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SHAPE GRAMMARS FOR HYBRID COMPONENT-BASED DESIGN

by

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Declaration

This thesis is the result of the author's original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.

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ABSTRACT

This thesis develops a methodology for deriving novel hybrid component-based designs using a shape grammar of heterogeneous antecedents. The grammar is used as both an interpolation and extrapolation tool with the results lying within or outside the range of their antecedents. An evaluation method is incorporated in the grammar rules to give feedback to the degree of innovation in a hybrid design enabling the shape grammar to be used as an assessment tool in addition to its use as an analytical and generative tool.

Analyzing antecedents is based on a grouping of the grammar rules of each component in a sub-class set to represent the different configurations of the same component among the heterogeneous designs in the corpus. Additionally, adding new hybrid rules to the original grammar rules increases the different options available to the grammar user and enhances the individuality of the generated hybrid design.

The derivation of feasible novel hybrid designs is ensured by the use of state labels and markers to control rule selection and rule application. The state labels constrain the rule selection via the user guide grammar of hybrid designs. The markers seek to generate feasible designs by directing the sequence of rules and controlling which rule is applicable and where it can be applied.

Innovation is assessed using variables derived from the internal structure of the grammar such as the number of antecedents in the corpus having the same rule and the number of rules in a sub-class set having the same geometry. The feedback signals are default values assigned to the rules as indicators of both the mixed character and individuality of rules. Feedback loops report the innovation metrics of generated hybrid designs immediately after each stage of rule application.

The methodology was implemented on a corpus of heterogeneous traditional minarets.

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1. INTRODUCTION

This chapter addresses the role of artificial intelligence in deriving novel designs from precedents, especially using heuristic methods such as rule-based and case-based procedures. The innovation processes in design computing are identified as adaptation techniques of old solutions to new design problems. Both shape grammar and case-based design are compared in order to establish the best approach. The research aims and objectives are defined.

1.1. Artificial intelligence in architectural design

The development of computational models founded on the AI paradigm has offered new methods and tools to architectural researchers to investigate the design process. Heuristic techniques adapted from AI have considerable achievements in architectural domain. In heuristic methods, knowledge is an essential constituent of intelligent performance that should be extracted from old experiences and put it into a computer to be used for design investigation. They established knowledge-based programs to represent knowledge acquisition systems that use analogical reasoning, case-based reasoning, and design rules to generate new designs within a known corpus of architectural works (Kalay, 2004, pp. 201, 256).

In architectural practice, the designer is guided by personal and professional expertise. Therefore, to benefit from the experiences of good designers and to avoid initiating new design from scratch, this research concerns the AI methods that use architectural precedents[1] to provide the designer with a starting point from which to develop a new design. In AI, the dominant design approaches that search for candidate solutions (precedents) to satisfy given problems are rule-based method and case-based method.

[1] A precedent is a specified design representation selected from a repertoire of past experience (Oxman & Oxman, 1992, p. 120). It has the characteristic of the unique knowledge such as conceptual points and relevant ideas embedded in a known design. The design knowledge in precedent includes the particular conceptual contribution to design which makes a prior design memorable as a precedent (Oxman, 1994, pp. 141-142).

AI motivated the development of rule-based production systems such as shape grammars, where knowledge gathered from highly experienced design professionals was encoded into (if – then) rules with geometrical constructs to describe the action to be taken when a certain condition is encountered (Kalay, 2004, pp. 201, 202). Additionally, case-based reasoning, a subfield of AI, assumes the existence of a collection of selected cases, represented in a complete and discrete form to be adapted to a new environment, or a new functional description without recreating the entire model. It relates the present situation to the closet experience in memory and uses that experience to solve an existing problem. In case-based design (CBD), the retrieved case may either match the current situation exactly or it may need modification (Schmitt, 1993, pp. 11-12).

The aim of this study is to examine AI tool such as shape grammars (SG) or case-based design (CBD) to generate novel designs from architectural precedents. However, both rule-based and case-based methods are accused of a lack of novel solutions (Kalay, 2004, p. 202). Therefore, the research needs to investigate the possibility of these methods to generate innovative designs. Accordingly, the next paragraphs aim firstly to explore the techniques that design computing uses to generate novel designs, and then to look into the use of these techniques in both shape grammars and case-base designs.

1.2. Design computing techniques for generating novel designs

One of the arguments in design computing concerns its ability to generate design solutions which might be considered sufficiently innovative. The need for computational design system that could generate creative solution is the legitimate goal for a design system to assist human designers by coming up with possibilities they may not have thought of themselves. (Kalay, 2004, p. 292)

Innovation involves the combination of inputs in the creation of outputs. The inputs to innovation are characterised, while the outputs in innovation are unpredictable and difficult to characterize, especially before the process is complete. Early literature in design computing classified new designs derived using computational tools as routine, innovative or creative. The former is the result of working within the constraints and prescribed activities that constitute similar precedents. In routine design "all the variables and their applicable ranges as well as the knowledge to compute their values are all directly instantiable from existing prototypes" (Gero, 1990, p. 34). Innovative design is defined as the improvement by extending the boundaries of particular precedents. It "is produced by manipulating the applicable ranges of values for variables"; therefore it has a familiar structure but with a different appearance because of the unfamiliar variables (Gero, 1990, p. 34). Creative design is the emergence of a totally new product by radically changing particular precedents to bring something new into existence. It is the result of using "new variables producing new types and as a result extending or moving the state space of potential designs" (Gero, 1990, p. 34).

Different methodologies are used to achieve each class of design. According to Coyne et al., the precedent (prototype or instance) is refined in a routine design. The refinement technique involves prototypes/instances whose performance corresponds to the goals of the new design. The new design varies according to the allowed values using parameterized design descriptions or design generators. (1990, pp. 32, 75)

Innovative design results from the precedent (prototype or instance) adaptation. It is the case whereas the prototype/instance is inadequate to meet the new design goal. Therefore, the class of designs will be extended by "adjusting the concepts that define the space of design". (Coyne et al., 1990, pp. 33, 78)

Creative design is the emergence of a new prototype or instance. It is "an entirely new vocabulary and new syntax combination" in which a new state space is produced. It occurs in response to changes in building styles or the need to find new methods for satisfying the design goals. (Coyne et al., 1990, pp. 78, 81)

Generating creative designs is more likely to occur in product designs than in architectural designs. Architects often draw on personal and professional experience rather than initiating new designs from scratch. The search for new and different designs in architecture, the definition of innovation, is better described as the renewal or improvement of already existing precedents using adaptation techniques, with novelty as a consequence[2]. Therefore, the research in the next paragraph investigates the use of adaptation techniques in shape grammar and case-base design tools to decide the more capable of them to generate novel design from precedents.

1.3. Shape grammars (SG) versus case-based design (CBD)

Both rule-based and case-based approaches rely on the explicit representation of knowledge based on precedents to solve new design problems. The main distinction between rule-based design and case-based design can be attributed to their reasoning schemes which depend on generalization in the former and specific episodes in the latter (Maher, 1990). In rule-based design the reasoning is the process of retrieving knowledge as abstract principles which is stored as generalized rules. In contrast, the reasoning in case-based designs depends on concrete instances in which the knowledge is stored as individual instances. Therefore, rule-based reasoning is the appropriate approach for simple defined tasks in which a limited number of rules is sufficient. On the other hand, case-based can be used for complicated and poorly understood problems such as designs with highly technical requirements.

In this section, the comparison between shape grammars as a rule-based method and case-based design method aims to assess their abilities to derive innovative designs. In rule-based reasoning the new wholes result from combining well defined elements, while in case-based reasoning the new whole results from modifying the details of a well-known whole solution (Kalay, 2004, p. 272). Shape grammars allows the transformation of a whole language of design while case-based design modifies only one case (Ahmad, 2009, p. 2) .

The process of adaptation in case-based designs includes a recognition of the differences between the selected case and the new problem and decisions regarding what aspects of the case are changed to fit the new design problem. According to Schmitt, deriving innovative design using the case-based process is questionable. He argues that the geometric and topological adaptation procedures in case-based design transform the original case into a new design resulting in creative solutions, but a routine design solution is more probable. He also asserts that case combination will

[2] http://en.wikipedia.org/wiki/Innovation

more likely lead to creative solutions but with few guarantees that the positive qualities of the original cases will be maintained (1993, pp. 18, 19). Also, Kalay raises doubts about the success of case-based design which depends on whether the adaptation of old case to the new problem will not damage the characteristics that made the case valuable (2004, p. 202). Additionally, other disadvantage of case-based design concerns the question of copyright and other legal problems which may arise. Schmitt refers to the worst case scenario in which the reasoning with cases might cause plagiarism and the inappropriate combination of elements (1993, pp. 18-19).

On the other hand, Schmitt (1990) believes that shape grammar surpasses case-base in its ability to achieve creative results within the schematic design phase. Shape grammar possesses a benefit over other methods of expressing design precedents by having the power to transform a grammar to create new languages of design options with small changes in grammar rules (X. Li & Schmidt, 2004). Cagan supposes that shape grammars, within bounds, can support both routine and creative designs (2001, p. 84). He asserts that "grammars can be modified 'on-the-fly' providing the potential for generation of creative solutions" (1994, p. 192). Additionally, Knight (1998) refers that a predefined grammar may be one that defines an entirely new type or style of designs. She considers the creativity or inventiveness in shape grammar is "not an abstruse, unconscious process, but an informed and deliberate manipulation of known or given information". The new languages of designs are defined by extending, modifying, or incorporating parts of known design languages (Knight, 1981, pp. 216, 237). Creativity in rule-based design lies in the creation of rules which can be modified and expanded at every stage of a design process (Colakoglu, 2005). The designers are able to transform or delete the existing rules and add new rules to formulate their own design language. In addition "shape grammars allow for emergence, that is, the ability to recognize and operate on shapes that are not predefined but rather emerge, or are formed, from any parts of shapes generated through rule applications" (Kalay, 2004, p. 274). This emergence gives the grammars the ability to create non-routine designs (Agarwal & Cagan, 2000, p. 438).

Built on the aforementioned studies, gaining novel designs from old solutions seems more likely using shape grammars than case-base designs. Other reasons for preferring shape grammar can be attributed to its ability to be easily modified using a clear and visible representation. Therefore, the research concentrates on adaptation using shape grammars as a method of generating novel designs from precedents.

1.4. Research problem

The research problem focuses on two axes:

- To derive novel designs from precedents using adaptation method in shape grammar.
- To verify the novelty of the generated design using innovation measures attached to shape grammars.

1.4.1. Adaptation method in shape grammars

The innovations in shape grammars are generally the results of carefully reasoned adaptation of the inherited design languages (Knight, 1983, p. 126). Adaptation techniques in shape grammar provide the potential for deriving novel solutions by expanding the design space and introducing a new information into the knowledge base (Cagan, 1994, p. 192). These techniques are based on changes in the grammatical structure and vocabulary of existing language of design which can modify it to a variety of other related languages of designs. For example, transformation techniques modify grammar structure using rule addition, rule deletion, and rule change (Knight, 1994).

Shape grammar practices apply a wide range of adaptation techniques. They need further investigation to identify their possible contribution to achieve the research aim as well as deficiencies in them that require further investigation.

1.4.2. Innovation assessment in shape grammars

Evaluation systems are used in shape grammar practices as design constraints, design specifications such as costs, or aesthetic assessments. They satisfy different design goals such as generating valid designs or assessing the generated designs. The study here seeks to add innovation metrics to shape grammar to verify the degree of novelty of the new adapted design.

1.5. Research objectives

The aim of this research is to generate novel designs from precedents using one of the adaptation techniques of shape grammar. To achieve the research aim, four objectives are defined as follows:

- To define adaptation techniques, strategies and tools in shape grammars; and to identify the technique that has not received sufficient attention to be the focus of this research.
- To propose a detailed approach for the definition of a chosen adaptation technique in shape grammars.
- To develop an assessment method for the innovation degree of generated designs in shape grammars to be attached to the adaptation technique.
- To validate the approach feasibility by implementing it to a case study on architectural precedents.

1.6. Thesis overview

The structure of this thesis is shown in Figure 1.1. The first section of chapter two reviews shape grammar method, its accompanied themes and its applications. The second section identifies adaptation techniques in design computing as transformation, substitution and hybridization. Then, in the third section, these techniques are scanned in shape grammar theory and practices. In sections four and five, a framework of adaptation methods in shape grammars and a comparative analysis of their techniques, strategies, tools, and outputs are crystallized, which reveal the limitation of hybrid adaptation technique in shape grammars. The sixth section includes a review of evaluation methods in shape grammars and concludes guidelines for adding innovation assessment to shape grammars. Lastly, the research problem and research methodology are addressed in the end of chapter two.

Chapter three reviews the definition of hybrid adaptation and concludes the characteristics of innovative hybridity. Then, a framework of shape grammar for hybrid component-based designs is put forward in three phases: the analysis phase, the synthesis phase and the evaluation phase.

To test the reliability of the proposed framework, a minaret design has been chosen as an architectural component-based model for case study. Accordingly, chapter four, in the first section, reviews the historical and morphological characteristics of minarets in the Islamic regions and presents the corpus of minarets to be undertaken in this study. The second section of this chapter determines the specifications of the analysis and synthesis phases of shape grammars for hybrid designs to satisfy the minarets under test.

Chapter five presents the implementation and results of a shape grammar for hybrid minarets in three stages. Examples of hybrid minarets using original rules and hybrid rules are shown to validate the proposed method. The verification of the innovation measures is demonstrated in the second stage by comparing the innovation metrics of copies of antecedents with hybrid designs. The third stage identifies the factors that contribute in generating more innovative hybrid designs. Finally, chapter six discusses the aspects of strength and weakness in hybrid adaptation using shape grammars. Additionally, the contribution of this research and recommendation for future work are identified.

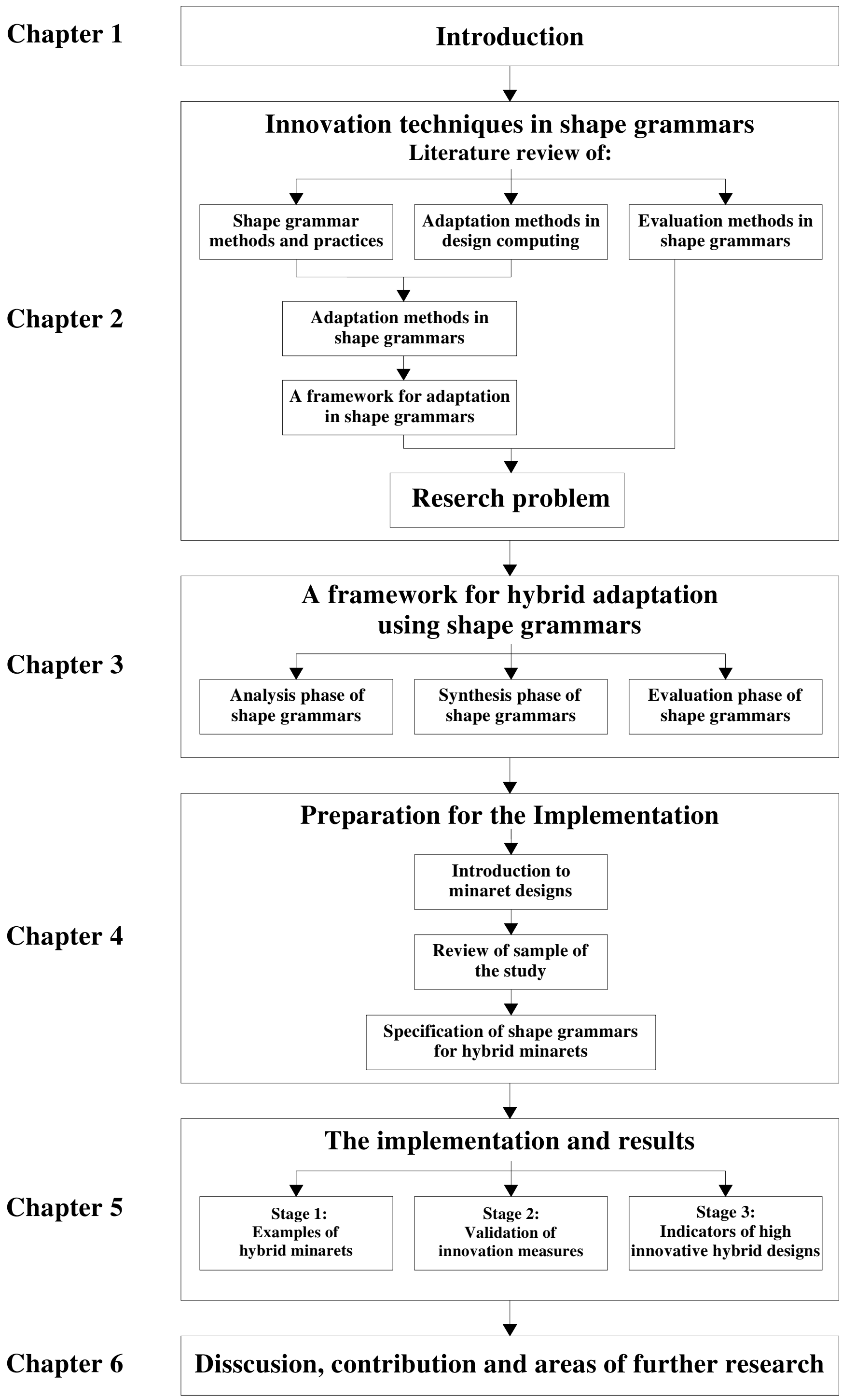


Figure 1.1: Structure of the thesis

2. INNOVATION TECHNIQUES IN SHAPE GRAMMARS

Chapter one defined the research area as generating innovative designs using an adaptation method in shape grammars. This chapter reviews shape grammar theory and practice, and investigates adaptation techniques to derive novel designs from precedents.

Topics are divided into three parts. The first part (section 2.1) provides an introduction to shape grammar methods and explores its applications in architectural, product and engineering design. The second part includes sections 2.2 to 2.5. Section 2.2 defines the adaptation concept in design computing and its use in transformation, substitution and hybridization of designs. Sections 2.3, 2.4 and 2.5 define a framework for adaptation in shape grammar theory and practice. The aim is to identify the possible contribution of shape grammar adaptation techniques in generating novel designs as well as their deficiencies that require further investigation. The third part is section 2.6 which investigates the feasibility of innovation assessment of the adapted design using shape grammar. Finally, the research problem and its methodology are presented in section 2.7 which reveals that hybridization techniques have received less attention than substitution and transformation in both shape grammar theory and practice. This problem, in addition to assessing the degree of innovation in the generated design using shape grammar are defined as the gaps in knowledge to be filled by the research.

2.1. Shape grammars - Introduction

Shape grammar is a rule-based algorithmic method invented by Stiny and Gips in 1972 as a production system that specifies a set of designs. Shape grammars are classified into analytical and original. The first category concerns describing existing buildings such as the Palladian grammars (Stiny & Mitchell, 1978). The grammar rules are used to generate designs in a corpus as well as new designs in the same language. The second approach creates new rules from scratch to derive a new language of designs such as Fredrick Froebel's kindergarten method (Stiny, 1980b). The research here concerns the first approach where shape grammars deal with known designs from architectural precedents. The following paragraphs review the theory and applications of analytical shape grammars.

2.1.1. Shape grammars theory

Shape grammars consist of four components (*S*, *L*, *R*, *I*). *S* is a finite set of shapes whereas a shape is an arrangement of lines in two or three dimensions. *L* is a finite set of symbols to limit the ways that rules apply such as state labels and spatial labels (markers). *R* is a finite set of shape rules of the form $\alpha \rightarrow \beta$ which consist of vocabulary of shapes and a set of spatial relations (arrangements of shapes). Finally, *I* is called the initial shape. (Stiny, 1980a, p. 347)

Shape grammars describe architectural precedents in terms of shapes, shape rules, symbols and initial shape to generate known and new designs. Shape rules can be applied to the initial shape and to shapes produced by previous rule applications whenever the shape or spatial relations in the left hand side of the rules matches a shape or shapes in the design. A shape matches another shape "whenever there is a Euclidean transformation[3] that makes the first shape equal to the second shape". In this case, the matched shapes on the left hand side of the rule is erased from the design and replaced with the shapes and spatial relations on the right hand side of the rule. (Knight, 1994, pp. 44, 45, 51)

Shape grammars vary in their rule format and rule ordering. Rule format can be addition rules only, subtraction rules only or both of them. Figure 2.1 clarifies deriving designs by the recursive application of addition and subtraction rules. Rule ordering in turn varies between deterministic and nondeterministic. Deterministic grammars have a restriction on rules to be ordered and applied in a controlled sequence. On the other hand, rules in nondeterministic grammars can be applied in any sequence. (Knight, 1998)

[3]Euclidean transformations are translation, rotation, reflection, scale, and combinations of these.

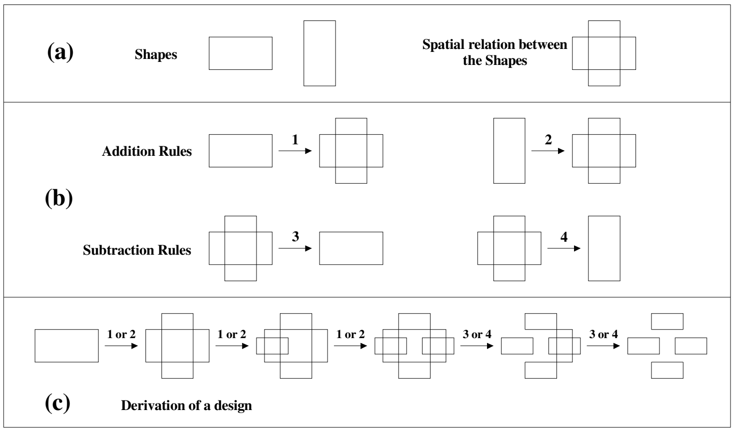


Figure 2.1: Example of shape grammars (Knight, 1994, p. 53)

Both rule format and rule ordering can be controlled using labels such as spatial labels (markers) and state labels (Knight, 1994, 1999b). The former labels manage where and how the rules apply by adding markers to shapes to limit what parts of a design and under what Euclidean transformation the rule can be matched as shown in Figure 2.2.

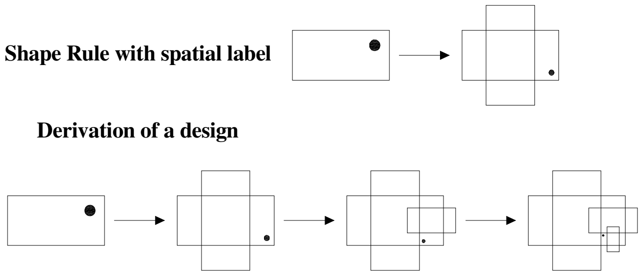


Figure 2.2: Using spatial labels to control rules application (Knight, 1994, p. 58)

On the other hand, state labels control when rules apply to designs. They control the sequence in which rules apply and the number of times rules repeat, as shown in Figure 2.3.

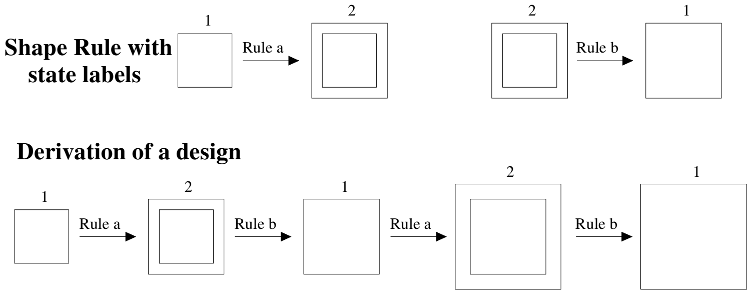


Figure 2.3: Using state labels to control rules sequence (Knight, 1994, p. 60)

In addition, labels and symbols are used in functional grammars and attribute grammars to produce realizable and functional designs by attaching symbols with behavioural and functional properties (Mitchell, 1991; Rinderle, 1991).

Shape grammars are associated with themes such as: parametric grammar, parallel grammar, descriptive grammar, and emergence as follows.

2.1.1.1. Parametric shape grammars

Shapes can be parametric in that the geometric scale of the shape can be varied. There are two types of shape grammars. The first is standard grammars in which rules are defined explicitly by a pair of shapes separated by an arrow. The second is parametric grammars in which rules are defined implicitly by rule schemata. Parametric grammar "embodies the formalism of a standard grammar and extends it to allow the computation of shapes with the same topological form but various dimensions". It is "commonly used by grammarians because it is flexible in shape recognition and able to generate more design variants than a basic grammar" (Chen, 2005, pp. 56, 78).

Parameterization of shapes increases the scope of shape grammars enormously and model shapes that could not otherwise be realistically modeled (Cagan & Mitchell, 1994, p. 175). A shape in rule schemata represents different shapes. It varies in specified ways by having varied lengths, angles, positions and so on. The varied aspects of shape are the variables of shape schema. The variables are defined by a range of values to be assigned to shape schema to determine the actual shape, as shown in Figure 2.4 left. In addition, the shape schema forms the spatial relation which in turn can be defined by assigning values to the variables in the spatial relation schema, as shown in Figure 2.4 right. (Stiny, 1985, p. 8)

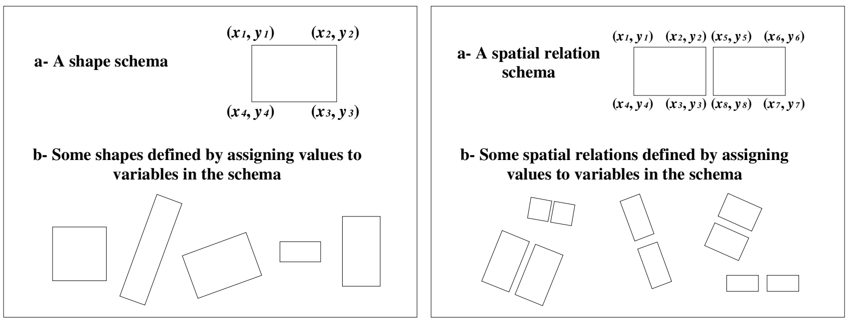


Figure 2.4: Left - A shape schema that defines a family of rectangles; Right - A spatial relation schema that defines a family of spatial relations (Knight, 1994, pp. 67, 68)

In this case, the spatial relations are fixed in standard grammars, while they are varied in parametric grammars.

2.1.1.2. Parallel shape grammars

Shape grammars can be singular or compound grammars. Compound grammars form parallel grammars consisting of shape grammars and other types of grammars that compute with texts, numbers or symbols. Parallel grammar is a network of two or more grammars that operate dependently or independently. The rules in the former can be linked so that the application of a rule in one grammar requires the simultaneous trigger of a rule in another grammar. On the other hand, in the latter parallel grammar, the rules in different grammars can apply separately of one another. (Knight, 2003)

Examples of parallel grammars are the coffeemaker grammar which links shape rules with manufacturing cost rules (Agarwal, Cagan, & Constantine, 1999); and Chinese building parallel grammars which consist of 16 different grammars to generate the visual representations of designs – plans, sections and elevations in addition to numerical and verbal descriptions of designs (A. Li, 2001). Part of the derivation process in Li's parallel grammars is shown in Figure 2.5.

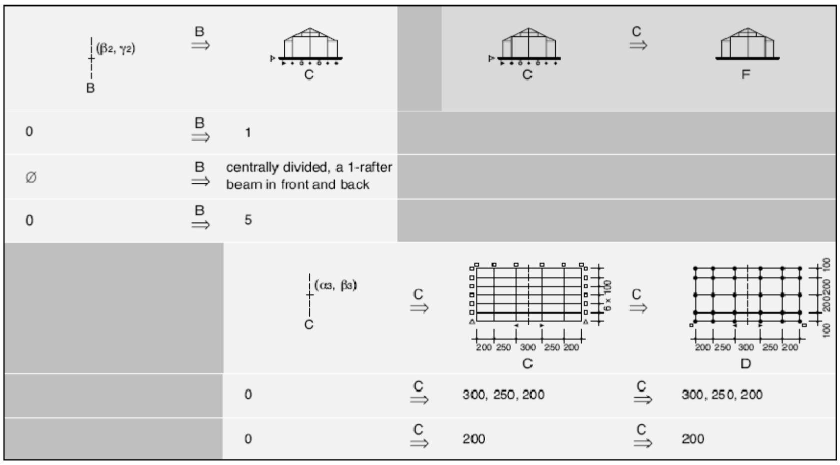


Figure 2.5: Parallel shape grammars (A. I.-k. Li, 2001)

2.1.1.3. Descriptive shape grammars

In addition to composing spatial elements, grammar rules can specify symbolic descriptions of designs to satisfy different goals basing on the application domain such as function or meaning (Stiny, 1981). The description functions "apply systems of categories to describe designs in languages defined by shape grammars" (Stiny, 1985, p. 15). They are chosen by grammar authors to interpret the properties of the designs at hand as shown in Figure 2.6.

