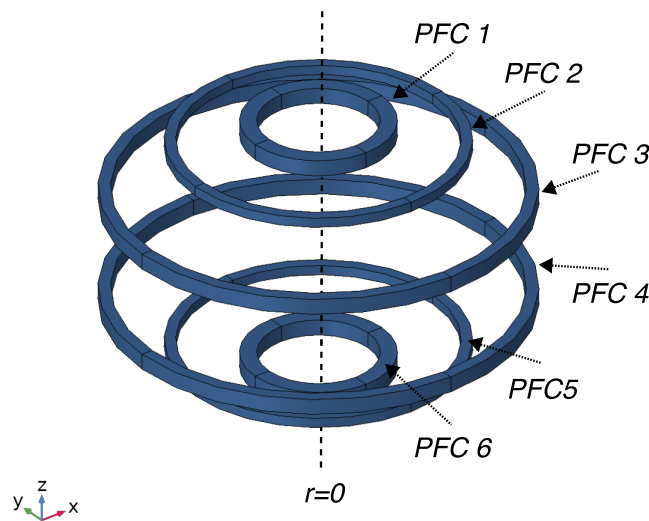


Figure 5.1: Poloidal field coil system of an ITER-like fusion reactor

PFCs must also be designed to withstand mechanical and thermal stresses due to the high magnetic field strength and the plasma's high temperature. The thermal stresses may cause thermal expansion, which can lead to deformation or even failure of the coils. The operating conditions of PFCs depend on the specific Tokamak fusion reactor design.

The design and optimization of the PFC configuration is a crucial step in the development of Tokamak reactors for fusion energy. The first models to evaluate the PFC system requirements involved the derivation of differential equations to calculate the poloidal magnetic field along the cross-section of magnetic field surfaces [199]. Nowadays, models can take into account the local magnetic field on the superconducting strands and the inter-strand contact resistances which depend on the "trajectories" of the strands within the cable [200]. Generally, the procedure for optimizing the design of the PFC in Tokamaks begins with the determination of the desired shape of the plasma, followed by the calculation of the optimal number and locations of the PFCs attached to the toroidal field coils. This is achieved by matching the external magnetic flux that keeps the plasma in magnetohydrodynamic (MHD) equilibrium with the magnetic flux generated by the PFCs at the desired plasma boundary [201].

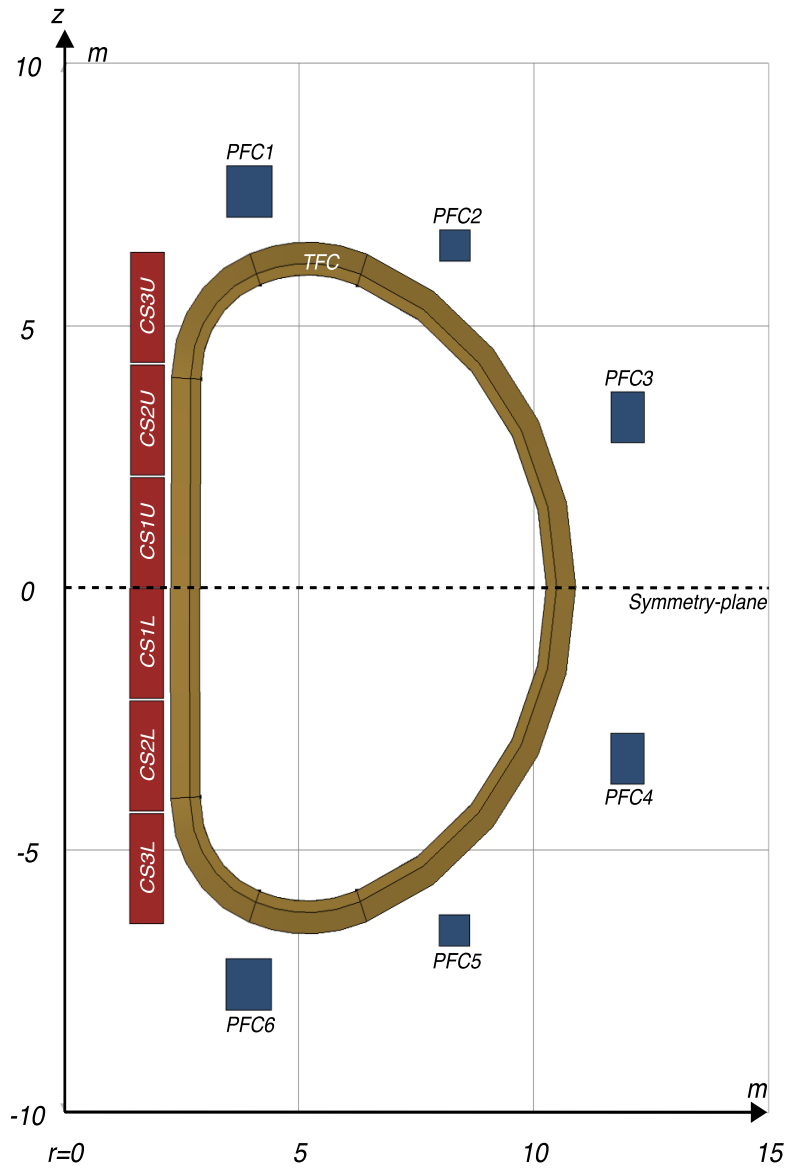
Here, the design of the coil configuration for the poloidal field coils using HTS is investigated, based on the ITER design. The magnets are designed to achieve a target magnetic field at the coil following the requirements for ITER. The final coil design is used to simulate the poloidal field in different operation scenarios and the corresponding losses during each phase are evaluated. For the ramp phase, an additional study is conducted regarding the losses during the initial increase of the current to identify an optimised ramp time.

5.2 Geometry

In this thesis, the available geometric design data for the ITER reactor is used to simulate the PFC system using HTS materials. The ITER magnets utilize superconductors that have a high current-carrying capacity. The PFCs are operated in pulsed mode, and their maximum operating currents vary depending on the coil number. The PFC system consists of six ring coils, PFC 1 through PFC 6, which are responsible for stabilising the position and controlling the shape of the plasma in the Tokamak. PFC 1 and 6 can carry up to 48 kA, PFC 2, 3, and 4 can carry up to 55 kA, and PFC 5 up to 52 kA. The PFCs range in diameter from 8 m to 24 m

Table 5.1: Size and position of PFCs [203]

	R_{centre} (m)	Z_{centre} (m)	ΔR (m)	ΔZ (m)
PFC 1	3.943	7.557	0.968	0.976
PFC 2	8.319	6.530	0.649	0.595
PFC 3	11.997	3.265	0.708	0.966
PFC 4	11.967	-2.243	0.649	0.966
PFC 5	8.395	-6.730	0.820	0.945
PFC 6	4.263	-7.557	1.633	0.976



*Figure 5.2: Cross-section of an ITER-like fusion reactor
Showing the size and position of the CS, TFC and PFCs*

and are all wound with NbTi CICC. The winding pack consists of stacked double pancakes

with joints only on the outer radius to allow for access and repair, as well as to minimize eddy current and AC loss. [202]

The PFCs are located outside the toroidal field coils starting with PFC 1 at the top of the reactor (Figure 5.2). The position of each coil and the corresponding area are shown in Table 5.1.

In the ITER reactor, the coils are not perfectly symmetrically distributed, which will be neglected in this study and therefore, only the upper three PFCs (PFC 1, PFC 2, and PFC 3) are considered. This enables the use of a symmetry condition in the mid-plane of the reactor. The ITER PFC system will be redesigned using HTS to meet the ITER design criteria in terms of their magnetic field strength. The geometric design data was taken from [203] and the location is visualized in Figure 5.2. A detailed report about the PFC and support system of the ITER reactor is presented in [204].

5.3 Model setup

The model is set up in such a way that each PFC rotates around the central axis, exploiting the symmetrical nature of ring coils. Therefore, only the cross-section of each coil needs to be modelled in 2D. In the midplane of the reactor, a perfect magnetic conductor boundary is used

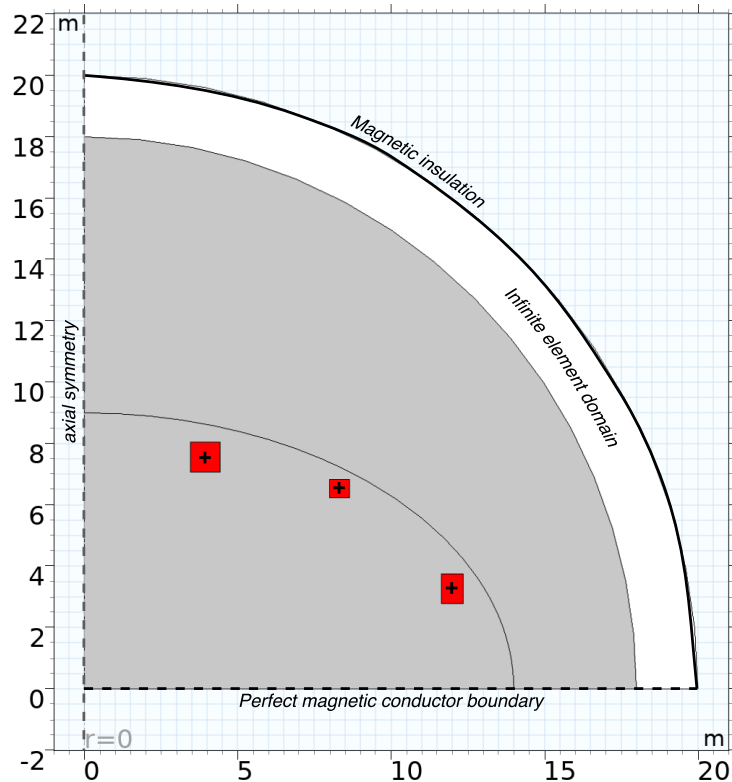


Figure 5.3: PFC model setup in FEM software

to simplify the model further, reducing the need of modelling all six PFCs by assuming a symmetrical distribution. An infinite element domain is added to avoid any retroactive effect of the magnetic field from the magnetic insulation boundary, which encloses the numerical domain. Furthermore, an elliptical line is used to enable a selection of a domain for the evaluation of the magnetic field within the reactor. All boundary conditions for the global domain, including the direction of the applied transport current are indicated in Figure 5.3.

5.4 Mesh

For an efficient simulation, it is crucial to define a mesh with as few elements as possible, while still retaining a high resolution in the areas of interest. Therefore, the HTS domains of each PFC and the small air gaps in between the PCs are meshed with structured, quadrilateral elements. Along the width of the pancake, 20 elements are distributed logarithmically to generate a denser mesh at the edges of the pancake, where the magnetic field will start to penetrate the coil. The height of each pancake features 60 elements while the air gaps will be meshed with 6 elements. In the infinite element domain on the outside of the geometry, 5 elements along the width are chosen to avoid numerical artefacts. The remaining geometry is

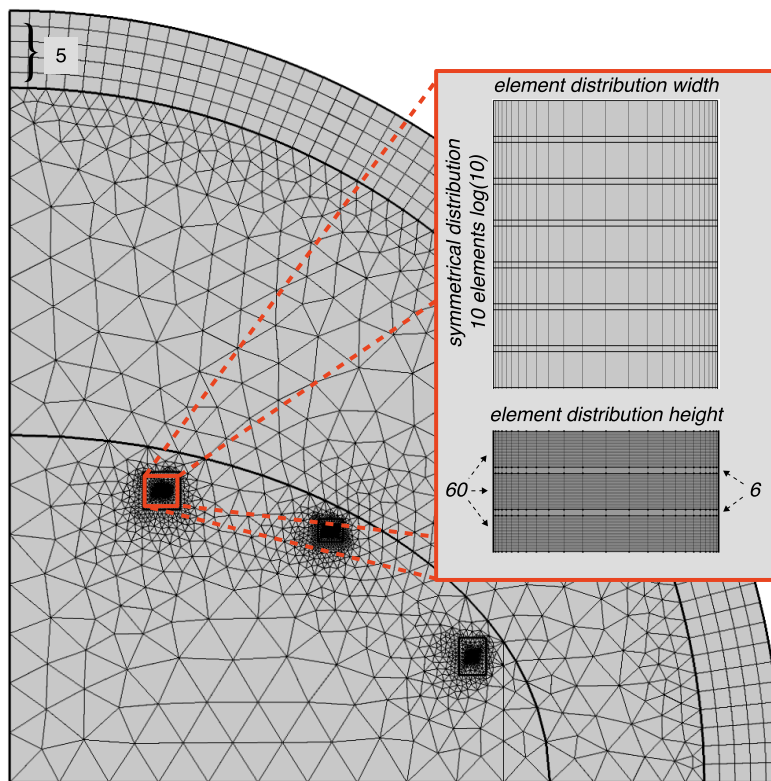


Figure 5.4: PFC mesh in FEM software

meshed with a free triangular mesh where the maximum growth rate of the elements is limited to 1.3. The resulting mesh can be seen in Figure 5.4. The total number of mesh elements for the PFC model is 136964 with 430279 resulting DOFs.

5.5 Operation scenario

The simulated operation scenario for the poloidal field coils of an ITER-like reactor consists of the initial ramp phase, AC cycles (normal operation), a fault scenario and the ramp-down phase. This load profile is shown in Figure 5.5 below.

In the ramp phase at the reactor start-up, the transport current is ramped up to the operating current. The rate of the ramp is determined in the ramp study. After the ramp phase the normal operation starts, where a dynamic magnetic field is used to control and shape the plasma. Here we consider the case of an alternating current with an amplitude of 1% of the operating current and a frequency of 5 Hz to simulate 10 cycles after the ramp phase. For normal operation, any waveform (sinusoidal, triangular, pulsed) for the current can be used to generate the desired dynamic magnetic field. The fault scenario simulates the coil response if a fault is detected and instead of ramping the coil down, the operational current is dropped down to the fault current ($75\% I_{op}$). After a stabilisation period the current is ramped up again to the operational current and normal operation is resumed. In this simulation, instead of continued normal operation, the current remains constant for a short period. The final operation of the poloidal field coils is the ramp-down phase, where the current is decreased from the operational current to zero.

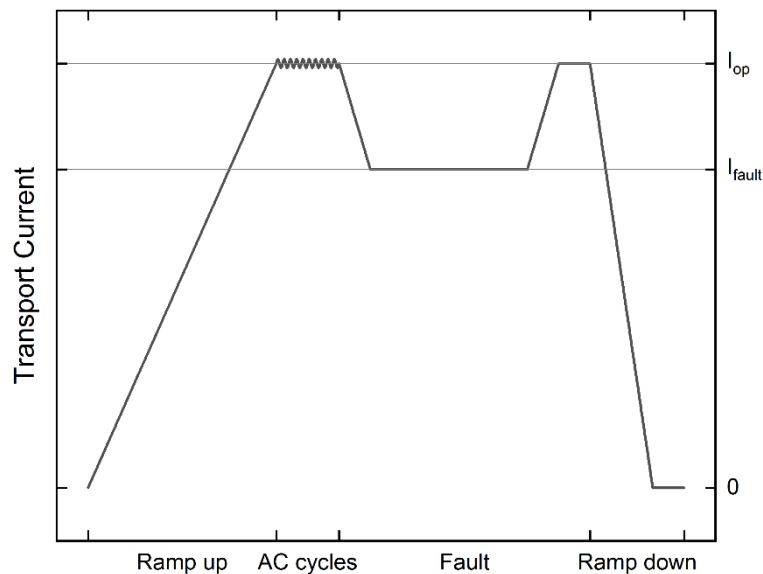


Figure 5.5: PFC operation scenario

5.6 Design process

HTS conductors have the advantage that they can be directly wound into a coil without the need for further processing. The simplest method to generate an electromagnetic coil from HTS tapes is to wind them into a coil by stacking the tapes into a pancake configuration. Multiple pancakes can be used to stack on top of each other to generate a solenoid coil. In practical applications, the tapes are not directly wound into pancake coils since this would result in an inductance that can be too large for pulsed operation. Nevertheless, for the purpose of this study, this method of analysis can be used. The magnetic field of the solenoid can be increased by either increasing the number of tapes per pancake or the number of stacked pancakes. Both configurations are limited in the amount of magnetic field they can generate. A single stack of pancakes can be considered a winding pack that is made up of pancakes with the same radial distance to the centre of the coil. Increasing the number of winding packs, namely stacking winding packs next to each other can further increase the maximum magnetic field strength of a magnet.

Since the magnetic field is dependent on the current of the coil, the critical current of the coil sets another limit for the maximum field. While for single tapes, the characterization of field-dependent critical current data can be used directly to simulate the electromagnetic behaviour, the critical current in a pancake configuration will be reduced by the tape-tape magnetic field interactions. The self-field of the coil will influence the critical current characteristic. Furthermore, stacked pancake configurations that can achieve high magnetic fields will reduce the critical current of the coil assembly further. Therefore, it is crucial to determine the operational current limit of the magnet. This can be done by investigating the weakest turn of the configuration, which is the innermost turn in the radial direction of the outermost pancake along the height. This can be explained by the fact that the current will penetrate the HTS conductor from the edges towards the centre in a single tape. The same is true for a pancake coil, where the penetration will start at the corners of the configuration and the flux fronts will gradually move towards the centre of the pancake. This is also true for a stacked pancake configuration since the current penetration will start at the corners of the outermost pancakes. Full penetration will be first achieved in the pancakes at the outside of the coil, while the centre pancakes will only experience a minor current penetration. Although the pancakes in the centre would be able to carry more current, the surrounding pancakes are already fully penetrated and therefore enter the resistive state. For this reason, the critical current of the coil is a crucial

parameter to identify the current carrying capabilities of the coil, which in turn will limit the maximum magnetic field that can be achieved.

To design a high-field magnet, an iterative design process is needed. First, it is necessary to define the limiting design parameters, such as the target magnetic field, power supply, cooling capabilities, and the size of the magnet. Another aspect is the selection of the HTS tape, which will be influenced by the availability of the required tape length or a certain tape configuration that can only be provided by a specific manufacturer. A lot of the research is focused on improving the current carrying capabilities by modifying the layer of an HTS conductor. As the manufacturing process matures, longer and longer tape lengths will be available giving a wider option for high-field magnet use. This is because high-field coils require long lengths of tape if they are wound into a pancake. With the design parameters and selected tape, the critical current of the prospective coil configuration will be iteratively investigated. Hereby, the critical current will be evaluated for the target magnetic field for the tape, the pancake and lastly the coil. When designing a magnet made of HTS conductors one must consider a safety margin, to avoid any part of the coil entering the resistive state. In this study, a safety margin of 80% of the critical current is considered, which in a real design might not be sufficient, but this should be considered on a case-to-case basis.

Starting with the 80% of the critical current of a single HTS tape at the target field as the operational current one can set up a model that changes the number of tapes per pancakes, pancakes per winding pack and winding packs per coil to find a suitable configuration. The next step is to investigate the selected pancakes by evaluating the critical current of a single pancake. This critical current will be lower than the one for the single tape because of the induced field due to tape-tape interactions. Using 80% of the critical current, the previous model can be adapted to identify the coil configuration that will generate the target field. When the pancake is selected and identified as well as the coil configuration in terms of the number of pancakes and winding packs are selected, the critical current of the whole coil needs to be identified. Modelling the superconducting current distribution will be a time-consuming task especially if the number of tapes and pancakes is large. Finally, if the target magnetic field can be achieved when the coil is operated at the safety margin limit of the current, the operational performance of the coil can be investigated. It should be noted that there will be multiple options in terms of viable coil configurations and the final design will be largely influenced by the design criteria in terms of HTS availability, operating condition, and available space.

The design process is conducted for each poloidal field coil due to the different target magnetic fields in each coil. Based on their position and available area the shape of the coil configuration is determined.

In the first step, the critical current of the tape is evaluated at the corresponding magnetic field. Then for each coil, a 2D infinitely long model is built and a parametric sweep study is conducted to identify potential coil configurations at 80% of the critical current of the tape. Since only the maximum magnetic field of the coil configuration is of interest the coils are modelled as conventional copper coils where each pancake is modelled as a homogenised coil with multiple turns. The sweep is conducted for the number of tapes per pancake, pancakes per winding pack and winding packs per coil. After identifying the possible configurations, the next step is to evaluate the critical current of the selected pancake configuration and re-run the study for 80% of the critical current of the pancake.

In Figure 5.6 the magnetic field at the centre of the coil for different configurations of the poloidal field coils is shown. The different line styles correspond to the number of winding packs (WP) and the coloured lines indicate the number of tapes per pancake. One can see from the figure above that a pancake of 50 tapes cannot generate more than 1.5 T, independent of how many pancakes are added. The magnetic field can be increased by adding either more tapes to the pancake or adding another winding pack to the coil configuration.