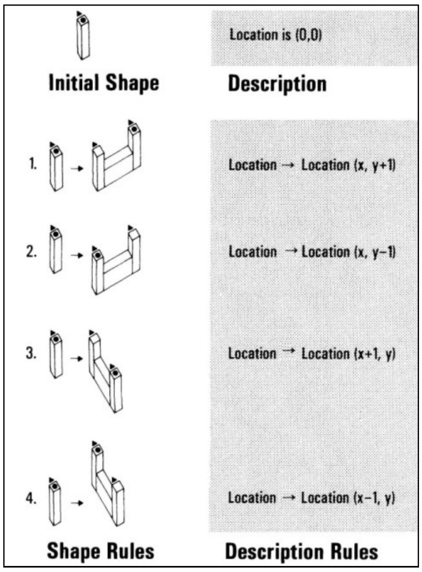


Figure 2.6: Descriptive shape grammars (Stiny, 1985, p. 17)

The grammar provides a link between languages defined by shape grammars and descriptions in other languages defined by rule systems (March & Stiny, 1985). The description rules can be correlated with shape rules to form a parallel grammar.

2.1.1.4. Emergence in shape grammars

Lastly, emergent shape in shape grammars is a shape or part of a shape in a computation that is not predefined by a grammar, but one that evolves from the shapes generated and added by a previous application of rules, as clarified in Figure 2.7. It can be generated and recognized by the designer and then fed back into a computation. (Knight, 2003, p. 127)

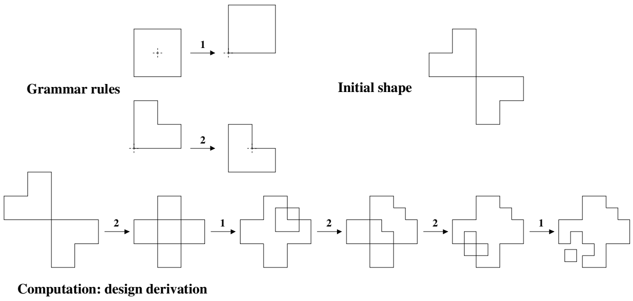


Figure 2.7: Emergent shape in shape grammars (Knight, 2003, p. 129)

2.1.2. Shape grammars applications

This paragraph lists some shape grammars practices in architecture, product, and engineering design. In architecture, many of these practices are focused on analyzing architectural styles. Their aim is to extract the rules necessary to determine if a design is an instance of the style and to use these rules to generate the corpus of existing designs and new designs in the same language. Examples of these grammars include Palladian villas (Stiny & Mitchell, 1978), the bungalows of Buffalo (Downing & Flemming, 1981), the Queen Anne style (Flemming, 1987), Frank Lloyd Wright Usonian houses (Knight, 1994), the traditional vernacular Taiwanese dwellings (Chiou & Krishnamurti, 1995, 1996), the traditional Turkish houses (Cagdas, 1996), the traditional Hayat houses in Bosnia (Colakoglu, 2001), the Siza houses at Malagueira (Duarte, 2001, 2005), and the Chinese style of Yingzao Faashi (A. Li, 2001).

There are many examples of shape grammars practices in product design, such as the grammar of the chair-back (Knight, 1980), coffeemaker grammar (Agarwal & Cagan, 1998), motorcycle grammar (Pugliese & Cagan, 2002), vehicle grammars (McCormack & Cagan, 2004; Orsborn, Cagan, Pawlicki, & Smith, 2006), and mobile phone grammar (Ahmad, 2009; Ahmad & Chase, 2006).

Shape grammar practices in engineering design have explored mechanical, civil and electromechanical disciplines, such as the Lathe grammar (Brown, McMahon, & Sims-Williams, 1994), robot arm grammar (Wells, 1994), and truss grammar (Shea & Cagan, 1997).

In addition to the aforementioned fields, some shape grammar practices explored the designs of fine arts and ornaments, such as the Chinese ice-ray lattice grammar (Stiny, 1977), the De Stijl art grammar (Knight, 1994), the geometric Islamic ornament grammar (Cenani & Cagdas), and the grammar for Louis Sullivan's ornamentation system (Phillips, 2008).

These examples reflect the wide range of design fields in which shape grammars practices are explored. The research, in the next paragraphs, concerns deriving innovative designs using adaptation method in shape grammar. The adaptation concept is firstly examined in design computing to specify their techniques in general, and then further investigation of these techniques in shape grammar theory and practice is carried out.

2.2. Adaptation method in design computing

Adaptation is the process of changing precedents to suit new conditions or needs. The new state of adapted design is called adaption. Adaptation techniques are procedures by which an adaptation is accomplished.

In the early literature of design computing, varied views on generating new designs using adaptation techniques and tools were presented. Oxman & Oxman (1992, pp. 124, 125) defined adaptation as a process by which a precedent is modified to generate another design. They identified three strategies of adaptation: elemental adaptation, schema adaptation and hybrid[4] adaptation. The first strategy uses transformational operations upon the elements of a prior design. It modifies the form and geometry of the design elements while the topological relations of the design scheme are maintained. The second is the schema adaptation which modifies all or some of the design topological characteristics using simple operations such as addition or substitution [5] of elements or using complex operations such as transformations of the description underlying the schema. The third is the hybrid adaptation in which a new design is derived from incorporating multiple precedents. In hybridization, the existing schema of each of the precedents may be maintained, or modified to generate an eclectic schema.

Kolodner (1993) considers that an old solution can be adapted to be made applicable to new situations. Adaptation methods in case-based design are classified under three main headings: substitution methods, transformation methods, and other methods. Substitution can be done using different processes such as re-instantiation and parameter adjustment.

Schmitt (1993, p. 16) makes a distinction between adaptation and combination in case-base design. According to him, adaptation will change the character of the

[4] Hybrid: an offspring of two animals or plants of different races, breeds, varieties, species, or genera. Also, it is something heterogeneous in origin or composition. (http://www.merriam-webster.com/dictionary/hybridity)

[5] Substitution: "is the process of choosing and installing a replacement for some part of an old solution". (Kolodner, 1993, p. 397)

original case by modifying its structure, behaviour and geometry. While with case combination, “features from radically different cases may be combined to form a new and definitively different design solution”. The study considers that case combination makes it easier to achieve innovative design solutions than case modification which will normally produce solutions close to routine design solutions.

Schnier & Gero (1998, p. 208) present computational processes to produce new designs from varied precedents. They determine two operations to adapt the design knowledge towards different design conditions and to “make the new designs more ‘interesting’ and ‘surprising’”. The first operation combines elements from different sources, and the second operation transforms elements into a different domain. The study defines “hybrid” as the crossbreeding[6] between different races that results from combining two different groups by which the resulting offspring includes features from both groups.

Coyne et al (1990, pp. 33, 78) consider that designing by precedent adaptation results in new variables for the prototype/instance. They specify processes of adaptation as modifying the prototype “vocabulary, syntactic knowledge, interpretation or interpretative knowledge”.

These various different definitions of adaptation methods in design computing can be summarized (Table 2.1).

Table 2.1: Adaptation techniques in design computing

<table>
<tr><th>Author</th><th colspan="4">Tools of deriving innovative design</th></tr>
<tr><td>Oxman & Oxman</td><td>Transformation
1. Change element
2. Change topology</td><td>Addition</td><td>Substitution</td><td>Hybridization</td></tr>
<tr><td>Kolodner</td><td colspan="2">Transformation</td><td colspan="2">Substitution
1. Re-instantiation
2. Parameter adjustment</td></tr>
<tr><td>Schmitt</td><td colspan="2">Modification</td><td colspan="2">Combination</td></tr>
<tr><td>Schnier & Gero</td><td colspan="2">Transformation</td><td colspan="2">Combination (hybridisation)</td></tr>
<tr><td>Coyne et al</td><td colspan="4">Modification of vocabulary, relations or meanings</td></tr>
</table>

[6] Crossbreeding in (genetics) is the act of mixing different species or varieties of animals or plants and thus to produce hybrids. http://dictionary.reference.com/browse/crossbreeding

The research proposes that adaptation methods are the best process to achieve new designs from precedents. The main adaptation techniques are transformation, substitution and hybridization. They can be summarized as follows:

- Transformation techniques consist of tools that modify an existing design by adding new parts, deleting or changing existing parts, modifying its interpretation or adjusting a design's parametric values.
- Substitution techniques generate innovative design through replacing parts of a specific design case by other parts.
- Hybridisation techniques achieve innovative design by mixing parts (crossbreeding[7]) that relate to a heterogeneous corpus of designs.

The next section discusses adaptation methods in shape grammar theory and practice to identify their role in generating novel designs as well as their deficiencies that require further investigation.

2.3. Adaptation method in shape grammars

The literature review of adaptation in shape grammars showed the contribution of their adaptation techniques in deriving new designs. The next paragraphs review the conception and implementation of adaptation in shape grammar theory and practice.

2.3.1. Adaptation method in shape grammar theory

The theory of adaptation, especially using transformation and substitution in shape grammars, were crystallised by Terry Knight. Her model of grammar transformations uses "mappings of grammars" to define individual designs in varied styles. New designs evolved from known ones "enhance our understanding of existing styles by linking them in different evolutionary chains, and increase our ability to create original styles of designs on the basis of our understanding of known ones" (March & Stiny, 1985, pp. 50, 51). According to Knight (1999a), a known style is analysed in the first step to define its grammar, and then grammar rules are transformed to become the basis for a new grammar and a style. For example, a new grammar rule

[7] Crossbreeding: in (genetics) the act of mixing different species or varieties of animals or plants and thus to produce hybrids. http://dictionary.reference.com/browse/crossbreeding

can result from transforming spatial relations in an existing shape rule, changing conditions of existing parameters attached to shapes in a spatial relation, or modifying the set of labels associated with shape rules (Knight, 1981, p. 217). This model can be used to innovate new styles from given ones and to distinguish the historical evolution of known styles into following ones.

Knight's book (1994) "Transformation in Design: A formal approach to stylistic change and innovation in the visual arts" has detailed definitions of transformation tools in shape grammars. In this book, she classified three processes of transforming shape grammars as shown in Figure 2.8. They are rule addition that adds rules to a grammar, rule deletion that subtracts rules from a grammar, and rule change that changes the constructive mechanisms underlying rules. The rule change can be done by modifying rule state labels, rule spatial labels, and rule spatial relations. The latter happens when new shapes are introduced or existing shapes are either resized or repositioned. Each process in itself or in combination with other processes can modify the original grammar to produce a new grammar.

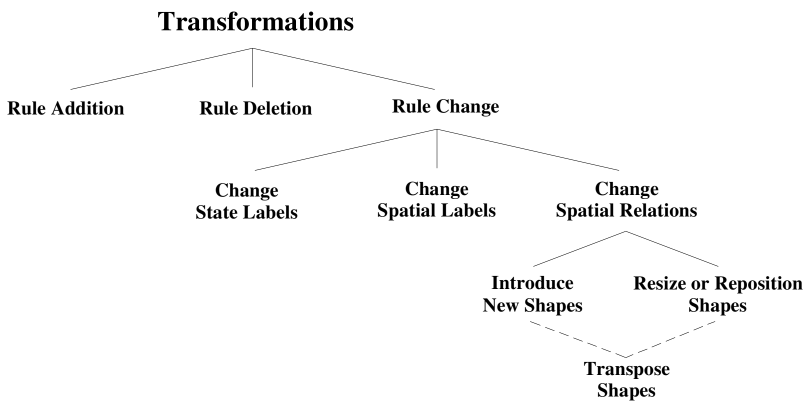


Figure 2.8: The ways of transforming a shape grammar (Knight, 1994, p. 105)

Parametric variation is also put forward in shape grammar. A given shape rule schema can be modified to a new one by changing conditions on the parameter associated with it. Different spatial relation can be derived by parameterizing the shapes occurring in a rule, or by changing conditions on existing parameters. In addition, changing the parameters of shapes in a spatial relation vary either the shapes themselves or the shapes disposition with respect to each other. The new spatial relations in shape rule schemata lead to different languages of design. (Knight, 1981, p. 217)

Substitution in shape grammar is defined as the shape replacement in spatial relations. For example, a rectangle surrounding a round arch in a particular spatial relation can be replaced with a round arch to produce a spatial relation between two round arches (Knight, 1981, p. 218). Lastly, Chase and Ahmad (2005) define “composite grammar” as a hybrid adaptation technique that generates a new grammar to derive new or known hybrid designs by merging two or more existing shape grammars. The rules in this grammar result from merging all or parts of shapes and spatial relations in the original grammars.

The next paragraph reviews shape grammar applications of adaptation techniques.

2.3.2. Adaptation method in shape grammar practice

In shape grammar practice, the adaptation concept is used to generate new designs only or both a new grammar and new designs from the original grammar. Transformation and substitution techniques have received widespread attention. Some examples from the past 15 years (from 1994 – 2009) in the fields of architectural design, product design and engineering design, are briefly presented.

Knight (1994) proves shape grammar’s efficiency to generate new innovative designs by transforming grammars of existing designs. She applies adaptation using grammatical transformations to characterize the historical evolution of known styles, such as the stylistic changes in the work of Frank Lloyd Wright, in De Stijil paintings, and in the ornamental design of ancient Greek pottery. Knight shows how the stylistic changes of Wright’s Prairie style into the Usonian style can be achieved by deleting, changing and adding rules to the Prairie grammar. The study modifies a simplified version of a parametric Prairie houses grammar, written by Koning and Eizenberg (1981), to derive a family of new grammars including a grammar that defines the language of polliwog Usonians. A subset of the Prairie grammar consisting of the inisial shape and some rules is transformed to determine a new grammar. A number of rules from the Koning and Eizenberg original grammar are deleted, and other rules are changed by modifying their spatial relations and labels. For instance, the spatial relations between a fire place and a living zone are changed by repositioning shapes and introducing new shapes, “the fireplace is moved from the border of a living zone into the interior of a living zone”. In addition, the spatial relation between a living zone and a service zone in a core unit of a rectangular Prairie house is modified by rotating 90º either a living or a service zone. Another adaptation technique is the substitution of the rectangular core unit of a Prairie house with an L-shape for a Usonian house to produce a new core unit. Also, some changes include the function of zones which become bedrooms only instead of bedrooms and bathrooms. Lastly, spatial labels and state labels are also modified and new ornamentation rules are added to Prairie grammar to suit the derivation of Usonian houses. (Knight, 1994, pp. 232, 233)

Other shape grammar authors followed Knight’s approach to transform an existing grammar to derive novel designs. One such example is Colakoglu (2001, 2005) who defined Bosnian Hayat houses in terms of parametric shape grammar. The original rules are defined to synthesize the existing prototypes whilst new rules are added to modify the existing designs to derive new hayat houses which have different functional, structural and aesthetic properties. The new rules use both adaptation techniques: transformation and substitution to change the observable configuration of form via different means such as changing the parametric values of the variables of component objects, adding new shapes to the grammar rules, replacing the vocabulary elements of the existing form with new ones, subtracting existing shapes from the grammar rules and changing the functions of existing component objects.

Duarte (2001, 2005) defined a shape grammar to derive the customized Alvaro Siza’s patio houses at Malagueira to generate both existing prototypes and new variations of Siza’s prototypes. A set of constraints on design features is attached to the grammar rules to generate a house matching given functional and dimensional criteria such as “the number and type of spaces, adjacency relations, widths and areas”. The grammar of existing prototypes is adapted to increase the number of customized designs by expanding the functional requirements in the initial designs. This is done by transforming rules and adding new rules to deal with new geometric problems and new programmatic features such as a different orientation of the staircase or a new location of the laundry room.

Ahmad and Chase (2006, 2007), and Ahmad (2009) defined shape rules for both mobile phones and Greek temple facades using a transformed grammar to derive new designs. These studies focused on grammar transformations using a style description scheme to aid the changes in design styles by facilitating the comparison of different design characteristics generated by the modified grammar rules. The adaptation here depends on rule modification tools such as addition, deletion and replacement to change either shapes or spatial relations and their descriptions.

Other examples from product design use the adaptation approach to shape grammar to derive new designs from existing ones. Agarwal and Cagan (1998) utilize shape grammar to describe a language of an existing class of coffeemakers and to introduce new designs. The characteristics of the new designs result from the use of adaptation tools such as small changes in the selection of rules and in the choices of the parametric values. Also, McCormack and Cagan (2004) use parametric shape grammars to encode and capture the brand identity of Buick cars from 1947 – 2002 and to use the grammar to recreate known Buicks and also to introduce new Buick concepts. The sample of new novel Buicks was adapted using different parameters for key elements such as the hood and the fenders.

Shea and Cagan (1999a, 1999b) used parametric shape grammars in engineering design. They introduced a truss design language to derive optimal and innovative designs using the adaptation techniques of transformation and substitution. The grammar rules define the design space and new designs using transformation rules that create geometrical and topological transformations of the original truss design. The geometrical transformations include tools for shape modification, such as changing the location of a single joint in a design, and also size modification, such as changing the cross-section area of a single member. Rules of topological transformation were also used to modify triangles according to the form-function stability relationship of the trusses.

Chase and Liew (2001a, 2001b) described a methodology for adapting an original grammar to produce new designs that meet a new set of requirements. They transform the functional, behavioural and structural (FBS) characteristics of the design using rule replacements in the grammar rules. The original design is firstly derived using rules (with associated graphs) to define the FBS description, as shown in Figure 2.9. Then, new rules are selected from the library of rules based on modified or additional requirement for the FBS properties of the original design. In this case, the newly selected rules replace the original rules in the grammar resulting in the adaptation of the original grammar which is used to generate a new design.

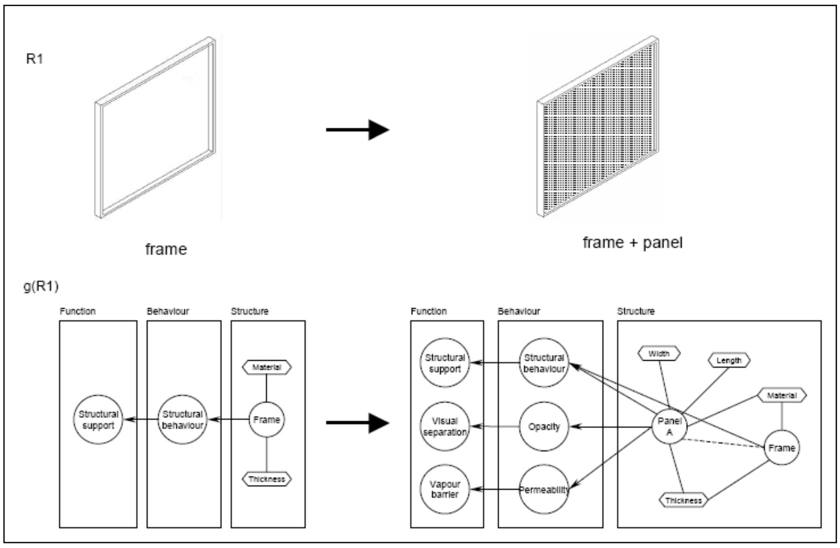


Figure 2.9: Example of rule with FBS description (Chase & Liew, 2001a)

Li and Schmidt (2004) present an adaptation method for modifying grammar rules of Epicyclical Gear Trains (EGT). They generate a new grammar by transforming the functional schemes defined in a graph representation. The adaptation tools add a new graph grammar rule to the original EGT rules and apply rules in a different sequence to generate novel designs. This is done without eliminating the original EGT grammar to maintain its capacity to generate all of its previous designs.

There are two studies that have implemented hybrid adaptation in shape grammars. The first study, in architectural design, defines a "composite grammar" as a technique that generates a new grammar to derive new and known hybrid designs by merging two or more existing shape grammars. The study applied this methodology to derive a composite grammar for Iranian caravanserai from both Roman fortification and Persian houses grammars. In addition to composite rules and a composite initial shape, the generated grammar was also subjected to other adaptation tools such as adding new rules to it, and assigning new forms to existing rules (Chase & Ahmad, 2005).

The second study, in product design, defines a parametric shape grammar for a heterogeneous corpus composed of three classes of vehicles including Coupes, Pickups and SUVs (Orsborn et al., 2006). The grammar consists of vehicle characteristic rules and general modification rules. The characteristic rules include "class-specific rules" and universal rules. The former rules are applicable to a certain class of vehicles; while the latter rules are adaptation tools which can be applied to any class (to be differentiated through parametric transformations). Other adaptation tools used are the modification rules which can change any curve shape in any view during the derivation process. The new, hybrid, vehicle is derived by mixing and combining rules to generate modified shapes with simple parametric changes to alter the class to which the vehicle belongs.

In conclusion, shape grammar theory and practice has defined and applied adaptation using varied methods and techniques to generate novel designs. The study proposes a framework for adaptation in shape grammars by identifying its techniques, strategies, tools and grammars. This section has identified the contributions of various shape grammar practices to the framework; as well as their deficiencies that require further investigation. Accordingly, the next section completes the description of adaptation framework in shape grammars and concludes the gap in knowledge regarding the adaptation technique that received less attention in shape grammar practices.

2.4. A framework for adaptation in shape grammars

The review of the adaptation concept in shape grammar theory and practice has revealed variations in adaptation means and ends. The act of changing previous precedents to new situations depends on different adaptation inputs such as techniques, strategies and tools; leading to varied adaptation outputs, as shown in Table 2.1. The different aspects of the adaptation method are identified as follows:

2.4.1. Adaptation techniques

An adaptation technique is a practical method by which a precedent adaptation is achieved. The main adaptation techniques described in section 2.2 are transformation, substitution and hybridisation. The review of shape grammar theory and practice showed that the transformation is the prevailing technique of adaptation. Substitution has been implemented to some extent. However, hybridisation has received less attention.

2.4.2. Adaptation strategies

Adaptation strategies are plans of actions chosen to bring about a desired modification of precedents. They can be defined in accordance with two factors. The first factor concerns where to locate adaptation tools in a grammar - they may be either embedded in the original grammar rules, or attached to them in the form of additional rules. The second factor concerns how to build adapted designs. It varies between an incremental design approach which synthesizes modified designs using adaptation rules, and a morphing design approach which changes existing designs using adaptation rules. Each of these factors is discussed below.

2.4.2.1. Where to locate adaptation tools in shape grammars?

The locations of adaptation tools in the original grammar can be internal or external. The embedded adaptation exists internally in some original grammar rules and is able to generate both existing and new designs. Original rules in this case are adaptive rules having the ability to change to suit different conditions. The adaptation of precedents is done by modifying rule formats, rule orders or both. The rule format is concerned with shapes, spatial relations and labels which can be adapted using Euclidean transformations and parametric variation. For example, nondeterministic shape grammars allow the generation of varied designs from previous ones using different transformations (translation, rotation, reflection and scale) under which a particular rule may apply to a specific part of a design (Knight, 1999b, pp. 19, 20), as shown in Figure 2.10. In addition, both shapes and spatial relations in parametric rules have varied length, angles, positions, and so on. Lastly, the embedded adaptation is also utilised by selecting rules in varied orders such as altering the sequence of rule application or altering the number of times the rule repeats.

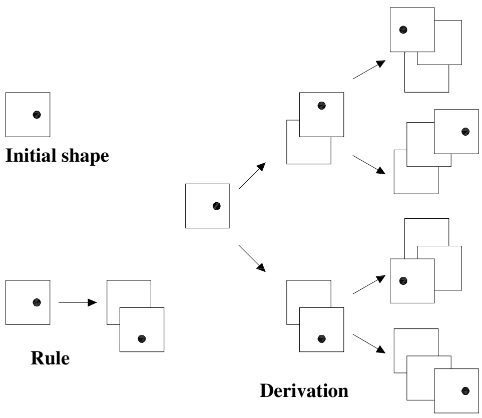


Figure 2.10: Different transformations under which a rule may apply (Knight, 1999b, p. 21)

The attached adaptation is added externally to original grammar rules. It results in a new adapted grammar and generates new designs and in some cases existing or known designs[8]. It is implemented using extra rules attached to the original grammar rules. The added rules are new rules or adapted rules which replace original rules to suit particular conditions. The adaptation of original grammar rules is accomplished by adding a new shape, replacing or deleting an existing shape, changing or adding spatial labels, and/or changing or adding state labels.

2.4.2.2. How to build an adapted design?

This factor reflects the different approaches to derive adapted designs as shown in Figure 2.11. In the first approach, adaptation rules are used to synthesize an adapted design; whereas in the second approach, rules are used to adapt an existing design, as follows:

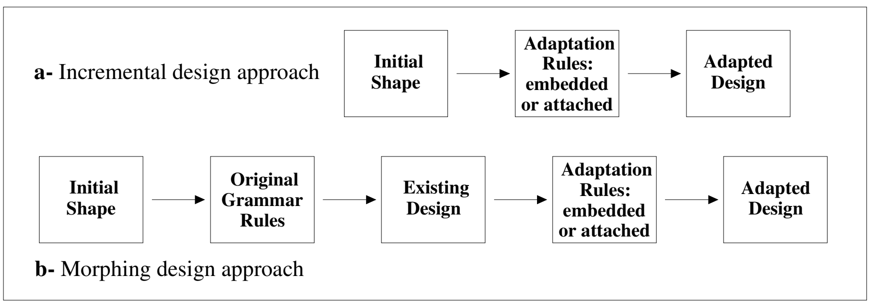


Figure 2.11: The difference between syntheses modified designs and change existing designs

[8] Existing designs are precedents in the corpus from which the original rules are derived, while known designs consist of both existing designs and other precedents not included in the corpus of grammar's antecedents.

The incremental design approach is an additive method to synthesize adapted designs using adapted rules only, adapted and new-found rules, or a mixture of original rules, new-found rules and/or adapted rules. In this case, the adaptation rules, embedded or attached, are applied gradually to an initial shape to derive a new adapted design. This is done using production rules to add shapes, spatial relations, and/or labels to the generated design.

The morphing design approach starts with the derivation of existing designs using original rules and changes them. In this case, the new rules, adapted rules, and/or adaptive rules, embedded or attached, are applied to modify the design generated from the application of original grammar rules. They have the ability to change shapes, spatial relations, and/or labels of the generated designs to suit desired conditions. For example, shape annealing is used to generate structural essays of a planar and three dimensional truss as “a set of designs within a structural language that satisfy problem specifications and constraints while exploring tradeoffs among design goals for a particular structural design application” (Shea & Cagan, 1998, p. 373). This method defines the design language of valid planar truss structures firstly. Then, the interpretation of this language is defined to identify the desired structural purpose to be included in the design essay. Lastly, generating a range of appropriate structures is done by modifying the design language or the interpretation. (Shea & Cagan, 1998, p. 369)

2.4.3. Adaptation tools

In shape grammars, adaptation tools are the means of performing and facilitating design adaptation. They are used to modify different parts of grammar such as the rule format, rule order, and grammar structure. Firstly, the rule format has been adapted using tools of embedded adaptation such as Euclidean transformation and parametric variation. Euclidean operations change the shape’s location, orientation, handedness, scale, or some combination of these changes in rules into geometrically similar ones. These changes correspond to translation, rotation, reflection, scale or combination these tools (Stiny, 2006, p. 194). Parametric variation can transform both shape and spatial relations of precedents to satisfy varied values. In addition, the tools of attached adaptation modify the rule format in the original grammar by adding a new shape to a rule, replacing an existing shape with a new one, merging existing shapes from different rules in a combined one, deleting an existing shape from a rule, or adding or changing the spatial labels. The embedded adaptation tool may be used to modify the rule order by changing the free sequence of rules if there is no restriction on the order of rules application, while the attached adaptation tool adds or modifies the state labels if the rule sequence is restricted. Lastly, the structure of the whole grammar can also be adapted by attached adaptation tools such as adding new rules to a grammar, replacing original rules with adapted rules, or deleting some original rules from a grammar.

2.4.4. Adaptation outputs: a grammar and generated designs

A grammar that performs adaptation processes can be an original grammar or a new adapted grammar. The former has embedded adaptation tools such as Euclidean transformation, parametric variation and changing the order of rules, therefore it can derive both existing designs in the corpus and new adapted designs. The new adapted grammar, on the other hand, consists of adapted rules only, new added rules and adapted rules, or a combination of whole or part of original grammar rules with new added rules and/or adapted rules. The designs derived by a new adapted grammar can be new designs only, new designs and existing designs in the corpus of grammar antecedents, or new and known designs, whereas known designs include both existing designs and other precedents not existed in the corpus of grammar antecedents from which the original rules are extracted. An example of deriving known designs is Knight's adaptation of original grammar rules belonging to Wright's Prairie style (existing designs) to generate Wright's Usonian style (known designs).

2.5. Comparative analysis of adaptation in shape grammar

The main stream of adaptation methods in shape grammar practices can be seen in the comparative analysis presented in Table 2.2.