The horizontal lines at 6 T, 5 T and 4 T indicate the target field requirement for PFC 1, PFC 2 and PFC 3, respectively. Subsequently, the possible coil configurations for each field coil are

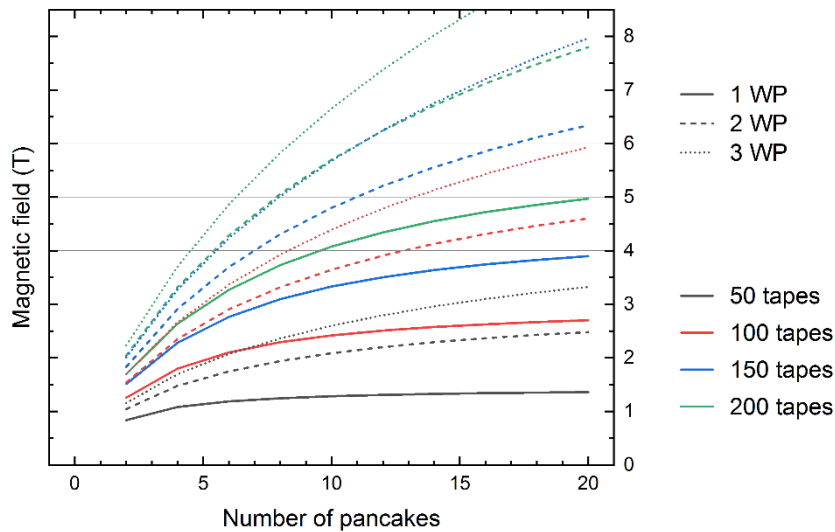


Figure 5.6: PFC magnetic field for different configurations

Table 5.2: Design options PFC 1 for a target field $B_{PF1} = 6T$

Design	#WPs	#PCs/WP	#tapes/PC	#HTS
#1	3	7	200	4200
#2	3	9	150	4050
#3	2	9	200	3600
#4	2	13	150	3900
#5	3	14	100	4200

Table 5.3: Design options PFC 2 for a target field $B_{PF2} = 5T$

Design	#WPs	#PCs/WP	#tapes/PC	#HTS
#1	3	6	200	3600
#2	3	7	150	3150
#3	2	7	200	2800
#4	2	9	150	2700
#5	3	10	100	3000
#6	1	14	200	2800
#7	2	16	100	3200

Table 5.4: Design options PFC 3 for a target field $B_{PF3} = 4T$

Design	#WPs	#PCs/WP	#tapes/PC	#HTS
#1	3	4	200	2400
#2	3	5	150	2250
#3	2	5	200	2000
#4	2	6	150	1800
#5	3	7	100	2100
#6	1	8	200	1600
#7	2	9	100	1800
#8	1	13	150	1950
#9	1	20	100	2000

evaluated. By calculating the total number of HTS tapes required for each design option the most effective design can be selected.

Table 5.5: Target magnetic field, maximum magnetic field, and magnetic field at the centre of PFCs

B (T)	B_{target}	SuperPower				SuperOx			
		25K		35K		25K		35K	
		B_{max}	B_{centre}	B_{max}	B_{centre}	B_{max}	B_{centre}	B_{max}	B_{centre}
PFC 1	6	6.98	6.06	6.88	6.36	7.04	6.12	6.98	6.26
PFC 2	5	5.72	4.79	5.59	5.06	5.76	4.87	5.71	5.01
PFC 3	4	4.53	3.49	4.38	3.68	4.54	3.57	4.48	3.69

The poloidal field coil at the top/bottom of the reactor has the highest field requirement of the PFC system. The target field of 6 T is achieved in five different designs listed in Table 5.2. The most viable option is design #3, which uses the least number of tapes.

The target field requirement for PFC 2 is 5 T which is generated in seven different coil configurations, listed in Table 5.3. Here, the least number of tapes is achieved in design #4, but in this case, design #3 is better suited due to the resulting shape of the coil, which fits best in the available coil area and matches the selected number of tapes per pancake of PFC 1.

The design target for PFC 3, which is located closest to the midplane of the reactor is 4 T. Out of the nine design options listed in Table 5.4, design #6 requires the least number of tapes, uses the same number of tapes per pancake as the other two coils and is suitable for the available coil area.

After selecting the coil configurations, the design choices are tested in a 2D model with field-dependent critical current data. Each field coil is modelled separately with the critical current data for tapes from SuperPower and SuperOx at 25 K and 35 K for an operating current of 800 A. The maximum magnetic field generated at the coil as well as the magnetic field at the centre of the coil is listed in Table 5.5 below.

For each field coil the target magnetic field is achieved by the maximum field at the coil. The central magnetic field is expected to be lower than the peak field since the highest magnetic field will be generated at the corners of the magnet. Also, the magnetic field from the SuperOx tape is higher at the corners of the magnets while the SuperPower tape has a higher magnetic field at the centre of the coils.

The last step of the design process is to identify the safety margin of the operating current. In Table 5.6 the critical current of the weakest turn in each coil for tapes from SuperPower and

Table 5.6: Critical current of the weakest turn for the individual PFCs

	SuperPower				SuperOx			
	25 K		35 K		25 K		35 K	
	I_c (A)	I/I_c	I_c (A)	I/I_c	I_c (A)	I/I_c	I_c (A)	I/I_c
PFC 1	1250	64%	960	83.3%	1230	65%	1010	79.2%
PFC 2	1370	58.4%	1028	77.8%	1300	61.5%	1090	73.4%
PFC 3	1430	55.9%	1088	73.5%	1346	59.4%	1135	70.5%

SuperOx at 25 K and 35 K is shown. Additionally, the current ratio I/I_c for the operating current of 800 A is given.

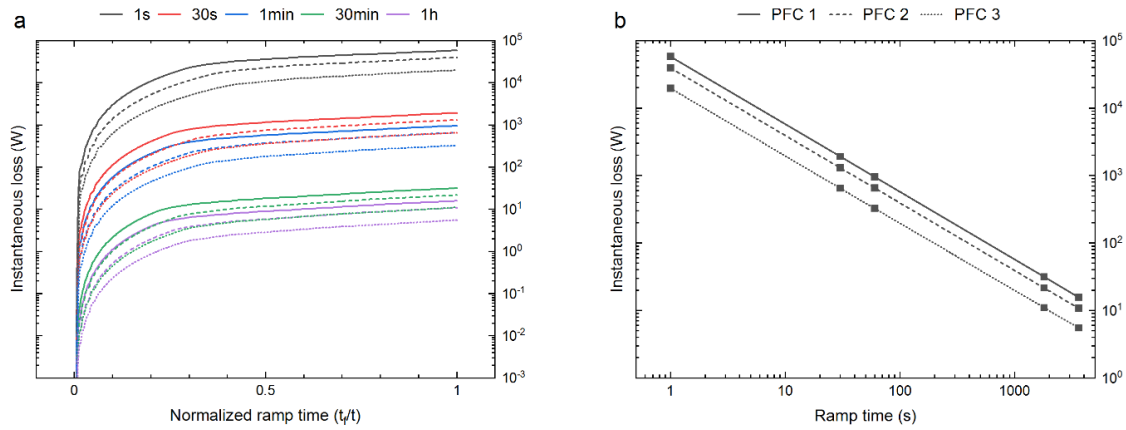
Since the critical current of the coil is dependent on the magnetic field, naturally PFC 1 which has the highest magnetic field, will also have the lowest critical current. At 35 K the SuperPower tape has a critical current of 960 A, which is 83.3% of the critical current when operated at 800 A. A safety margin of 80% is usually sufficient to avoid high losses and potential loss of superconductivity when the coil is operated close to the critical current. For PFC 3 the operating current is less than 60% of the critical current at 25 K for both tapes. The operating current could be increased further in these cases, but this would result in higher magnetic fields that are not required. Designing all coils to use 80% of the critical current would mean that each tape and temperature has a different design and operating current, which does not allow for a direct comparison in the subsequent studies.

5.7 Ramp time

For an efficient start-up of the poloidal field coil system, the losses during the ramp phase should be mitigated. Here we investigate the effect of ramp time on the power loss during the ramp-up of the PFCs. For this study, a constant critical current of 1000 A is used to enable a faster computation since a comparison of the different tape types and temperatures is not of interest for the evaluation of the ramp time. The ramp-up phase is simulated for 1 s, 30 s, 1 min, 30 min and 1 h up to the operating current of 800 A. It is expected that a longer ramp time will reduce the instantaneous loss since the flux flow within the conductor is slower. It should be noted that for short ramp times (i.e. 1 s and 30 s) the temperature would rise and reduce the critical current of the HTS and effect the losses. This could be investigated by electromagnetic-thermal coupled simulations, which are not within the scope of this study.

Table 5.7. Dissipated energy and dissipated power for different ramp times of PFCs

Ramp time	Dissipated energy (J)			Dissipated power (W)		
	PFC 1	PFC 2	PFC 3	PFC 1	PFC 2	PFC 3
1 s	3.18e4	2.02e4	9.80e3	57684	39374	19564
30 s	3.11e4	2.01e4	9.79e3	1907.8	1300.8	645.61
1 min	3.09e4	2.00e4	9.79e3	950.29	651.1	325.23
30 min	3.03e4	1.99e4	9.79e3	31.62	21.598	10.996
1 h	3.02e4	1.99e4	9.80e3	15.684	10.746	5.4994

Figure 5.7. PFCs instantaneous losses with constant critical current ($I_c = 1000$ A)

(a) loss distribution over normalized ramp time. (b) losses at the end of the ramp phase for different ramp times. The points correspond to 1 s, 30 s, 1 min, 30 min and 1 h of ramp time.

In Figure 5.7 (a) the instantaneous loss for each coil is shown over the normalized ramp time. The instantaneous loss is shown on a logarithmic scale to visualize the losses at lower ramp rates. The losses at the end of the ramp phase are shown in Figure 5.7 (b) for the selected ramp times.

The instantaneous power loss at the end of the ramp phase for 1 s is over 50 kW in PFC 1 while an increase in ramp time to one hour reduces the dissipated power to 15 W. The dissipated power loss as well as the dissipated energy during the ramp phase is listed for different times in Table 5.7.

While a lower ramp rate can effectively decrease the instantaneous loss of the coil, it may not consistently reduce the total energy dissipated during the ramp phase, as the actual total energy loss is contingent upon factors such as the temperature dependence of the critical current and whether the pancake is fully penetrated. Although the temperature dependence of the critical

current is not considered in this study, it should be mentioned that a slower ramp does not necessarily guarantee lower total dissipated energy. Based on the results, the ramp time for the study of the operation cycle is selected to be 30 min. At this time the instantaneous loss is already minimized and extending the ramp time has no major effect on the loss reduction and will only prolong the numerical simulation.

5.8 Current distribution

In this section, the current distribution of the individual poloidal field coils is evaluated for SuperPower and SuperOx tapes at 25 K and 35 K at the end of the ramp phase when the operating current is reached. At this point, the coil generates the target magnetic field and can be used for the plasma control scenarios. The current distribution is shown for the normalized current density J/J_c on a scale from -1 to 1.

In Figure 5.8 the current penetration of PFC 1 at the end of the ramp phase is shown. In all cases, the two outermost pancakes are penetrated, while at 35 K for both tapes even the third pancake from the top is penetrated.

The current penetration of PFC 2 is shown in Figure 5.9 below. Due to the lower magnetic field, the critical current of this coil is higher and therefore, the current penetration of the coil will be lower. At 25 K only the first pancake is penetrated and at 35 K the two outermost

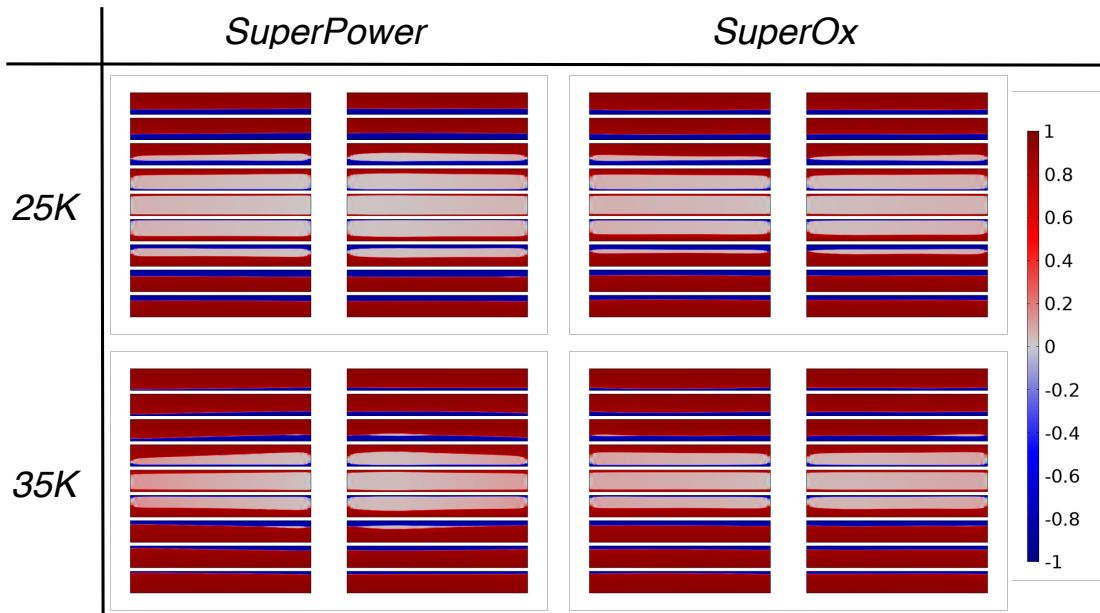


Figure 5.8: PFC 1 current distribution at the end of ramp phase

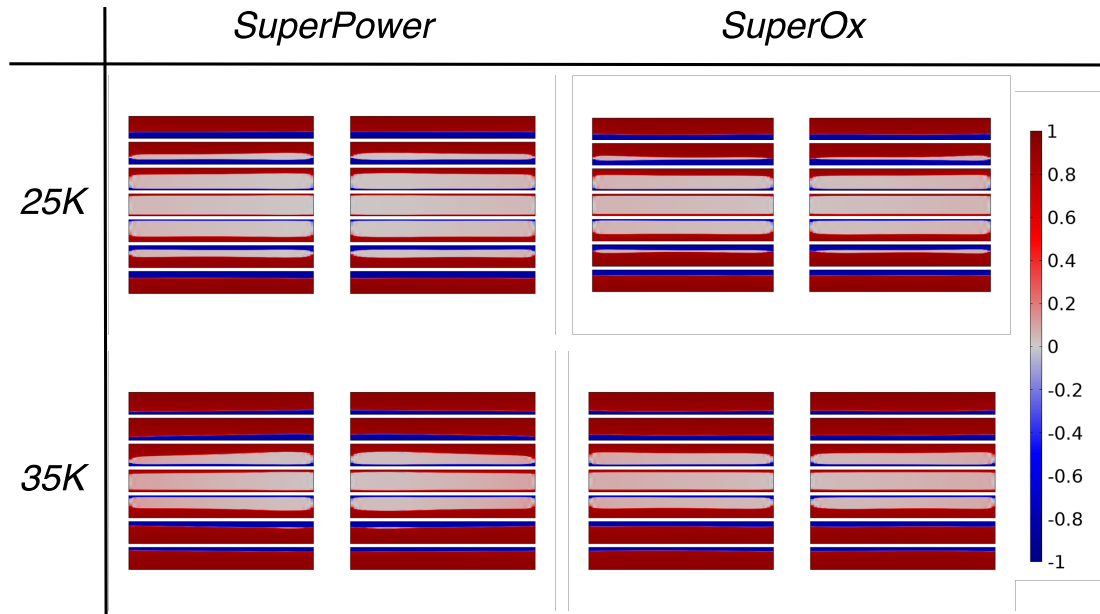


Figure 5.9: PFC 2 current distribution at the end of ramp phase

pancakes are fully energized. The distribution of the current is similar to PFC 1 due to the reduced number of pancakes.

Figure 5.10 shows the current penetration for PFC 3 at the end of the ramp phase. Here a difference in the current penetration for different tapes can be observed. Especially at 25 K the current penetration in the second pancake is more advanced in the SuperOx tape than in the SuperPower tape.

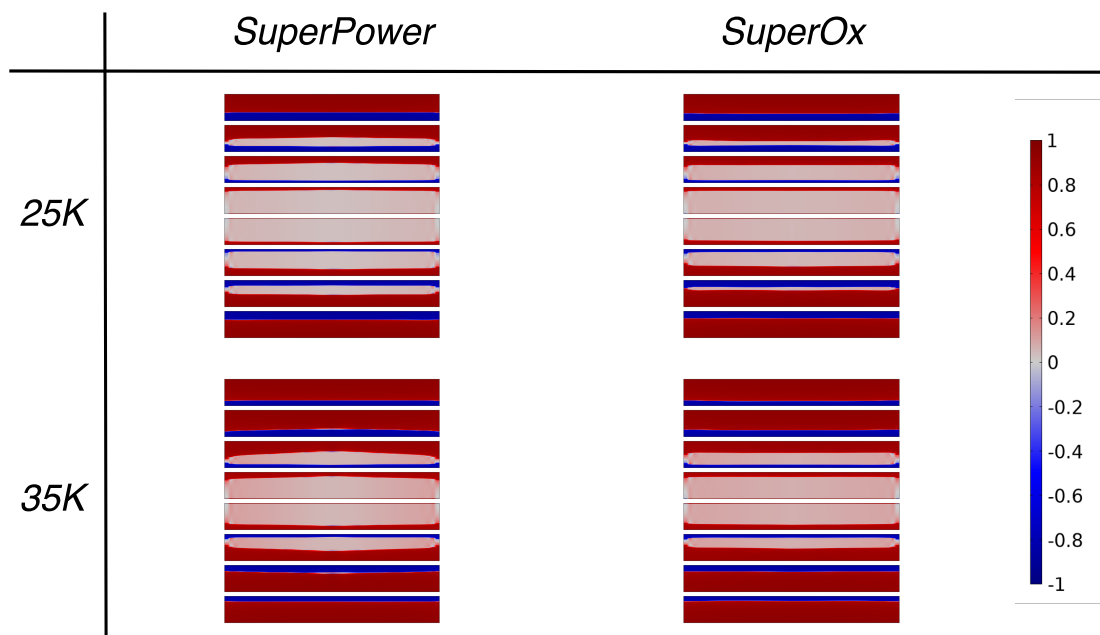


Figure 5.10: PFC 3 current distribution at the end of ramp phase

The investigation of the current penetration shows that both tapes, SuperPower and SuperOx, have a similar distribution. At 35 K the current penetrates further into the coil than at 25 K, which is expected due to the lower critical current at 35 K.

5.9 Magnetic field distribution

The magnetic field distribution is shown at the end of the ramp phase when the operating current is reached. In the figures below, the normalized magnetic field is shown in Tesla and the arrows indicate the direction of the field. The surrounding domain of the coil is the available coil area, which is used to visualize the compact design of an HTS coil.

Figure 5.11 shows the distribution of the magnetic field in the PFC 1 area, with the coil located at the centre. The target magnetic field of 6 T is reached along the left side of the coil which is oriented towards the symmetry axis. Since the PFC 1 is located on top of the reactor, it is closest to the symmetry axis compared to the other poloidal field coils resulting in a higher magnetic field on the inside of the coil. Due to the higher current penetration at 35 K, the magnetic field at the centre of the coil is higher than at 25 K.