Table 2.2: Comparative analysis of adaptation method in shape grammar practice

			Shape grammar practice using adaptation method										
			Knight / 1983, 1994	Colakoglu / 2001, 2005	Duarte / 2001, 2005	Ahmad & Chase / 2006, 2007, Ahmad / 2009	Agarwal & Cagan/ 1998	McCormack & Cagan / 2004	Shea & Cagan / 1999	Chase & Liew / 2001	Li & Schmidt / 2004	Chase & Ahmed / 2005	Osbrone et al / 2006
Adaptation techniques		Transformation	●	●	●	●	●	●	●	●	●	●	●
		Substitution	●	●		●			●	●			
		Hybridisation										●	●
Adaptation strategies	Where to locate	Embedded adaptation	●		●		●	●	●				●
		Attached adaptation	●	●	●	●			●	●	●	●	●
	How to build	Incremental design approach	●		●	●	●	●			●	●	●
		Morphing design Approach		●					●	●			●
Adaptation tools	Rule format	Euclidean transformation	●						●				
		Parametric variation	●	●	●		●	●	●				●
		Specifications variation	●	●	●	●				●			
		Add a new shape	●	●		●				●		●	
		Replace an existing shape	●	●	●	●			●	●			
		Merge existing shapes										●	
		Subtract an existing shape	●	●									
		Change or add spatial labels	●										
	Rule order	Change the free sequence of rules selection				●	●		●		●		●
		Change or add state labels	●										
	Grammar structure	Add a new initial shape	●									●	
		Add new rules	●	●					●		●	●	●
		Replace original rules	●		●	●				●		●	
		Delete original rules	●									●	
Adaptation outputs		New grammar for new designs											
		New grammar for new and existing designs		●	●	●			●	●	●		●
		New grammar for new and known design	●									●	
		Original grammar for new and existing designs					●	●					

The techniques of adaptation have been used to varying degrees. Transformation is the most dominant adaptation technique, followed by substitution then hybridization which are applied to support transformation. The prevailing strategies of adaptation are using the attached adaptation rather than the embedded adaptation and using the incremental design approach to synthesize new adapted designs rather than the morphing design approach to adapt the existing design. The most commonly used tools are parametric variation, specifications variation, adding a new shape and replacing an existing shape to modify rule formats; changing the sequence of rules to modify rule orders; and adding new rules and replacing existing rules to modify grammar structures. Lastly, in most of the shape grammar practices under investigation, the adaptation is achieved by implementing a new adapted grammar that derives new designs while maintaining its ability to generate the existing designs in the corpus of antecedents.

Hybrid adaptation is the least used technique in shape grammar practice. Both hybrid adaptation studies mentioned earlier used a hybridization technique beside a transformation technique. For example, Orsborn et al depend largely on varied parameters to reflect the different features of vehicles. However, in architecture the parametric variation of universal rules is not enough to represent the diverse characters of heterogeneous classes of designs. According to that, the research needs to investigate the application of hybrid adaptation in shape grammars apart from other adaptation tools as a gap in knowledge to be filled. In addition, the research aims to determine the characteristics of hybrid adaptation method such as its strategies, tools, and outputs in isolation from other adaptation techniques. Therefore, the next chapter focuses on defining a framework for hybridization strategies, tools, and outputs in shape grammar as a research problem.

The next paragraph investigates evaluation methods in shape grammars to determine their possible use in measuring the innovation of the generated design.

2.6. Evaluation method in shape grammars

In the end of the first chapter, the research aim is outlined as generating innovative designs using adaptation technique in shape grammars. The success of adaptation technique in deriving novel designs is questionable and needs evidence to support. Therefore, the adaptation technique in shape grammar should be associated with a method of innovation assessment to confirm the novelty of the derived design. According to that, the research, in this section, investigates the possibilities of applying evaluation techniques to shape grammars. The assessment methods in shape grammar practice are reviewed to establish their roles in shape grammar design process; the number of assessment criteria; and the possibility of adding an innovation assessment to shape grammars.

2.6.1. The role of evaluation method in shape grammars

Architectural design is not a random, trial-and-error search, but it is a goal-directed search. Evaluation of design is "the measure of how well a given or proposed design solution fits the set of goals it is intended to meet" (Kalay, 2004, pp. 295, 301). Generative grammars aim not only to define a language of designs but also to search this language for feasible and optimal designs (Shea & Cagan, 1999b). The central role evaluation plays in design concerns the feedback part of the design cycle. The grammar author controls form generation by explicitly defining the criteria for new designs (Colakoglu, 2005).

In shape grammar practice, evaluation systems have been used to satisfy different goals. Three approaches can be distinguished according to their roles in the design process. They are the generating role only, the assessing role only, or gathering both the generating and assessing roles.

2.6.1.1. The generating role of the evaluation system in shape grammars

In the first approach, generating valid designs only, the problem specific knowledge of the design goals is hard-coded in a grammar. The evaluation criteria are descriptions that follow compositions and drive the design derivation (A. Li, 2001, p. 47). They are associated to shape grammar in two manners, as shown in Figure 2.12. In both cases, the evaluation criteria are knowledge placed in grammar rules such as parameters, specifications, constraints or labels to restrict either the design derivation, or the selection algorithm of rules. The aim is to limit the design space by preventing the designers from searching the space of infeasible designs.

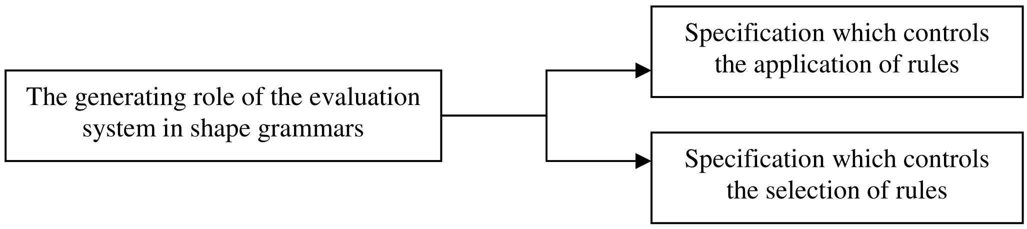


Figure 2.12: The role of evaluation systems in generating design in shape grammars

Example of this approach is Soman et al using shape grammar for the design of sheet metal components. In this study, a grammar user can only choose a rule from a list of applicable rules which is generated for every stage of a grammar. These rules are associated with “a framework of constraints to be verified while deciding the values for the parameters of the rules” (2003, p. 193).

2.6.1.2. The assessing role of the evaluation system in shape grammars

The second approach, assessing the generated designs only, occurs in two cases (Figure 2.13). In the first case, the evaluation criteria are used after each rule application to assess the generated designs through the grammar runtime. The second case seeks the optimal design within fixed configurations resulting from applying a shape grammar where the form of the solutions are known but specific values needed to be determined.

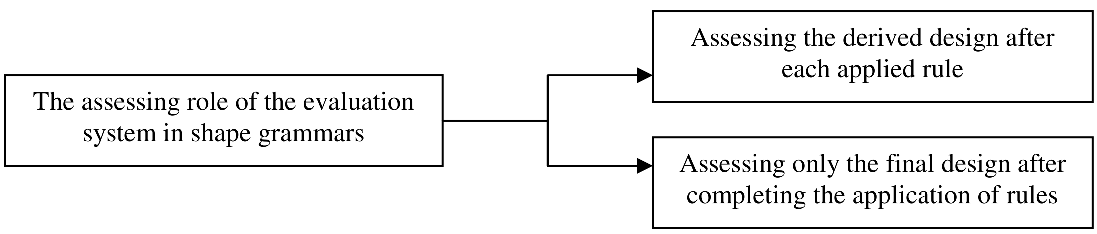


Figure 2.13: The role of evaluation systems in assessing design in shape grammars

Example of the former case is Agarwal et al (1999) who have incorporated costing evaluation into the coffee maker grammar. The current cost of the generated design is evaluated as soon as the rule is selected and applied, and its parametric values are defined. At each stage during the process, "partial designs of the final product can be used to provide feedback to the designer based on specific design objectives and thus suggest possible rule choices" (Agarwal et al., 1999, p. 253). On the other hand, example of the latter case is Stiny and Gips (1978) investigation of a set of evaluative criteria for Palladian villas ground plans generated by the use of parametric shape grammar developed in Stiny and Mitchell (1978). The measures are the aesthetic values of a generated design based on a relationship between the way it is generated by a shape grammar and the way it is described.

2.6.1.3. The generating and assessing roles of the evaluation system in shape grammars

In the third one, combining both the generating and assessing roles, rules are written to derive valid designs and then problem specific models of evaluation criteria are used to search the language for purposeful designs such as in the shape annealing method (Shea & Cagan, 1998). It is used as a directed stochastic search approach to generate designs using a combination between shape grammars and simulated annealing which selects shape rules and evaluates the quality of the resulting design (Cagan & Mitchell, 1994, p. 185). In this approach, the use of performance metrics along with a grammar-based generative system control the derivation of the design to meet the desired objectives and create a powerful feedback mechanism for the designer during the process of design generation (Cagan, 2001, p. 76).

2.6.2. The number of assessment criteria

In all three approaches, evaluation criteria are either inclusive and take into account many design factors such as efficiency, economy, utility, and elegance in shape annealing grammar or exclusive and adopt only one design factor such as the estimated cost in coffee maker grammar.

2.6.2.1. Multi-criteria assessment

Examples of these factors are design specification, constraints and goals estimation. In Shea & Cagan (1997, 1998), the shape annealing method is used as a directed stochastic search approach to generate designs using a combination between shape grammar and simulated annealing that controls the derivation of design to meet the desired objectives (Cagan & Mitchell, 1994, p. 185). This method is applied to derive the structural language of traditional and new innovative three dimensional domes (Shea & Cagan, 1997), and optimum three-dimensional truss structures consisting of linear members and joints (Shea & Cagan, 1998). The stochastic optimization is used to direct the search for incorporation of design goals of efficiency, economy, utility, and elegance. For example, they use metrics of the design efficiency such as minimum mass, the economy such as minimum number of distinct cross-sections areas and lengths, the utility such as maximum enclosure space and minimum surface area and lastly the aesthetic such as visual uniformity metric and golden ratio metric.

2.6.2.2. Mono-criterion assessment

An example of this is the cost estimation in Agarwal et al study (1999). In this study, the coffeemaker shape grammar introduced by Agarwal and Cagan (1998) is developed to associate expressions with their rules that model manufacturing costs. The cost of manufacturing the product is realized with each application of a shape grammar rule at each stage of the design process. “The designer has an indication of what the overall cost of the product will be and how the selection of one grammar rules over other influences the final cost”. The total manufacturing cost is given to the designer once the complete product is generated. The designer can understand the implications of decisions made in the early steps of rule application. (Agarwal et al., 1999, p. 253)

In conclusion, evaluation systems in shape grammar practice are used as mono or multi-assessment criteria in different situations. It is merged in grammar rules to derive feasible designs as purposeful knowledge placed in grammar rules or restrictions within rule selection. On the other hand, it is used as a test control after each rule application excluding unfeasible designs or after the grammar finishes assessing the resulting designs.

2.6.3. Innovation assessment in shape grammars

In this study, the goal of using shape grammar is to derive a new adapted design with high measures of innovation. In the literature there are many methods of assessing design novelty depending on the characteristics of the generated design, the process of generating the design, or both.

For example, shape annealing grammars (Shea, 1997; Shea & Cagan, 1997) have two aesthetic models for the design of truss roofs depending on the characteristics of the generated designs. The first model uses the golden ratio as a proportional system to measure the relative lengths of the individual shape members. The second aesthetic model concerns the design uniformity which aims to proportion the size of the design members to be near in length. The aesthetic value is calculated for a design from a standard deviation of the length of all members of a design, such as “a design with a more uniform breakdown and thus a lower standard deviation of length is considered to be of greater aesthetic value than a design with a more random breakdown”.

The evaluation system in Stiny and Gips (1978) depends on both the characteristics of the design process and the characteristics of the generated design. It proposes the aesthetic values of Palladian villa ground plans generated by the use of parametric shape grammar based on a relationship between the way it is generated by a shape grammar and the way it is described. The criteria used are based on the arrangements of rooms in plans and the shapes and sizes of the rooms in terms of the underlying grid. For example, the first principle ensures the extension of only the central room in a plan from the east exterior wall to the west exterior wall. And the second principle ensures that the exterior rooms on the north-south axis of symmetry are among the largest in the plan. The aesthetic value for an individual Palladian plan is calculated as the ratio of the length of a sequence of symbols to describe the plan to the length of a sequence of symbols to specify the information needed to generate the plan. (pp. 199, 200)

2.6.3.1. Innovation assessment of adapted design in shape grammars

In shape grammars for innovative adapted design, the innovation assessments need to be built on both the process of design derivation and the generated design itself. Incorporation of both the generating and assessing roles of evaluation method is required. Within the generating role, feedback signals are used to guide the grammar user to choose rules that achieve the best innovation measures. Additionally, innovative assessments of generated design are associated with each applied rule to give a feedback loop to the grammar user at each stage of design derivation.

However, in shape grammar literature, there is no mechanism for the comparison and evaluation of novelty of adapted designs generated by various grammar rules. The research needs to put forward objective measurable conceptions obtained from both the process and the design to assist in generating and assessing innovative adapted designs using shape grammars. It aims to use mathematical measures to quantify the innovative characteristics of design in each grammar rule on one hand, and in the generated design on the other hand.

2.7. Research problem and research methodology

Built on the aforementioned studies, the research needs to investigate the application of hybrid adaptation apart from other adaptation techniques such as transformation and substitution. In addition, innovation assessment needs to be associated with shape grammar for hybrid designs. The research problem is identified as:

"Generating and evaluating innovative design using hybridisation techniques in shape grammars"

The research put forwards the following enquiries:

- How can grammar rules be defined to facilitate hybridity? To answer this question the research need to investigate which adaptation strategies can the hybridisation technique use? Is it embedded or attached adaptation, with incremental design approach to synthesise modified design or with morphing design approach to modify existing design? And what adaptation tools can be used to achieve hybridisation, and in which part of a grammar they can be applied.
- How can rule selection be controlled to derive hybrid designs and not existing designs? To answer this question the research needs to investigate what type of output grammar can the hybrid adaptation generate? Is it a new grammar for new designs only, a new grammar for new and existing designs,

a new grammar for new and known designs, or an original grammar for new and existing designs?

- How can the innovation measurements be added to shape grammars to contribute effectively in both generating hybrid designs with desired metrics and assessing them? Innovation assessment is needed here to guide the generative process towards achieving the stated objective (deriving innovative design) by indicating grammar rules that could be selected to improve the innovation values of the generated design. In addition, evaluation metrics have assessment role to verify shape grammar capability to derive novel hybrid designs.

According to that, the research aims are:

i. To develop shape grammar methods to derive hybrid designs from a corpus of heterogeneous antecedents.

ii. To develop an innovation assessment associated with shape grammars.

iii. To apply the method above on a simple architectural configuration such as a corpus of heterogeneous minaret designs.

2.8. Chapter summary

From the previous sections, it can be said that adaptation has achieved a great success in generating innovative design using shape grammars. The framework of adaptation presented in this chapter gives a comprehensive picture of the concept of adaptation in shape grammar. The variation in adaptation methods in shape grammars has been defined in terms of their techniques, strategies and tools on one hand, and the outputs as grammars and their derived designs on the other hand. The framework reveals that transformation and substitution techniques are used widely in contrast to hybridisation which received less attention. In fact, hybrid adaptation practice mentioned earlier used hybridisation technique beside transformation technique. Therefore, the research problem is crystallised as generating innovative designs using hybrid adaptation in shape grammars. The research needs to establish hybrid adaptation strategies, tools, and outputs.

In addition, shape grammar for innovative design needs an evaluation system having metrics derived from both process and design characteristics. The evaluation system is required to have both generating and assessing roles to provide information on what rule can be chosen to improve the innovation measures of the generated design. A computational assessment method is needed that provide feedback on the effects of rule decisions made through the generation process and allows the grammar user to adjust these decisions to achieve an adapted design with better innovation values.

According to that, the research aim is to develop a shape grammar method for hybrid adaptation. The evaluation system of the degree of innovation in hybrid designs is needed here to guide both the generating and assessing processes. Lastly, the research seeks to apply a shape grammar for hybrid adaptation on a corpus of heterogeneous traditional minaret designs.

3. A FRAMEWORK FOR HYBRID ADAPTATION USING SHAPE GRAMMARS

Chapter two focused on shape grammar applications in two contexts: the use of adaptation techniques and the use of evaluation methods. The role of adaptation techniques in generating novel designs using shape grammars has been highlighted. A framework of adaptation in shape grammars was identified which revealed that hybrid adaptation needs more research investigation. On the other hand, the investigation of adding assessment methods to shape grammars showed the potential use of innovation assessment of hybrid designs.

This chapter presents hybridization as the adaptation technique for generating innovative designs using shape grammars. Firstly, an introduction to hybrid concepts which defines the main characteristics of hybrid designs is presented in section 3.1. Secondly, a framework of hybrid adaptation within the analysis and synthesis phases of a shape grammar is described in section 3.2. The third section, 3.3, puts forward a method of assessing the innovation in hybrid designs using shape grammars. The objectives and requirements of implementation are revealed in section 3.4. Lastly, the chapter is summarised in section 3.5.

3.1. Definition of hybridisation

The term hybrid, originating in biology, describes the offspring of two different species (Yessios & Pantelidou, 2006). It has been generalised to refer to “any recognisable entity that is made up of elements drawn from multiple sources”. A hybrid “is of particular interest where its elements are derived from heterogeneous sources, or it is composed of elements of a different or seemingly incongruous kind” (Clarke, 2005). Treizidis (2003) defines a hybrid object as a combination of the characteristics of both parent objects. It is composed of the topology of one object and the geometry of the other. Additionally, Stross defines the cultural hybrid as "a culture, or elements of culture, derived from unlike sources, that is, something heterogeneous in origin or composition". He states that "the parents of a hybrid are internally homogeneous and differ in composition from one another" (1999, pp. 254, 258).

Accordingly, the research adopts the definition of hybrid design as a new design whose elements are derived from a class of unlike designs with heterogeneous compositions.

3.1.1. Characteristics of hybrid designs

Clarke (2005) identifies the main characters of a hybrid entity. He considers that, without these features, "it is inappropriate to talk of hybridisation having occurred". The key characteristics are as follows:

- The new entity must be recognised, which has existence distinct from its progenitors[9].
- The new entity must exhibit elements from two or more progenitors.
- There must be a significant difference between the new entity and its progenitors to justify the use of the term hybridisation.
- There must be an integration or fusion of some features of one entity with some form of at least one other entity, to produce a new entity.

Terzidis (2003) considers that a hybrid is an object in disguise. Although it is topologically identical to one parent, it resembles the geometry of the other parent.

Based on Clarke and Terzidis, it can be concluded that the two main characteristics of a hybrid design are **being a mixture of elements** of its ancestors on one hand, and **having the individuality** which makes it distinct from its ancestors on the other hand.

3.1.2. Process of hybridization

Hybridisation is a technique for creating new entities. Terzidis defines two processes of deriving hybrid objects depending on the degree of difference between parents. He

[9] Progenitor is a direct ancestor. (http://dictionary.reference.com/browse/progenitor)

considers that the hybrid object derives its structure from its isomorphic[10] parents through formal interpolations, where “interpolation is a method for estimating values that lie between two known values”. On the other hand, the lack of homogeneity among the heteromorphic[11] parents leads to a selective process of omissions and inclusions of elements between the two parent sets. He defined the extrapolation of hybrid form as a method for estimating “values outside a known range from values within a known range” (2003, pp. 58, 60).

Stross (1999, p. 264) identifies the differences between biological and cultural mechanisms of hybridity. In the former, a hybrid is created through mating which involves two heterogeneous parents. The process includes a combination between half of the father set of paired chromosomes and half of the mother set of paired chromosomes. While in the latter, the cultural hybrid can have more than two parents. Cultural hybrids are created through processes such as diffusion or borrowing.

Yessios & Pantelidou (2006) consider mixing to be the main process for hybridisation. A hybrid solution is the mixture of two or more previous solutions to a problem, keeping the benefits whilst avoiding the negatives.

Accordingly, the research concludes that omissions and inclusions, combination, fusion and mixture are the main processes for generating hybrid design in architecture.

3.1.3. Hybrid adaptation in shape grammars

From this section onwards, the study considers that a hybrid design has heterogeneous antecedents which are internally homogeneous having something common such as function or structure, and externally heterogeneous having various components. The main characters of a hybrid design can be identified as:

- Being a mixture of its ancestors on one hand.
- Having individuality that differentiates it from its ancestors on the other hand.

[10] Isomorphic: different in ancestry, but having the same structure. (http://www.answers.com/topic/isomorphic)
[11] Heteromorphic: differing from each other in shape, structure, or magnitude. (http://dictionary.reference.com/browse/heteomorphic)

The research adopts a component-based generative design system to define hybrid design in shape grammars. The hybridisation technique is a process of mixing, combining and fusing rules belonging to a class of heterogeneous antecedents. The next paragraph concentrates on developing a shape grammar to derive hybrid designs from a corpus of heterogeneous antecedents.

3.2. A framework for hybridisation in shape grammars

This section puts forward an approach to component-based design hybridization using shape grammars. The method is clarified within two main phases of shape grammar: the analysis phase and the synthesis phase. The former phase is done by the grammar author, while the latter phase can be run by the grammar user.

3.2.1. Analysis phase of shape grammars for hybrid designs

This section investigates the means that facilitate deriving hybrid designs through the analysis phase of shape grammars. This phase concerns the steps of defining grammar rules to describe known component-based designs. According to Knight, there are four steps in analysing precedents (1994, p. 28) as follows:

- A vocabulary of shapes and a set of spatial relations common to design are distinguished.
- Shape rules that fix the occurrences of spatial relations in designs are defined.
- An initial shape to begin the generation of designs is given.
- A shape grammar is specified in terms of the shape rules and initial shapes.

Shape grammars, as defined above, generate known designs as well as new designs in the same language. The theorists of shape grammar assert that there is no mandatory way to define rules. Stiny refers to the fact that, in shape grammars, there are "different ways of arranging lines and different categories for describing designs" which establish different languages of designs (1985, p. 8). According to Knight, shape grammar allows for designs to be generated in more than one way. She considered that "For any corpus of designs, there are always alternative ways to define a shape grammar" (2003, p. 140). Shape grammars define equivalence and ordering relations that "allow for designs to be grouped or linked together in terms of the way they are understood, and thus reveal similarities and differences of structure and properties in the composition and description of designs" (Stiny, 1981, p. 266).

Stiny and Knight confirm that the grammar author is able to define grammar rules in multiple ways. However, defining rules for shape grammars of hybrid component-based designs should be constrained to satisfy the main characteristics of hybrid design, as defined in the previous section, such as the mixed character and the individuality. The hybrid component-based design results from a mixture, combination and fusion of elements belonging to more than one varied ancestor. The study considers that ancestors are a class of building type which consists of heterogeneous designs having the same components but varied configurations.

In architectural design, representing different features of a same component in a heterogeneous class of designs using only parametric variation of rules is not always possible. Therefore, shape grammars for hybrid designs need to identify a way of defining multi-choice rules to reflect the different shape features of the same component of the existing designs in the corpus. This multi-choice rule facilitates the mixture of rules from varied designs. In addition, finding a way to enhance the differences between a generated hybrid design and its ancestors is recommended. A hybrid design has the individuality that differentiates it from its ancestors. This can be done by fusing the original rules to derive new hybrid rules. According to that, the shape grammar needs to adopt two main operations in generating the hybrid designs. They are:

- **Mixing rules** – a combination of grammar rules that belong to varied designs in the corpus. This process enhances the mixed character of the generated hybrid design.
- **Fusing rules** – a merging of original grammar rules that belong to varied designs in the corpus to produce new hybrid rules. This process enhances the individuality character of the generated hybrid design.

Based on the above, the study suggests two approaches to be taken into account in the analysis phase of shape grammars for hybrid designs. The first approach is compatible with the embedded adaptation in which deriving a hybrid design depends on mixing rules by modifying their order. In this approach, at each step of rule application, shape grammar facilitates a free choice of rules belonging to varied designs in the corpus. The second approach is compatible with the attached adaptation in which new hybrid rules are added to replace original grammar rules. The fusion of original rules into hybrid rules increases the variations between the generated hybrid designs and the existing designs in the corpus. In addition, adding new hybrid rules to the original grammar rules increases the options of mixture available to grammar users.

In conclusion, in a shape grammar for hybrid component-based design, analysing a corpus of heterogeneous existing designs should take into account the followings adaptation strategies:

- Using embedded adaptation by defining the original grammar rules of each component in a subclass rule set to allow free choice of rules for grammar users. In this case, the characteristic of hybrid design as a mixture of rules is facilitated.
- Using attached adaptation by adding new hybrid rules to the original grammar rules to enhance the differentiation of hybrid design and to increase the available choices for hybridity. In this case, both individuality and mixed properties in hybrid design are facilitated.

Detailed descriptions of each approach are given in the following paragraphs.

3.2.1.1. Embedded hybrid adaptation by arranging rules in sub-class rule sets

Orsborn et al. consider that "our understanding of the differences between classes of products, and the possibilities to merge them would be a useful application in the current trend of cross-over innovation" (2006, p. 218). The grammar author analyses the heterogeneous class of designs to define the similarities and differences among antecedents in the corpus. Similarities and differences among typological designs generate rules which can be placed into three categories: generic rules, sub-class rules and instance rules.

Generic rules reflect the similarities among the existing designs in the corpus. They have the same value of the same attribute for all members of the class. These rules are compulsory rules which apply to derive all existing designs in a language. The configuration in the left hand side (LHS) of the generic rule has only one possible configuration in the right hand side (RHS), that allows for one derivation or multi derivations in the design process. The user role is limited to the selection of different Euclidean transformations and parametric variations under which the generic rules may apply.

Sub-class rules are a set of rules consisting of more than one rule which have different values for the same attribute among the existing designs in the corpus. They are optional (multiple choice) rules having identical configurations on their left sides (LHS), but with different configurations on their right hand side (RHS). In this case, the grammar user can choose from the different possible configurations of the right hand side of these rules. Therefore, these rules are useful in deriving hybrid designs as they facilitate the mixing of rules relating to different designs.

Instance rules are a set of rules that define the different attributes of the same component using different configurations in both the left hand side (LHS) and the right hand side (RHS) of the rules. In addition, an instance rule can be a single rule which defines an attribute of one or more existing designs in the corpus. These rules do not represent a typological attribute, therefore cannot be helpful in deriving hybrid designs.

Examples of generic, sub-class and instance grammar rules that represent three existing ornamental patterns are shown in Figure 3.1. Generic rules 1 and 2 apply to all three patterns A, B and C to embody the common features among the existing ornamental patterns. On the other hand, each rule in the sub-class set 3, 4 or 5 and the set of instance rules 6, 7 or 8 applies to one of the existing patterns. Rules in sub-class rule set have the same shape in their left hand side and represent the different options of inscribed shapes inside the square. Therefore, in the derivation of new ornamental patterns, the sub-class rules set offers multiple options to grammar users to choose from. In contrast, the instance rules have different shapes in their left hand side. Therefore, they only offer a single choice to grammar users in the derivation process.

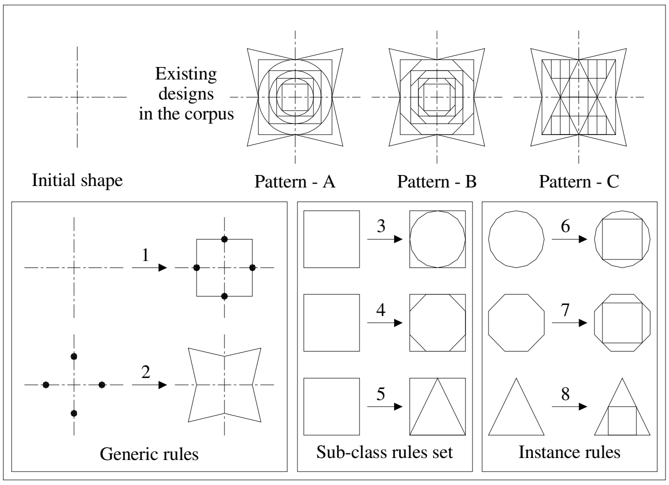


Figure 3.1: Examples of generic rules, sub-class rules and instance rules (Source: Author)

Preference for a set of sub-class rules on the instance rules can be attributed to the fact that the grammar user, in case of the latter, cannot compare the different options, choose one of them and apply it at the same time because each instance rule requires different rules to be applied in advance. For example, most of the instance rules of interior space layout of Palladian villas written by Stiny and Mitchell (1978) (Figure 3.2 left) can be restructured in a set of sub-class rules (Figure 3.2 right) where all rules have the same configuration based on 3 × 3 grids in their left hand side (LHS) to facilitate rule comparison, selection and application simultaneously.

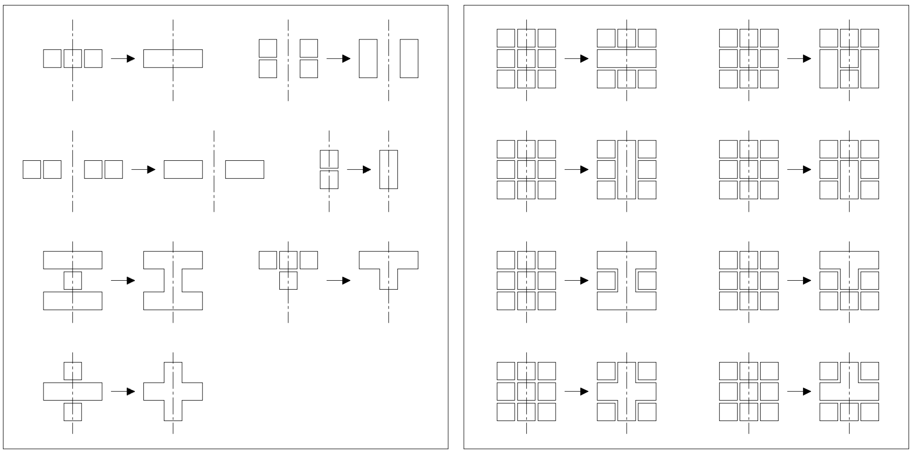


Figure 3.2 (Left): Instance rules of interior layout of Palladian Villas (Stiny & Mitchell, 1978), (Right): Sub-class rule set for some interior layout rules of Palladian Villas (Source: Author)

Accordingly, organizing rules in a sub-class rule set enhances the mixed character of the hybrid design. Varied rules can be grouped in the subclass rule set to reflect the different configurations of each component of the heterogeneous existing designs in the corpus. This set contains more than one rule having the same shapes, spatial relations and labels in their left hand side (LHS), and different shapes, spatial relations and labels in their right hand side (RHS). The grammar user is able to compare the available choices easily, to select one of them and apply it directly.

Examples of using sub-class rule sets to facilitate the derivation of new designs are clarified in Figure 3.3. The rules in the sub-class set of entrance location (1a, 2a, 3a and 4a) use markers to indicate the varied numbers and locations of entrances. They offer four options to a grammar user to choose from. Additionally, the rules of sub-class set of entrance shapes (1b, 2b, 3b and 4b) replace the entrance marker with varied configurations. A grammar user has the freedom to choose the form of the entrance for each marker. This free choice in both sub-class sets enables a grammar user to derive varied new hybrid designs as shown in the bottom of Figure 3.3.

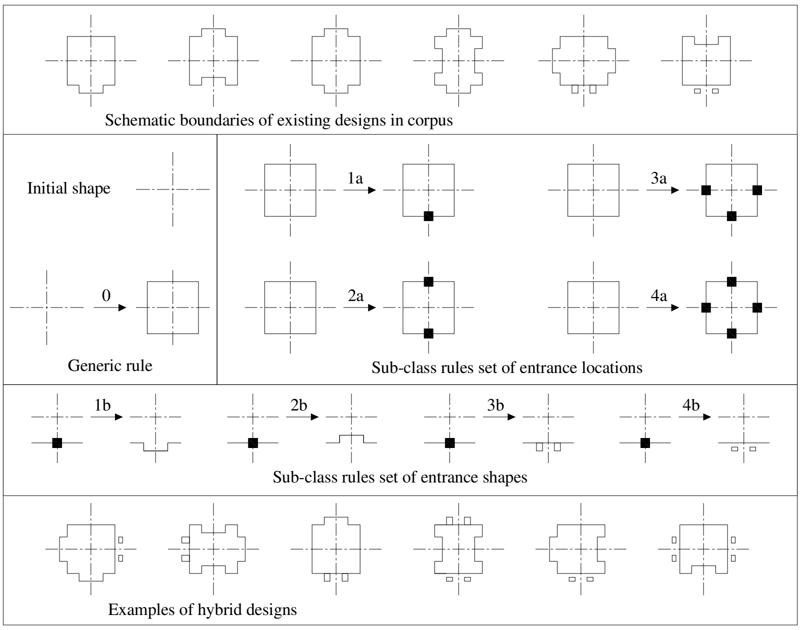


Figure 3.3: Example of deriving hybrid designs by mixing rules which belong to sub-class sets (Source: Author)

3.2.1.2. Attached hybrid adaptation by adding new hybrid rules to original grammar rules

New hybrid rules can be added to original grammar rules to enhance the individuality of the hybrid design. In addition, they boost the mixed character of hybrid design by increasing the available options in the subclass set of rules and converting the instance rules to sub-class set. The hybrid rule combines two or more original rules (parents) which belong to one sub-class rules set or instance rules. If the parents' rules belong to the same sub-class rules set, then they have the same left hand side (LHS) configuration and the hybrid rule keeps this LHS. Otherwise, the parents' original rules with different LHS configurations are identified as the host and guest(s) rules; whereas hybrid rule keeps the same LHS configuration of the host rule. In all cases, the configuration of the right hand side (RHS) of the hybrid rule results from merging all or part of the shapes, spatial relations and markers of the right hand sides (RHS) of the parents' rules.

Figure 3.4 shows three rules 1, 2 and 3 of the principal entrances in Palladian villa grammars written by Stiny and Mitchell (1978). Hybrid rules are derived by merging the host rule 1 with the guest rule 2 to generate the hybrid rule 1a, and the guest rule 3 to generate the hybrid rule 1b. The addition of two new hybrid rules to the original instance rule (1) converts it to sub-class rules set composed of the rules (1, 1a and 1b).

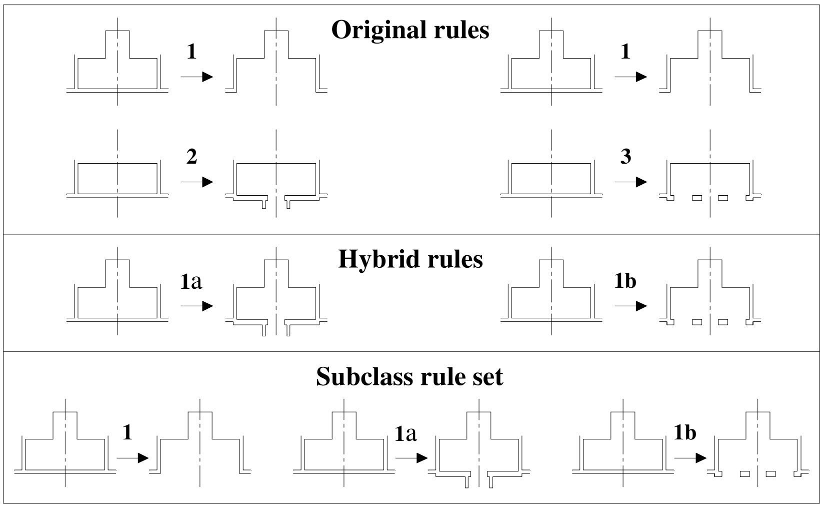


Figure 3.4: Hybrid rule derived from merging two original rules (Source: Author)

The configuration of the RHS of the hybrid rules results from the interpolation or extrapolation processes (Terzidis, 2003). In the former process, the elements of the RHS of the hybrid rule correspond to a sum or an average of their parents' characteristics. The right hand side (RHS) of the hybrid rule has the same shapes and spatial relations as its parents but in a different configuration. In the latter process, the elements of the RHS of the hybrid rule are outside the range of their parents. In this case, new shapes and/or spatial relations are introduced to formulate a new configuration of the RHS of the hybrid rule (as shown in Figure 3.5). At the left bottom of Figure 3.5, the interpolation in the RHS of the hybrid rule is the sum of the RHS of the original rules at the top. On the other hand, at the right bottom of Figure 3.5, the extrapolation in the RHS of the hybrid rule results in a new shape by merging half of the circle and rhomboid shapes of the RHS of the original rules.

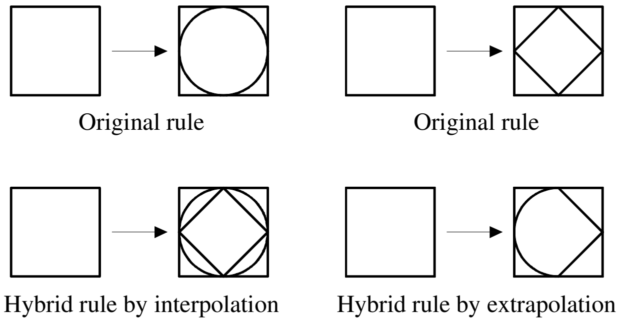


Figure 3.5: Deriving hybrid rules by interpolation or extrapolation (Source: Author)

The Composition of hybrid rules in shape grammars depends on the type of parents' rules. These may be simple, compound, labelled or parametric rules. An introduction to deriving hybrid rules from each type of rules is presented in the following paragraphs.

3.2.1.2.1. Deriving hybrid rules from simple rules

Simple rules are defined in the form of A→B, where A and B are shapes made up of solids, planes, lines, or points. A rule specifies that whenever a shape A is found in a design, it can be replaced with the shape B. A grammar consisting of simple rules in the form of sub-class rules or instance rules can be hybridized to produce a new design language. This is done by defining new shapes and spatial relations from ones given in the existing language of designs. Two operations are used to derive the hybrid rules. They are:

- ***Merging operation*.** The merging of rules entails combining all or parts of the shapes and/or their spatial relations of the parents' rules.

- ***Replacement operation.*** Allows replacement the shapes and/or their spatial relations in the host rule with other ones from the guest rule. In this case, "the exact location of the shape replacing another shape must be specified with respect to the Cartesian coordinate system in which the original shape is defined". (Knight, 1981, pp. 217-223)

Both merging and replacement operations can lead to interpolation (Figures 3.6 - 3.9) or extrapolation (Figure 3.10) of hybrid rules. Examples of deriving hybrid rules using merging operations are shown in Figures 3.6 and 3.7. Deriving hybrid rules by combining two original rules of sub-class rules set is shown in Figure 3.6. The hybrid rules keep the same left hand side (LHS) configuration of the parents' rules while their right hand sides (RHS) configurations are the result of combining whole of the shapes and their spatial relations in the left hybrid rule, and part of the shapes and their spatial relations in the right hybrid rule (Figure 3.6).

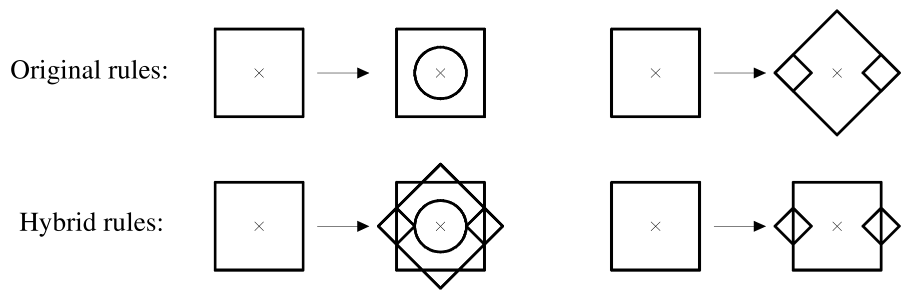


Figure 3.6: Hybrid rules combine whole or part of the original subclass rules set (Source: Author)

In case of instance rules, the hybrid rule keeps the left hand side (LHS) configuration of the host rule, while merging whole or part of the shapes and their spatial relations in the right hand side (RHS) configurations of both the host and guest rules, as shown in Figure 3.7.

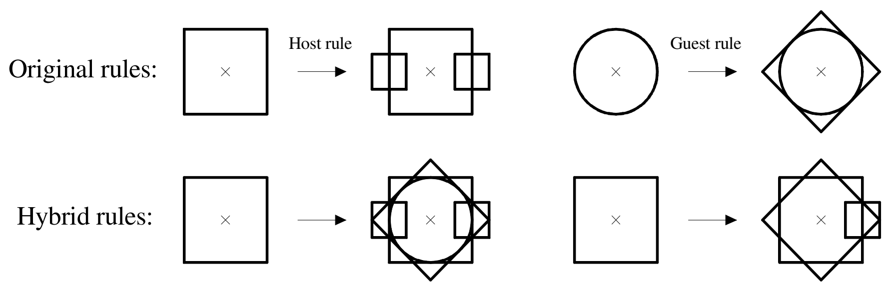


Figure 3.7: Hybrid rules combine whole or part of RHS original instance rules (Source: Author)

Deriving hybrid rules using the replacement operation is shown in Figures 3.8 and 3.9. Shapes and/or their spatial relations in the right hand side (RHS) configurations of the original sub-class rule or instance rule are substituted with shapes and/or their spatial relations from the right hand side (RHS) configuration of the other original sub-class rule or instance rule. In Figure 3.8, the RHS configuration of the left hybrid rule is the result of replacing the rhombus in the left original rule with the circles from the right original rule; while the right hybrid rule is the result of replacing the circles in the right original rule with the rhombus from the left original rule.

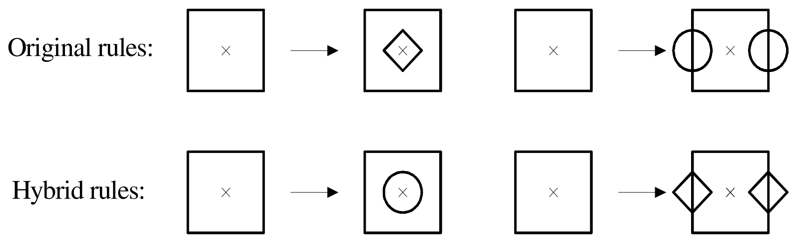


Figure 3.8: Hybrid rules result from replacing shape of a rule with shape of other rule (Source: Author)

In case of instance rules, a hybrid rule retains the left hand side (LHS) configuration of the host rules while the right hand side (RHS) is derived by replacing shapes and/or spatial relations between the host and guest rules, as shown in Figure 3.9.

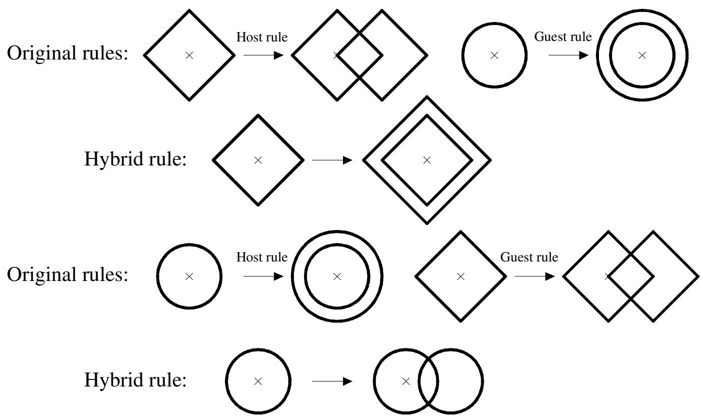


Figure 3.9: Hybrid rules result from replacing spatial relations of RHS host rule with spatial relations of RHS guest rule (Source: Author)

Other possible hybrid rules can result from an extrapolation process such as replacing part of one shape in the rule with part of a different shape in the other rule within the same Cartesian coordinate system, as shown in Figure 3.10.

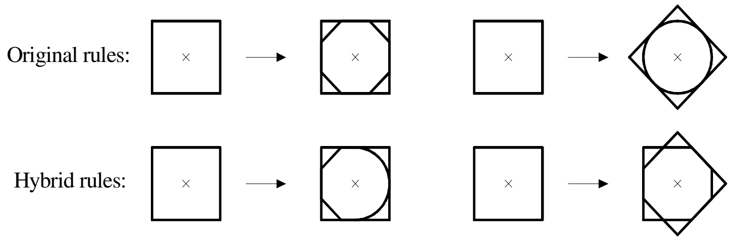


Figure 3.10: Hybrid rules result from replacing parts of one shape with other (Source: Author)

The possibilities of deriving hybrid rules from simple grammar rules using mergence and replacement operations are shown in Table 3.1.

Table 3.1: Possible combinations between simple host and guest rules

	Original Simple rules							
	RHS of Host rules				RHS of Guest rules			
	Whole shapes	Whole Spatial relations	Parts of Shapes	Parts of Spatial relations	Whole shapes	Whole Spatial relations	Parts of Shapes	Parts of Spatial relations
Hybrid Rules Possibilities	*	*			*	*		
			*	*			*	*
	*					*		
		*			*			
	*	*			*			
	*	*			*			*
	*	*						*
	*	*					*	*
	*	*					*	
	*	*				*		
	*	*				*	*	
	*			*				*
	*			*			*	*
	*						*	*
		*	*				*	
		*	*				*	*

3.2.1.2.2. Deriving hybrid rules from compound grammar rules

The relations between grammars with compound rules may be dependent or independent. In the former, any adaptation in one grammar affects the other grammar(s). While in the latter; one grammar can be adapted without changing the other grammar(s) (Stiny, 1990, p. 102).

Hybrid rules can result from merging whole or part of shapes and/or spatial relations of original parallel rules. In Figure 3.11, a parallel grammar combines the rules of plan and elevation. The hybrid rule is the result of merging the whole shapes of both the plans and elevations of the original rules.

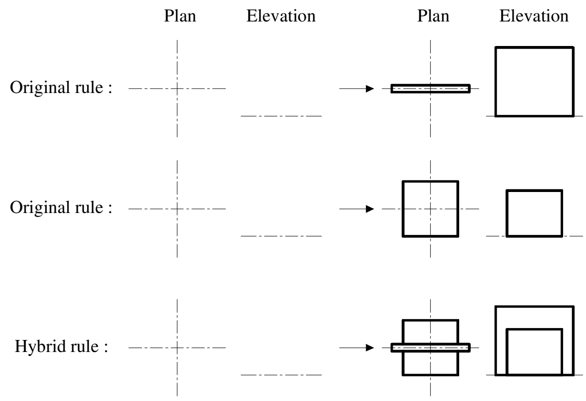


Figure 3.11: Hybrid rules result from merging process in parallel rules (Source: Author)

In addition, possible hybrid rules can be derived in parallel grammars by replacing shapes or part of shapes and/or their spatial relations between two original rules using interpolation and extrapolation. Example of the former, hybrid rules can be generated by replacing the elevation boundary in one original rule with the elevation boundary in the other original rule, as shown in the middle of Figure 3.12. On the other hand, example of the extrapolation in hybrid rules can happen when new plans are prompted as a result of replacing the whole elevations between original rules, as shown in the bottom of Figure 3.12.

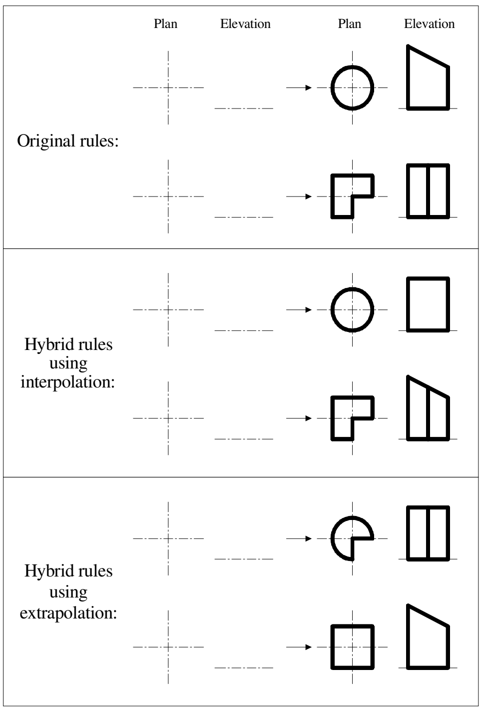


Figure 3.12: Hybrid rules result from replacement process in parallel rules (Source: Author)

3.2.1.2.3. Deriving hybrid rules from labelled rules

In the labelled grammar, “the computation in the shape grammar combines two computations – one with shapes and one with sets of labelled points – that are carried out in parallel” (Stiny, 1990, p. 101). Hybrid rules can be derived by merging a shape grammar from a host rules with a label grammar from a guest rules. In Figure 3.13, the hybrid rule 1 combines the shapes and spatial relations of the original rule 1 with the markers of the original rule 2, and the hybrid rule 2 combines shapes and spatial from the original rule 2 with the markers of the original rule 1. Examples of the designs derived by each rule show the effect of replacing labels in the hybrid rules.

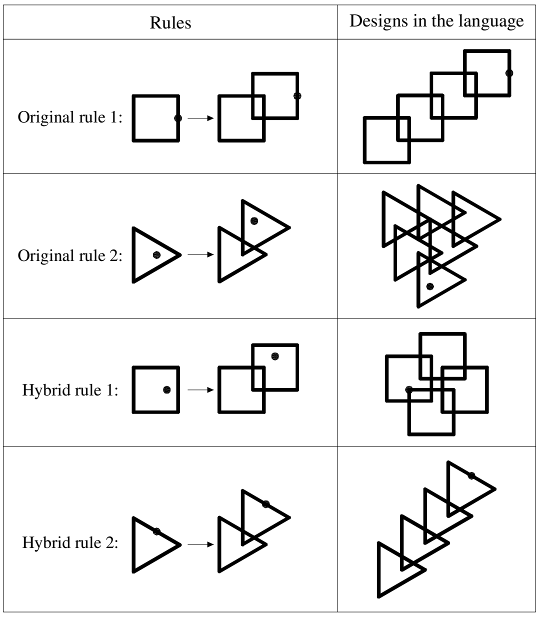


Figure 3.13: Hybrid rules result from replacement process in labelled rules (Source: Author)

3.2.1.2.4. Deriving hybrid rules from parametric rules

Another form of hybrid rule results from merging different parametric rules. Grammars describing shapes having varied geometric scale are called parametric shape grammars (Cagan & Mitchell, 1994, p. 175). Knight considers that “new shape rules may be defined from a given one by parameterizing the shapes occurring in it” (1981, p. 217). According to Cagan, “the parametric nature of shape grammars enables the grammar to concisely represent large (sometimes infinite) variation within a class of designs” (2001, p. 83). Design attributes such as parameters can be used to differentiate designs having similar shapes. A shape in a rule can have dimensions which are fixed or parametric. A parametric shape is commonly called a 'schema' in which some of its details or characteristics are fixed and others are varied. The characteristics that vary are called the variables of the shape. Each variable has a range of conditional variation to be satisfied by the values assigned to them (Knight, 2003, p. 135). Spatial relation can be varied by "parameterizing the shapes occurring in it or by changing conditions on existing parameters". The parametric shapes in a spatial relation can vary either the shapes themselves or the disposition of these shapes with respect to each other (Knight, 1981, p. 217).

In a parametric grammar, there are different ways to derive the hybrid rules. Firstly, in the same way as with simple rules, hybrid designs can result from combining parametric shapes belonging to different designs. Secondly, a hybrid rule can be introduced by merging a shape from the host rule with a shape parameter in the guest rule, as shown in Figure 3.14. In this example, the shape parameters of the upper original rules are replaced with the shape parameters of the original rule at the middle to constitute a new hybrid rule at the bottom. In this case, the large square in the upper original rule where the length is equal to width (L=2a, W=2a) becomes a rectangle in the hybrid rule having the same parameters of the ellipse in the middle rule in which the width (c=3a) is twice the length (b=1.5a).

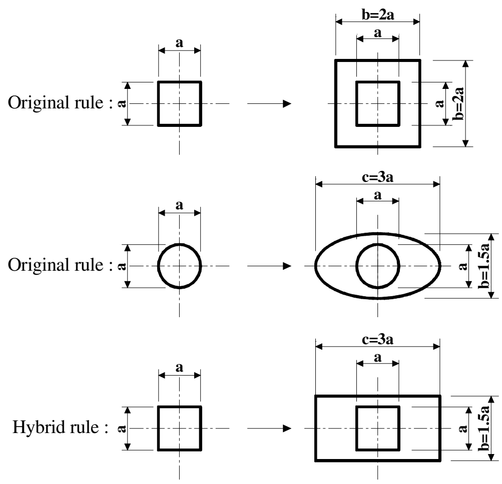


Figure 3.14: Hybrid rules result from parameters replacement (Source: Author)

The grammar user can mix the parameters of different shapes in different rules relating to different designs to reflect the parametric variations of the same component amongst existing designs in the corpus.

3.2.2. Analysis phase of shape grammars for hybrid design – Conclusions

The last section established the principles for writing rules in shape grammars for hybrid component-based designs. The proposed procedures aim to achieve the hybrid design requirements of being a mixture and having individuality. Knowledge about the components of precedents should be represented in terms of sub-class rule sets to be easily mixed. In addition, adding new hybrid rules to the original grammar rules was proposed to enhance the individuality as well as the mixture by increasing the options available to the grammar user at each stage of rule application.

The set of sub-class rules embodies the different configurations of the same component among existing designs in the corpus. It consists of rules having different right hand side (RHS) configurations, while their left hand sides (LHS) are the same. Therefore they are applicable to the same part of the generated design. It plays a dominant role in the derivation of hybrid design with a mixed character because it provides the grammar user with multiple-choice rules belonging to varied existing designs. Accordingly, organising rules in a sub-class rule set facilitates the mixing of rules relating to varied precedents in the corpus.

On the other hand, adding new hybrid rules to original grammar rules enhances the individuality by increasing the differences between the generated hybrid design and its antecedents. Additionally, these rules can be added to both the sub-class rules set and the instance rules set to enhance the mixture in the hybrid design. In the sub-class rules set, the new hybrid rules will increase the available options of rules in each set. Furthermore, a new hybrid rule added to an instance rule will modify it to a set of sub-class rules which, in turn, offers multiple choice rules.

3.2.3. Synthesis phase of shape grammars for hybrid designs

This phase of a shape grammar is controlled by both the system and the grammar user. It aims to identify the form of a design solution as a mixture of components from varied designs in the corpus. It has two main operations: the selection of rules and the application of rules. In shape grammars for hybrid designs, the adaptation tool depends on changing the original sequence of rules by selecting rules from a variety of designs in the corpus. To derive hybrid designs and not existing designs, the synthesis phase provides an interactive system for a grammar user to select eligible rules from different antecedents in the corpus.

The first act of synthesising a hybrid design is for the system to direct the selection of rules to a specific sub-class rule set. The second act in the selection process is guided by the system and run by grammar users to control the process of searching for alternative rules within the search space of the sub-class rules set. The third act in the application process is run by the system to constrain the possibilities of applying a rule by restricting its execution to derive only valid designs.

Based on the above, state labels and spatial labels (markers) are required to restrict the format of rules and the sequence order of their applications (Knight, 1994, 1998). The computation in a labelled shape grammar "combines two computations - one with shapes and one with sets of labelled points - that are carried out in parallel and influence one another mutually" (Stiny, 1990, p. 101). They are used to constrain both the selection and application of rules to derive only feasible hybrid designs (Table 3.2). The rule selection is controlled by state labels and markers while the rule application is controlled by markers only. State labels are the alphanumeric characters attached to the rules to ensure hybridisation by mixing rules derived from varied existing designs in the corpus. Markers are symbols attached to shapes in rules to ensure the valid derivation of a hybrid design by restraining the formal and functional compatibility of its components. The markers control the sequence of rules from one sub-class rule set to the others, constrain the location where the rule applies, and restrict the relation between the shapes in the generated design.

Table 3.2: The synthesis phase in shape grammar for hybrid designs

	Type of process		**Type of controller**	**Control tools**
Synthesis Phase	Selection process	Sub-class rules selection	Controlled by system	Markers
		Rule selection	Controlled by user	State labels
	Application process	Determine the location	Controlled by system	Markers
		Determine the parameters	Controlled by user	Parameters

3.2.3.1. Rule Selection in shape grammars for hybrid designs

The selection of rules in shape grammars for hybrid component-based design requires firstly determining the component type by deciding the sub-class rules set from which the rules are to be chosen. In a computer implementation of an interactive shape grammar system, the user is prompted with the available rules for application. A rule must firstly be selected before applying it. The typical questions in shape grammars for hybrid designs are:

- Does the grammar system restrict the sub-class rule set from which the next rule can be chosen?
- If the user is free to select any rule from the sub-class rule set, then the second question is: Does the grammar restrict which rules can be selected to achieve hybrid designs and to avoid the derivation of existing designs?

To answer these questions, the following paragraphs focus on the selection of sub-class rules set and the selection of rules in the synthesis process of shape grammar for hybrid component-based design.

3.2.3.1.1. Selection of sub-class rule set

This process is run by a grammar system using markers as a technical mechanism to direct the proper sequence of components in the derivation of hybrid component-based designs. Their task within the selection process is to ensure the functional compatibility of the selected rule with the generated design by directing the sequence of design stages properly. To prevent the improper sequence of components, each rule inherits the sequence of a next component from its antecedent(s). In this case, the markers restrict the type of component in the next sub-class rule set from which the next rule can apply.