The target magnetic field of PFC 2 is shown in Figure 5.12. Due to the increased distance from the symmetry axis, the distribution of the magnetic field is more uniform around the coil. Nevertheless, the highest magnetic field is oriented towards the inside of the coil. Again, the

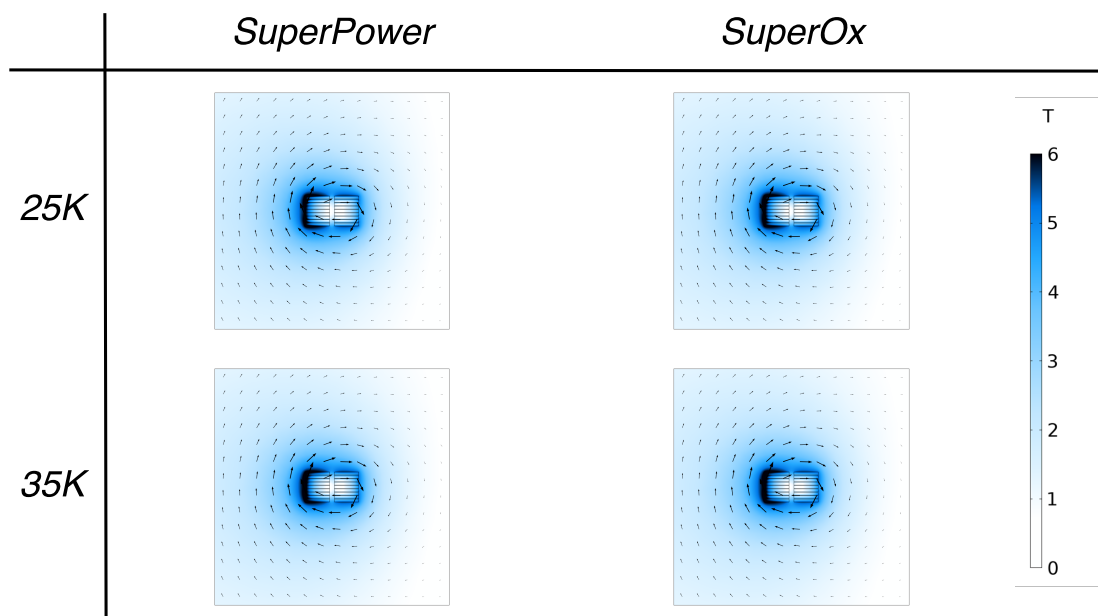


Figure 5.11: PFC 1 magnetic field distribution at the end of ramp phase

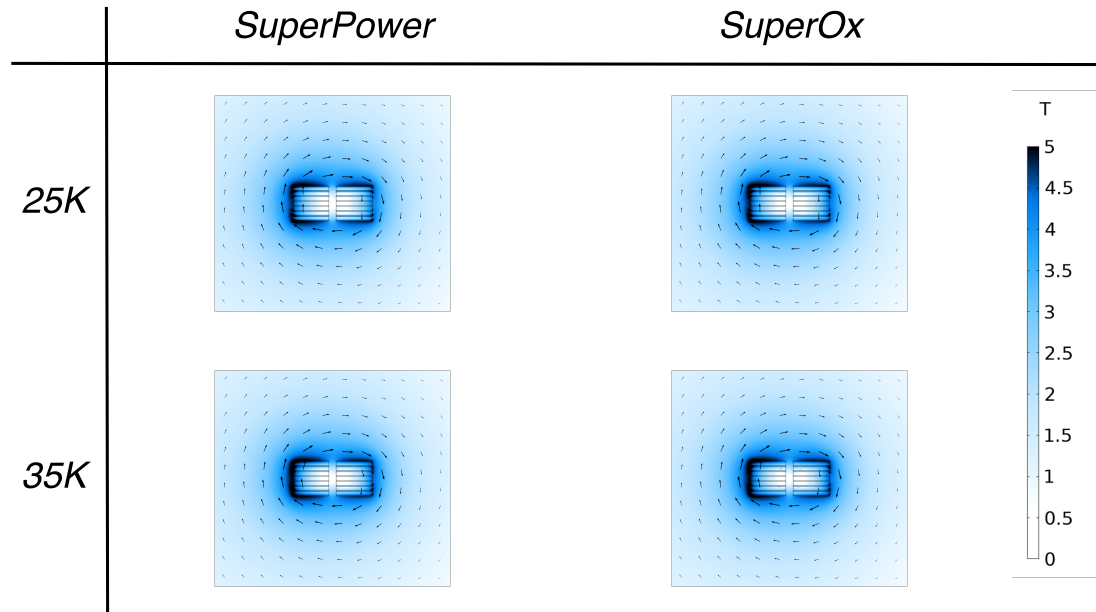


Figure 5.12: PFC 2 magnetic field distribution at the end of ramp phase

higher current penetration of the coil at 35 K results in a higher magnetic field at the centre of the coil.

The smallest of the PFCs is shown in the centre of the PFC 3 coil area in Figure 5.13. The magnetic field for this coil is highest at the top and bottom of the coil, which can be explained by the large distance towards the symmetry axis at the centre of the coil, the proximity to the symmetry plane in the midplane of the reactor and the coil configuration with one winding pack.

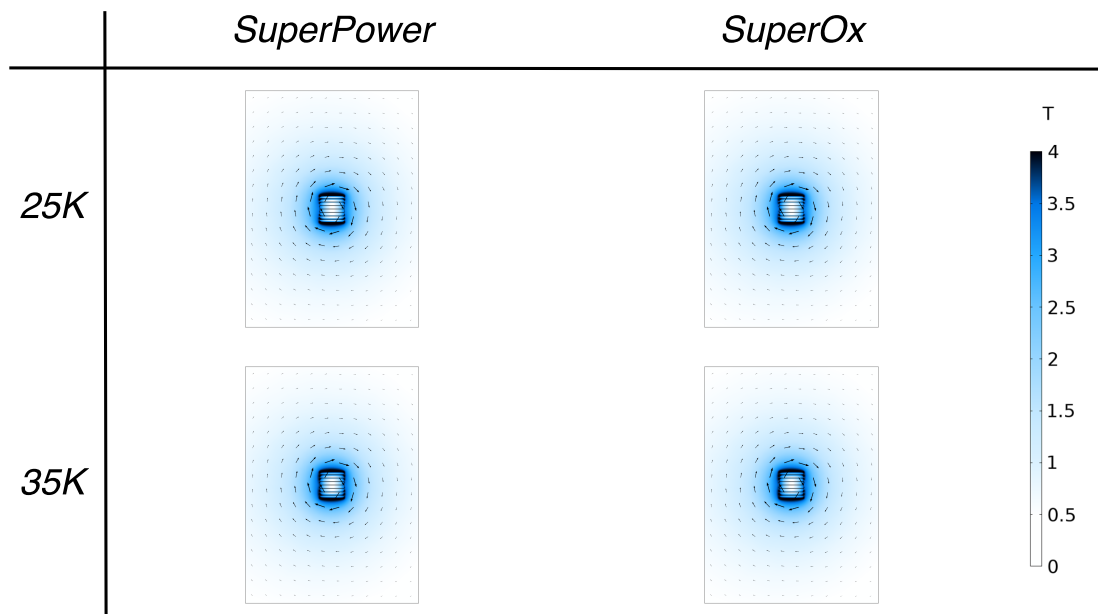


Figure 5.13: PFC 3 magnetic field distribution at the end of ramp phase

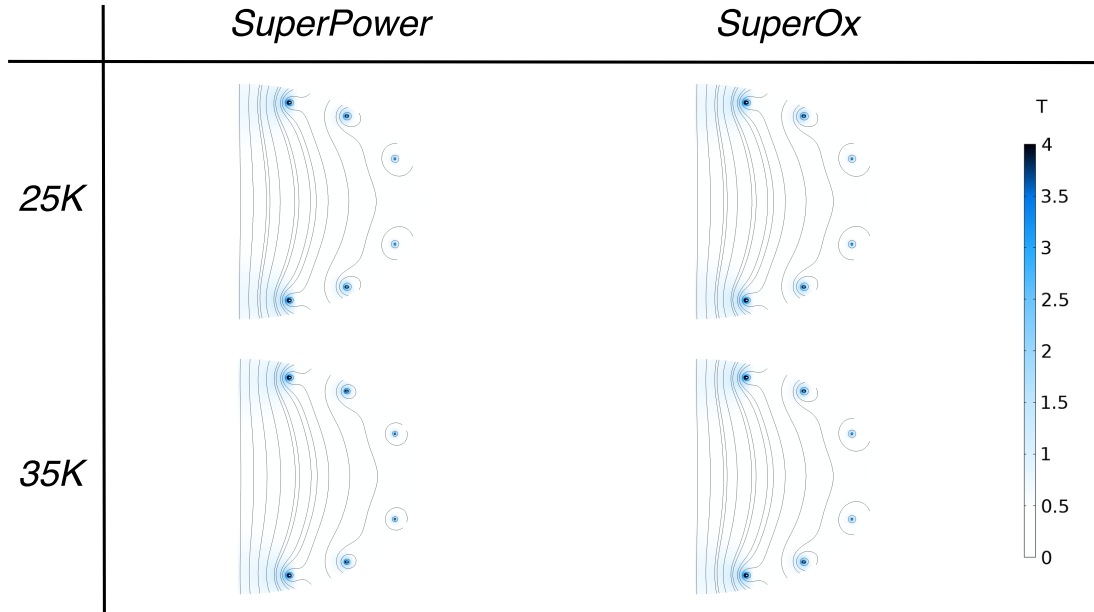


Figure 5.14: PFCs magnetic field distribution at the end of ramp phase

In Figure 5.14, the magnetic field generated by the poloidal field coils is shown for half of the reactor. The magnetic field is scaled to a maximum of 4 T, which corresponds to the target field of the smallest field coil (PFC 3). The flux lines show the shape of the magnetic field generated by the coils. Both tapes at 25 K and 35 K meet the target field requirements and show similar distribution.

5.10 Losses

In the last section of the poloidal field coil study, the losses during the different operation scenarios are evaluated. Before the losses of the whole coil for each PFC are investigated, a closer look at the losses in the selected pancakes during the ramp phase is necessary. In Figure 5.15 the cross-sectional losses of each PFC are shown for the selected pancakes (PC #1, PC #2, and PC #3). The pancakes are numbered according to their position in the coil configuration starting from the pancake of the innermost winding pack at the top of the coil and descending downwards.

The first pancake of the innermost turn experiences the maximum magnetic field and has the highest current penetration of the whole configuration. Therefore, the losses in this pancake are the major contribution to the total losses of the coil. For each tape at both temperatures, a peak in the losses can be observed. At the peak of hysteresis losses, the superconductor is fully magnetized with vortices uniformly distributed. Beyond this point, hysteresis losses decline

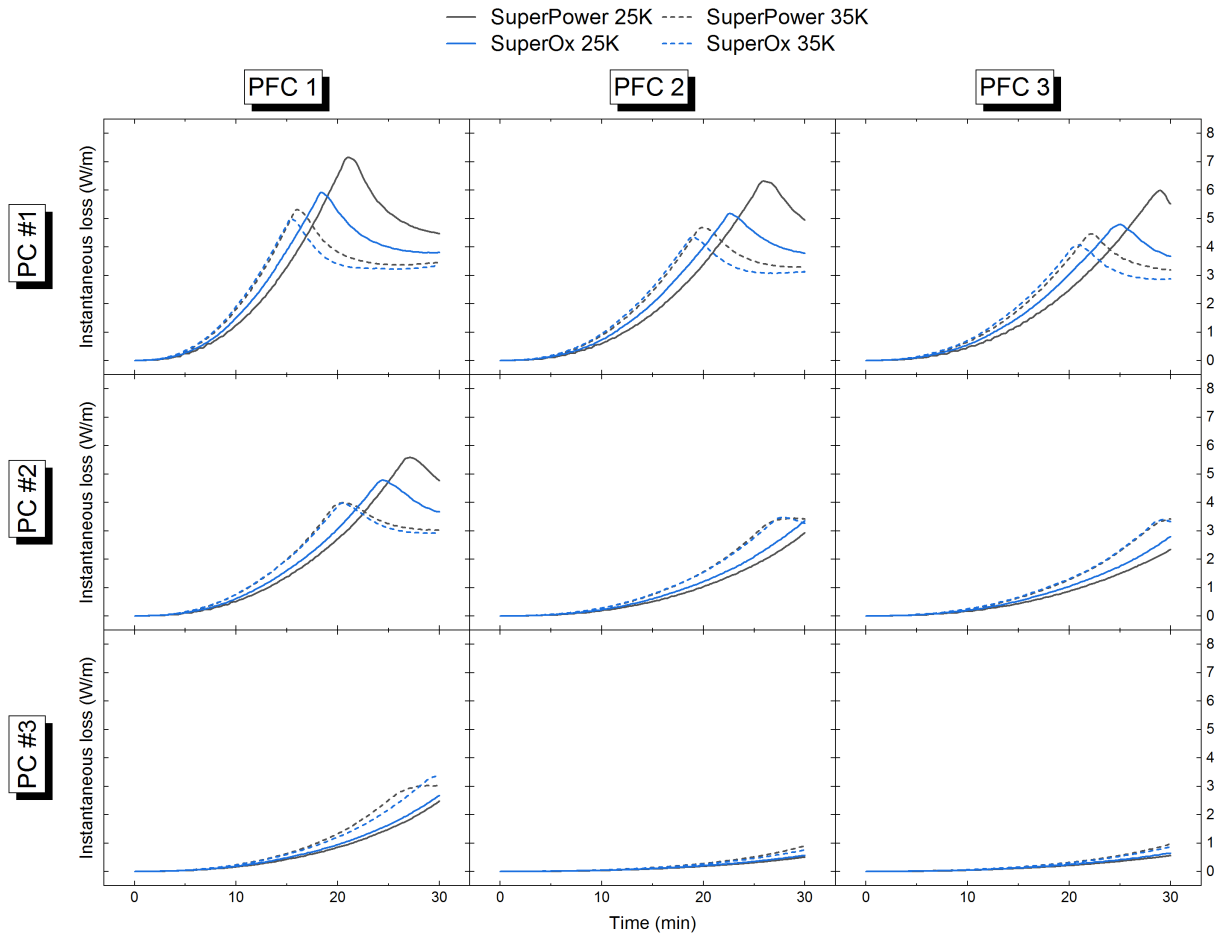


Figure 5.15: PFCs cross-sectional losses during ramp phase
for the PFCs in the selected pancakes #1, #2 and #3

The numbering starts with the innermost pancake at the top of the coil.

due to the saturation of pinning centres and the resulting decreased effectiveness of vortex pinning, which reduces incremental energy dissipation. The losses will climb again as the transport current gets closer to the critical current. Due to the lower critical current at 35 K, less current is required to establish full current distribution within the pancake. Therefore, the effect occurs earlier than at 25 K and can be linked to the critical current of the tape. In all PFCs, for all tapes and temperatures, the first pancake is fully magnetised during the ramp phase. Because PFC 1 generates the highest magnetic field, the magnetisation is happening faster than in the other PFCs.

The second pancake of the configuration experiences less magnetic self-field and therefore, full magnetisation is not achieved in PFC 2 and PFC 3 at 25 K. The high field generated by PFC 1 can even magnetise the third pancake at 35 K when the current is ramped up to the operating current.

It should be mentioned that the total losses of the coil will not decline due to the full magnetisation of the individual pancake. When the first pancake of the coil configuration is magnetised, the other pancakes are still generating losses, but the increase of the total loss is slowed down.

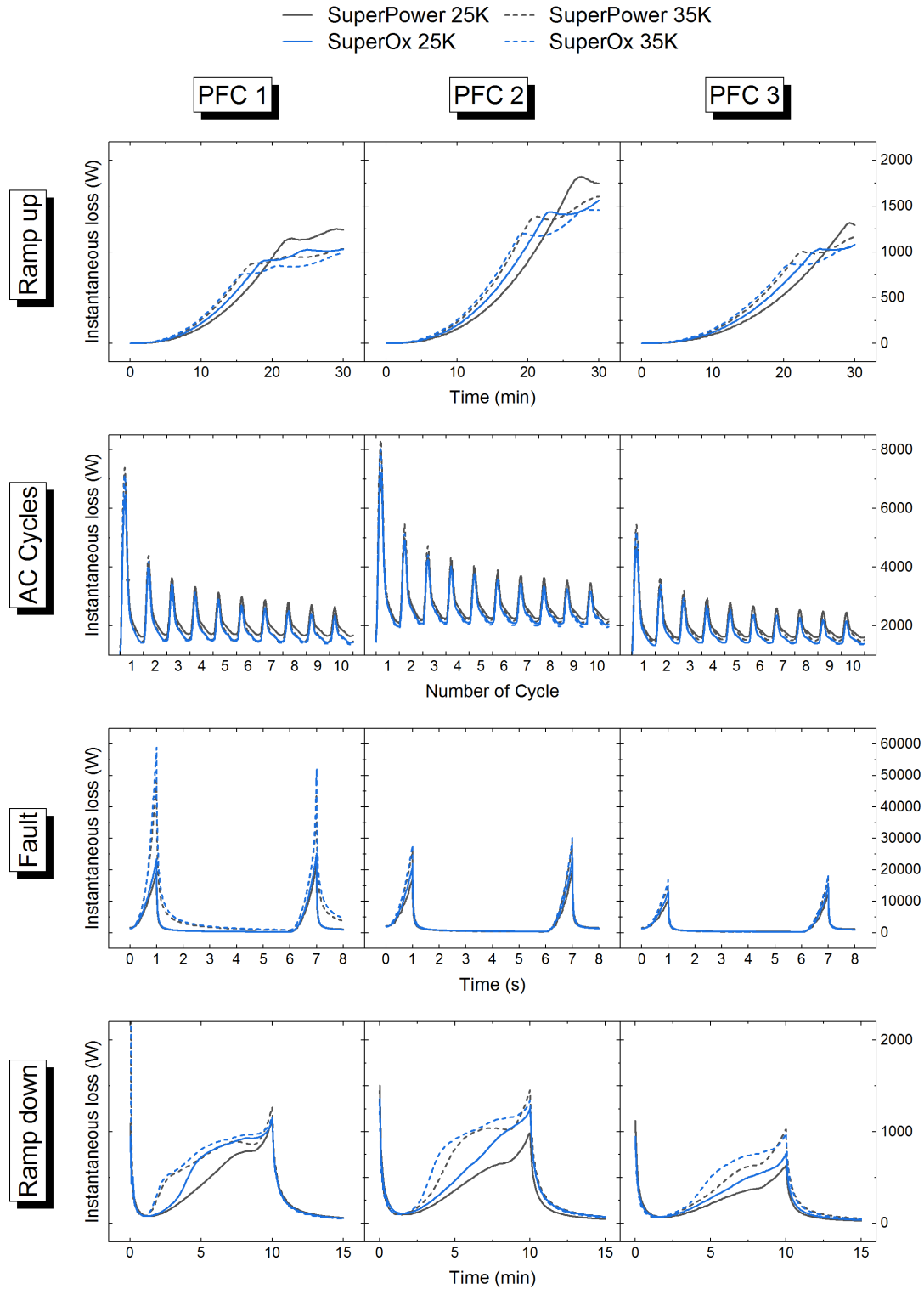


Figure 5.16: PFCs instantaneous losses during different operation scenarios

Table 5.8: Dissipated energy (J) during operation PFC 1

Scenario	SuperPower		SuperOx	
	25K	35K	25K	35K
Ramp up	1.03e6	1.02e6	9.88e5	9.62e5
AC cycles	#1	577	600	542
	#2	474	457	428
	#3	448	424	400
	#4	433	405	383
	#5	422	391	372
	#6	413	380	362
	#7	405	371	354
	#8	398	363	347
	#9	392	356	341
	#10	387	349	335
Fault	1.97e4	4.76e4	2.24e4	5.44e4
Ramp down	3.36e5	4.68e5	4.07e5	4.93e5

Figure 5.16 shows the instantaneous loss for the whole coil during each operation scenario for all PFCs. The corresponding dissipated energy of each phase is listed in the subsequent tables for each PFC separately.

As previously established, the losses will increase initially due to the ramping current in the coil, with the main contribution coming from the weakest pancake of the configuration. Once this pancake is fully magnetized, the losses will increase further, mainly due to the second pancake.

After the initial ramp phase, 10 AC cycles are simulated. The high losses at the end of the ramp phase would decrease exponentially if the operating current would be constant without generating additional losses since there is no time-varying force acting on the flux flow. The instead applied alternating current will give rise to AC losses as shown in Figure 5.16. The losses are highest during the first cycle and decrease in each cycle thereafter due to the progressive immobilization of magnetic flux lines by pinning centres, leading to a more stable and loss-reduced state over successive cycles. It can be noticed that the AC losses at 35 K are higher for the first cycle than at 25 K, spiking to 7390 W and 6415 W respectively, for the

Table 5.9: Dissipated energy (J) during operation PFC 2

		SuperPower		SuperOx	
		25K	35K	25K	35K
Scenario					
Ramp up		1.21e6	1.33e6	1.23e6	1.27Ee6
	#1	691	750	675	711
	#2	599	615	567	574
	#3	574	581	539	541
	#4	559	561	523	521
	#5	549	547	511	507
AC cycles	#6	540	535	501	495
	#7	532	525	493	485
	#8	526	516	486	476
	#9	519	508	479	468
	#10	514	500	473	461
Fault		1.95e4	2.56e4	2.23e4	2.69e4
Ramp down		3.01e5	4.84e5	4.00e5	5.19e5

SuperPower tape in PFC 1. Due to the highest losses after the ramp phase in PFC 2, the AC losses during the 10 cycles are also the highest of the three PFCs.

During the fault scenario, the initial drop of the operating current and the ramping after the stabilisation will cause the losses to reach a maximum. The highest peak is reached in PFC 1 at 35 K for the SuperOx tape (59 kW). During the stabilisation phase of five seconds, the losses are exponentially decreasing towards zero after which the second spike occurs during the ramping phase where the current is increased back to the operating current. After the operating current is reached the losses decline again since the current is kept constant.

The final operating scenario is the ramp-down phase, where the current is decreased to zero over a period of 10 minutes. The losses generated during this phase are due to the change in the direction of the current. Similar to the ramp phase, the losses increase due to the current penetration, but when the current is completely reversed within the first pancake, the losses rise faster until the current is ramped down to zero. During the final 5 minutes, no current is applied, and the losses decrease towards zero.

Table 5.10: Dissipated energy (J) during operation PFC 3

Scenario	SuperPower		SuperOx	
	25K	35K	25K	35K
Ramp up	7.55e5	8.84e5	8.07e5	8.58e5
AC cycles	#1	478	506	444
	#2	422	423	378
	#3	406	402	361
	#4	397	390	351
	#5	390	381	343
	#6	385	374	337
	#7	380	367	332
	#8	376	362	328
	#9	372	357	324
	#10	368	352	320
Fault	1.26e4	1.56e4	1.40e4	1.65e4
Ramp down	1.88e5	2.91e5	2.32e5	3.27e5

The instantaneous losses can be used to calculate the dissipated energy for each operation scenario. Here the individual PFCs are evaluated for the total energy loss of the tapes from SuperPower and SuperOx at 25 K and 35 K. In Table 5.8 the dissipated energy for the whole coil during each operation scenario is listed.

During the ramp phase, the SuperOx tapes dissipated less energy, while both tapes performed better at 35 K than at 25 K of the respective tape. During the AC cycles the losses decreased after each cycle, but interestingly the SuperOx tape generated the same amount of losses after cycle #9 at 25 K and 35 K. During the fault scenario as well as the ramp-down phase, the SuperPower tape at 25 K shows the least dissipation of energy and performs better at both temperatures than the respective SuperOx tape.

Due to the larger radius of PFC 2, the instantaneous losses as well as the corresponding dissipated energy are higher than for PFC 1. Here the SuperPower tape at 25 K has the lowest losses during the ramp phase, while at 35 K the losses are the highest (Table 5.9).

While the losses during the first AC cycles are higher at 35 K for both tapes, the dissipated energy after the 5th cycle is lower than at 25 K. During the fault scenario and the ramp-down phase, again, the SuperPower tape generates fewer losses than the SuperOx tape at the

respective temperatures. While the initial ramping losses are higher for PFC 2 than PFC 1, the fault scenario and the ramp down dissipated less energy.

PFC 3 is the smallest coil of the PFC system and although this coil features the largest radius, the losses due to the single winding pack and the comparably small number of tapes, result in the lowest energy dissipation for all operation scenarios, as shown in Table 5.10.

The ramp-up phase with the SuperPower tape at 35 K generates the highest losses, while at 25 K the SuperPower tape dissipates the least amount of energy. It is interesting to see that the SuperPower tape at 35 K has lower losses after the 3rd AC cycle compared to the same tape at 25 K. The SuperOx tape on the other hand will perform better at 25 K throughout all 10 cycles. Also, in PFC 3, the SuperPower tape has the lowest energy dissipation during the fault scenario and the ramp-down phase compared to the SuperOx tape at the respective temperatures.

5.11 Discussion

In this chapter the poloidal field coil system consisting of three poloidal field coils was investigated. The coils were designed to meet a minimum target magnetic field of 6 T, 5 T and 4 T for PFC 1, PFC 2, and PFC 3, respectively. All coils achieved higher fields at the operating current of 800 A, corresponding to 83% of the critical current in the weakest turn of PFC 1 at 35 K, which generates the highest field and will therefore naturally have the lowest critical current. The design was accomplished by iteratively recalculating the coil configuration and re-evaluating the critical current limit. The coil design could be further improved to meet the exact target value by iteratively narrowing down the number of tapes per pancake to an exact configuration that will produce the desired field. The coil configuration itself can also be modified in terms of the gaps between the pancakes and winding packs. Here a narrow gap of 2 mm between pancakes and 2 cm between the winding packs is used to achieve the highest field with the least number of tapes. Depending on cooling methods, additional material needed for insulation and structural support needed to prevent the coil from moving, this can be optimized further.

One of the goals of this study was to achieve or exceed the target magnetic field with a compact HTS magnet configuration within the available space of the ITER-like fusion reactor. In Table 5.11, the required surface area for the ITER-like reactor design and the PFCs using HTS are shown. It must be pointed out that in the magnet design the cooling method was not considered.

Table 5.11. Comparison of required surface area for the magnet [203]

	PFC 1	PFC 2	PFC 3
ITER-like magnet	9447.7 cm ²	3861.5 cm ²	6839.3 cm ²
HTS magnet	272.8 cm ²	211.2 cm ²	110 cm ²

In the ITER reactor, CICC's are used, where the channels for forced flow cooling are already included in the required area. For the HTS magnet simulation, the temperature is assumed to be constant and modelled by the field-dependent critical current at the selected temperature. Therefore, additional space would need to be allocated for the cryogenic cooling of the magnet as well as structural materials to support the magnet.

By selecting the same number of tapes per pancake for each coil, a direct comparison in terms of the critical current and losses between the coils can be made. In a reactor designed for the use of HTS magnets, the coils will need to be designed to meet the exact requirements. This means that each coil most likely will not have the same configuration of tapes per pancake. This will require a more detailed investigation of the magnet design by considering various aspects, such as forces acting on the magnet, connection to power sources, cooling system, and so on.

In Table 5.12, the required conductor lengths for the magnet design are listed. It should be noted that the conductor length between LTS and HTS magnets cannot be directly compared. While the required HTS conductor length here, is the length of HTS tape needed to wind this coil, the LTS conductor consists of up to 1440 superconducting strands, depending on the coil. Nevertheless, it can be said that the length required for the HTS magnets exceeds the currently available tape lengths for HTS.

The operation scenarios of the HTS magnet are used to simulate the behaviour of the magnet under different conditions. The current penetration, magnetic field distribution and losses for the different scenarios were evaluated for the SuperPower and SuperOx tape at 25 K and 35 K. The initial ramp study was conducted with a constant critical current to evaluate the effect of the ramp time on the losses. It was found that in this case extending the ramp time to 30 minutes is an effective way to reduce the losses. The magnetic field distribution when the operating current was reached after the ramp was similar for all tapes and temperatures. Due to the lower critical current at 35 K, the current penetration was higher for both tapes at this temperature. This also had an interesting effect on the dissipated power, especially during the ramp phase,

Table 5.12: Comparison of required conductor length for the PFC magnets [203]

	PFC 1	PFC 2	PFC 3
ITER-like magnet	11.418 km	11.480 km	13.988 km
HTS magnet	14.195 km	23.293 km	19.195 km

where the magnetization of the top pancakes reduced the instantaneous ramping losses. While the energy dissipation at 25 K was still lower than at 35 K, this phenomenon could be used for future magnet design to reduce the losses and operate the magnet efficiently. It should be pointed out that a coil should be designed for a specific operating temperature since at 25 K the operating current was around 60% of the critical current for the SuperPower tape. In this study, using the same coil configurations for both tapes at all temperatures was an effective way to compare the results.

6 Toroidal field coils

6.1 Introduction

In this chapter, the toroidal field coil system of an ITER-like fusion reactor is investigated. The TFC system consists of 18 coils symmetrically distributed around the torus of the reactor. The TFCs generate a constant magnetic field to confine the plasma in a toroidal direction. The straight legs on the inside are placed as close as possible to each to achieve a homogenised field distribution resulting in the unique D-shape of the coils.

In the first step of the study, a small D-shaped coil is investigated in 3D using the homogenisation technique for the **T-A** formulation. Most of the information presented in section 6.2.1 has been published as part of this thesis [205]. Then a case study is conducted using 2D modelling approaches where the model setup and operational conditions are outlined. Then the coil is designed to achieve a target magnetic field at the coil which is based on the requirement of the ITER TFCs. Using two different modelling approaches, the current and magnetic field distribution as well as the losses during a baseline operation scenario are evaluated. The ramp phase is additionally investigated to identify the influence of time on the losses during the ramping of the current.

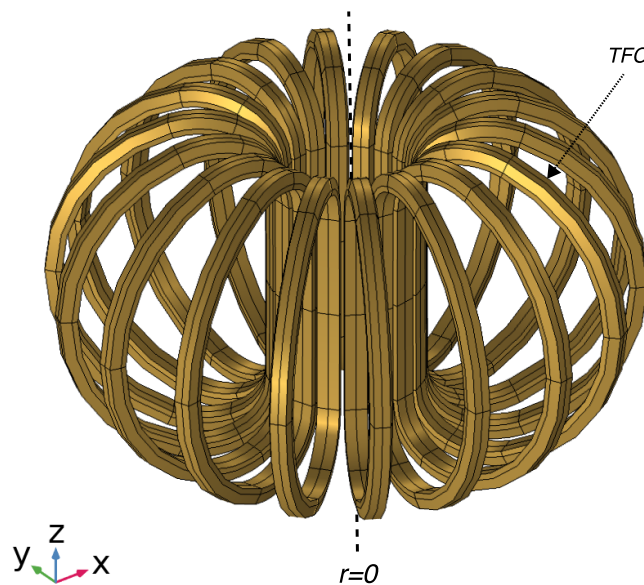


Figure 6.1: Toroidal field coils of an ITER-like fusion reactor

6.2 D-shaped coil study

6.2.1 3D homogenised T-A formulation

Although 2D numerical models can reflect the electromagnetic properties of superconducting devices in many cases, e.g., infinitely long conductors, it is not considered trustworthy enough to predict the behaviour of a 3D superconducting device in a specific shape. For example, when the ratio between the thickness of a racetrack coil and its diameter cannot be neglected, a 3D numerical model is necessary to accurately quantify the losses. The homogenisation technique assumes that the superconducting tapes that are wound in coils can be represented by an anisotropic bulk that can reproduce the overall electromagnetic behaviour of the coil. This allows reducing the number of degrees of freedom and the computation time. Therefore, a similar approach is followed to extend the homogenisation into the 3D analysis of superconducting coils.

Subsequently, the procedure presented in section 3.2.4 will be outlined again in more detail. Starting from the arrangement of tapes modelled with the **T-A** formulation in 3D as sheets, a cell unit is defined around them (with the same height as the superconducting tape and thickness Λ). The main characteristics of the tape will be impressed in this cell unit to transform the stack of tapes into a block. The scaled current in the homogenised bulk is defined according to the definition for 2D homogenisation. In principle, the homogenisation makes use of

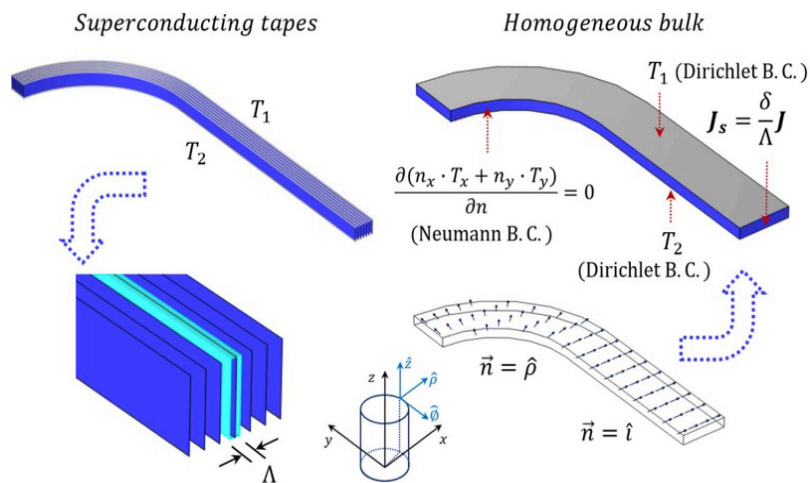


Figure 6.2: **T-A** homogenisation in 3D

The superconducting sheets are replaced by a homogeneous block. The scale current density J_s is introduced as a source term and boundary conditions T_1 and T_2 are applied to the upper and lower boundaries [205]

Maxwell's equations in the same way as the 3D **T-A** formulation. Therefore, the thin strip approximation of the tape is kept which allows reducing the current vector potential into a scalar quantity. This means that even if the tapes are replaced by a homogeneous bulk, the current can only flow in the plane parallel to the original superconducting sheets. For example, in the zoom presented in the bottom left corner of Figure 6.2, the current can only have J_y and J_z components and the current vector potential only has a T_x component. The homogenised block can be seen as a highly compressed group of superconducting tapes.

The magnetic vector potential is calculated in all the domains, and we solve the current vector potential only in the superconducting domain by using Maxwell-Faraday's law, where the normal vector $\mathbf{n}$ can be easily determined in the **T-A** formulation because it is the vector perpendicular to the superconducting sheet. Therefore, it is usually defined by default in commercial software like COMSOL Multiphysics. However, once the stack of tapes is replaced, it is not easy to define $\mathbf{n}$ inside the bulk. We no longer have a reference surface from which we can define the normal vector. A similar issue can be found in the 2D **T-A** homogenisation when complex (rotated or curved) cross-sections are modelled in 2D. Two approaches can be used to identify normal vectors within the superconducting domain. One option is to analytically calculate $\mathbf{n}$ by following the geometrical path of the tape before homogenisation as presented in equations (3.17) and (3.18) for more complex geometries as the one shown in the right corner of Figure 6.2. In this case, $\mathbf{n} = \mathbf{i}$ in the straight section and $\mathbf{n} = \boldsymbol{\rho}$ in the circular one. However, these analytic expressions can become very complex if we change the geometry of the coil, for example by introducing twisted or curved non-circular sections. Therefore, a second and more general solution is a curvilinear coordinate (CC) system defined in the superconducting domain. This approach creates a local coordinate system with curved lines that follow the shape of the superconducting domain. The unit vectors of this local coordinate system can be defined for all possible geometries and used as normal/tangential vectors. This approach can be implemented in COMSOL Multiphysics by using the CC module.

Since the homogenised bulk represents a densely packed group of HTS sheets, each one of these sheets should transport the same current as its original counterpart. Therefore, Dirichlet boundary conditions are applied to the boundaries that correspond to the edges of the tape, as expressed in equation (3.13). Neumann boundary conditions are applied to the boundaries that correspond to the flat side of the tape according to equation (3.19).

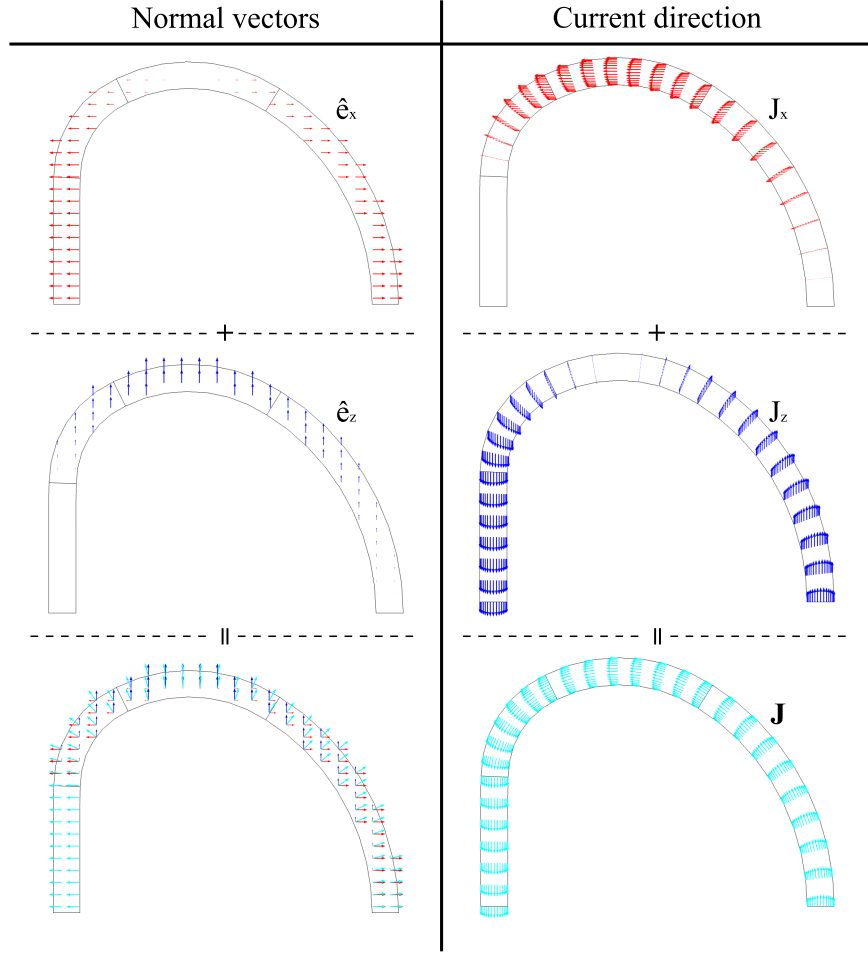


Figure 6.3: Distribution of normal vectors and current components inside the D-shaped coil
 From top to bottom: x-component (J_x), and z-component (J_z). The resulting normal vector and total current is depicted in light blue arrows at the bottom

In the specific geometry of a D-shaped coil, the current vector has an x-component and a z-component (Figure 6.3). In the straight section, the current flows parallel to the z-axis. The curved sections will have a current flowing in the z and x directions following the winding and position of the tapes. Due to the complex behaviour of the current density in 3D geometries, the cartesian components (J_x, J_y, J_z) are not suitable for representation purposes. A possible way to overcome this problem is to use the norm of $\mathbf{J}$. However, with the norm of $\mathbf{J}$ it would not be possible to appreciate the two fronts of currents in opposite directions during the AC cycle. Therefore, the dot product between the current density vector and the tangential vector parallel to the winding direction ($\mathbf{e}_t$) can be used:

$$\frac{\mathbf{J} \cdot \mathbf{e}_t}{J_c(B_{\parallel}, B_{\perp})} \quad (6.1)$$

The dot product between this unit vector, parallel to the longitudinal direction of the tape, and the current density keeps the same module and direction of $\mathbf{J}$, by having a positive/negative

value when the current density is in the same/opposite direction of $\mathbf{e}_t$. The parallel and perpendicular (to the wide face of the tape) components of the magnetic field required for the local computation of $J_c(B_{\parallel}, B_{\perp})$ can also be calculated in a more general expression with the dot product between the magnetic field and the other two-unit (base) vectors associated to the definition of the CC system:

$$B_{\parallel} = \mathbf{B} \cdot \mathbf{e}_{\parallel} \quad (6.2)$$

$$B_{\perp} = \mathbf{B} \cdot \mathbf{e}_{\perp} \quad (6.3)$$

6.2.2 Model setup

In this section, the geometry of a D-shaped coil, used in Tokamak fusion devices, is investigated [206], [207]. The shape and dimensions of the coil are shown in Figure 6.4 with the design parameters taken from [208]. The geometrical design of this type of coil features a straight inner leg and curved segments with different radii. This geometry is reduced to a quarter model by considering symmetry with the horizontal and vertical planes at the centre of the coil. In this study, the critical current characteristic outline in section 3.4.1 is used for two different HTS tape configurations. For the validation of the modelling strategy, 50 turns of a 4 mm HTS tape with a total width of 20 mm are used to compare the results with the 3D H

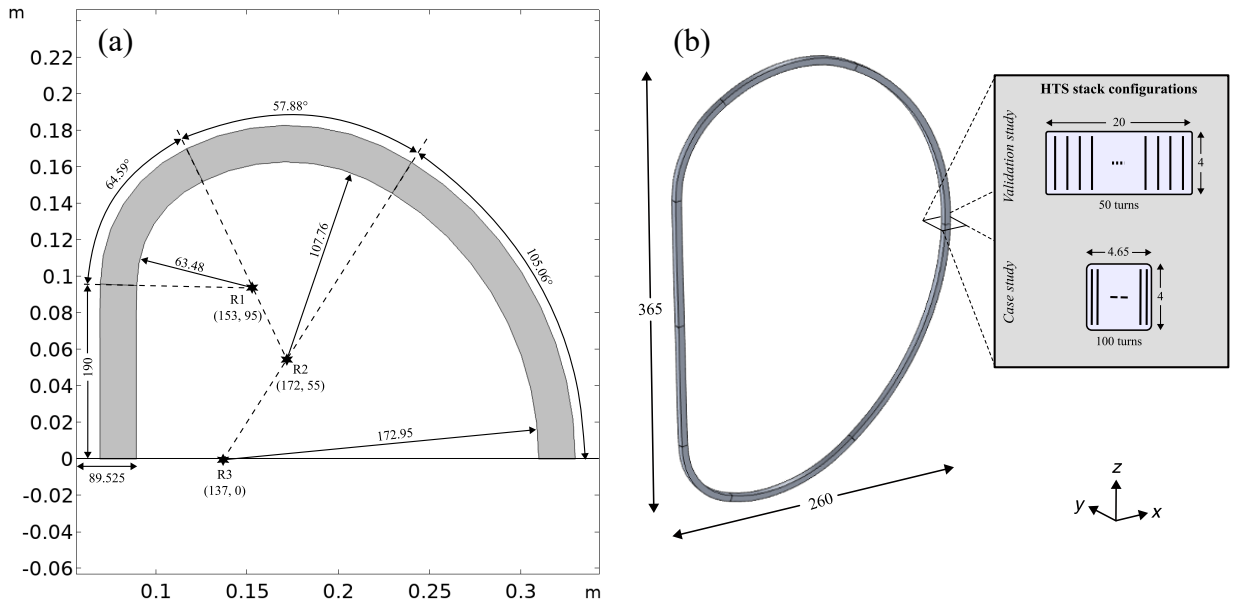


Figure 6.4: D-shaped coil geometry for a Tokamak fusion reactor
a) cross-section of the coil b) overall coil dimensions and tape configurations

homogenisation presented in [148]. In the case study, a more compact configuration of 100 turns with a width of 4.65 mm is investigated using the same critical current characteristics.