Accordingly, the sequence of rules application from sub-class rules set to others is guided using the marker which symbolizes the component type of the sub-class rules set. The marker in the left hand side (LHS) of a rule is an indicator to the sub-class rules set to which it belongs; while the marker in the RHS of a rule is an indicator to the next sub-class rules set from which the next rule can be selected. In a sub-class rules set, all rules share the same component marker in their LHS, while there are the same or different component markers in their RHS. For example, if laboratory chair designs are defined in terms of shape grammar for hybrid component-based designs, then each sub-class rules set define one of the main components of this type of chair[12] such as: bases, lifts, seats, backs and arms. The markers, in the LHS of the sub-class rules set of chair bases are the symbol of the base, while the marker in the RHS of these rules are the lift markers which direct the subsequent selection to the sub-class rule set of chair lifts, as shown in Figure 3.15.

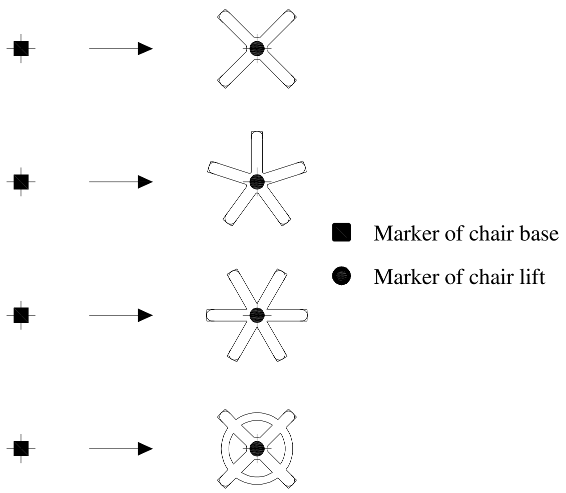


Figure 3.15: Using markers in the sub-class rule set of laboratory chair bases (Source: Author)

However, the markers in the RHS of the sub-class rules set of chair seats are varied. They direct the user either to end the grammars in case of the stool chair, or to add a chair back which in turn is followed by the chair arms or the end of the grammar in case of the non arm chairs, as clarified in Figure 3.16.

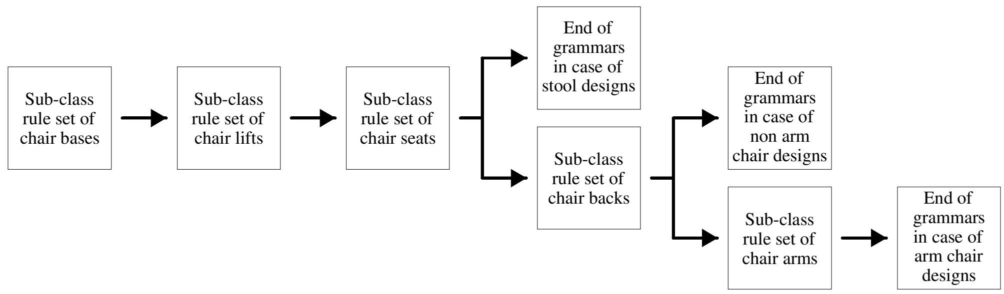


Figure 3.16: Sequence of rule application in case of laboratory chair design

According to that, in shape grammars for hybrid component-based design, the markers in the RHS of rules in each sub-class rules set represent the similarity and differences in the sequences of components among the existing designs in the corpus,

[12] The main components of laboratory chairs are concluded from a chair catalogue.

as shown in Figure 3.16. The use of markers to restrict the next choice of rules with specific components, based on precedents, leads to the derivation of valid hybrid design and prevents components from being randomly overlaid - which may lead to illogical designs.

3.2.3.1.2. Selection of rules

In a shape grammar for hybrid designs, the selection of rules is made by grammar users under the system constraints. In the theory and practice of shape grammar there are varied methods of selection. The selection technique of rules can be dependent on the search technique or independent of it. Two approaches of rule selection can be distinguished in which the user has a role in the selection process. They are either direct selection or indirect selection. The differences between them can be attributed to the use or non-use of the search technique. If it is used, then a further difference is in the ordering of the selection and the search techniques. In the first approach, users can select rules and choose matching conditions such as parameters and transformations. They may decide the location to apply a rule and the sequence of rules application. In this approach, selecting a rule precedes any search technique if it is needed, such as in the shape annealing grammars (Cagan & Mitchell, 1993). On the other hand, indirect selection of rules results from choosing criteria related to design constraints or optimisation that act as controls on subsequent rule selection and application. In this approach, search techniques precede any rule selection, such as the implementation of Siza's houses grammar (Duarte, 2001).

Methods of search in design can satisfy different criteria, the most relevant being:

- **Optimally directed design,** which seeks to achieve the functional, technical and domain requirements and constraints.
- **Goal directed design,** which seeks to achieve the user's wants, needs, motivations, and contexts.

Both optimally directed and goal directed searches are used in shape grammar practice. Shape annealing methods use optimisation criteria to control the derivation of shape. They apply any rule which has a matching condition and then tests it to decide if this rule is suitable or not according to the optimisation requirements (Cagan

& Mitchell, 1993). Examples of goal directed shape grammars are the consumer products which are "driven by a basic functional decomposition, but the products themselves are differentiated by form" to achieve style change (Cagan, 2001, p. 73) (Ahmad & Chase, 2007).

Based on the above, shape grammars of hybrid designs require an approach of rule selection which combines both direct and indirect approaches. On one hand, the user interaction is recommended to select rules in the same way as the direct selection approach. On the other hand, the choice of rules should be constrained in advance, in the same way as the indirect selection approach, to satisfy the design aim: deriving innovative hybrid design. Shape grammar for novel component-based hybrid design is a goal directed search in which the user should be able to select rules from a specific subset of rules. This subset is part of a sub-class rule set and includes rules which are eligible to derive the innovative hybrid design.

Accordingly two search mechanisms are suggested to help grammar users to achieve the innovative hybrid design. The first mechanism uses state labels to identify the antecedents in the corpus which have no rules or minimum rules in the previous stages of design derivation. In this case, selecting rules belonging to these antecedents leads to a hybrid design. The second mechanism attaches innovation metrics of hybrid designs to the shape grammar. The grammar user receives feedback on the degree of mixture and individuality each rule offers to distinguish the generated hybrid design from antecedents in the corpus.

The use of the innovation metrics as a search mechanism in the synthesis phase has been discussed in detail in section 3.3 of this chapter. The following section clarifies the ways in which the state labels can constrain the rule options for the grammar user to derive only hybrid designs.

3.2.3.1.3. State labels in shape grammars for hybrid designs

State labels in each rule are used as indicators of antecedents in two manners:

i. Current indicators of the rule sources. Rule sources are existing designs in the corpus from which the current rule is derived. These state labels are constant values attached to the left hand side (LHS) of each rule in the grammar.

ii. Predictive indicators of the possible next rule sources. Next rule sources are the existing designs in the corpus from which the next rule can be chosen to derive the hybrid design. These state labels are constant or variable values added to right hand side (RHS) of each rule in the grammar.

Thus the state labels in shape grammars for hybrid design can be both constant and variable values. All current indicators are constant state labels. Each antecedent in the corpus has a label to symbolize it. For example, if the corpus of antecedents is (*n)*; and there are 5 antecedents in the corpus, then each design has a symbol composed of the letter *d* and the number varies among (*1*-*5*) as follows:

$n = \{d1, d2, d3, d4, d5\}$

If the rule is derived from *d2* and *d5* then the current LHS state labels of this rule are: $\{d2, d5\}$. Therefore, all rules in the grammar have LHS state label(s) to indicate one or some of the antecedents in the corpus *n* as the rule sources.

On the other hand, predictive indicators have constant or variable state labels. Both of them are attached to the right hand side (RHS) of each rule in a subclass rule set. The predictive indicators are defined as either (*n1*) or (*nx*). (*n1*) is a constant state labels attached to the RHS of all rules which have the initial shape in their LHS as the first rules to be applied in a grammar. While (*nx*) is a variable state label attached to all rules of sub-class rule sets that do not have the initial shape in their LHS. The variable (*x*) is the stage number of rule application which is replaced at grammar runtime by an ascending integer starting from *2* to *y*. The variable (*n2*) replaces (*nx*) in the RHS of a second rule to be applied. The variable (*ny*) is attached to the RHS of the last rule to be applied; whereas (*y*) is replaced by an integer which represents the total number of rules required to derive a hybrid design. The use of variable state labels in the RHS of rules gives the grammar flexibility in applying different numbers of rules and with different sequences to derive hybrid designs. The same rule can have varied values of (*x*) if it is applied multiply to the same design, or applied multiply in different sequences to more than one design.

3.2.3.1.4. The user guide grammar in shape grammars for hybrid designs

It is proposed that the user guide grammar for hybrid design may be used to specify at rule application, the values of the predictive state labels: ($n1$ and nx). It is a parallel grammar as "a network of two or more grammars that operate simultaneously" (Knight, 2003). This guide is added to each rule in all sub-class rule sets to define automatically at grammar runtime, the values of the RHS state labels of the current rule. These values are the set of possible antecedents from which the LHS state labels of the next rule can be chosen. The constant values of state label ($n1$) exclude the LHS labels of the current first rule from the set of whole antecedents (n); while the variable values of state label (nx) of other rules exclude the LHS labels of the current rule from the set of state labels of the previous rule ($n(x–1)$).

$n1$ = {n\LHS labels of the current rule}, which is {n – (the designs from which the first rule is derived)}

nx = {$n(x–1)$\LHS labels of the current rule}

For example, if the value of n is:

$n = \{d1, d2, d3, d4, d5\}$

And if the first rule is derived from $d2$ and $d5$, then the value of $n1$ is:

$n1 = \{\mathrm{n} \backslash d2, d5\} = \{d1, d3, d4\}$

If the second rule is derived from $d3$, then the value of nx is:

$nx = \{n(x–1) \backslash d3\}$

$n2 = \{n(2–1) \backslash d3\} = \{n1 \backslash d3\} = \{(d1, d3, d4) \backslash d3\} = \{d1, d4\}$

The LHS state labels of a hybrid rule derived from two original rules is the sum of the state labels of original rules. For example, if the first original rule has the state labels {$d1$, $d5$}, and the second original rule has the state label {$d2$}, then the state labels of the hybrid rule is{$d1$, $d2$, $d5$}.

However, there are two cases in which the value of the variable state label (nx) can be {Ø}. The first case results from the non-matching condition between (nx) values and the LHS labels of all applicable rules in the next subclass rules set. The other case happens when the state labels of all antecedents are exhausted in the previous steps of design derivation. In both cases the value of (nx) is replaced by (nx^*) which excludes the current LHS labels from the set (m). The set (m) is part of the set of whole antecedents (n), ($m \in n$), and includes state labels of antecedents that have a minimum number of applied rules in the previous stages of design derivation.

If $nx = \{\emptyset\}$, then

$nx = nx^*$

$nx^* = \{m\backslash \text{LHS labels of the current rule}\}$

Additionally, if nx^* is $\{\emptyset\}$, then nx^* is replaced by nx^{**} which is all antecedents (n) that are not in both sets (nx and nx^*), as follows:

If $nx^* = \{\emptyset\}$, then

$nx^* = nx^{**}$

$nx^{**} = \{n\backslash(nx+nx^*)\}$

If the generation of hybrid design has a limited number of rules, a specific component to end the derivation, or both of them, then other constraints on state labels should be added. In the case of grammars which generate designs with a limited number of rules y, a state label (ny) on the RHS of last rule in design derivation is defined in the user guide grammar as $ny = 0$.

If $nx = ny$, then $ny = 0$

The only rule in the grammar that has a state label (0) in its LHS is the termination rule. This rule aims to stop the generation process by removing state labels, markers, or unwanted lines.

On the other hand, generating designs with a specific component to end the derivation requires two constraints on state labels to be taken into account. The first constraint ensures that the final component is added to the generated design while the second constraint ends the derivation using RHS label $ny = 0$. With regard to the former constraint, if the grammar can add the final component at any stage, then the constraints on rules are as follow:

- In the grammar rules, adding the symbol (′) to the LHS state labels of all rules that have the final component markers in their RHS, such as *d1′*, *d2′*.
- At grammar runtime, adding a symbol (′) to the values of variable state labels $nx = n(y - 2)$ defined by the user guide grammar, such as:

$nx = n(y - 2) = \{d5', d8', d9'\}$

The latter condition constrains the choice of rules at the penultimate stage of design derivation to only rules that add the final component markers.

Accordingly the significant role of the state labels in selecting eligible rules to derive hybrid designs and not copies of existing is determined. Each rule has current and predictive indicators defined using constants and variables to identify antecedents in the corpus from which the current rule is derived and the next rule can be derived. They also play a role in ending the design derivation using a specific rule to add the final component, or a required number of rules to complete the derivation. The next paragraph concerns the constraints on rule application in the synthesis phase of shape grammars for hybrid designs.

3.2.3.2. Rule application in shape grammars for hybrid designs

In shape grammars for hybrid design, the application of a mixture of rules to generate the hybrid product requires the use of markers beside state labels to obtain feasible designs. The rule in the sub-class rule set may apply to different portions of the same design, as well as to the different designs in the corpus. In addition, different grammar rules can also apply to the same portion of the design. In these cases, markers as the spatial labels ensure that the derived design is valid by constraining the rule application to specific contexts.

In the application process, markers are technical mechanisms to control "where and how rules apply to designs by distinguishing spatial aspects of rules and designs" (Knight, 1994, p. 52). They have two main roles in the synthesis of hybrid design using shape grammars. The first role in the selection process, as discussed in the preceding section, ensures the functional compatibility between the selected rule and the generated design. While in the second role, markers play an important role in the application process by ensuring the formal compatibility between the selected rule and the generated design. They control where the rules apply to the designs and which formal constraints can be applied. The location of markers maintains the proper spatial relations between the shapes in the generated hybrid design. Figure 3.17 show how the mixing of grammar rules (without markers) to derive the hybrid designs could lead to a nonsense composition. The spatial relations between shapes derived by applying rule 2 then rule 3 are meaningless.

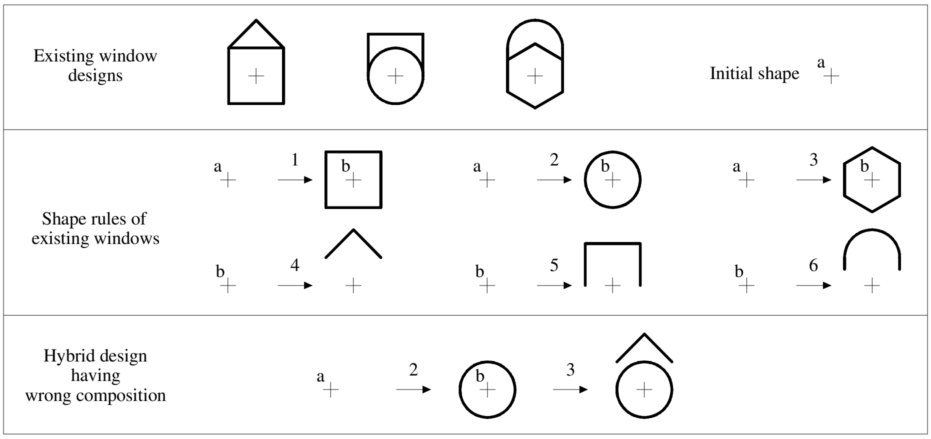


Figure 3.17: Hybrid design results from improper adjacency relations between shapes (Source: Author)

The use of markers to control the generation of proper spatial relations between shapes in hybrid designs is demonstrated in Figure 3.18. The role of markers in this example is to restrict the location where the rule can apply.

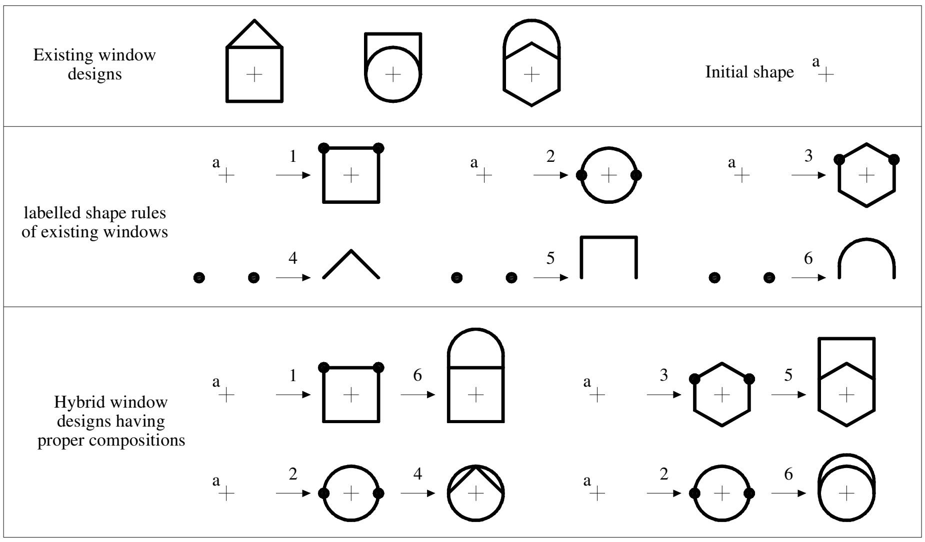


Figure 3.18: Using markers to derive hybrid designs with proper adjacency relations (Source: Author)

Based on the above, markers in shape grammars for hybrid designs ensure valid designs by maintaining the formal and functional compatibility between the selected rule and the generated design. In the selection process, they ensure the proper sequence of components in design stages by directing the derivation process from one sub-class rule set to others. Additionally, in the application process, they ensure that each component is generated and placed in a reasonable relationship with other components.

3.2.4. Synthesis phase of shape grammars for hybrid design - Conclusions

In the last section, the tools for synthesising rules in shape grammars for hybrid component-based designs using state labels and markers were defined. State labels help to derive hybrid design by directing the user to choose rules from the varied antecedents in the corpus. Markers in shape grammars for hybrid designs ensure valid designs by controlling the functional and formal compatibility between components in the generated hybrid design, as shown in Figure 3.19.

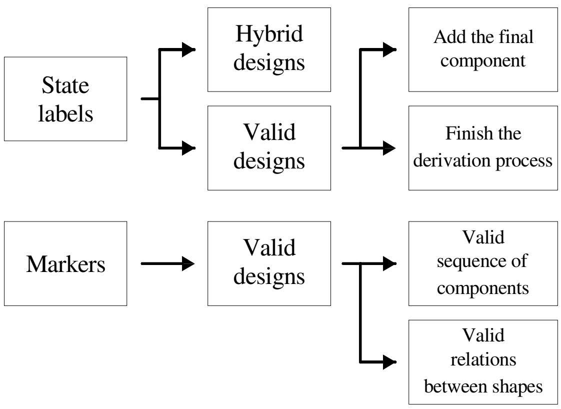


Figure 3.19: Roles of labels in the synthesis phase of shape grammars for hybrid design

To derive novel hybrid designs, the state labels in each rule have dual functions. On the LHS of rules, the state labels are current indicators having constant values to represent the antecedents in the corpus from which the current rule is derived. On the RHS of rules, the state labels are predictive indicators of possible antecedents from which the next rule can be chosen. With the exception of the first and last rules in the derivation, the values of state labels in the RHS of all other rules are variable. The values of RHS state labels are defined by the user guide grammar for hybrid design. In addition, state labels participate in deriving proper design by controlling the addition of final component and ending the derivation process. Markers aim mainly to derive valid designs by directing the proper sequence of components, and the proper positioning and relations between shapes in the generated hybrid designs. Their purposes can be summarised as follows:

- Guiding the proper sequence from sub-class rules set to the others.
- Constraining the location where rules are applicable.
- Controlling the proper spatial relations between shapes in the derived design.

Built on the analysis and synthesis phases of shape grammars for hybrid designs, the next paragraph concludes the characteristics of hybrid adaptation in shape grammars.

3.2.5. Hybrid adaptation in shape grammars

This paragraph concludes the hybrid adaptation strategies, tools and outputs. There are three types of possible hybridisation using shape grammars depending on the type of rules used to derive the hybrid designs, as follows:

- Hybrid designs derived using original rules only.
- Hybrid designs derived using hybrid rules only.
- Hybrid designs derived using both original and hybrid rules.

There are similarities and differences in adaptation strategies among these types as clarified in Table 3.3. All three types use the incremental adaptation approach to derive the hybrid designs. On the other hand, hybrid designs composed of original rules only depend on both embedded and attached adaptation; hybrid designs composed of hybrid rules only depend on attached adaptation; and hybrid designs composed of both original and hybrid rules depend on embedded and attached adaptation together.

Table 3.3: Hybrid adaptation in shape grammars for component-based design

			Hybrid adaptation using shape grammar: Hybrid adaptation using original rules only	Hybrid adaptation using shape grammar: Hybrid adaptation using hybrid rules only	Hybrid adaptation using shape grammar: Hybrid adaptation using both original and hybrid rules
Adaptation strategies	Where to locate	Embedded adaptation	●		●
		Attached adaptation	●	●	●
	How to build	Incremental design approach	●	●	●
		Morphing design approach			
Adaptation tools	Rule format	Euclidean transformation			
		Parametric variation	●	●	●
		Specification variation			
		Add a new shape			
		Replace an existing shape		●	●
		Merge existing shapes		●	●
		Subtract an existing shape		●	●
		Change or add spatial labels	●	●	●
	Rule order	Change the free sequence of rules selection	●		●
		Change or add state labels	●	●	●
	Grammar structure	Change an initial shape			
		Add new rules			
		Replace original rules		●	●
		Delete original rules		●	
Adaptation outputs		New grammar for new designs only	●	●	●
		New grammar for new and existing designs			
		New grammar for new and known designs			
		Original grammar for new and existing designs	●		

The adaptation tools of hybridisation using original rules works on rule format and rule orders. The former changes the rules' spatial labels (markers) if they exist, or adds them to the grammar to ensure the derivation of valid designs. The latter changes the free sequence of rules or adds state labels to ensure the mixture of rules belonging to a variety of designs in the corpus. Other adaptation tools are used to generate hybrid rules by combining original rules. The format of original rules is adapted using tools such as the parametric replacement, replacing an existing shape, merging existing shapes, subtracting an existing shape and changing or adding spatial labels. The rule order in hybrid designs composed of hybrid rules only is adapted by changing or adding state labels, and the grammar structure is adapted by replacing the original rules with hybrid rules and deleting the original rules from the grammar. Lastly, the hybrid adaptation tools of rule format and rule order in hybrid design composed of both original and hybrid rules gather tools of both hybrid adaptation using original rules only and hybrid rules only, while the adaptation tool of grammar structure is the replacement of some original rules with hybrid rules.

Tow outputs of hybrid adaptation in shape grammar can be identified. The first output of all three types of hybrid adaptation results from the attached adaptation strategy. It is a new grammar for new hybrid designs composed of either adapted original rules only, hybrid rules only, or both adapted original rules and hybrid rules. The adapted original rules result from changing or adding state labels and/or spatial labels. The second output of hybrid adaptation using original rules only results from the embedded adaptation strategy. It is an original grammar for both new hybrid designs and existing designs when the hybrid adaptation tool depends only on changing the free sequence of original rules.

The next section develops the method for assessing the innovation in hybrid component-based designs using shape grammars.

3.3. Evaluation method of innovation in hybrid designs

The research in shape grammars for innovative hybrid design aims to incorporate both the generating and assessing roles of evaluation. In the former, a grammar user, influenced by feedback signals from grammar rules, is able to choose the rules that achieve the best innovation measures. In the latter, the degree of innovation in the generated design is calculated for each applied rule to form feedback loop for a grammar user at each stage.

To achieve the roles above, shape grammars for hybrid design are associated with evaluation descriptions that follow compositions and drive the design derivation. The proposed system is parallel grammars that compute with text and numbers. The assessment uses calculation as "a method that is applicable whenever prediction is based on some concise list of simple components, each of which is unambiguous and certain, and when the dependencies between the components are well known" (Kalay, 2004, p. 318). It works on two levels: the rule evaluation and the generated design evaluation. By linking the evaluation criteria to each rule in the grammar, two types of feedback are provided to the user. They are:

i. Rule evaluation values give feedback to a grammar user before choosing the rule. They are default values added to each rule during the writing of the grammar rules.

ii. Generated design evaluation values give feedback to a grammar user after applying the rule. Their values are computed and triggered automatically at shape grammar runtime depending on the previous choices of shape rules in the derived design.

The feedback obtained from the values of evaluation of rules helps the grammar user to control the generated design evaluation values which in turn are feedback that may indicate preferences for the next rule choices. The evaluation criteria in both levels use metrics of the degree of innovation in the hybrid component-based design.

The innovation metrics of the rules and generated designs take into consideration the two main characteristics of the innovative hybrid design which are inferred from the preceding sections, as follows:

i. **Being a mixture**: the hybrid design combines and blends features from the antecedents in the corpus. This character can be quantified by measuring the variety and density of hybrid design antecedents used.

ii. **Having individuality**: the hybrid design shows differences from the existing designs in the corpus. This character can be quantified by measuring the degree of matching to and difference from the antecedents in the corpus.

The next paragraphs clarify how the characteristics of innovation in the hybrid designs can be measured using rules and grammar.

3.3.1. Innovation measurements via shape rules

It is proposed that a hypothetical definition may be made of the innovative hybrid design as having high value of variety and density of antecedents in its mixture, and high value of differentiation from all antecedents in the corpus. The degree of innovation in the generated hybrid design can be predicted using the grammar rules which give feedback signals on the mixed character and individuality of each rule. These characters are the criteria for rule selection to be quantified in rule assessment method, as follows:

- Rule prevalence value (RPV) is the measure of the mixed character of the grammar rule.
- Rule geometrical difference value (RGDV) is the measure of the individuality of the grammar rule in terms of the difference in the rule format of the current rule from the other rules in the same sub-class rules set.
- Rule sequential difference value (RSDV) is the measure of the individuality of the grammar rule in terms of the difference in the rule order of the current rule from the other rules in the same sub-class rules set.

The values of these criteria are assigned by default to each rule in the subclass rule set.

3.3.1.1. Rule prevalence value (RPV)

The rule prevalence value (RPV) is a measure of the frequency of the rule in the corpus of antecedents. It is an indicator to the degree of mixture as the innovation character of the generated hybrid component-based design. Using rules with high rule prevalence values to derive hybrid design enhances the mixed character of the generated design. In this case, a large number of antecedents participate in the derivation of a hybrid design. It is calculated as a ratio of the number of existing designs that the current rule is derived from to the total number of existing designs in the corpus (n), as follows:

Rule prevalence = the number of existing designs that the current rule is derived from / the total number of existing designs in the corpus

The rule prevalence value will vary between 1/n and 1. The maximum rule prevalence value 1 means that the rule is derived from all existing designs in the corpus. This case is rare in shape grammars for hybrid designs because the heterogeneity of existing designs means only a small percentage of (or even none) rules are shared between all antecedents. The minimum rule prevalence value occurs when the rule is derived from only one existing design in the corpus.

3.3.1.2. Rule geometrical difference value (RGDV)

In shape grammar rules, the individuality of the generated hybrid design can be measured in terms of its rules format. Rules in the same sub-class rule set can have the same format but different orders which make them separate rules. In these cases, rule prevalence value cannot be an indicator of the difference in the rule format. Accordingly, there is a need to express the individuality in the rule format of the generated hybrid design via the difference in the rule geometry.

The rule geometrical difference value (RGDV) measures the degree of dissimilarities in shapes and/or their spatial relations of rules in the same sub-class rule set among the antecedents in the corpus. It is the ratio of existing designs in the corpus having different geometries from the current rule to the total number of existing designs in the corpus. It is calculated as the result of one minus the ratio of existing designs having rules in the same sub-class rule set with a similar geometry to the current rule to the total number of existing designs in the corpus, as follows:

Rule geometrical difference = 1 – (the number of existing designs in the same sub-class rule set having similar geometry / the total number of existing designs in the corpus)

High rule geometrical difference value enhances the individuality in the rule format of the generated hybrid design. It results from a rule having the least number of designs with a similar rule format in the same sub-class rule set. Maximum rule geometrical difference value results from an original rule which is derived from one existing design and having none similar rule format in the same sub-class rule set, as follows:

Maximum rule geometrical difference = 1 – (1/ the total number of existing designs in the corpus)

Hybrid rules derived from merging the formats of two original rules have new geometries which boost the individuality of hybrid design. They have a maximum rule geometrical difference value 1 because no existing design in the corpus has a similar geometry to the new geometry of a hybrid rule, as follows:

Hybrid rule geometrical difference = 1 – (0/ the total number of existing designs in the corpus) = 1

In contrast, minimum rule geometrical difference is rare. Its value is 0 which results from original rule in the sub-class rule set belonging to all antecedents in the corpus or having similar rule format to other rules in the same sub-class set which belong to all antecedents in the corpus.

Minimum rule geometrical difference = 1 – (the total number of existing designs in the corpus / the total number of existing designs in the corpus) = 0

3.3.1.3. Rule sequential difference value (RSDV)

The individuality in hybrid component-based design can also be measured in terms of rule order. The rules inherit from their precedents the sequences of the next sub-class rule set from which the next rule can be chosen. Different rules in the same sub-class rule set can have the same rule order, defined in terms of markers, but different formats. They direct the user to choose the next rule from the same sub-class rule set. Different rule orders can be assessed using rule sequential difference value to reflect the dissimilarities in rule order in the same sub-class rule set among the existing designs in the corpus. Rule sequential difference measures the ratio of existing designs in corpus that a rule has a different sequence from them to the total number of existing designs in corpus. It is calculated as the result of one minus the ratio of designs having rules in the same sub-class rule set with a similar sequence to the current rule to the total number of existing designs in the corpus, as follows:

Rule sequential difference = 1 – (the number of existing designs in the same subclass rule set having a similar sequence / the total number of existing designs in corpus)

High rule sequential difference value enhances the individuality of the generated hybrid design. It results from a rule having the least number of designs with similar rule order in the same sub-class rule set. Maximum rule sequential difference value results from a rule derived from one design and having none rules with similar sequence in the same sub-class rule set.

Maximum rule sequential difference = 1 – (1/ the total number of existing designs in corpus)

In contrast, minimum rule sequential difference value is 0 which results from original rule belong to all the antecedents in the corpus or having similar rule order to other rules in the same sub-class rule set which belong to all antecedents in the corpus.

Minimum rule sequential difference = 1 – (the total number of existing designs in the corpus / the total number of existing designs in the corpus) = 0

Based on the above, the rule assessments criteria are feedback signals that direct the grammar user to generate innovative hybrid designs with high mixture and individuality in both rule format and rule order.

3.3.2. Innovation measurements via grammar application

The innovative characters of hybrid component-based design such as being a mixture and having individuality can be quantified in the generated and final designs via metrics such as diversity, abundance, matching degree, design geometrical difference and design sequential difference. Diversity is the measure of the variety in hybrid design mixture, while abundance measures the density in the mixture of hybrid design. The other metrics such as matching degree, design geometrical difference and design sequential difference are all measures of individuality in the generated hybrid design. Matching degree is an inverse measure of individuality. On the other hand, the design geometrical difference and sequential difference are direct measures of individuality in the generated design. In the case of the former, it is the measure of individuality in the rule formats of the generated design, and in the latter, it is the measure of individuality in the rule orders of the generated design.

The innovation criteria are based on comparisons between the generated designs and the antecedents in the corpus, as follows:

3.3.2.1. Diversity value

Diversity measures the variety of antecedents in the mixture of hybrid design. It is calculated as the ratio of the number of existing designs participating in the generated design to the total number of existing designs in the corpus, as follows:

Design diversity = the number of existing designs that the generated design is derived from / the total number of existing designs in the corpus

Maximum diversity is 1 which results from applying rules belonging to all antecedents in the corpus. On the other hand, minimum diversity is the case when all applied rules in the generated design are derived from only one existing design.

Minimum design diversity = 1 / the total number of existing designs in the corpus

3.3.2.2. Abundance value

Abundance measures the density of the presence of antecedents in the mixture of generated hybrid design. Its value takes into account the repetition of antecedent in more than one rule, in contrast to diversity value which the repetition of antecedent does not consider. Its value represents the average number of antecedents in each rule of the generated design. It is calculated as the ratio of the sum of existing designs in the applied rules to the number of applied rules in the generated design, as follows:

Design abundance = the sum of the numbers of existing designs in the applied rules / the number of applied rules

Maximum abundance value is equal to the total number of existing designs in the corpus. It is the case when each of the applied rules is derived from all existing designs in the corpus.

Maximum design abundance = the total number of existing designs in the corpus

Minimum abundance value is 1. It is the case when each of the applied rules is derived from only one existing design in the corpus.

3.3.2.3. Matching degree value

Matching degree is an inverse measure of the individuality in the hybrid design. It measures the highest ratio of similarity between one antecedent in the corpus and the generated design. It is calculated as the ratio of the highest number of rules derived from one existing design to the number of applied rules, as follows:

Matching degree = the highest number of rules derived from one existing design / the number of applied rules

In the case of designs derived from hybrid rules only, or a combination of hybrid and original rules, each hybrid rule is multiplied by (0.5) for matching degree calculation. The reason is the new format of hybrid rule results from merging two formats of the original rules. Therefore, the hybrid rule has less resemblance to existing designs than the original rules. The decrease of matching using hybrid rules enhances the individuality of hybrid designs derived by hybrid rules.

Maximum matching degree is 1, which results from all applied rules belonging to one existing design in the corpus. This case leads to minimum individuality in the generated hybrid design. On the other hand, minimum matching degree and maximum individuality results from applied rules belonging to varied existing designs in the corpus in which each existing design has one or none applied rule.

3.3.2.4. Design geometrical difference value

Geometrical difference of the generated design measures the individuality in the formats of applied rules. It is calculated as the average of rule geometrical difference values of applied rules in the generated design, as follows:

Design geometrical difference = the sum of rule geometrical difference values of the applied rules / the number of applied rules

Maximum geometrical difference in the generated design leads to maximum individuality. It is resulted from applying rules with maximum rule geometrical difference values which are derived from one existing design and have no similar rule format in the same sub-class rule set. On the contrary, minimum geometrical difference in the generated design is 0 which results from applying rules with minimum rule geometrical difference values 0. Each rule, in this case, is either derived from all the antecedents in the corpus or having similar rule formats to all or some rules in its sub-class rules set which in total are derived from all existing designs in the corpus.

3.3.2.5. Design sequential difference value

Sequential difference measures the individuality in the rule orders of hybrid design. It is calculated as the average of rule sequential difference values of the applied rules in the generated design, as follows:

Design sequential difference = the sum of rule sequential difference values of the applied rules / the number of applied rules

Maximum sequential difference value of the generated design is the result of using rules with maximum sequential difference values. These rules are derived from one design in the corpus and have no similar rule order in the same sub-class rule set. On the other hand minimum sequential difference value is 0 which results from applying rules with minimum rule sequential difference values 0. Each rule, in this case, is either derived from all the antecedents in the corpus or having similar rule orders to all or some rules in its sub-class rules set which in total are derived from all existing designs in the corpus.

3.3.3. Innovation assessment of hybrid design - Conclusion

These metrics provide a measure of the innovation of hybrid component-based designs using shape grammars. The main characteristics of innovative hybrid design are defined as a mixture of components of antecedents and having the individuality that distinguishes it from these antecedents.

The grammar user has both feedback signals and feedback loop to derive a hybrid design with high values of innovation. Feedback signals are default values associated to all grammar rules to measure the mixed character of rule using rule prevalence value (RPV), and the individuality in both the rule format and rule order using rule geometrical difference value (RGDV) and rule sequential difference value (RSDV) respectively.

The feedback loop, on the other hand, is the innovation measures of a hybrid design which are updated values added to the generated design after each applied rule. In every feedback loop, the values of innovation measures resulted from applied rule became the input data with the feedback signals of the next applicable rules. The innovation measures are the diversity value which measures the variety of hybrid design mixture, the abundance value which measures the density of hybrid design mixture, the matching degree which is an inverse measure of the individuality of hybrid design, the geometrical difference value which measures the individuality in the rule formats of the generated hybrid design, and lastly the sequential difference value which measures the individuality in the rule orders of the generated hybrid design.

3.4. Implementation – objectives and requirements

The study needs to implement shape grammars for hybrid component-based designs to achieve three main objectives:

- Firstly, to verify that the proposed method is able to generate hybrid designs from the corpus of heterogeneous antecedents.

- Secondly, to verify the validity of the proposed measures of innovation in hybrid designs.
- Thirdly, to identify the indicators that the grammar user can take into consideration to derive hybrid design with high innovation values.

The study aims to implement shape grammars for hybrid component-based design on corpus of simple architectural configuration such as traditional minaret designs. The reasons for choosing minaret as antecedents can be attributed to the following matters:

- Minaret is component-based design composed of easily recognised elements with superimposed relations that make it a clear and tangible example of hybridisation.
- Minaret can be analysed apart from other mosque components.
- The function in minaret is marginal. Therefore, the grammar can focus exclusively on the formal characters of minaret components.
- Minaret is a simple formal composition of several components which can be analysed in a limited number of original rules.
- There are varied designs of minaret which have heterogeneous configurations.
- According to historians, some of the historical minarets were already generated using hybridisation.

3.5. Chapter summary

This chapter has defined shape grammar for hybrid designs as a bottom-up [13] component-based modelling approach. The procedures to derive hybrid design from a corpus of antecedents using shape grammars have been developed within the analysis and synthesis phases. Additionally, measurements of innovation in hybrid design are proposed as indicators to give feedback to grammar users in an evaluation.

[13] In bottom-up approach, designs are built from known components in anticipation of satisfying functional requirements.

In the Analysis phase, to facilitate the mixed character of hybrid component-based designs, grammar rules are defined in sub-class rule sets to represent the different configuration of the same component among heterogeneous designs in the corpus. Each sub-class rule set has multi-choice rules with the same left hand side (LHS) to enable the grammar user to select from. In addition, the multi-choice rules can be expanded by adding new hybrid rules to original grammar rules to enhance the individuality character in the generated hybrid design.

The synthesis phase, on the other hand, ensures the derivation of feasible and novel hybrid designs by controlling the rule selection and rule application using state labels and makers. The former is used to constrain the rule selection to generate only hybrid designs via the user guide grammar. While the latter seeks to generate feasible designs by directing the sequence of rules from sub-class rule set to other, and controlling which rule is applicable and where it can be applied.

The evaluation of the innovation degree in hybrid designs is proposed to give feedback to grammar user on two levels: feedback signals and feedback loop. The former are default values assigned to rules as indicators of both the mixed character and the individuality in rules. They are the rule prevalence value (RPV), the rule geometrical difference value (RGDV), and the rule sequential difference value (RSDV). The latter, on the other hand, is the innovation metrics of the generated hybrid designs which are triggered immediately after each stage of rule application. They measure the variety and density of hybrid design mixture using the values of diversity and abundance respectively. In addition, they measure the individuality of hybrid design using the matching degree, the individuality of rule formats in the generated hybrid design using the design geometrical difference value, and lastly the individuality of rule orders in the generated hybrid design using the design sequential difference value.

4. PREPARING FOR THE IMPLEMENTATION

Chapter three defined a framework of shape grammars for hybrid component-based designs within two phases of design: the analysis phase and the synthesis phase. In addition, an assessment method is associated with the derivation of the hybrid design to measure the degree of innovation in the generated hybrid design.

This chapter presents the initial stages of implementation in three sections. The first section, 4.1, reviews the morphology of minaret designs in Islamic architecture and defines the main components and characteristics of minaret design. The second section, 4.2, identifies the study sample and gives a brief description of each. Lastly, the specifications of the analysis and synthesis phases to satisfy shape grammars for hybrid minaret designs are presented in section 4.3. A brief summary of the chapter is included in section 4.4.

4.1. Introduction to minaret design in Islamic architecture

The reason for choosing minarets as the subject for investigating shape grammar techniques for deriving hybrid designs can be attributed to its being a simple formal component-based type. Minarets "seem unrelated to its function of *adhan* (calling the faithful to prayer)" (Hillenbrand, 1994, p. 129). The functional and behavioural attributes of minaret can be neutralised, therefore the minaret can be studied apart from the mosque's other components. The role of the grammar user can be interactive to control the formal and structural attributes in a critic mode. The user can also explore a variety of syntactical representations of minarets in a way that leads to hybrid results.

This section reviews minaret design in Islamic architecture. It looks at the different styles of this building type which varies widely according to the region and time period. The syntactical anatomy of the minaret components and their relations is explained: this reveals the aspects of similarities and differences between varied minaret designs. In addition, it concludes the design constraints and allowances that the new hybrid minaret designs can take into account or ignore.

4.1.1. Minarets: terminological definition in Islamic culture

The minaret is the most prominent architectural feature of mosques. Minaret is a word ultimately derived from the Arabic word meaning "sign" or "mark". The idea of a minaret first arose during the Umayyad dynasty in Syria (J. M. Bloom, 1991, p. 55). It is the principal vertical feature of most mosques which “provides a local landmark as well as allowing the voice of the *muezzin*[14] to carry over a considerable distance when calling the faithful to prayer” (Frishman, 1994, p. 24). However, other studies consider that the call to prayer can be adequately accomplished from the roof of the mosque (Hillenbrand, 1994, p. 129). Grabar argues that “minarets did not appear systematically until the twelfth century and that their function was not initially restricted to a purely liturgical purpose connected with prayer, for in some instances the minaret may also have served as local landmarks or as lighthouses” (Garbar, 1994, p. 243). Bloom also refers to the use of minarets in some Islamic period to mark the pilgrimage road (1989, p. 159).

Styles of various minaret designs can be distinguished in different regions and periods of Islamic history which reflect heterogeneous compositions. In addition, according to historians, there are already hybrid models which combine characteristics of minarets of different regions and times. These two reasons, along with the fact that minaret has simple syntactical configuration in which the function is ineffective, have led to the choice of minarets for implementing shape grammars for hybrid component-based designs.

4.1.2. The historical development of minarets

The minaret, as one of the mosque components, is either free standing or attached to the mosque roof. The review of minarets in this section focuses on the minaret itself apart from its position in the context of the mosque, and its relations with the other mosque components.

[14] Muezzin derived from Arabic which means the man appointed to call to prayer.

Architectural styles of the minaret down the centuries have been widely different in various regions (Frishman, 1994, p. 24). Minarets take many shapes – square, cylindrical, polygonal, spiral, or a combination of several – with each region developing its specific formal type (Scerrato, 1976, p. 6). In Egypt in the 7th century and Syria until the 13th century, the minarets are square towers, sitting at the four corners of the mosque as shown in Figure 4.1[15].

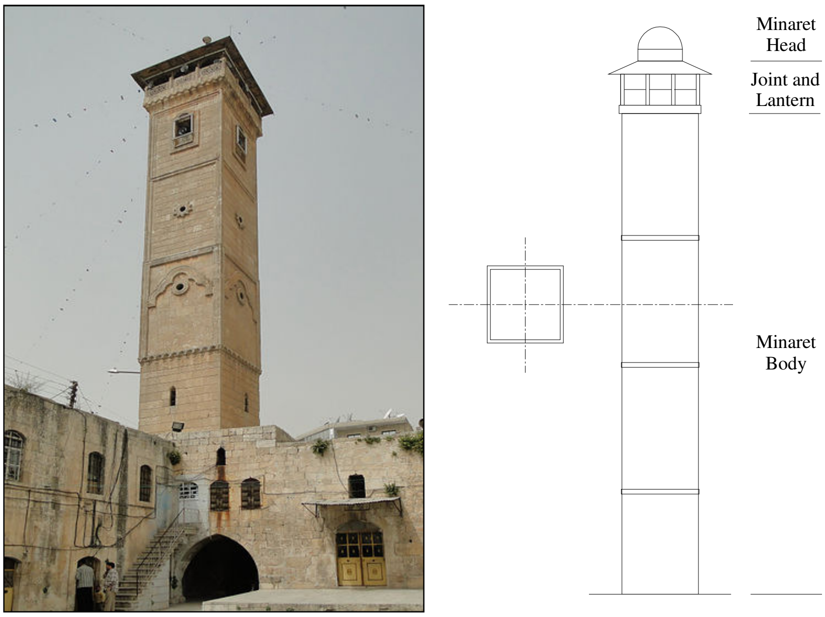


Figure 4.1: Minaret of Great Mosque of Ma'arrat al-Numan in Syria in 1099 AD

After that, Egyptian minarets are developed to be squares at the base and circular in the upper registers, "the shafts were interrupted at intervals by very rich ornamented balconies over *muqarnas*[16] cornices. Surface patterns consisted of niches, arches and decorative panels". In general, the decorative and multilayered appearance of the minarets in Egypt was maintained (Figure 4.2)[17], demonstrating in an obvious manner their originality (Kuban, 1994, p. 97).

[15] Figure 4.1 left: http://en.wikipedia.org/wiki/File:Great_Mosque_of_Ma%27arrat_al-Numan_03.jpg, Figure 4.1 right: analytical drawing has been done by the author.
[16] *Muqarnas*: "honeycomb or stalactite vaulting made up of individual cells or small niches; often used as a bridging element". (Hillenbrand, 1994, p. 599)
[17] Figure 4.2 left: http://lexicorient.com/e.o/ill/minaret02.jpg, Figure 4.2 right: analytical drawing has been done by the author.

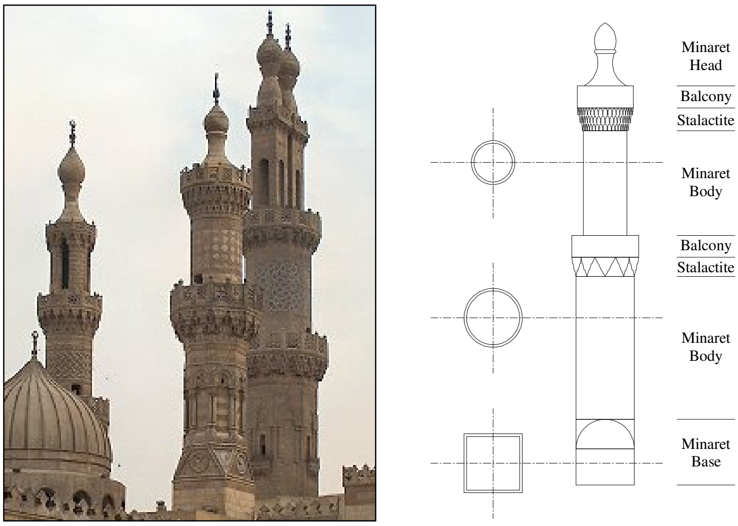


Figure 4.2: Al-Azhar minarets in Egypt (10th century AD)

Other examples of Egypt minarets are the two minarets of Al-Hakim's mosque. That at the north corner is a tall cylindrical shaft on a nearly cubical base; the other at the west, is a tall square shaft, "surmounted by a series of receding octagonal courses" (Creswell, 1926b, p. 257). The former has the typical Egyptian style which has multiple divisions of parts. The lowest part is a cylinder resting on cube. Then there is "an octagon shaft with blind arch and windows on each side, which give way to a heavy band of a *muqarnas* decoration in three distinct tiers and a fluted keel-shaped dome crowns the whole" (Figure 4.3)[18]. In the western minaret, there is "a reduction in the size of the octagonal *muqarnas* zone and the square lower shaft is pierced by a double tier of arched windows" (Hillenbrand, 1994, p. 166).

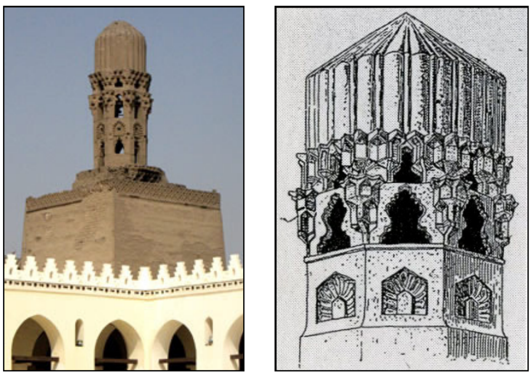


Figure 4.3: Al-Hakim's northern minaret in Egypt (990-1003 AD)

[18] Figure 4.3 left: http://www.oldroads.org/images/hakim13.jpg, Figure 4.3 left right: (Hillenbrand, 1994, p. 506).

In the Fatimid period (909–1171 AD) in Egypt, minarets share common features with different proportions. They display the characteristic division of the minaret into three separately conceived superposed tiers. They have square bases often of stone; tapering, cylindrical brick shafts, and domed lanterns, often of three stories (Figure 4.4)[19] (J. M. Bloom, 1984, p. 163). The Isna minaret (Figure 4.4 left) has "a square base some thirty five feet high, generously articulated by windows, rises a plain tapering truncated cylinder capped by an open pavilion whose eight concave sides bear a diminutive hexagonal domed aedicule" (Hillenbrand, 1994, p. 165).

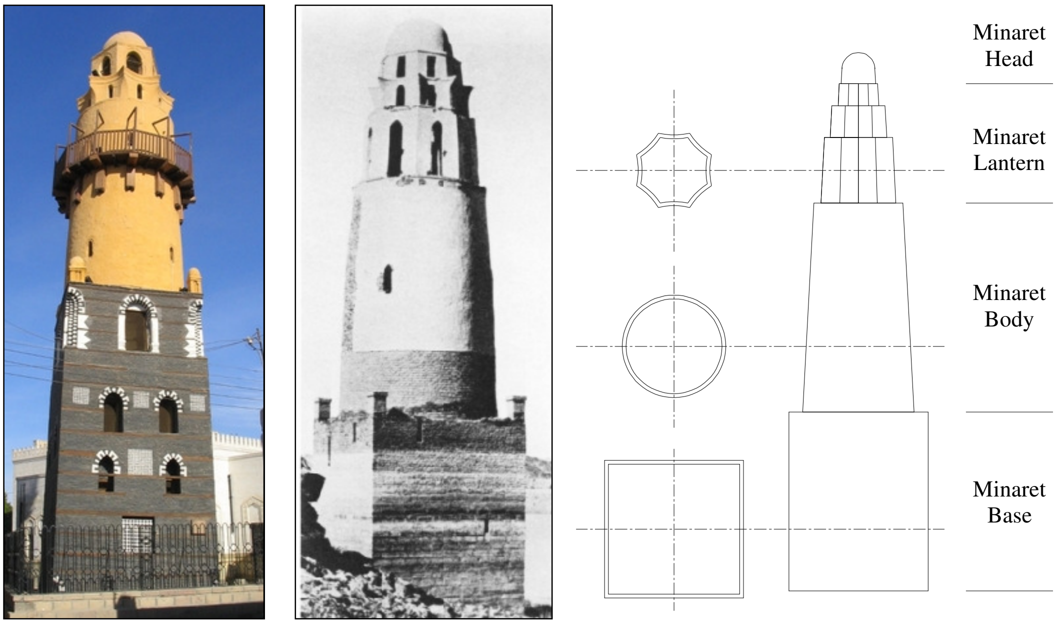


Figure 4.4: Minarets of Isna and al-Mashhad al-Qibli in Egypt (late 11th century)

In Maghrib and Spain, the ninth-century minarets typically have tall square-shafted minarets (Figure 4.5)[20]. The minarets there consist of multi-tiered elevations which tend to be rather smaller in size than the previous one. Three sides have blind windows, and the fourth has real windows to admit light to the staircase. (Frenandez-Puertas, 1994, p. 102)

[19] Figure 4.4 left: Isna minaret from http://www.panoramio.com/photo/5792962, Figure 4.4 middle: Al-Mashhad al-Qibli minaret (J. M. Bloom, 1984, p. 163), Figure 4.4 right: analytical drawing has been done by the author.
[20] http://www.sacred-destinations.com/spain/cordoba-mezquita-pictures/slides/d80_2_375.htm

Figure 4.5: Minaret of Mezquita Cordoba (9th century)

In the Seljuk era (form 11th century), the minarets in Iran had seen the emergence of a tall, smooth cylinder broken only by small balconies. Iranian minarets in the 12th century were capable of generating surprising variety of forms. They include low plinths that are flanged, lobed or combination of both (Figure 4.6)[21]; others are "octagonal with elaborate blind arcading, or that are square in ground-plan but pylon-like in elevation". In some cases, a circular shaft rests on an intermediate octagon which is carried by a very plain square plinth. The plinth is quite plain in contrast with the richly textured upper elevation. Sometimes, the plinth extends to such a height that it rivals the cylindrical shaft in importance" as in the Kerat minaret (Figure 4.7). (Hillenbrand, 1994, p. 153)

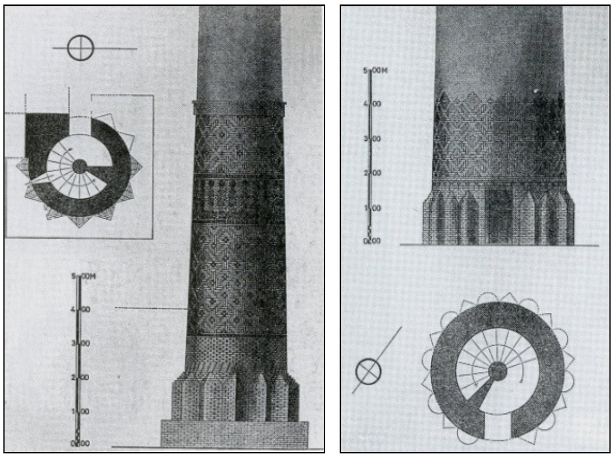


Figure 4.6: Bases of Iranian minarets

[21] Figure 4.6: from (Hillenbrand, 1994, pp. 503, 504)

According to Creswell, Persian minarets, down to the early part of the thirteenth century, may be divided into three groups. Firstly, the two minarets at Ghazna (Figure 4.8) [22] had no descendants. Secondly, the tapering cylindrical shaft is the most prevalent type such as the minaret at Semnan built in (1170-1171 AD) which is without base (Figure 4.9)[23]. Thirdly, the octagon-cylinder type is the only one that apparently did not stand free, such as Kerat minaret which is an octagonal-cylindrical type with octagonal shaft of I5 meters in height, surmounted by a cylindrical upper storey of 9 meters (Figure 4.7)[24]. (1926c, p. 292)

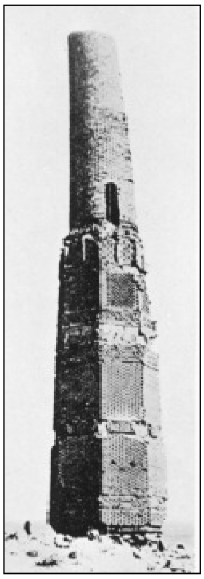


Figure 4.7: Kerat minaret

Figure 4.8: Ghazna minaret

Figure 4.9: Semnan minaret

In Mesopotamia, minarets first appeared in the twelfth century and the cylindrical shaft was popular in the early examples. There are two types of form chosen for the lower part: a square and an octagon, as follows:

- The first type is a tall, cylindrical shaft on cubic base. One of the examples is the Great Mosque minaret in Mosul (Figure 4.10)[25]. Its cylindrical shaft rests on a lower storey of 8.80 meters square and 15 meters high, with total height of 45 meters.

[22] Figure 4.8: from http://static.panoramio.com/photos/original/6271155.jpg
[23] http://www.risaalaat.com/pictures/sitepic/city/Semnan/Damqan%20Jame%20Mosque%20Minaret,%20Damqan.JPG
[24] (Creswell, 1926b, p. 259)
[25] http://static.panoramio.com/photos/original/9349869.jpg

- The second type is cylindrical tower on octagonal lower storey (Figure 4.11[26] and 4.12[27]). Both Erbil and Ta'uq minarets have octagonal lower storey with arched panels, surmounted by a tall, cylindrical shaft. (Creswell, 1926c, pp. 293, 295)

Figure 4.10: Great Mosque minaret in Mosul

Figure 4.11: Great Mosque minaret in Erbil

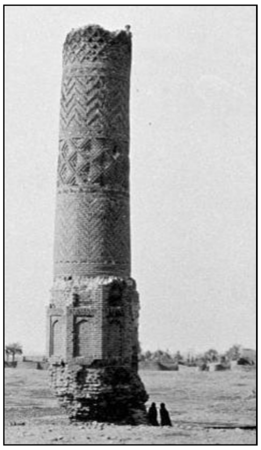


Figure 4.12: Ta'uq minaret

In Central Asia, architects experimented with another variation, characterised by more massive proportions and a markedly tapered shaft, as in the Kalyan minaret and Vabkent minaret in Bukhara built in 1196 AD (Figure 4.13)[28]. (Hillenbrand, 1994, p. 148; Scerrato, 1976, p. 74)

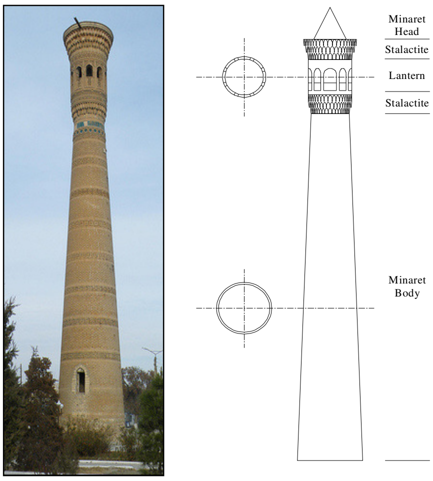


Figure 4.13: Vabkent minaret in Bukhara

[26] http://portal.unesco.org/culture/en/files/14243/10644187603irakmb_II_18_minaret_Erbil.jpg/irakmb_II_18_minaret_Erbil.jpg
[27] http://www.gerty.ncl.ac.uk/photos_in_album.php?album_id=17&start=180
[28] Figure 4.13 left: http://www.dwoodworks.com/travel/CA/page48/files/page48_2.jpg, Figure 4.13 right: analytical drawing has been done by the author.