6.2.3 Mesh

Due to the high ratio of the coil dimension to the HTS conductor cross-section, the model has a large number of degrees of freedom. To keep the simulation within a manageable timeframe, the mesh of the geometry needs to be carefully chosen. It is important to use as few elements as possible while at the same time not compromising the accuracy of the model. This limits the number of elements that can be used to mesh the HTS conductor cross-section. By using a structured, quadrilateral mesh in the superconducting domain (10 elements equally spaced along the tape width and 8 elements along the width of the stack) and a free tetrahedral mesh in the surrounding air domain, the whole geometry can be meshed with 22822 elements in total, resulting in 293477 degrees of freedom.

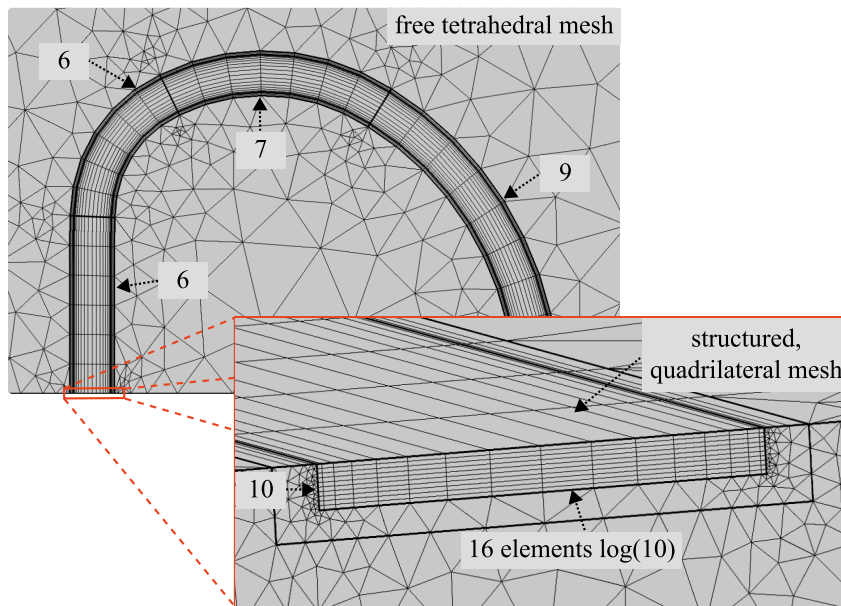


Figure 6.5: D-shaped coil mesh in FEM software

6.2.4 D-shaped coil validation

In the first step, the modelling approach with the D-shaped geometry is validated by using the same HTS cross-section presented in the previous section. For the validation, the transport losses of the 3D model are compared to a 2D axial symmetric model using the tape model and the homogenisation technique. The mean radius is calculated from the height and width of the 3D geometry ($R_m = 145$ m) to approximate the D-shaped coil in 2D.

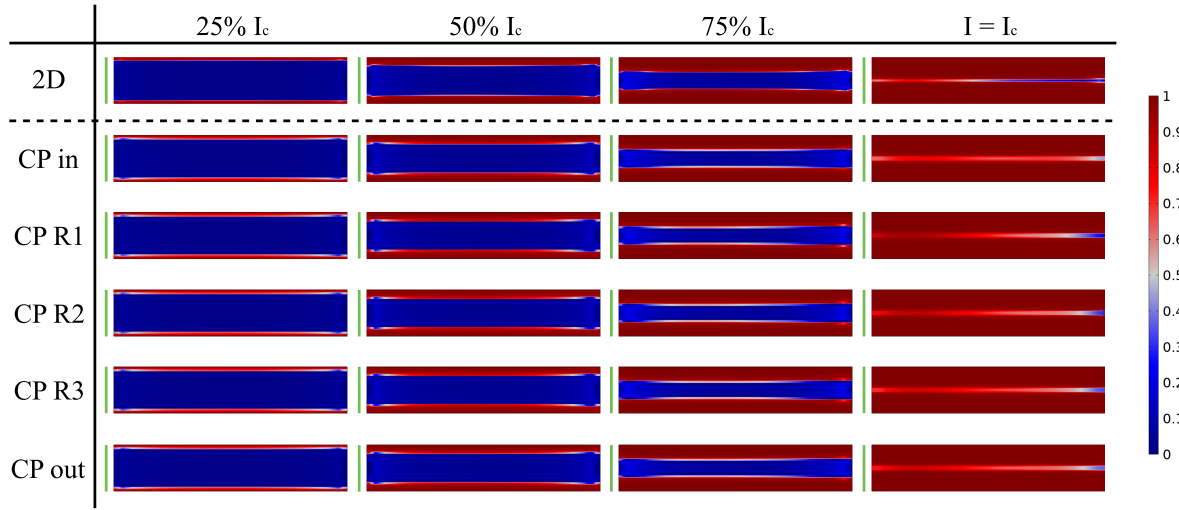


Figure 6.6: D-shaped coil current distribution in the validation study

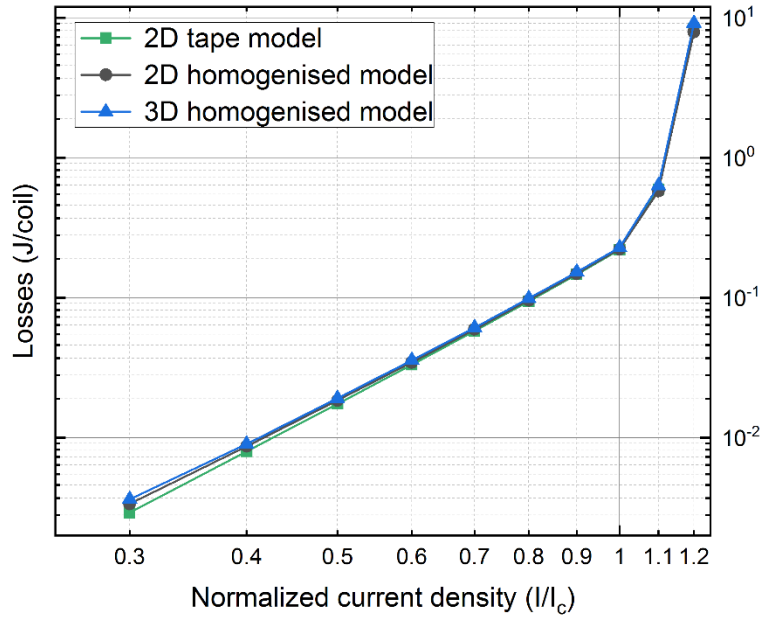


Figure 6.7: Transport losses for the homogenised D-shaped coil model in 3D

The results are compared to the simplified axial symmetric 2D model using the tape model and the homogenisation technique. The 2D model uses the mean radius of the D-shaped coil

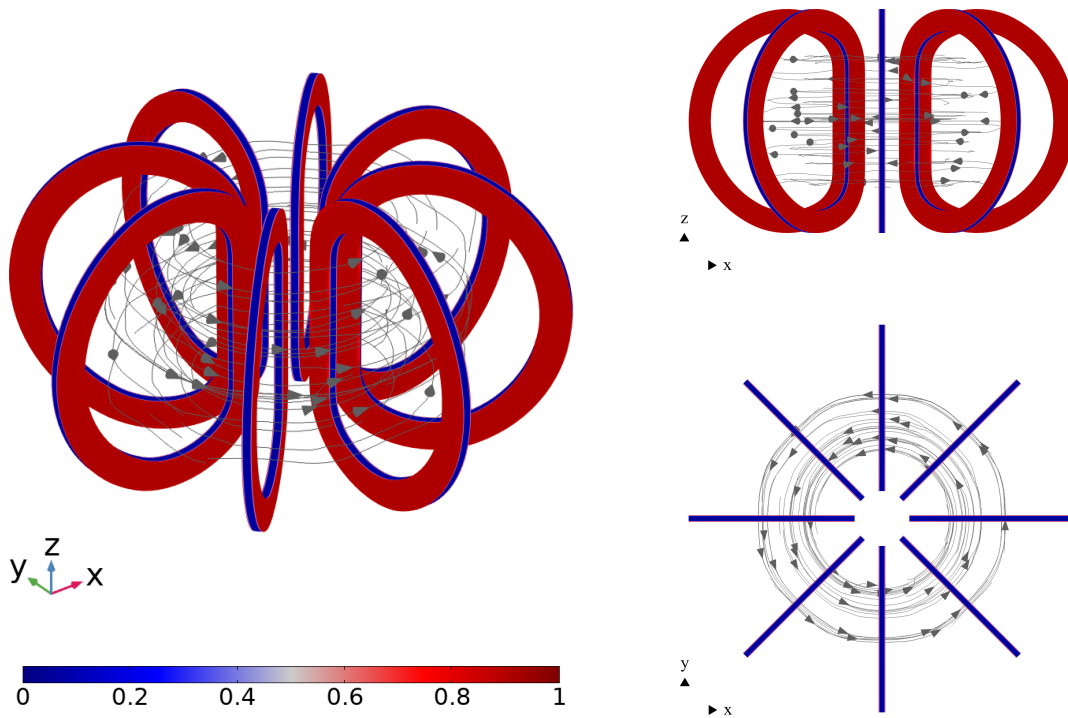


Figure 6.8: 3D D-shaped coil current and magnetic field distribution in a revolved geometry

The results for the comparison of transport currents ranging from 30 A to 120 A are shown in Figure 6.7. It can be seen that the agreement between the models is very good, with a maximum relative difference of 12.9%.

In this model, the D-shaped coils are placed inside a cylindrical domain. By taking advantage of the symmetrical arrangement of the coils as well as the symmetry of the coil geometry the model can be reduced to $1/16^{\text{th}}$ of the full-scale model. Reducing the complexity of the geometry requires the implementation of adequate boundary conditions as discussed in section 3.2.3 and 6.2.1. Although the simulation time increases due to the low-quality elements at the centre of the revolved geometry, it is possible to model the influence of the magnetic field from the neighbouring coils which is shown in Figure 6.8.

6.2.5 D-shaped coil case study

Subsequently, a study case for a load cycle is evaluated by using the HTS cross-section shown in Figure 6.4. The critical current for this coil is 60 A. In the first second of the load cycle, the transport current is ramped up from 0 to 50 A, followed by a 2 s relaxation time. Then the

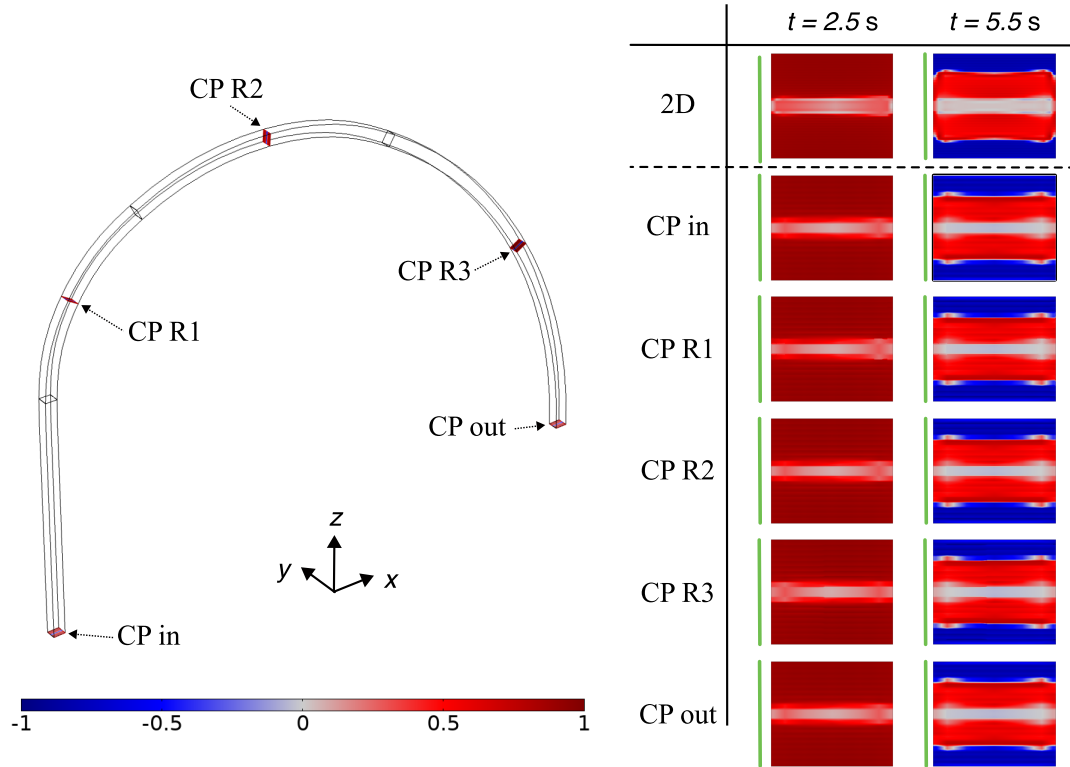


Figure 6.9: D-shaped coil current penetration in the case study

current is ramped down over 1 s from 50 to 0 A, and at the end, another 2 s are given as relaxation time.

In Figure 6.9, the current penetration is shown at 2.5 s (during the stabilisation after the initial ramp phase) and 5.5 s (after the ramp down). On the left side of the figure, the locations of the cut planes (CP) are indicated in the geometry. The current distribution is shown in the respective CP of the 3D model and compared to the solution from the 2D axial symmetric model. In CP in and CP out, the distribution is almost identical to the 2D model. The largest difference can be observed in the CP of the segment with the smallest radius (CP R1). The current penetration at the inside of the coil is higher than at the outside of the coil. This can be explained by the influence of the self-field, which is larger in the inner part of the segment with the highest curvature.

In Figure 6.10 the load cycle is shown where the applied transport current is indicated by the red dashed line. The blue and green dashed lines correspond to the power dissipation calculated with the 2D, and 3D models respectively. There are two distinct peaks in the losses, which occur at the end of each ramp phase (up/down).

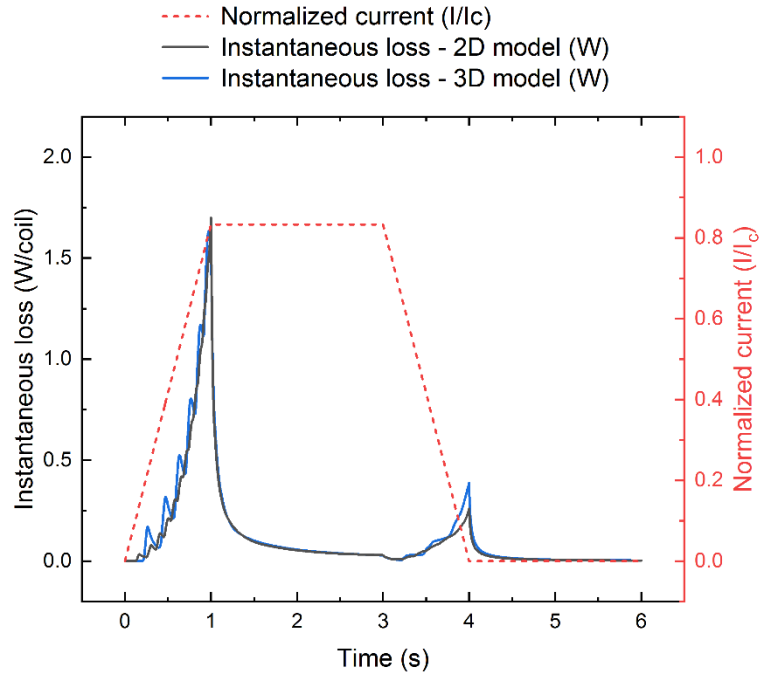


Figure 6.10: Current ramp-up/down process (load cycle) and instantaneous losses in the D-shaped coil model computed with the 3D and 2D *T-A* homogenisation models

During the ramp-up of the current, the losses of the 3D model show a ripple effect, which can be attributed to numerical (mesh distribution) and physical (coil geometry) factors, where the coil geometry has a major impact on the mesh distribution. Due to the low number of mesh elements used for simulation feasibility, the ripple effect is particularly noticeable, as fewer elements are available to accurately capture the physical behaviour. This behaviour is amplified at lower currents, if the same mesh is used, and it can also be observed in coarsely meshed 2D models.

The instantaneous losses at the end of the ramp-up phase at 1 s are 1.69 W for the 2D model and 1.61 W for the 3D model. For the second peak at the end of the ramp-down phase at 4 s, the instantaneous losses are 0.26 W and 0.39 W, for 2D and 3D respectively. The evaluation of the losses for the whole load cycle shows that there is a discrepancy, and the 2D model underestimates the losses for this complex geometry by 11.54%. The losses amount to 0.69 and 0.78 J for the whole load cycle, for the 2D and 3D models respectively.

In summary, the 2D and 3D models show a good agreement. The computation time for a 1 s ramp phase is about 20 h for the 3D model, while the 2D model finishes within minutes. For a detailed investigation of a D-shaped coil, the 3D model can provide a better understanding of the electromagnetic behaviour, although it is limited in terms number of turns. Especially if multiple pancakes need to be simulated, the required number of mesh elements and resulting

DOF exceed the available computational capabilities of this thesis. Therefore, the 2D modelling approach will be used for the following case study of the TFC system.

6.3 Toroidal field coil case study

In a Tokamak fusion reactor, the toroidal field coils are an essential component for confining the plasma in a toroidal shape and are responsible for generating a strong magnetic field that is used to maintain the plasma's stability and prevent it from touching the walls of the reactor. The magnetic field generated by the toroidal field coils creates a toroidal confinement region where the plasma can be sustained and heated to the high temperatures required for fusion to occur. The toroidal field coils are a set of discrete coils that are arranged around the torus of the reactor. The shape of the coils is engineered to produce a uniform magnetic field throughout the confinement region. The coils are typically made of a superconducting material to achieve high magnetic field strengths and must be cooled to maintain their superconducting state. The design of the toroidal field coils is critical to the performance of the Tokamak reactor. The coils must be able to withstand the large Lorentz forces, which arise due to the high currents and magnetic fields. This results in high mechanical loads that can lead to catastrophic stress and affect the performance of the magnet system. Therefore, the numerical investigations of TFCs are focusing on magneto-mechanical simulations [209]. Additionally, the coils must be designed to minimize the amount of electrical current required to generate the magnetic field, as this current can generate significant amounts of heat at the power suppliers and leads as well as connections, that must be removed from the system in addition to the heat coming from the high temperatures of the plasma. Therefore, the cooling system for the toroidal field coils must also be carefully monitored to ensure that the coils remain at their superconducting temperatures and do not overheat. The operation conditions of the toroidal field coils are controlled to ensure the stability and safety of the Tokamak reactor. Close attention must be paid to the regulation of the magnetic field strength to maintain the plasma in a stable configuration since any disturbances or fluctuations in the magnetic field can cause the plasma to become unstable and potentially damage the reactor.

Here, the electromagnetic behaviour of a TFC using HTS is investigated. The required number of tapes to achieve the required target field at the coil is determined and the current as well as the magnetic field distribution of the resulting coil configuration are evaluated.

6.3.1 Geometry

In this part of the chapter, the design of the ITER reactor is used for the investigation of the electromagnetic design and requirements. The design of the ITER TFCs has been reported in parts in several publications [210] [211]. The most detailed report from the ITER reactor [211] provides the basis of the design.

Due to the incomplete data on the geometrical design of the TFC, the mathematical expression for the curvature of the TFC geometry [212] was used to calculate the missing points. At the straight leg, the radius ρ is defined as,

$$\rho = kr. \quad (6.4)$$

The local radius of the curvature at any point in the $r - z$ plane can be defined by,

$$\rho = \pm \left[1 + \left(\frac{dr}{dz} \right)^2 \right]^{\frac{3}{2}} / \frac{\partial^2 r}{\partial z^2}. \quad (6.5)$$

Using these definitions of the local radius, the following equation needs to be solved numerically.

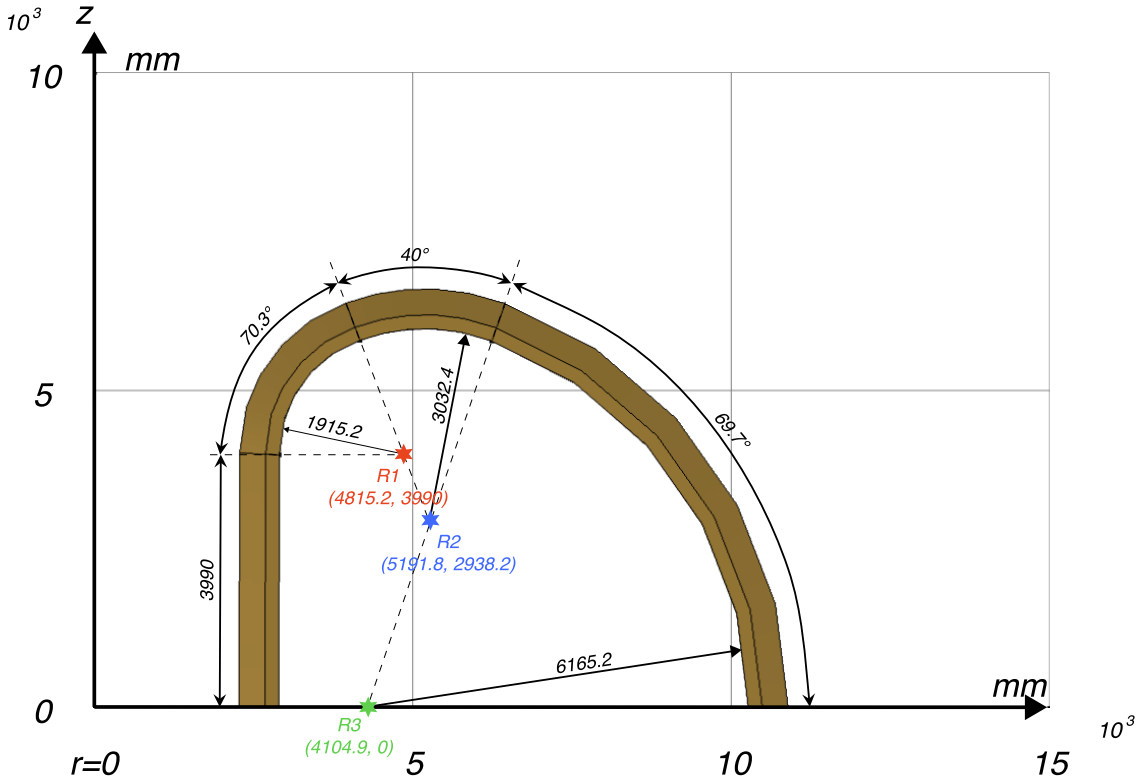


Figure 6.11: TFC geometry for an ITER-like Tokamak fusion reactor

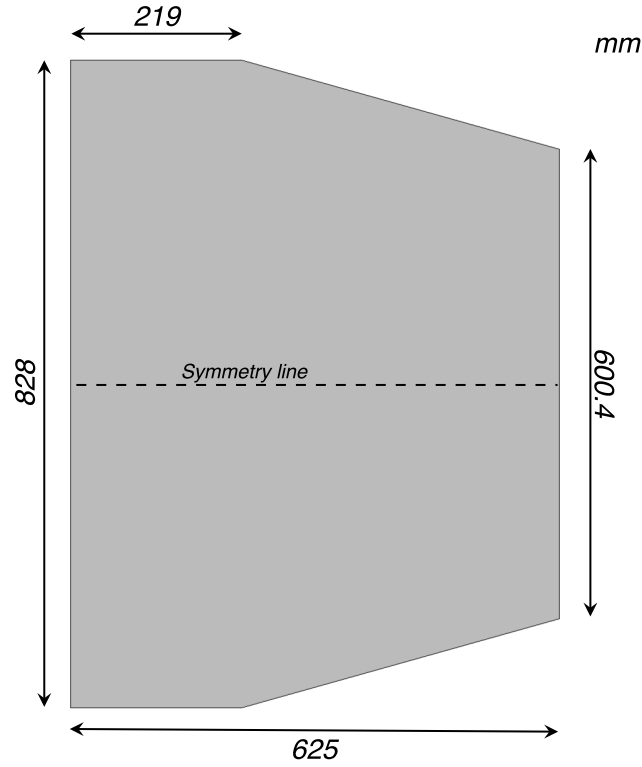


Figure 6.12: Cross-section of the available coil area of the ITER-like TFC in the midplane

$$r \frac{d^2 r}{dz^2} = \frac{\pm 1}{k} \left[1 + \left(\frac{dr}{dz} \right)^2 \right]^{\frac{3}{2}} \quad (6.6)$$

While this equation is of second order and therefore, requires two integrations, the first integration can already provide information about some of the limits of the curve. The resulting coil has a constant tension curve whose local radius of curvature is proportional to the distance from the z-axis. The curve begins and ends tangentially to a cylinder section which supports the net force on the coil towards the z-axis. The large dimensions from the machine axis are given by,

$$r_1 = r_0 e^{-k} \quad (6.7)$$

$$r_2 = r_0 e^k \quad (6.8)$$

and

$$k = \frac{1}{2} \log_e \left(\frac{r_2}{r_1} \right) \quad (6.9)$$

The curve segment of the coil is completely determined by specifying k and either r_1 , r_2 or r_0 .

By using this set of equations, it is possible to calculate the shape of a toroidal field coil. In this case, only specific points of the geometry were calculated to obtain the D-shaped geometry.

The geometric design of the cross-section is presented in [211]. The area shown in Figure 6.12 represents the area available for the conductor material. While in the ITER design the whole area is exploited using LTS cable strands, the HTS coil will only use a fraction of this area, leaving more space for structural material.

6.3.2 Model setup

The symmetric distribution of the TFCs allows for reducing the investigation to only one coil. Based on the results of the 3D study of the D-shaped coil, a 2D axial-symmetric model can be used to simulate the electromagnetic behaviour with sufficient accuracy. Using the height of 11.941 m and width of 7.8 m of the TFC, the D-shape can be approximated by the mean radius of 5.2478 m. Since the cross-section of the coil is also symmetrical, the model can further be simplified to a quarter model, using the perfect magnetic conductor boundary condition along the r-axis. To reduce the overall modelling geometry, an infinite element domain is added outside of the coil as shown in Figure 6.13. In this artificial domain, the coordinates are scaled to stretch the physical domain in a radial direction. The magnetic insulation boundary condition on the outside of the infinite element domain is therefore applied far from the area of interest. To use this feature in a 2D axial-symmetric model, the infinite element domain needs to use spherical coordinates with the origin in the centre of the geometry.

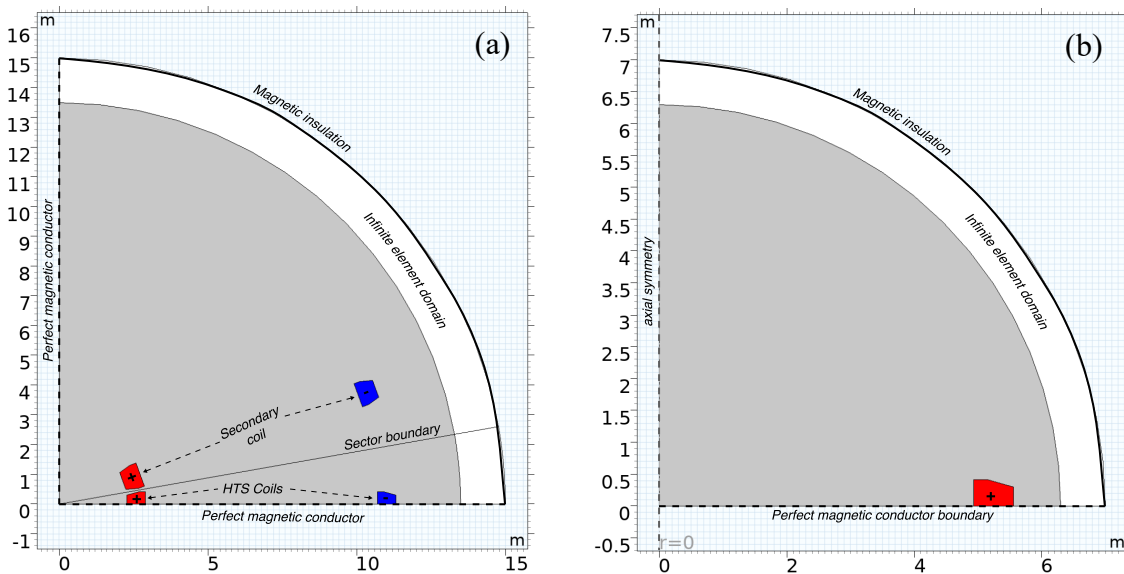


Figure 6.13: TFC model setup in FEM software

(a) 2D axial-symmetric (b) 2D infinitely long

While the 2D axial-symmetric model can approximate the behaviour of a single TFC, the magnetic field influence of the neighbouring coils is neglected in this approach. Especially at the inner leg, this will impact the current and magnetic field distribution due to the proximity of the coils. In the 2D infinitely long model, the cross-section of the inner and outer leg of the TFC is modelled. For the HTS coil, symmetry can be exploited to simulate only half the coil and use a perfect magnetic conductor boundary along the horizontal axis. The sector boundary shown in Figure 6.13(b) is used for the evaluation of the magnetic field distribution later in the study. The narrow region close to the centre of the model affects the modelling results if only the sector of the HTS coil is modelled with adequate boundary conditions. Therefore, the domain is increased, and a secondary coil is rotated $1/18^{\text{th}}$ to simulate the magnetic field generated by the next closest coil. This can be done using the coil feature in the magnetic field formulation and defining the corresponding current directions. The vertical axis of the model is selected to be a perfect magnetic conductor and an infinite element domain is defined on the outside of the physical model.

6.3.3 Mesh

The mesh structure of the 2D axial-symmetric model is shown in Figure 6.14 and will be used for all subsequent studies. The HTS pancakes in each model are meshed with a structured quadrilateral mesh with 60 elements along the height of the pancake and 24 elements symmetrically distributed along the width with an element ratio of 10. The air gaps in between the pancakes are refined with 6 elements for the height.

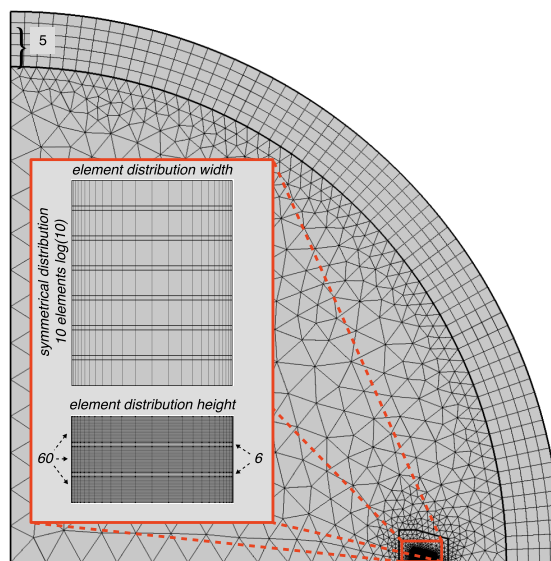


Figure 6.14. TFC mesh in FEM software for the 2D axial-symmetric model

The infinite element domain uses the same type of mesh with 5 elements along the radial direction and the number of elements on the outside boundary are selected in such a way that the resulting mesh elements are close to squares. The remaining area uses a free tetrahedral mesh which is additionally refined in the narrow regions of the 2D infinitely long model to avoid simulation errors. For both models, the cross-sectional and axial symmetric model, the coil area is using the same mesh, but due to the added complexity of the cross-sectional model, the resulting number of total mesh elements is higher. This results in 226'239 and 96473 mesh elements for the cross-sectional and axial symmetric models, respectively. As the number of mesh elements directly impacts the number of DOF for the model, the cross-sectional model has a total of 732536 DOF, while the axial symmetric model only has 320521.

6.3.4 Operation scenario

The TFC system generates a homogeneous magnetic field in a toroidal direction to confine the plasma. The operation scenario is like the D-shaped coil study, but with increased time scales. During the initial ramp phase, the current in the coils is increased to the operational current. The optimal ramp rate for the TFC is determined by studying the losses for different ramp times.

Once the operational current is reached, the TFCs are operating in DC conditions to generate a stable magnetic field on the plasma. At the end of the operation, the current is ramped down to zero.

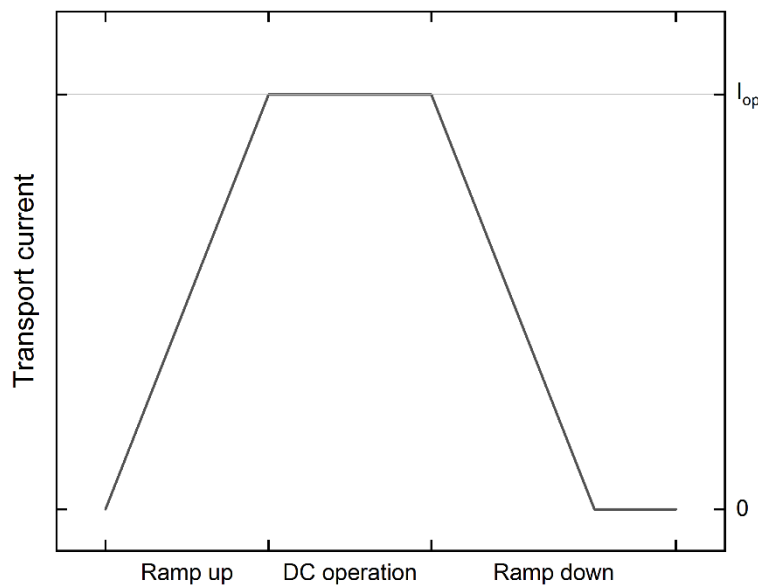


Figure 6.15: TFC operation scenario

6.3.5 Design process

To reduce the proximity of the TFCs at the inner legs, the cross-section of the coil area is trapezoidal so that the coils are not intersecting each other. The geometric shape of the available coil area also affects the design of the TFC. High-field solenoids are designed to have large coils on the outside, while their height decreases towards the innermost coils. This design choice allows for peak fields at the centre of the innermost WP and tape of the coil. For the

Table 6.1: Design options TFC with the same number of PCs for all WPs for a target field $B=11.8T$

Design	WPs	PCs/WP	tapes/PC	#HTS
#1	4	11	400	17600
#2	5	12	300	18000
#3	3	14	400	16800
#4	4	14	300	16800
#5	5	18	200	18000
#6	3	18	300	16200
#7	2	20	400	16000
#8	4	23	200	18400
#9	2	35	300	21000
#10	3	40	200	24000

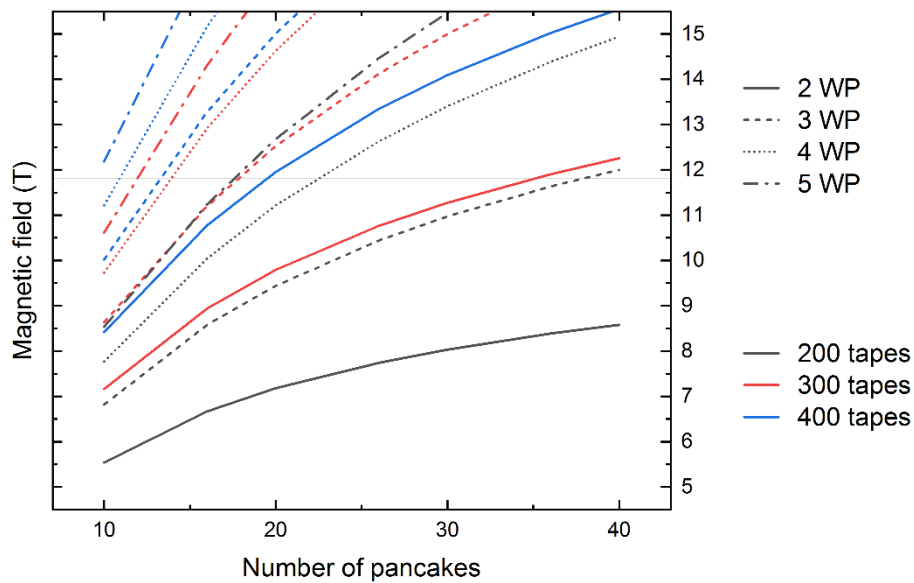


Figure 6.16: TFC magnetic field for different configurations

The same number of PCs is used for all WPs

TFC the design can be reversed, placing the largest stack of pancakes closest to the plasma facing side.

For this design process, the critical current of an initial tape configuration for the pancakes needs to be evaluated at the target magnetic field. Then an axial-symmetric model with the coil located at the mean radius is built. Using a parametric sweep, potential coil configurations to generate the required magnetic field at 70% of the critical current are evaluated. For this evaluation, only the maximum magnetic field is required and therefore, a conventional coil model can be used to model the pancake as a homogenised multiturn. After identifying possible configurations, the critical current of the selected pancake is recalculated, and the parametric study is conducted again for a variation in the number of tapes per pancake, pancakes per winding pack and winding packs per coil, shown in Figure 6.16. In this first step, the same number of pancakes is used for all winding packs.

Table 6.2. Design options TFC with the varying number of PCs for each WPs for a target field $B=11.8T$

Difference PCs/WP	PCs/WP1	PCs/WP2	PCs/WP3	#HTS
const.	18	18	18	16200
2	20	18	16	16200
4	22	18	14	16200
6	26	20	14	18000
8	28	20	12	18000

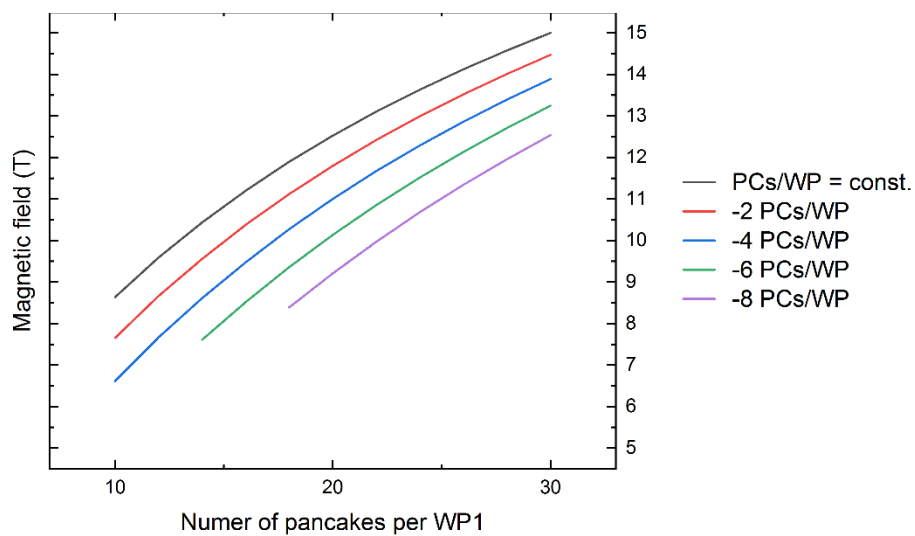


Figure 6.17: TFC magnetic field for selected configurations

The number of PCs varies in each WP

The horizontal line in Figure 6.16 indicates the target magnetic field of 11.8 T. A pancake with 200 tapes requires at least three winding packs to achieve the field target. As shown in the figure above, an increase in the number of winding packs has a great effect on the maximum magnetic field but also greatly increases the total number of HTS tapes in the coil. To identify the best coil configuration, all viable designs are listed in Table 6.1.

Although, design option #7 requires the least number of tapes, the configuration with two WPs shows a quite elongated coil geometry. The design choice is #6 using 3 WPs and 300 tapes/PC which will be used to conduct the second part of the coil design. Because of the trapezoidal shape of the coil area, the coil itself is designed to have the largest stack of pancakes facing the plasma. The number of pancakes in the other winding packs is reduced consecutively by 2, 4, 6 and 8 pancakes. The maximum magnetic field generated by these coil configurations is shown in Figure 6.17, where the black line represents design #7.

While each configuration can achieve the target magnetic field, reducing the number of pancakes in each winding pack requires a higher number of pancakes in the first winding pack. The results from the figure above are listed in Table 6.2.

Decreasing the number of pancakes by four for each consecutive winding pack does not increase the total required HTS tapes to achieve the target field. Also, the geometry of this coil follows the gradient of the trapezoidal area of the available area while keeping the configuration as compact as possible.

The selected coil configuration is then tested in a 2D model using the field-dependent critical current data to evaluate if the target field is achieved. This is done for tapes from SuperPower and SuperOx at 25 K and 35 K for an operating current of 500 A. In Table 6.4, the maximum magnetic field at the coil as well as the magnetic field at the centre of the coil are listed.

The target magnetic field is achieved with each tape at both temperatures. The maximum magnetic field is expected to be at the corners of the coil due to the higher current penetration at the outermost pancakes. This results in a lower magnetic field at the centre of the coil. While the SuperOx tape can achieve slightly higher magnetic fields, the central field of the SuperPower tape at 35 K is the highest. This can be explained by identifying the safety margins of the coil at the operational current for the different tapes and temperatures (Table 6.3).

Table 6.3: Critical current of the weakest turn for the individual TFC

	25K		35K	
	I_c (A)	I/I_c	I_c (A)	I/I_c
SuperPower	702	71.2%	558	89.6%
SuperOx	815	61.3%	704	70.9%

The weakest turn of the coil configuration is the top pancake of WP1, which is the largest stack consisting of the most number of pancakes and located closest to the plasma. A safety margin of 70% is chosen in this study although it should be noted that for stable DC operation and to prevent quenching a higher safety margin could be required. Due to the low critical current of the SuperPower tape at 35 K, the ratio of I/I_c is almost 90%. Although the coil is still superconducting, the operation at an operating current of 500 A is very close to the critical limit and explains the high magnetic field at the centre of the coil. Since the current is close to the critical current, the current penetration of the coil is higher than in the SuperOx tape or the SuperPower tape at 25 K and will be closest to the centre of the coil, resulting in a high central magnetic field.