The minaret of Jam (1191-1198 AD) (Figure 4.14)[29] in Afghanistan is a sixty meters tower which "consists of an octagonal base supporting three superposed tapering shafts separated by *muqarnas* cornices crowned by a lantern" (J. Bloom, 1989, p. 173).

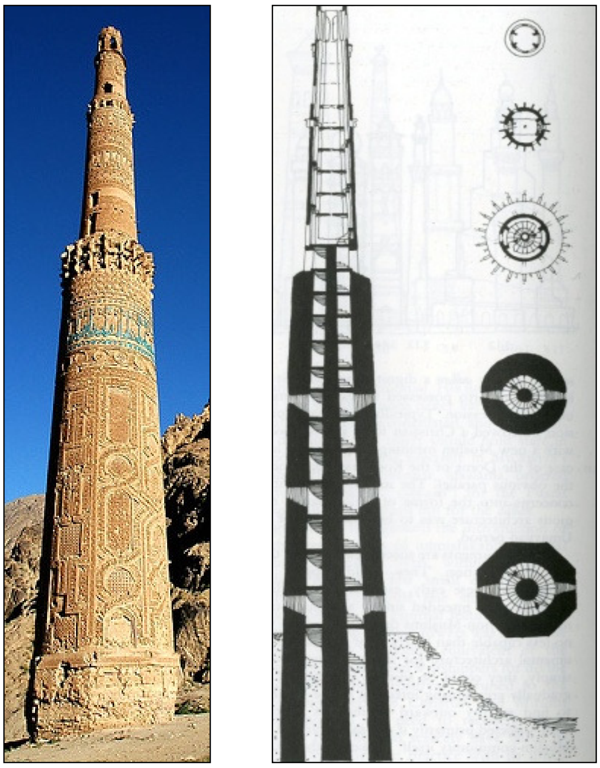


Figure 4.14: Jam minaret

In India and Anatolia, minarets commonly have their shafts enhanced by semi-cylindrical grooves (Figure 4.15)[30]. The details of Qutb minaret in Delhi (1191-1198 AD) (Figure 4.16)[31], acknowledge the influence of eastern Iranian minarets (Hillenbrand, 1994, p. 158). It "consists of five superimposed tapering shafts separated by balconies resting on *muqarnas* corbels. The whole stands on a twenty-four-sided polygonal base. The lowest story has twenty-four flanges, alternatively semi-circular and angular; the second story has only semi-circular flutes, the third story only angular ones. The fourth story is plain and the fifth has semi-circular flutes" (J. Bloom, 1989, p. 172).

[29] Figure 4.14 right: (Hillenbrand, 1994, p. 132); Figure 4.14 left: http://www.flickr.com/photos/7152100@N04/421954029/sizes/m/in/photostream/

[30] (Hillenbrand, 1994, p. 502)

[31] http://www.planetware.com/i/photo/old-qutub-minar-delhi-ind008.jpg

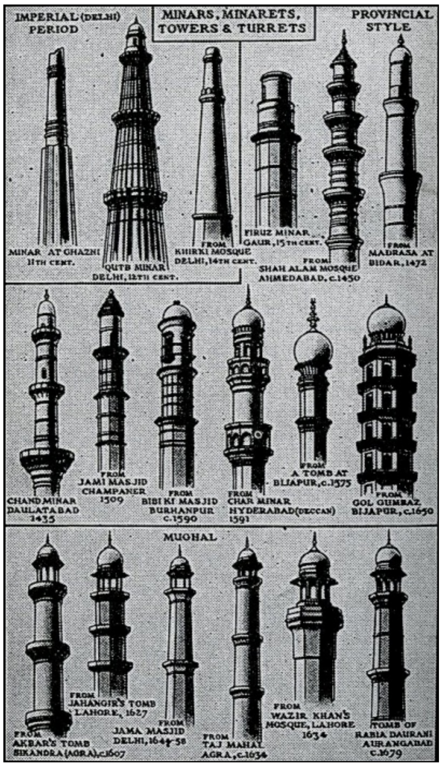


Figure 4.15: Indian type of minaret

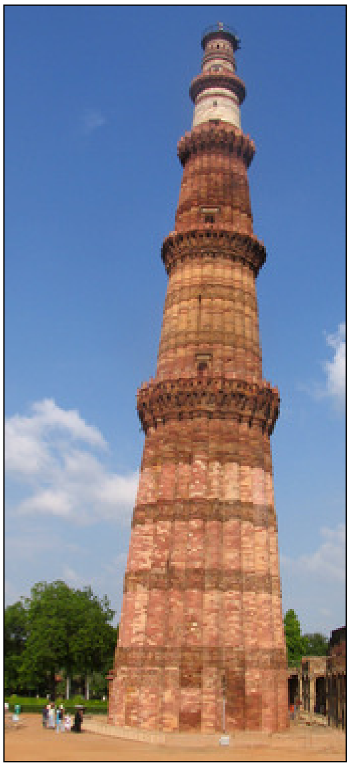


Figure 4.16: Qutb minaret in Delhi

The elevations of Bibi-ki Masjid minaret (Figure 4.17)[32], are the later minarets in India in 1590 AD at Burhanpur. They are "octagonal, hexadecagonal, cylindrical and domed, with balconies on brackets separating the various stages". (Hillenbrand, 1994, pp. 158, 159)

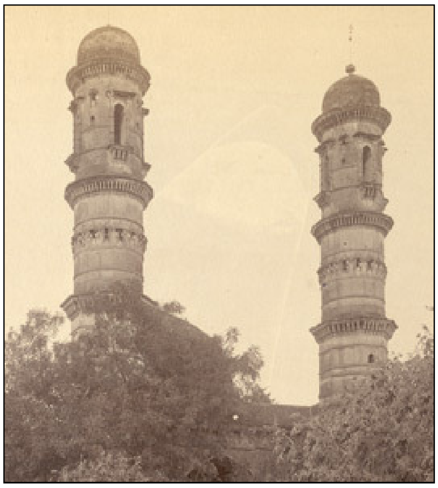
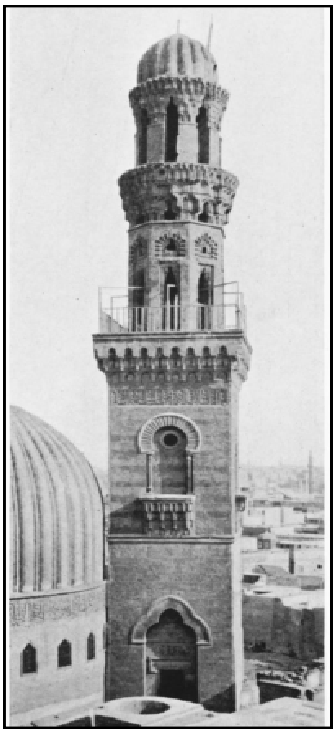


Figure 4.17: Bibi-ki Masjid minaret in Burhanpur (1590 AD)

[32] http://www.columbia.edu/itc/mealac/pritchett/00routesdata/1500_1599/akbar/burhanpur/cousens1892.jp

The shafts of the Mamluk minarets in Egypt have a characteristic appearance which is made up of a number of richly decorated segments. In the older types, a square base carries an octagonal shaft. The last segment is cylindrical in form covered by a ribbed cap and connected to the rest by a stalactite cornice. In the more recent types, a series of dimensioning octagons are used. The octagonal base section was surmounted by two upper sections, smaller in size to end with a tiny pavilion. The stalactite-connecting elements were incorporated into balconies (Scerrato, 1976, p. 89). The octagonal minarets, surmounted by small lanterns, became popular under the influence of these minarets that were relatively common in the eastern lands of Islam in the period before the Mongol conquest. (J. M. Bloom, 1991, p. 55)

The first example of the square-octagonal-circular type is the minaret of Sangar al-Gawly (1303-1304 AD) (Figure 4.18)[33], which is "a marked elongation of the two top storeys at the expense of the shaft; moreover, the lantern, instead of being octagonal, with a dome circular in plan above it, is itself circular and the octagon forms a separate storey" (Creswell, 1926b, p. 257).



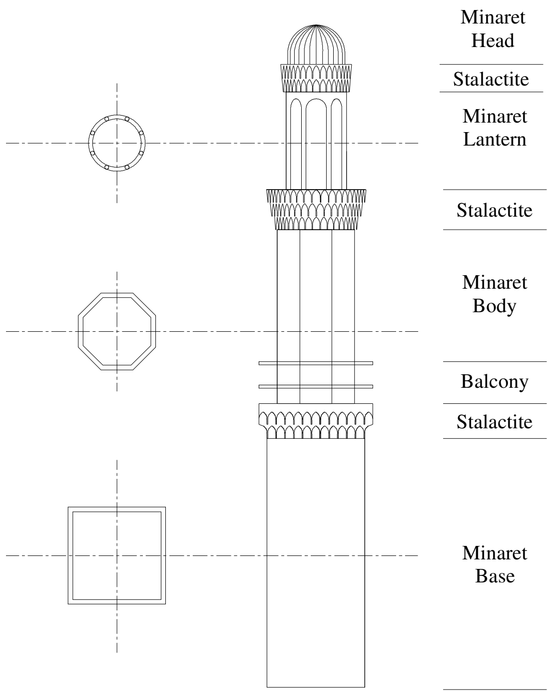


Figure 4.18: Sangar al-Gawly minaret

[33] Figure 4.18 left: (Creswell, 1926b, p. 259), Figure 4.18 right: analytical drawing has been done by the author.

Other minarets are regarded as octagonal minarets surmounted with a small lantern, which is no longer a mabkhara but a little dome supported on columns, such as al-Maridani (I340 AD) (Figure 4.19)[34]. In these minarets the square shaft shortens so much that only its bevelled-off top corners show above the mosque roof, and the visible part of the minaret commences with an octagonal shaft (Creswell, 1926b, p. 258).

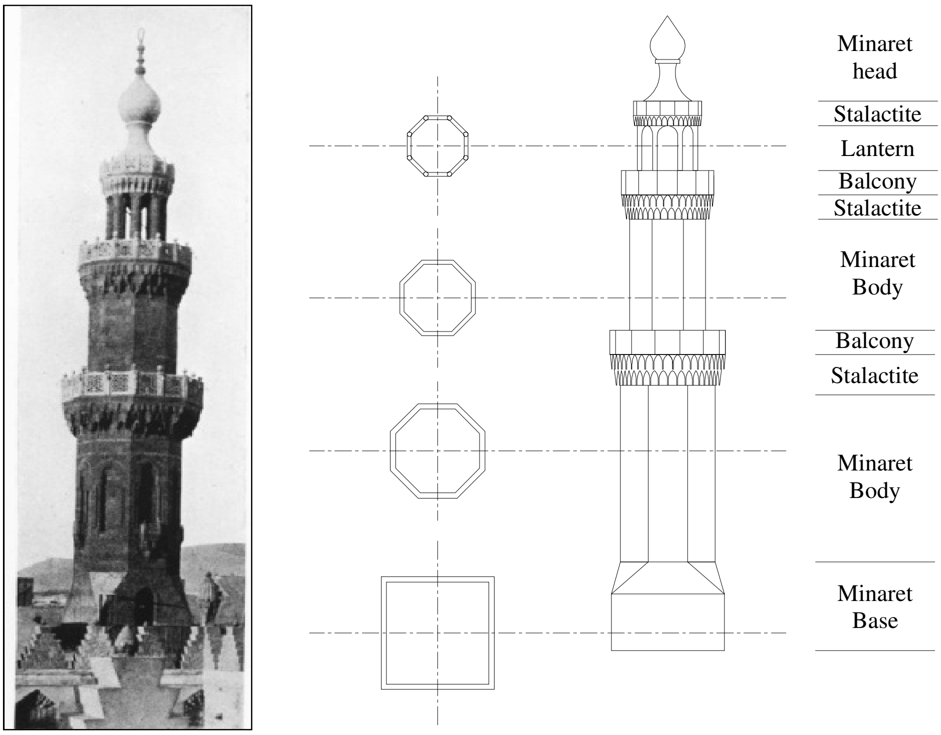


Figure 4.19: Al-Maridani minaret

In Ta'izz (south of Yemen) (Figure 4.20)[35], the minaret style is a free interpretation of Ayyubid[36] and Mamluk[37] styles: typical features are polygonal multi-layered minaret towers, richly articulated with niches and topped with small domes. (Kuban, 1994, p. 99)

[34] Figure 4.19 left:(Creswell, 1926b, p. 259), Figure 4.19 right: analytical drawing has been done by the author.
[35] Figure 4:20: http://girlsoloinarabia.typepad.com/photos/uncategorized/2007/03/28/alashrafiyya.jpg
[36] Ayyubid period (1169 -1250 AD)
[37] Mamluk period (1250 -1517 AD)

Figure 4.20: Al-ashrafiyya minaret in Ta'izz

Timurid and Safavid minarets, in the 15th century follow established precedents. "*Muqarnas* cornices often five or six-tired, are perhaps denser than before or developed bolder contrast of solid and void than their predecessors". Also, there are slight changes in the form of the balcony in this period, developing a distinctive overhanging canopy above the railing (Figure 4.21)[38]. In Safavid times, the form of a tapering shallow-domed cylinder is the standardised topmost storey of the minaret (Hillenbrand, 1994, pp. 157, 158).

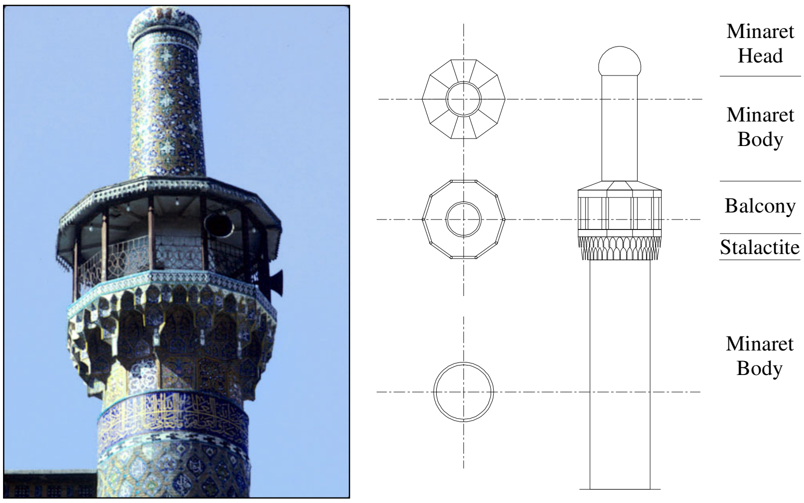


Figure 4.21: Minaret of mosque of Gauhar Shad in Mashhad

[38] Figure 4.21 right: analytical drawing has been done by the author, Figure 4.21 left: from http://archnet.org/library/images/one-image.jsp?location_id=5877&image_id=12001

In Turkey, among variant forms of pre-Ottoman minarets, there are "square bases with blind arcades or with chamfered upper corners, intermediate octagonal drums with blind arcades, and various types of gadrooning[39] applied to the main shaft". Yivili minaret at Antalya (early 13th century) is the best illustration of the latter feature (Figure 4.22)[40], where "a cannular flange divides the engaged columns from each other". The minaret of Hoca Hassan mosque in Konya is the most curious version of the theme, whose "square shaft has a semi-circular buttress at the center of each side, and similarly placed buttresses on the octagon above" (Figure 4.23)[41]. (Hillenbrand, 1994, pp. 163, 164)

The distinctive sign of Ottoman Islamic rule is the "sharpened pencil"-like minarets with conical lead caps (Figure 4.24)[42]. Minarets are related to the size of the mosque. They are slim, circular of equal cross-section.

Figure 4.22: Yivili minaret

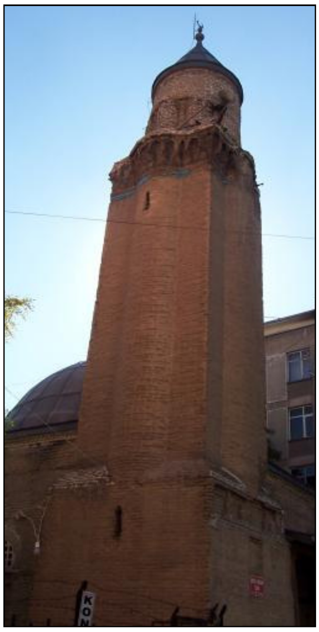


Figure 4.23: Hoca Hassan minaret

Figure 4.24: Blue Mosque minaret

[39] In architectural decoration, gadrooning is surfaces worked into a regular series of (vertical) concave grooves or convex ridges, frequently used on columns. http://www.britannica.com/EBchecked/topic/211529/fluting-and-reeding

[40] http://www.archnet.org/library/images/one-image-large.jsp?location_id=14341&image_id=139794

[41] http://www.fotothing.com/Zodyak/photo/0a700e537b977c977e7a25b44b39fb08/

[42] http://www.sacred-destinations.com/turkey/images/istanbul/blue-mosque/resized/blue-mosque-minaret-cc-Cybjorg.jpg

Ottoman minaret rose from a square or polygonal base. The plinth was commonly square in plan but in elevation its walls sloped sharply inward. The main cylindrical shaft is "punctuated by one, two or even three circular balconies carried on *muqarnas* vaulting. Elongated conical roofs, sheathed in lead and ending in finials, caped the shafts". Often the shaft is a polygon and not a cylinder, whereas the angles are so obtuse that have the visual effect of a cylinder. (Hillenbrand, 1994, p. 164)

4.1.3. Other models of minarets

At the same time, there are "non-standard" models of minarets such as Samarra (Figure 4.25) [43] and Ghazna (Figure 4.8). Samarra (847 AD) in Iraq, is an "extraordinary helicoid minaret" that had an impact upon Ibn Tulun spiral minaret (Figure 4.26)[44] in Egypt (Hoag, 1968, p. 18). In addition the Ghazna minarets (Figure 4.8) of Bahramash and Masud III in Afghanistan (1099-1115 AD) are tall and slender having the high lower unique storey in the form of star-shaped polygons.

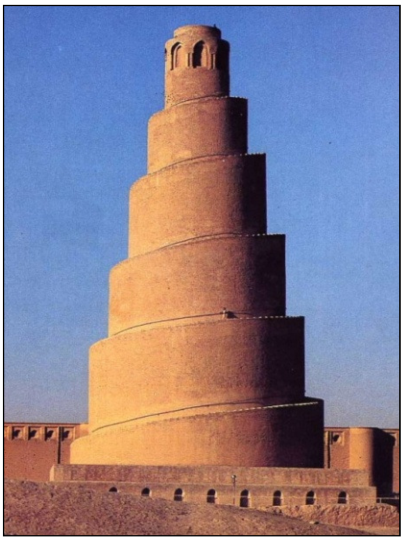


Figure 4.25: Samarra minaret

Figure 4.26: Ibn Tulon minaret

4.1.4. Historical hybridisation in mosque and minaret designs

The literature on Islamic architecture reveals the role of hybridization in generating some monuments of Islamic architecture including mosques. The fusion of elements from several styles can be seen which reflects "the Islamic ideal of a refuge formed of

[43] http://rubens.anu.edu.au/raider6/iraq_heritage/photos/samarra0.jpg
[44] http://www.lifeinthefastlane.ca/wp-content/uploads/2008/10/mosque_ibn_tulun_4sfw.jpg

endlessly repeated, hypothetically identical, shapes and spaces" (Hoag, 1968, pp. 19, 20). The Mamluk architecture was primarily derived from the Syrian tradition, but it showed a composite of other contributions that varied from the Iranian to the Anatolian and from the Maghrebian to Romanesque-Gothic (Scerrato, 1976, p. 88). Moreover, India, under the Muslim Mogul emperors, developed a mosque type in the seventeenth century that reflects a hybrid of the other major types derived from Turkish and Iranian mosques (Scerrato, 1976, p. 7).

The literature also suggests that there was an artistic influence in minaret designs which moved from west to east and vice versa. For example, the minaret of Ibn Tulun in Cairo (Figure 4.26), a work of the late ninth or early tenth century is a spiral, circular in plan, with a staircase outside, a type derived from Samarra (Figure 4.25). It has the following stylistic features in common with those of the Andalusian Emirates such as the proportion in which its height is twice its width; the horseshoe arches with or without frame; and having double blind arches divided by a central column on its facades. Also, the external staircase around a cylindrical shaft imitates the Abbasid manner. In addition, the mosque itself, al-Qatai built in (876-79), has Abbasid characteristics (Frenandez-Puertas, 1994, p. 103) (Creswell, 1926b, p. 257). Also, in Ottoman Tunis, there is a type of octagonal minaret, "each face richly tiled and the whole crowned by projecting balcony and steeped pavilion". It reflects "a blend of the local tradition with the slender pencil-shaped Turkish minaret, and manages to forfeit the distinctive qualities of both" (Figure 4.27)[45]. (Hillenbrand, 1994, p. 142)

Figure 4.27: Hammuda Pasha minaret in Tunis

[45] http://www.trekearth.com/gallery/photo147558.htm

4.1.5. Syntactical analysis of minaret designs

Minarets components can be classified in the following categories: main components, joints and secondary components as follows:

- **Main components** of minaret comprise five elements. They are the base (plinth or socle), the body (shaft), the balcony, the lantern[46] and finally the head (such as dome, cupola) as shown in Figure 4.28. Both lantern and head are called the *mabhkara* [47]. Some minarets are composed of a body and a head only; others have all the five components. In addition, in some minarets, there are multi bodies and balconies unlike the other components such as the base and head which should be singular.
- **Joints** are transitional elements between main components, especially body and balcony such as muqarnas as the stalactite-like components and cornices.
- **Secondary components** comprise niches and openings such as entrance and windows, arcades and ornamental patterns such as Arabic calligraphy, geometric tracery and floral band.

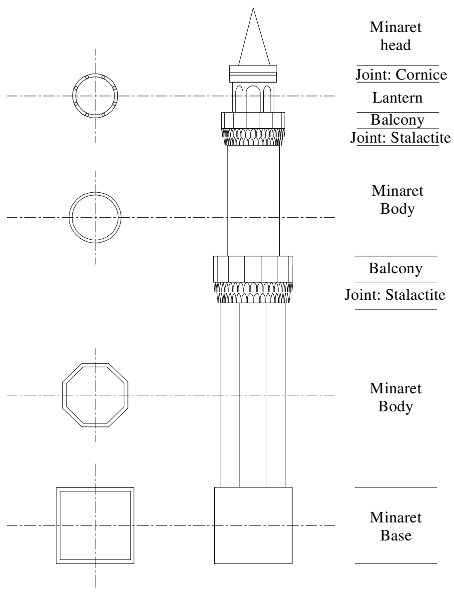


Figure 4.28: Main components of minaret (Source: author)

[46] Lantern is the connector between body and head. Its diameter is almost narrower than the body's diameter underneath it. It has openings such as arcades or windows which facilitate its function as a source of light.
[47] *Mabkhara* is "two separate storeys, whose formal and decorative independence from each other is underlined by the use of different ground-plans: an octagonal storey giving way to a circular one which bears the crowning dome and finial". (Hillenbrand, 1994, p. 170)

In this implementation, the syntactical analysis of the corpus of minarets focuses only on the main components of minaret: base, body, balcony, lantern and head, in addition to the joints between them. The secondary components are ignored to simplify the application by reducing the number of original rules.

4.1.5.1. Formal differences in minaret designs

The main differences in traditional minaret designs that characterise the heterogeneous corpus of minarets can be determined by:

- The number of components in the minaret. This differs from one minaret to the others.
- The sequence of main components from the base to the head. This differs from minaret to others, as in the following sequences from the bottom to the top:

  Base → body → head
  Base → body → lantern → head
  Base → body1 → body2 → body3 → head
  Base → body → balcony → lantern → head
  Base → body1 → balcony1 → body2 → balcony2 → lantern → head

- Different configurations of each main component which have varied sections such as standard geometrical shapes: square, circle, octagon, polygon of 10, 12 or 16 sides; in addition to other shapes such as: stellar and lobular shape. Also, the main components may have varied elevations even if the sections are the same, such as the difference between spherical dome and conical dome. The minaret heads also may vary in both sections and elevations such as hemispherical dome, lobed dome, bulbourethral form dome and ribbed cap.
- Different proportions of the main components among minarets are found (Figure 4.29)[48] including the following:

  * The proportion of width to height of the one minaret differs than other minarets.

  * The proportion of width to height of each component differs from one minaret to others.

[48] (Hillenbrand, 1994, pp. 130, 131)

* The proportion among components' diameters of each minaret is different from one minaret to others.

* The proportion of components' height of each minaret is also different from one minaret to others.

* The proportion of each component diameter to the total minaret diameter varies from minaret to others.

* The proportion of each component height to the total minaret height varies from minaret to others.

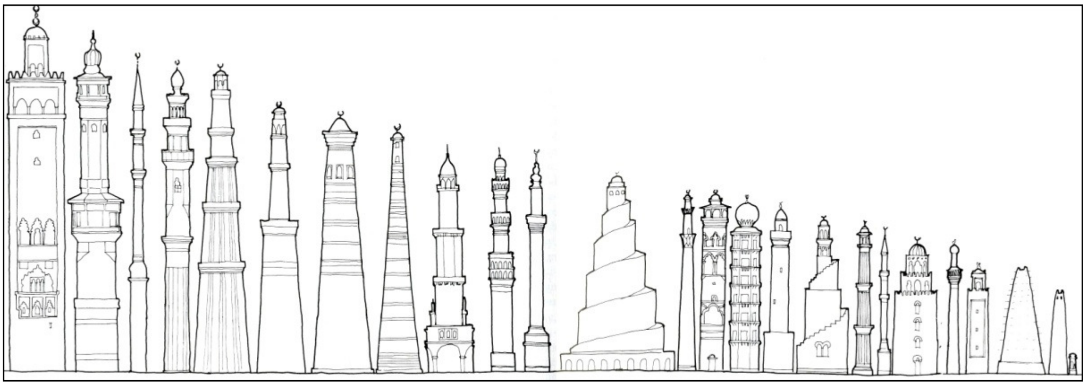


Figure 4.29: Different proportions of minaret designs (Hillenbrand, 1994, pp. 130, 131)

- Different shapes of joints can be distinguished, such as Muqarnas (stalactites) and cornices.
- Different shape of secondary components such as openings, arcades, ornaments, etc.

4.1.5.2. Constraints and variations in minaret designs

The hybridity in minaret design should take into account the fact that the generated designs do not deviate from the main stream of traditional minaret designs. The new design should meet design constraints induced from the old minarets. At the same time, it can benefit from design variations that the old designs have.

The constraints can be discussed in relation to the number of components, the geometry of components, and the sequence of components.

4.1.5.2.1. Constraints on the number of minaret components

Despite the fact that there is no specific constraint on the maximum number of minaret components, for practical reasons this number cannot be infinite. To constrain this matter, it is suggested that the maximum number of minaret components in the new hybrid designs should be determined by the maximum number of components in the analyzed corpus of antecedents. The minimum number of components in the minaret design is two - a body and a head which are the essential components of any minaret.

4.1.5.2.2. Constraints on the geometry of minaret components

The constraints regarding the components' shapes that can be induced from the antecedents are:

- In most cases, minarets have a symmetrical composition.
- If the minaret has multi-bodies of different diameters, then these diameters are either the same or decrease gradually when ascending.
- If the minaret has multi-balconies of different diameters, then the diameters are either the same or decrease gradually when ascending.
- The section of the body should be surrounded by the section of the base, joint or balcony which precedes it, and the sections of joint, balcony which follows it.
- The section of the joint can be identical to or surrounded by the balcony or lantern which follows it.
- The section of the lantern can be identical to or surrounded by the section of the joint or balcony which follows it.
- The head should be surrounded by the section of the body, joint, balcony or lantern which precedes it.

4.1.5.2.3. Constraints on the sequences of minaret components

The constraints regarding the components' sequences that can be induced from the antecedents are:

- The lowest component of any minaret is either the base or the body.
- The minaret base should be followed by a body directly or after a joint and/or balcony.

- The minaret lantern or head should be preceded by a body.
- The minaret lantern cannot be followed by a body.
- The upper component of any minaret is a head.