The critical current of the SuperOx tape for a target field of 11.8 T is about 10% higher at 25 K than the SuperPower tape. Even at 35 K, the SuperOx tape has the same critical current performance as the SuperPower tape at 25 K. In those three cases, the safety margin is achieved, and the coil operation can be conducted safely.

For the SuperPower tape at 35 K, the coil should be redesigned, and more pancakes added to achieve the target field with the safety margin. The design of the coil with the SuperOx tape at 25 K can be optimised and the total number of tapes reduced. To allow a direct comparison, in terms of the current and magnetic field distribution as well as the losses, the selected design is evaluated for all tapes and temperatures.

Table 6.4: Target magnetic field, maximum magnetic field, and magnetic field at the centre of TFC

B(T)	B_{target}	25K		35K	
		B_{max}	B_{centre}	B_{max}	B_{centre}
SuperPower	11.8	12	10.68	11.8	11.06
SuperOx		12.3	10.82	12.2	10.64

6.3.6 Ramp time study

Before evaluating the performance of the coil, the ramp time at the start of the reactor operation is evaluated. The ramp phase for a duration of 1 s, 30 s, 1 min, 30 min and 1 h is conducted using a constant critical current density of 625 A. The current during this operation is ramped up to the operating current of 500 A. The corresponding instantaneous losses in the cross-section of the coil are shown in Figure 6.18a over the normalized ramp time. The vertical axis is scaled logarithmically to show that the losses at lower ramp rates are significantly reduced. The instantaneous loss is lower for longer ramp times because the gradual increase in current helps maintain a higher critical current density by allowing more effective flux pinning and reducing vortex movement, thus minimizing energy dissipation.

Table 6.5: Dissipated energy and dissipated power for different ramp times of TFC

Ramp time	Dissipated energy (J)	Dissipated power (W)
1 s	1.81e5	283810
30 s	1.71e5	9185.9
1 min	1.69e5	4557.8
30 min	1.61e5	150.44
1 h	1.60e5	74.99

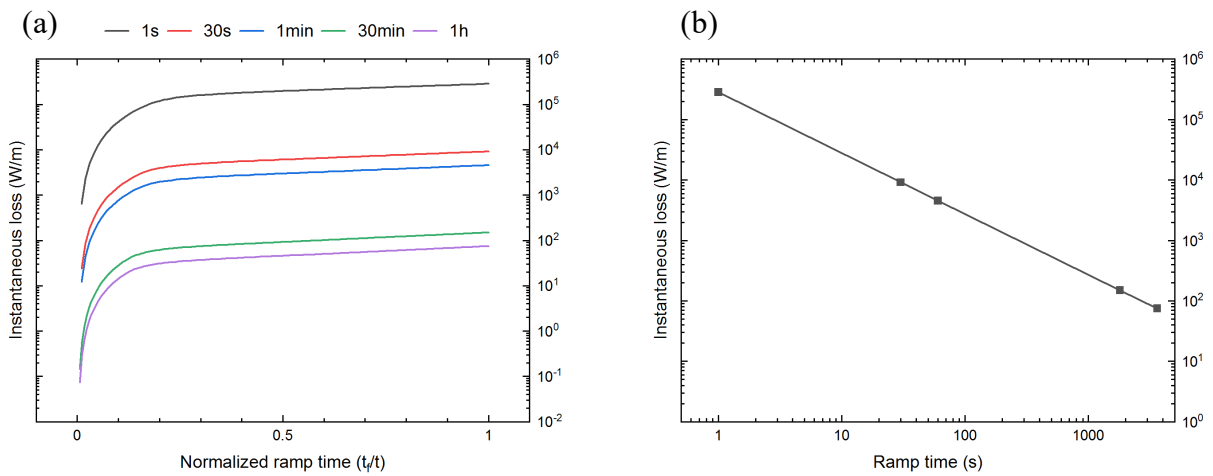


Figure 6.18: TFC instantaneous losses with constant critical current ($I_c = 625A$)

(a) loss distribution over normalized ramp time. (b) losses at the end of the ramp phase for different ramp times.

In Figure 6.18b, the dissipated power at the end of the ramp phase is shown for the selected ramp times. While the power loss at the end of the ramp phase for 1 s is almost 30 kW, extending the ramp time to 1 h reduces the losses to less than 75 W. The values for the instantaneous losses at the end of the ramp phase as well as the dissipated energy during the ramp phase are listed in Table 6.5. As already mentioned, the dissipated power is greatly reduced as the ramp time increases. This also affects the total amount of energy dissipated during the ramp phase. The ramp time selected for the case study is 30 min since at this point the instantaneous losses are already greatly reduced and the change in dissipated energy is minor. Extending the ramp time further will only prolong the simulation and not have a major impact on the losses since it is economically not feasible to ramp the current over days before starting operation.

6.3.7 Current distribution

In this section, the current distribution at the end of the ramp phase is evaluated. The normalized current density J/J_c is shown for the SuperPower and SuperOx tape at 25 K and 25 K for the 2D axial-symmetric model and the cross-sectional model in Figure 6.19 on a scale from -1 to 1. In all cases, the current is penetrating deep into the coil leaving only the eight innermost pancakes of each winding pack free of fully developed flux fronts. As previously indicated, the current penetration for the SuperPower tape at 35 K is the highest due to the low critical current, where all but the four innermost pancakes show a fully distributed current. It is interesting to see that the current distribution is not greatly influenced by the proximity of the neighbouring coil in the cross-sectional model.

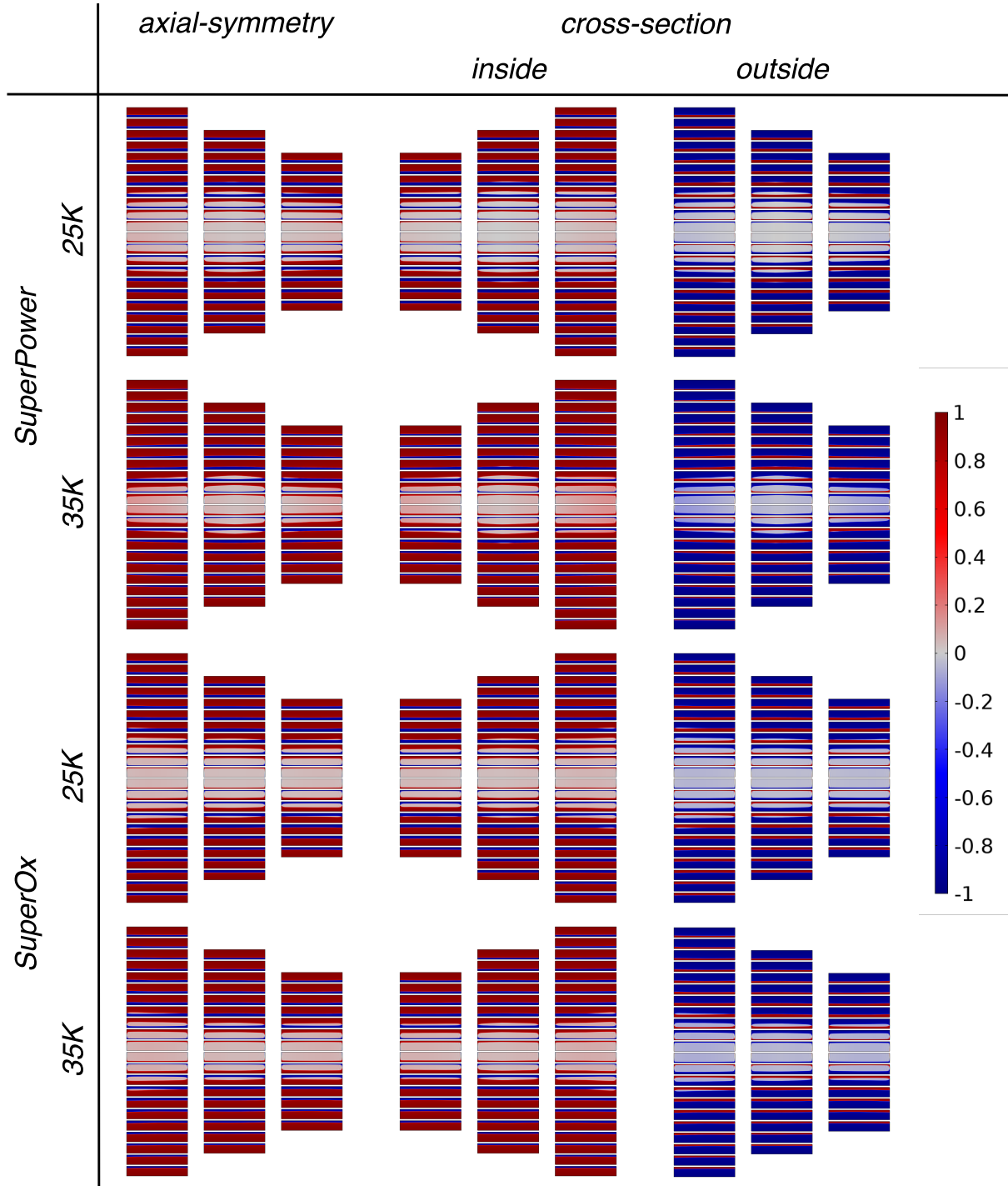


Figure 6.19: TFC current distribution at the end of the ramp phase for axial-symmetric model and cross-sectional model (inside and outside)

6.3.8 Magnetic field distribution

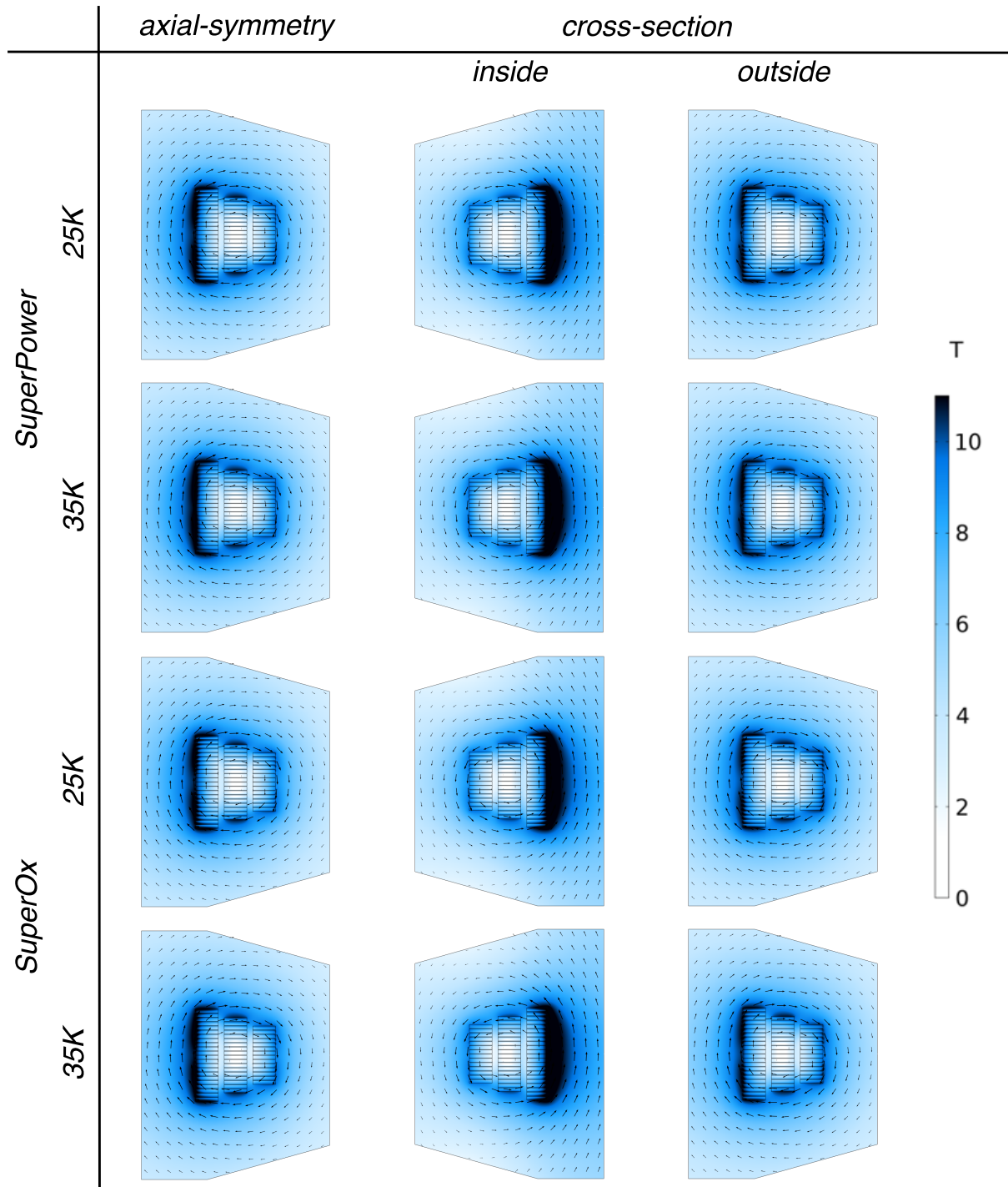
In Figure 6.20, the magnetic field distribution for all tapes and temperatures in the axial-symmetric and cross-sectional model is shown on a scale from 0 to 11 T. The magnetic field is shown in the available coil area and visualizes again the compact design that can be achieved using HTS conductors. Here, the difference between the two modelling approaches is shown. In the axial symmetric model, the isolated coil has a large radius and does not consider any external magnetic field generated by the neighbouring coils and therefore, is only affected by the self-field generated due to the operating current. The distribution of the magnetic field is similar to the one in the cross-sectional model of the outer part of the coil. This can be explained by the larger distance between the outer parts of the coil. At the inner leg of the coil, there a major impact on the magnetic field distribution can be observed. The magnetic field is greatly influenced by the neighbouring coil resulting in a higher parallel field component. The field is developed towards the centre of the coil, aiming at the plasma. Due to the higher parallel field at the inner leg, the critical current is not degraded, and the penetration as shown above is not much affected by the higher magnetic field. In Table 6.6, the maximum field values for each model are listed.

The SuperOx tape can generate slightly higher fields compared to the SuperPower tape at the respective temperatures. As shown in Figure 6.20, the magnetic field at the inside leg of the cross-sectional model is much higher due to the influence of the coils in proximity. While the axial-symmetric model is useful to evaluate the magnetic field of an isolated coil, the cross-sectional model can show the distribution, especially at the inside of the coil, more accurately.

The cross-sectional modelling approach for the TFC has another advantage in the visualization of the magnetic field distribution. The sector defined for the simulated TFC can be mirrored and then rotated around the centre point to visualize the field lines for the whole TFC system.

Table 6.6: Maximum magnetic field for 2D axial-symmetrical and cross-sectional model

Model	SuperPower		SuperOx	
	25K	35K	25 K	35K
axial-symmetric	12T	11.8T	12.3T	12.2T
cross-section (inside)	13.3T	13.1T	13.4T	13.3T
cross-section (outside)	11.6T	11.4T	11.9T	11.8T



*Figure 6.20: TFC magnetic field distribution at the end of the ramp phase
for 2D axial-symmetric and 2D cross-sectional model (inside and outside)*

This is shown in Figure 6.21, where the magnetic field is again shown on a scale from 0 to 11 T. The black lines show the magnetic flux distribution around the torus in the midplane of the reactor.

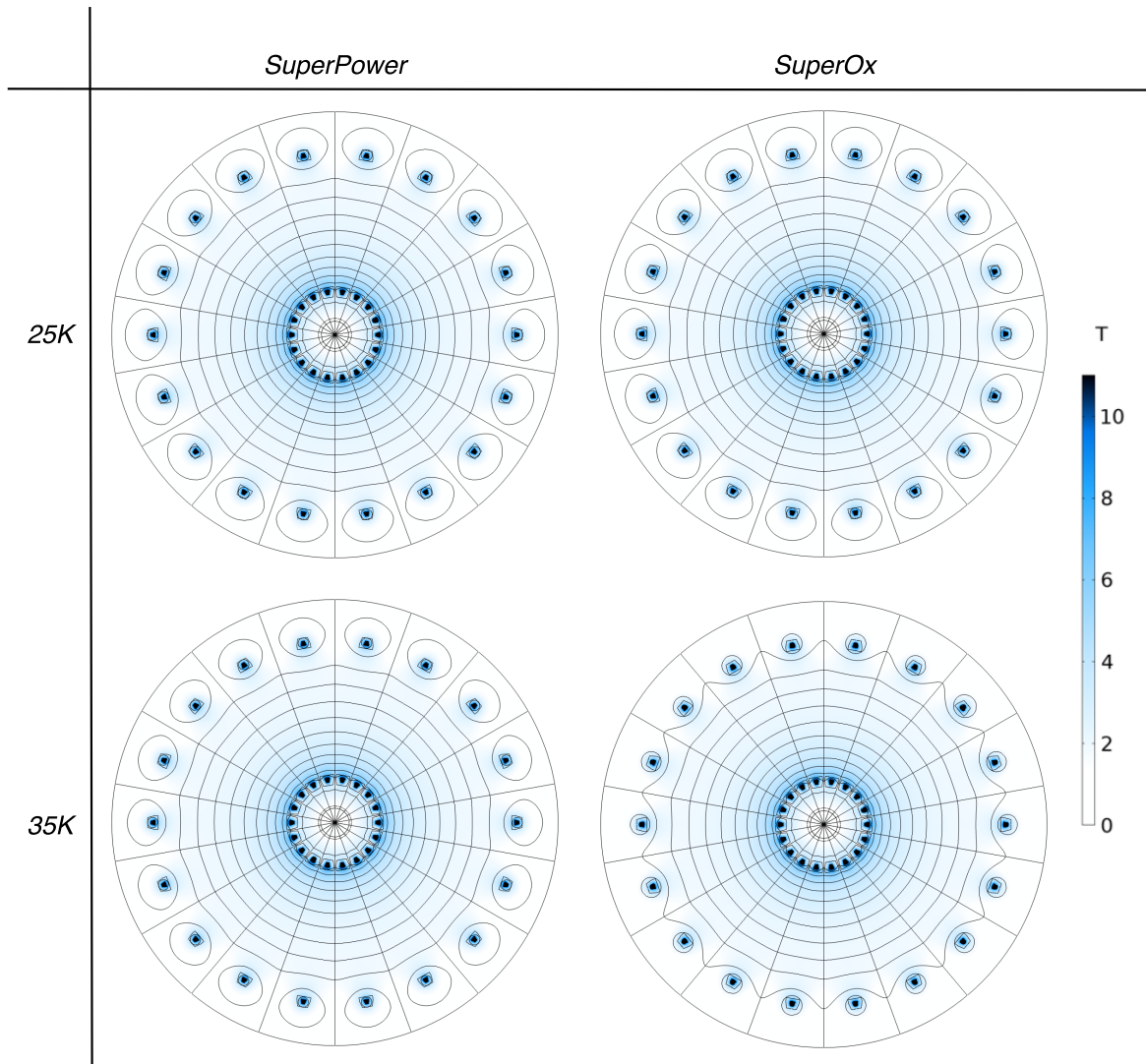


Figure 6.21: TFC magnetic field distribution at the end of ramp phase

Due to the gaps between the individual TFCs, the field lines close to the inside and outside of the coil show some ripple effect. Closer to the centre of the coil, the magnetic field shows a homogenous distribution. The field at the major plasma radius in the midplane is around 3 T, which is less than the 5.2 T achieved by ITER, but this model only considers the TFCs. Also, the reduced size of the HTS coil increases the radial distance from the coil to the centre of the plasma and therefore, reduces the achievable field.

6.3.9 Losses

The final evaluations of the TFC study are the losses during the different operation scenarios. At first, the losses for the cross-section of the coil area are shown in Figure 6.22 for the 2D

axial-symmetric model and the 2D cross-sectional model for the inner and outer parts of the coil. During the ramp phase, the transport current is increased linearly up to the operating current. As the current is increased the self-field of the coil generated by the transport current drives the magnetization of the coil. This means that the outermost pancakes, which experience the highest magnetic field will be fully magnetized, but the transport current is still below the critical current. At this point, the hysteresis losses reach a peak as the magnetic flux vortices are uniformly distributed in the fully magnetized pancake. Further increasing the transport current will lead to a decrease in the losses in the pancake due to the saturation of the pinning centres. As more pancakes become fully magnetized, the reduced effectiveness of vortex pinning in these pancakes reduces the energy dissipation of the whole coil. Interestingly, there is not much difference between the two modelling approaches and even between the inner and outer part of the coil. When looking closely, the losses of the inner coil section seem to remain constant, while in the axial-symmetric model and the outer cross-section, the losses are slightly declining. This can be explained by the direction of the magnetic field, which has a higher perpendicular component in the two latter cases.

After the initial 30 min ramp phase, the DC operation is simulated for 10 min where the operating current is kept constant, during which the losses decrease towards zero. After constant DC operation, the current is ramped down for another 30 min from the operating current level to zero. During this phase the current will reverse its direction which generates the losses shown in the figure above. The initial slow rise of the instantaneous loss during the ramp-down of the current can be attributed to shielding effects when the current increment reverses. Once the current increment direction in the pancakes starts to be completely reversed the losses will rise sharply until the current reaches zero. In the final 10 min, no current is applied to the coil and the losses decrease again towards zero.

Table 6.7 lists the dissipated energy for each operation scenario and model. The SuperOx tapes have lower losses than the SuperPower tapes at the respective temperatures during the ramp phase. The SuperPower tape at 25 K has the highest losses during the energization of the coil. When the coil is operated under DC conditions, the dissipated energy is one magnitude lower than for all tapes and temperatures. In the final ramp down phase, the SuperPower tape dissipates less energy than the SuperOx tape at the respective temperature.

After comparing the losses of each cross section, now the losses for the whole coil are evaluated. With the axial-symmetric model the losses can be directly evaluated for the circular

Table 6.7: Dissipated energy (J/m) during operation in the cross-section of TFC models

Model	Scenario	SuperPower		SuperOx	
		25K	35K	25K	35K
Axial-symmetric	Ramp-up	2.90e5	2.50e5	2.66e5	2.41e5
	DC operation	1.66e4	1.18e4	1.51e4	1.21e4
	Ramp-down	1.46e5	1.44e5	1.51e5	1.49e5
Cross-section (inside)	Ramp-up	2.71e5	2.36e5	2.46e5	2.24e5
	DC operation	1.69e4	1.17e4	1.49e4	1.20e4
	Ramp-down	2.27e5	2.22e5	2.30e5	2.27e5
Cross-section (outside)	Ramp-up	2.89e5	2.49e5	2.65e5	2.40e5
	DC operation	1.69e4	1.18e4	1.54e4	1.23e4
	Ramp-down	2.50e5	2.41e5	2.57e5	2.50e5

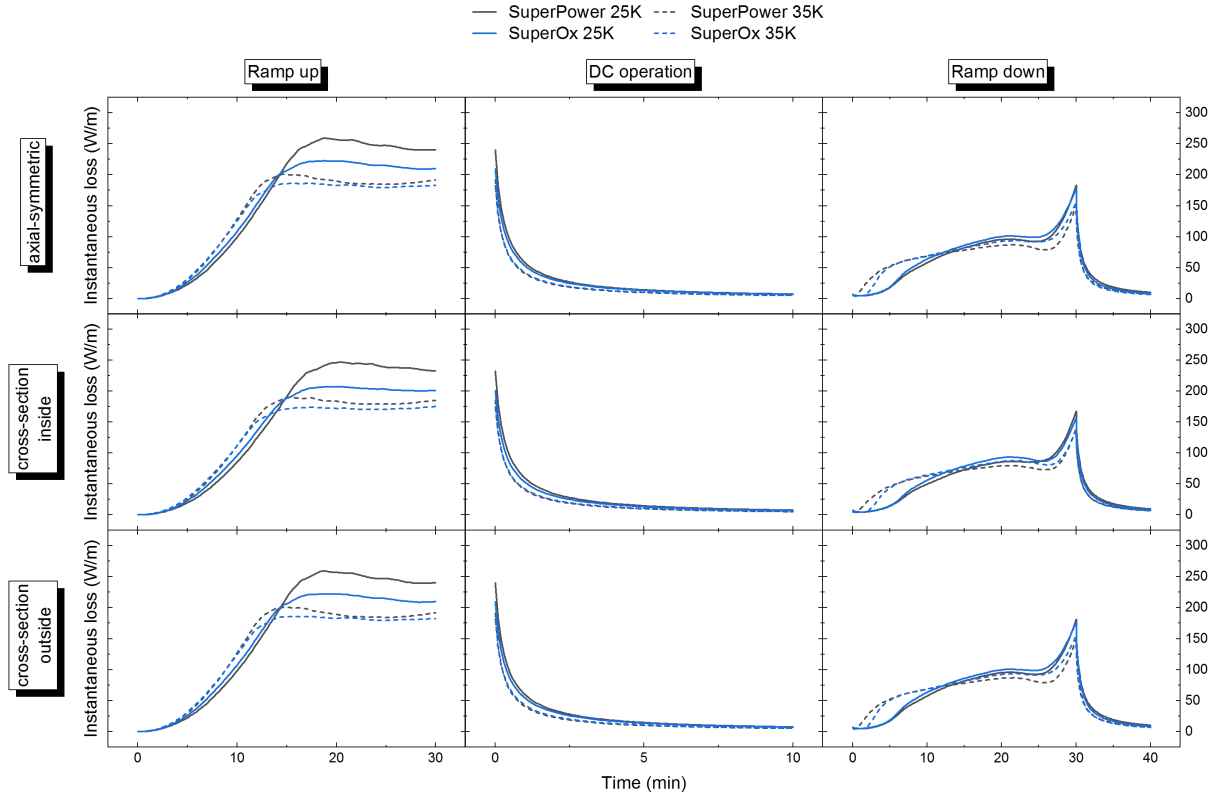


Figure 6.22: TFC cross-sectional losses during different operation scenarios for 2D axial-symmetric and 2D cross-sectional model (inside and outside coil)

coil. In the cross-sectional model, the losses in each cross-section are evaluated first as shown above. Then we assume that the half of the coil will be near the neighbouring coil while the other half will be farther apart. Therefore, we can calculate the total losses of the coil by adding

Table 6.8: Dissipated energy (J) during operation of TFC models

Model	Scenario	SuperPower		SuperOx	
		25K	35K	25K	35K
axial-symmetric	Ramp-up	9.56e6	8.24e6	8.78e6	7.94e6
	DC operation	5.47e5	3.88e5	4.99e5	3.98e5
	Ramp-down	4.83e6	4.73e6	4.99e6	4.91e6
cross-section	Ramp-up	9.23e6	7.99e6	8.43e6	7.65e6
	DC operation	5.57e5	3.87e5	4.99e5	4.00e5
	Ramp-down	4.58e6	4.52e6	4.72e6	4.68e6

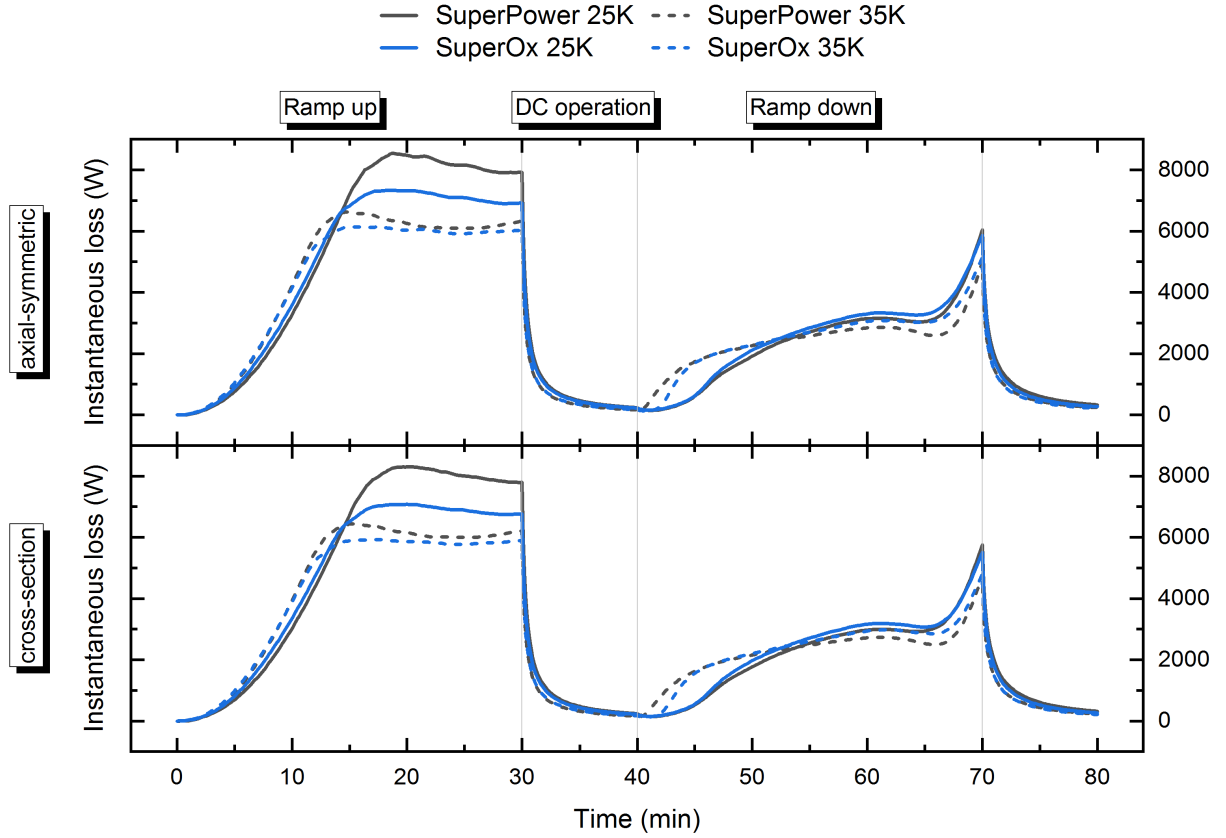


Figure 6.23: TFC instantaneous losses of the whole coil
for the axial-symmetric model and the cross-sectional model

the cross-sectional losses and multiplying with the circumference of the mean radius. The corresponding losses are shown in Figure 6.23.

Lastly, the total dissipated energy for each scenario and both models is listed in Table 6.8 above. While again, the SuperOx tape generates lower losses during the ramp-up phase, but higher losses during ramping down of the current, the axial symmetric model gives higher

losses for these scenarios. The losses for the whole TFC system can be calculated by multiplying the losses of the single TFC with the number of coils in the system.

6.4 Discussion

In this chapter, the 3D homogenised model for a D-shaped coil was evaluated. The validation showed that the homogenisation in 3D matches the results from the 2D models well, but in terms of the current distribution in the arc segment with the smallest radius, a slight discrepancy can be seen. To identify specific points in the coil where the current penetration varies from the simplified 2D approach, a 3D model is a useful tool. Furthermore, the full magnetic field distribution of a revolved 3D geometry can be visualised. In terms of the losses for the conducted case study, the 3D homogenisation again matches the results obtained with the simple 2D model. However, during the initial ramp-up phase, ripples in the losses can be observed due to the coarse mesh that is required to simulate within a reasonable time. For the simulation of a complete TFC magnet with multiple pancakes and an even higher number of turns per PC, the required number of mesh elements exceeds the numerical capabilities available for this thesis. Due to the elongated PC geometry and the narrow gaps between each pancake, the resulting geometry would become so complex that it is not possible to simulate the electromagnetic behaviour with current computational resources.

Since the 2D model matches the results of the 3D model very well, the next step was to simplify the modelling approach simulation of the ITER-like TFC system. This magnet system consists of 18 field coils symmetrically distributed along the torus of the fusion reactor, enclosing the plasma. Due to the symmetry involved only a single TFC was designed to achieve the target field of 11.8 T. While both, SuperPower and SuperOx tapes at 25 K and 35 K were able to produce the required field strength, the SuperPower tape at 35 K did not meet the safety margin requirements since it was operating at 90% I_c . The operating current was set to be 500 A for a selected coil configuration of 3 WPs where each WP had a total of four pancakes less than the previous one. While the coil was in the first stage designed with the same number of pancakes per WP for each WP, the coil design was modified in the next step to follow the trapezoidal shape of the available coil area. To achieve the target magnetic field for this coil, while keeping the design compact, the assumed thickness of the HTS tape was reduced from 50 μm to 25 μm . It should be mentioned that this could have impacts on the mechanical strength of the tape, but this is not considered in this study. The gaps between the pancakes are 2 mm and the gaps

between the winding packs were set to be 2 cm. With this design, the coil shape is emulating the available coil area in a much smaller area. In the ITER-like reactor the conductor cross-section is 4713 cm^2 , while the HTS design only uses 562.5 cm^2 . This effectively reduces by a factor of 8, not considering additional area required for cooling and structural support.

In this study the operation of a toroidal field coil using two different modelling approaches in 2D were compared. The operation scenarios simulated were the initial ramp phase, a stable DC operation and the final ramping down of the current. The effect of the time on the losses during the ramp phase was evaluated and a 30 min ramp time was identified to be the most effective approach. Then the current and magnetic field distribution as well as the losses during the operation of the magnet were evaluated using a 2D axial-symmetric model and a 2D cross-sectional model. The first model was already used to approximate the 3D coil shape of a TFC in 2D with high accuracy. Since this model cannot consider the influence of the neighbouring coils, a cross-sectional model was built to identify the influence of the coils in the proximity of the HTS coil. While the current distribution did not show much difference, the magnetic field especially on the inner leg showed that the magnetic field is directed more towards the plasma and have a higher parallel field component. This model also had the advantage of showing the magnetic field distribution of the full TFC system in the midplane of the reactor. Finally, the losses for the models using SuperPower and SuperOx tapes at 25 K and 35 K were evaluated. It was shown that the axial-symmetric model estimated the losses of the coil slightly higher than the cross-sectional model. Overall, the SuperOx tape performed better since at both temperatures, the safety margin of 70% was achieved. Also, the losses during the ramp phase were lower with the SuperOx tape at both temperatures.

Even though the 2D approximation is not an accurate depiction of the unique D-shape of the TFC, it allows for the simulation of large-scale magnet designs, which cannot be achieved in 3D due to the large aspect ratio of the D-shaped coil size to the conductor.

7 Conclusion

High-temperature superconductors are a promising technology that has the potential to significantly impact the development of commercial fusion reactors. While fusion has been the subject of research for several decades, the first reactor to achieve a sustained fusion reaction is still years away. Nevertheless, there have been major breakthroughs in the past decades, mostly due to the availability of new technologies such as high-temperature superconductors. Private companies are now acquiring a lot of funding for their novel magnet technology which is crucial in the design of compact reactors. A compact design will be required if fusion energy wants to compete with other sources of energy as the cost of the reactor increases with its size. By developing novel magnet technologies such as cable configuration, the production rate and volume of HTS is increasing, which will ultimately reduce the cost for these materials. Unfortunately, the high cost is a limiting factor at the moment, outweighing the benefits in terms of higher critical current density, higher magnetic field strength, higher critical temperature and the benefit of building simple and compact coils.

To decrease the cost of magnet development, numerical investigations can help to reduce the number and scale of experiments needed to test the technology. Due to the large size of fusion reactors which required high-field magnets that are one magnitude larger than experimental high-field solenoids numerical simulations are essential in predicting the electromagnetic behaviour of the superconducting coils. While system codes are used to investigate the overall reactor geometry, economic feasibility and configurations of subsystems, a detailed investigation of the magnet system, especially if HTS materials are used, is required. For the simulation of large-scale HTS applications, FEM software is used to investigate the current and magnetic field distribution as well as the losses of the coil configuration. Due to the anisotropy of HTS materials, various differential methods have been proposed to solve Maxwell's equation efficiently and accurately. While the $\mathbf{H}$ formulation can be seen as the most versatile model, the $\mathbf{T}$ - $\mathbf{A}$ formulation is advantageous for modelling large-scale applications, as in the case of a fusion reactor. By using the thin strip approximation, the HTS tape can be simplified to a 1D line element and by employing advanced modelling strategies such as homogenisation, models consisting of stacked tape configurations can be further

simplified. By considering a stack of pancakes as an anisotropic bulk, this model can be efficiently solved for a large number of turns.

To validate the approach and investigate the behaviour of HTS at a field higher than the full penetration field a study evaluating the losses in three different scenarios has been conducted: 1) applied transport current 2) external magnetic field 3) applied transport current in an external field. The study showed that if an external field is present, the losses will increase until the full penetration field is reached and then start to decrease. Only if a transport current is applied at the same time, the losses will start to rise again when the current approaches the critical current.

The next step was to investigate the PFC system of an ITER-like reactor, consisting of six ring coils with a diameter of up to 24 m. Here the design of each PFC magnet was conducted yielding the coil configuration with the minimum number of tapes required to achieve the target magnetic field at the coil. The ramp time was found to have an impact on the total losses, which will decrease if the current is ramped at a lower rate. The target magnetic field could be achieved for each coil and the current distribution showed that the coils remained superconducting. The corresponding losses were investigated for an operation scenario consisting of a ramp-up phase, normal operation (AC cycles), a fault scenario (current drop to 70%) followed by a ramp-down phase. It was shown that the previously investigated loss behaviour can be observed in the weakest turns of each pancake. The innermost turn of the outermost pancake will experience the highest magnetic field from the coil and therefore, have the lowest critical current and highest penetration. Nevertheless, the weakest turn was not fully penetrated with current although the flux fronts were fully established, resulting in the decrease of losses. Although the losses in the weakest turn decrease after the full penetration field of the coil is reached, the losses for the whole coil do not decrease, but change the slope of the increase in the losses or even remain constant. In addition to the different operation scenarios, two tape from SuperPower and SuperOx were investigated at 25 K and 35 K. The results showed that both tapes can achieve the required magnetic field strength with the same number of turns. Since the same coil configuration was used for all simulations, the losses at 25 K were significantly lower than at 35 K, which can be directly related to the critical current, which increases as the temperature is decreased.

In the final study, the TFC system was investigated. Due to the unique D-shape of the TFCs, a 3D model using the **T-A** homogenisation method was developed. The model was validated by a 2D approximation, and the accuracy of the model was confirmed. By using the 3D

homogenised **T-A** formulation it is possible to model complex geometries that cannot be simplified in 2D without further investigation. In the case of the D-shaped coil, the results showed that the behaviour of a simplified TFC geometry can be quite accurately represented in 2D. The 3D model has the problem that the number of mesh elements required to investigate large-scale coil configurations exceeds the computational resources and is therefore not possible to simulate. The high aspect ratio of HTS tapes on the one hand and the large aspect ratio of the whole coil compared to the HTS on the other are major issues that cannot be solved with the available resources. Nevertheless, the implementation of the 3D homogenisation technique using the **T-A** formulation will be beneficial for many applications such as electric machines and potentially full-scale fusion reactor magnet system models in the future. Here the TFC study for the ITER-like reactor design was then further conducted through 2D simulation, using two different approaches: 1) a simplified axial-symmetric model 2) a cross-sectional model. For the axial symmetric model, the mean radius of the D-shape was used to build a simple ring coil model as it was used in the validation of the 3D approach. The cross-sectional model is more complex since a cross-section of the midplane of the reactor is modelled. This means that the inner and outer legs of a single TFC are simulated in a sectional area which is revolved around the centre point of the reactor. The advantage of this model is that the magnetic field distribution in the midplane of the reactor can be modelled since the influence of the neighbouring coils is considered. After finding the coil configuration required to achieve the target magnetic field at the coil, the ramp study again showed that a lower ramp rate is beneficial in reducing the losses. In both models, the target magnetic field was achieved, while in the cross-sectional model, the magnetic field at the inner leg was much higher than on the outer leg, due to the close vicinity of the neighbouring TFCs. The operation scenario conducted for the TFC consisted of a simple ramp-up, normal DC operation followed by a ramp-down of the current. Both models showed a good agreement in terms of current distribution, magnetic field distribution and losses. The study was also conducted using the two HTS tapes from SuperPower and SuperOx which again showed a similar performance.

In this thesis, the modelling approach for the magnet system of a Tokamak fusion reactor using FEM was developed. Due to the complexity of a fusion reactor magnet system and the limited information available for the use of HTS such an application posed a significant challenge. Therefore, this work could potentially have a significant impact on promoting the availability of computational resources for the investigation of the use of HTS materials for fusion. The developed numerical method combined with the first of a kind approach to tackle the modelling

of a fusion reactor magnet system can help researchers and companies to improve their magnet design and accelerate the development of fusion reactors.

7.1 Future Work

Since this thesis was the first attempt at modelling a fusion reactor magnet system, there were a lot of issues associated with the project. With the presented approach it is possible to use a system code to identify a reactor design for the use of HTS. The design can then be optimized by investigating the electromagnetic behaviour of a magnet system designed for the use of HTS magnets with FEM simulations. Because of the limited computational resources available for this research project, particularly for the 3D modelling of the TFC, the investigation was restricted to analysing a single pancake, which serves as the foundation for the modelling approach. By using a computational cluster, to increase the computation performance, it could be possible to model a full TFC with the 3D homogenisation using the **T-A** formulation. This would enable a more detailed analysis of the electromagnetic behaviour of the individual segments of the coil. Ultimately, maybe with the use of supercomputers, the full magnet system could be modelled as whole to investigate the interaction of the individual PFC and TFC magnet systems and their effects on the performance.

Although it can be expected that the commercialization of fusion reactors will still take some time, further improvements in technology and the use of high-field magnets, specifically HTS magnets, will become a major factor for establishing fusion energy as the energy source of the future.

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