4.1.5.2.4. Variations of minaret design

Variations in the new minaret designs may be made in the following areas:

- In spite of the majority of minarets having a symmetrical composition, there are two paradigms of asymmetrical models: Samarra in Iraq and Ibn Tulun in Egypt as shown in Figures 4.25 and 4.26. This allows the derivation of asymmetrical hybrid designs.
- It is possible to have multi shapes of components' sections in the same design such as square, polygon (octagon) and circle in varied sequences. Creswell asserts that, "the circular part in the Egyptian minarets always comes above the octagonal, which in its turn rests on a square lower storey" such as Emir Qusfin minaret (Figure 4.30) (1926a, p. 140). However, other examples show different sequences using octagonal or eight-pointed-star polygonal parts above the circular part such as in the Kanqah of Faraj minaret in Egypt (Figure 4.31)[49] and Ghazni minaret (Figure 4.32)[50] in Afghanistan.

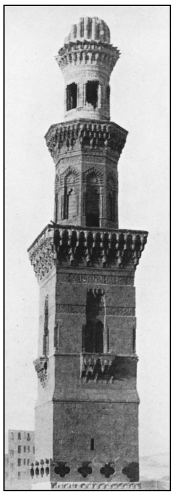


Figure 4.30: Emir Qusfin minaret

Figure 4.31: Kanqah of Faraj minaret

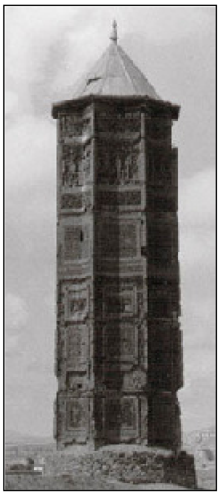


Figure 4.32: Ghazni minaret

[49] http://2.bp.blogspot.com/_BrntFFiXN8U/SPSKHfBG-YI/AAAAAAAACDc/8UlZoJKiu-E/s1600-h/sultan.JPG

[50] http://www.archnet.org/library/images/one-image-large.jsp?location_id=12583&image_id=128308

Accordingly different sections of base, bodies, joints, balconies lantern and head can be used in the same minaret design. Also, the section of each component can be differentiated from the sections of both the preceding and following components which take into account the constraints on surrounded and surrounding relations. In this case, the sequence of varied shapes is allowable within the constraints to derive hybrid minaret design.

- There is no canon governing the respective proportions of minaret, either the whole or parts (Figure 4.29). Similarly in minarets consisting essentially of multi-bodies, the proportional relationship between one body and others could vary quiet markedly (Hillenbrand, 1994, p. 154) . For this reason, the dimensional proportion of the analysed minarets can be parameterised to be easily manipulated in the new hybrid design.
- It is possible to have different types of joints such as muqarnas and cornices between the same or different minaret components.
- It is possible to have different openings and ornaments on base and body of the same minaret design.

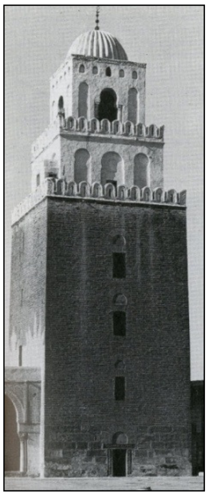
In conclusion, the review of minarets in the Islamic architecture reveals the actual presence of heterogeneous models. In addition, exemplars of hybrid historical minarets already exist. The study, in the next section, presents samples of twelve minarets to be the subject for implementing shape grammars for hybrid designs.

4.2. Sample of the study: the corpus of 12 minarets

Twelve traditional minarets have been selected to be the corpus of antecedents in shape grammar for hybrid minarets. To ensure the heterogeneity, the sample includes non-standard models beside typical ones. It consists of well known minarets which belong to different regions (Tunisia, Egypt, Iraq, Syria, Turkey, Iran, India, Yemen, Uzbekistan, and Afghanistan) and different Islamic periods. Each minaret was analysed syntactically in terms of its components and the relation between them as follows.

4.2.1. Minaret of the Great Mosque of Sidi 'Uqba in Tunisia

This is the Umayyad minaret in the Great Mosque of Sidi 'Uqba at Qayrawan, (724-727 AD). It has the tradition of the tall square-shafted minaret (Figure 4.33)[51]. The minaret consists of multi-parts of body and a head. The body is a stepped three-tier elevation, each tier smaller than the previous one. The square of the tall body is about 10m a side, and the total height of the minaret is about 35m. The second and third tiers are smaller replicas of the main shaft of the minaret, whereas the third one is crowned with a lobed dome. The triply-stepped silhouette of the Qairawan minaret was to remain something of a dead end in the later history of the minaret. (Creswell, 1926a, p. 138; Hillenbrand, 1994, pp. 138-140)



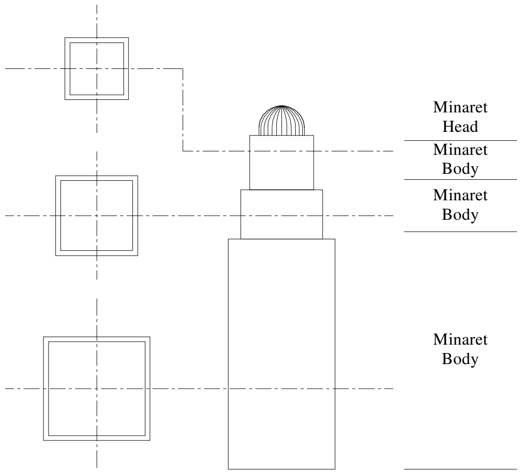


Figure 4.33: Sidi 'Uqba minaret

4.2.2. Minaret of Khanqah of Faraj b. Barquq in Egypt

The Minaret of Khanqah of Faraj b. Barquq, in Egypt (1398-1411 AD) is the square-circular-octagonal type. It consists of square base and circular body surmounted by octagonal lantern with an onion-shaped dome above it. The shafts were interrupted at intervals by very richly ornamented balconies over joints consisting of *muqarnas* cornices. The surface patterns contained niches, arches and decorative panels (Figure 4.34)[52].

[51] Figure 4.33 left: (Hillenbrand, 1994, p. 139), Figure 4.33 right: analytical drawing has been done by the author

[52] Figure 4.34 left: http://2.bp.blogspot.com/_BrntFFiXN8U/SPSKHfBG-YI/AAAAAAAACDc/8UlZoJKiu-E/s1600-h/sultan.JPG , Figure 4.34 right: analytical drawing has been done by the author.

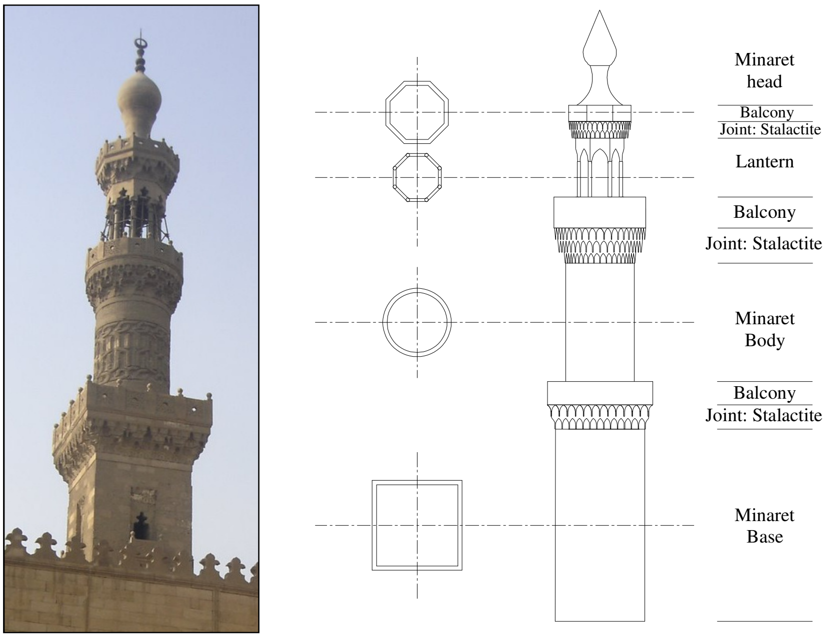


Figure 4.34: Khanqah of Faraj minaret

4.2.3. Minaret of Friday Mosque of Na'in in Iran

The Na’in minaret (960 AD) (Figure 4.35)[53] maintains the traditional square format in ground plan of the base only.

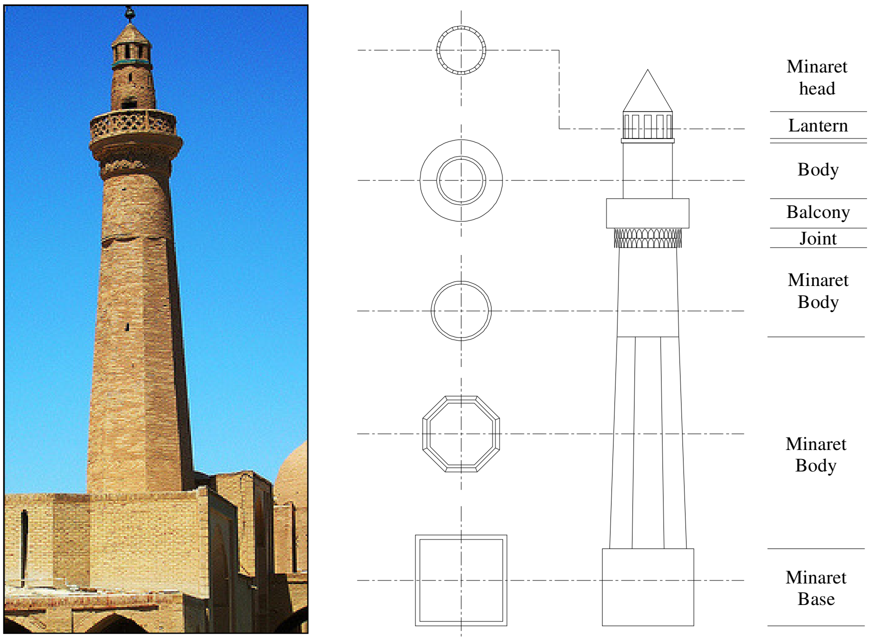


Figure 4.35: Na’in minaret

[53] Figure 4.35 left: http://www.flickr.com/photos/53047624@N00/708078800, Figure 4.35 right: analytical drawing has been done by the author,

The innovative body was generated by setting a tapering octagonal shaft on the square plinth. This shaft is two-thirds of the elevation, which thereafter becomes a tapering cylinder. The transition from octagon to circle is muted as to be scarcely noticeable. A cornice carries a substantial balcony and a small, cylindrical shaft surmounted by a domed lantern pierced by multiple apertures. Apart from a palmette designs on the cornice, the minaret is devoid of ornament. (Hillenbrand, 1994, p. 148)

4.2.4. Minaret of Jesus in the Umayyad Mosque in Syria

The original construction dates back to (706-715 AD). It is an amalgam of different architectural styles that characterize the changing political environment. It was renovated and restored by the Ayyubids, Mamluks and Ottomans after the earthquake of 1759. It has a Mamluk lower part and an Ottoman top[54]. The former is a tall square body, surmounted by a series of receding octagonal courses of small shafts separated by joints and balconies (Figure 4.36)[55].

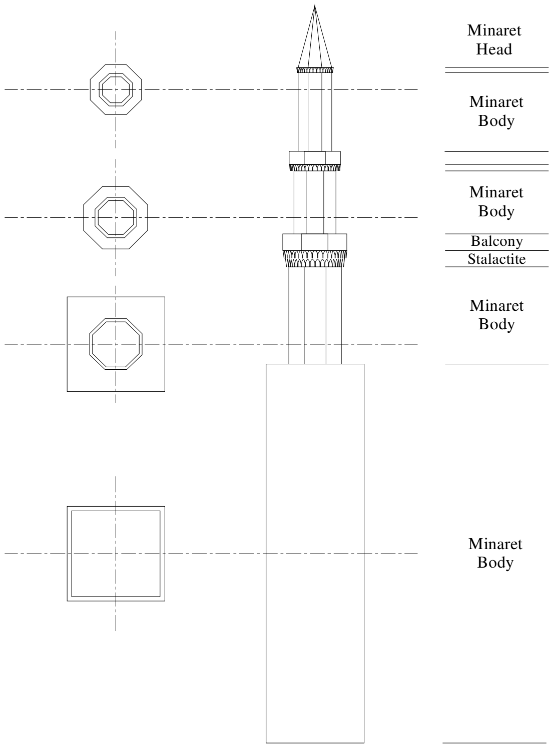



Figure 4.36: Jesus minaret

[54] http://www.archnet.org/library/sites/one-site.jsp?site_id=7161
[55] Figure 4.36 left: http://commons.wikimedia.org/wiki/File:Umayyad_Mosque_Jesus_Minaret.jpg, Figure 4.36 right: analytical drawing has been done by the author.

4.2.5. Anna minaret in Iraq

This is a freestanding tower which dates to the 11th century AD (Figure 4.37)[56]. It consists of a base, two bodies and a head. The octagonal base has an arched opening on the north side providing access to the interior of the minaret. Its octagonal shaft leans sidewise.

The minaret is decorated with eight rows of arched niches set in rectangular frames. Every row is composed of eight niches located on each of the eight sides of the octagon. Some of these sixty-four niches constitute windows to light the internal staircase. The shaft ends with an octagonal recessed shaft covered by a low dome. This recess creates a space for the terrace inscribed inside the minaret envelope; it is accessible through four arched openings situated on the sides of the octagonal spire below the dome. [57]

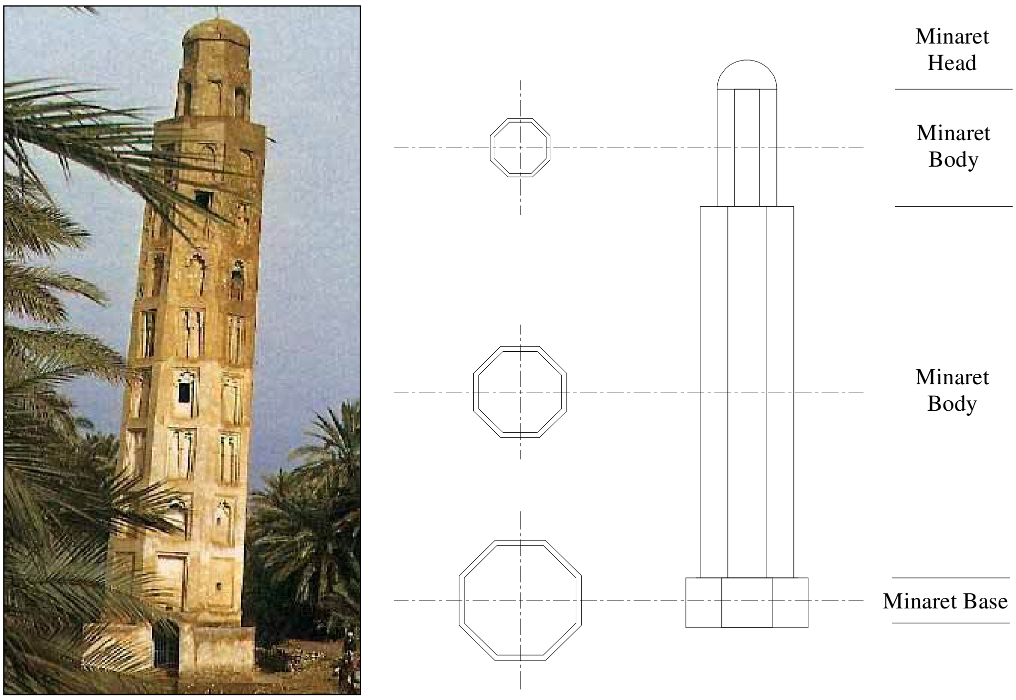


Figure 4.37: Anna minaret

[56] Figure 4.37 left: http://www.arabictourism.org/iq//modules/gallery/images/99e6037u2722a9jdbqwf07742.jpg, Figure 4.37 right: analytical drawing has been done by the author.
[57] http://www.archnet.org/library/sites/one-site.jsp?site_id=7825

4.2.6. Yivli Minaret in Antalya, Turkey

The minaret construction dates to the mid 13^{th} century AD (Figure 4.38)[58]. Its square stone base is six and a half meters tall and five and a half meters wide has chamfered upper corners to meet an octagonal transition body carved with a blind niche on each side. A narrow circular ring precedes the eight semi-circular flanks of the shaft which are followed by a circular band below the stone muqarnas and the balcony. The minaret ends with a lead-covered conical top to cap a simple cylindrical tower above the balcony. [59]

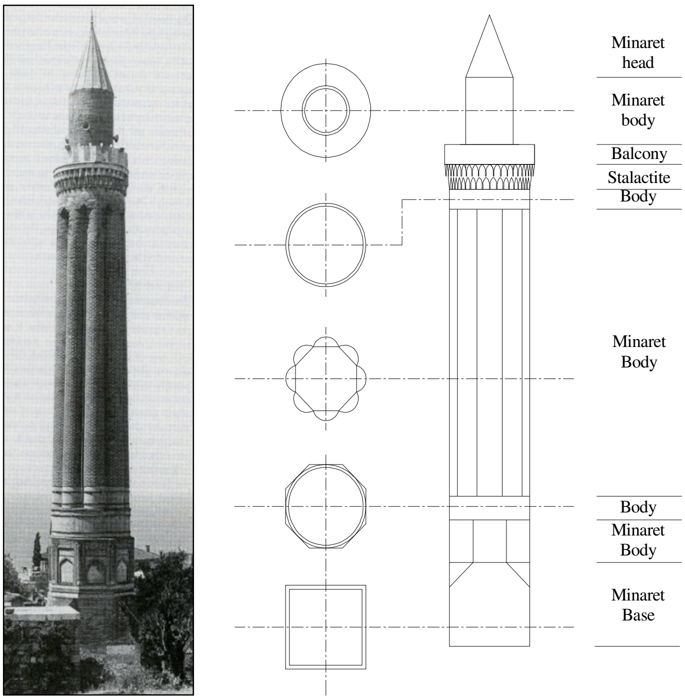


Figure 4.38: Yivili minaret

4.2.7. Minaret of Blue Mosque in Istanbul, Turkey

The minarets of Blue mosque (1609-1616 AD) are fluted, pencil-shaped minarets that have three balconies with stalactite corbels[60] (Figure 4.39)[61]. The shaft is considered cylindrical because the angles of polygon are so obtuse that the visual effect is that of

[58] Figure 4.38 left: http://www.archnet.org/library/images/one-image-large.jsp?location_id=14341&image_id=139794, Figure 4.38 right: analytical drawing has been done by the author
[59] http://www.archnet.org/library/sites/one-site.jsp?site_id=12121
[60] Corbel is a bracket of stone, wood, brick, or other building material, projecting from the face of a wall and generally used to support a cornice or arch. http://dictionary.reference.com/browse/corbels
[61] Figure 4.39 left: http://farm2.static.flickr.com/1216/1464301698_23478f1ed8.jpg?v=0, Figure 4.39 right: analytical drawing has been done by the author

cylinder. The base has a square plan but in elevation its walls sloped sharply inward (Hillenbrand, 1994, p. 164)

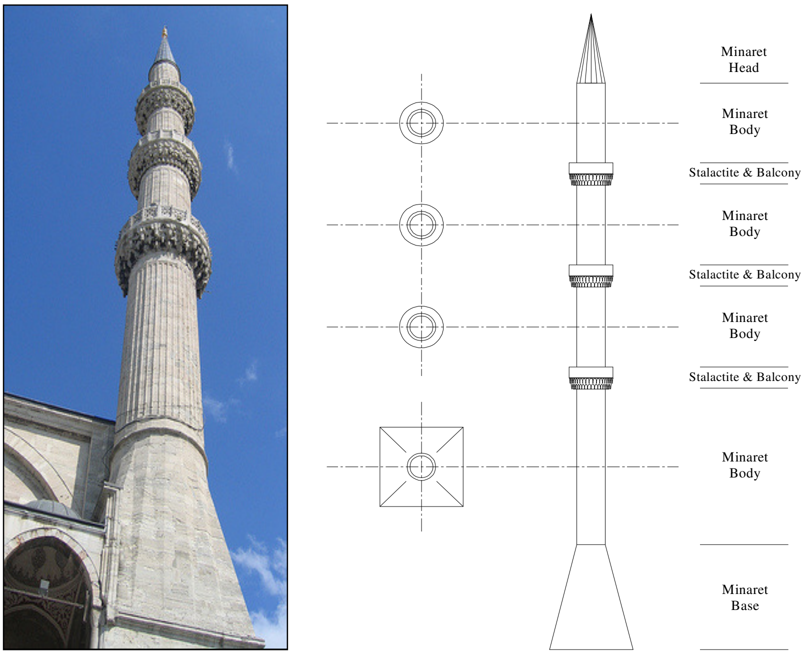


Figure 4.39: Blue mosque minaret

4.2.8. Minaret of Qubbat Talha in Sana'a, Yemen

The minaret was built in (1619-1620 AD) and reconstructed in (1831-1832 AD) (Figure 4.40)[62].

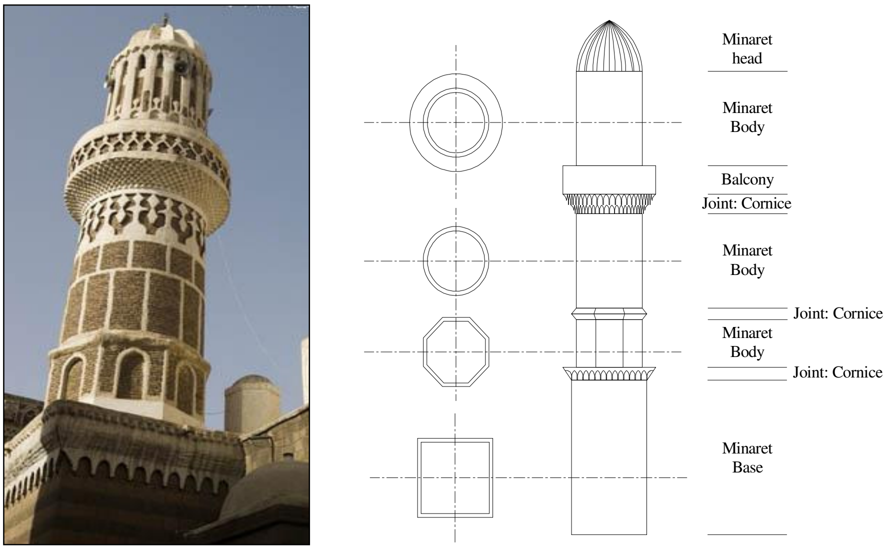


Figure 4.40: Qubbat Talha minaret

[62] Figure 4.40 left: http://www.phase.com/bmcmorrow/image/40669685 , Figure 4.40 right: analytical drawing has been done by the author.

It is a square high base topped by an octagonal and circular shaft, single balcony and a ribbed conical dome. A cornice was used at the top of the base and between the octagonal and circular shafts.

4.2.9. Minaret of the Wazir Khan Mosque in Lahore, Pakistan

The minaret dates back to (1634-1635 AD). It sits on a square base (Figure 4.41)[63]. It has an octagonal shaft rising to a height of over 30 meters. It consists of several stages which are capped with balcony and kiosk configuration terminated with fluted cupolas (Figure 4.42)[64], all profusely decorated with the tile mosaic. [65]

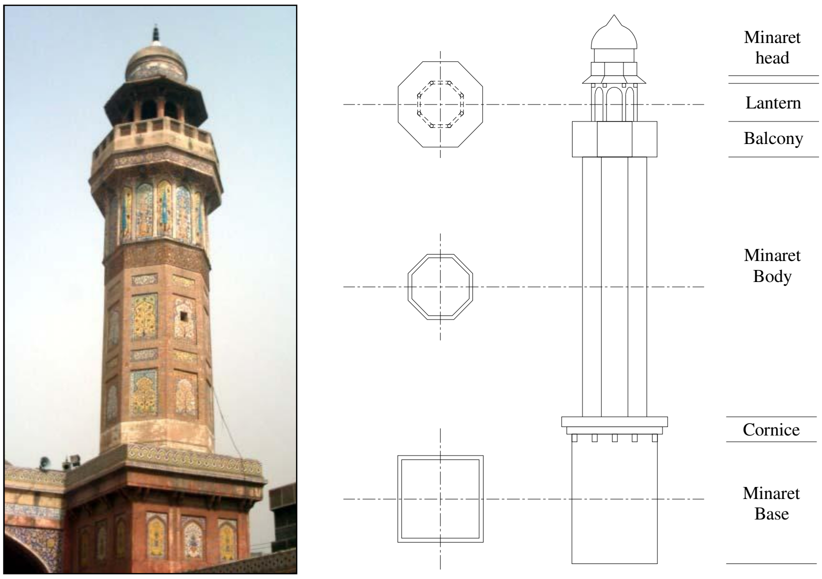


Figure 4.41: Wazir Khan minaret

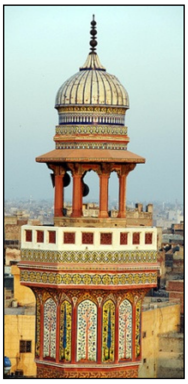


Figure 4.42: Lantern and head of Wazir Khan minaret

[63] Figure 4.41 left: http://www.flickriver.com/photos/rsarwar/sets/72157608139574104/, Figure 4.41 right: analytical drawing has been done by the author.
[64] http://www.flickr.com/photos/rsarwar/2951256897
[65] http://www.ualberta.ca/~rnoor/mosque_wazir_khan.html

4.2.10. Minaret of Abu el-Hajaj in Luxor, Egypt

The minaret is located in Aswan and is dated back to (1077-1082 AD) (Figure 4.43)[66]. It has a square tall base and a tapering cylindrical shaft covered by dome.

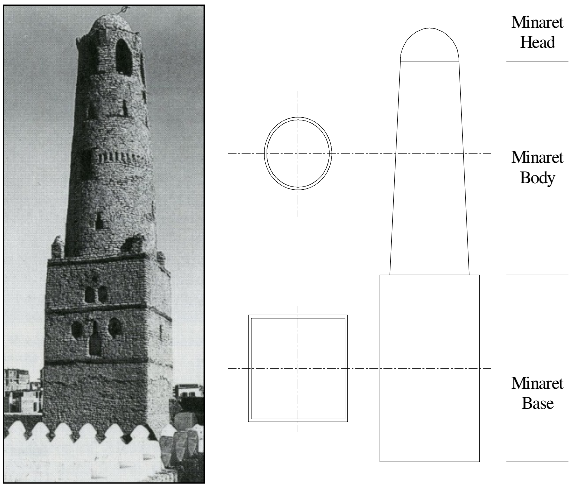


Figure 4.43: Minaret of Abu el-Hajaj in Egypt

4.2.11. Kalyan minaret in Bukhara, Uzbekistan

The minaret was built in (1127 AD). It has a cylindrical, tapering tower of 45.6 meters high and 9 meters diameter at the bottom and 6 meters overhead (Figure 4.44)[67].

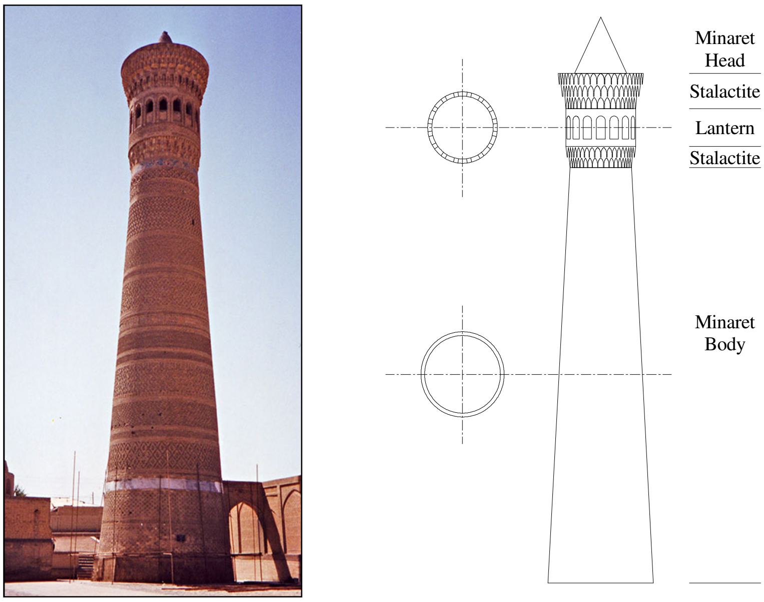


Figure 4.44: Kalyan minaret

[66]Figure 4.43 left: (Hillenbrand, 1994, p. 165), Figure 4.43 right: analytical drawing has been done by the author.
[67] Figure 4.44 left:http://www.dwoodworks.com/travel/CA/page48/files/page48_2.jpg , Figure 4.44 right: analytical drawing has been done by the author.

The body of the minaret is topped by a lantern-like rotunda with 16 arched fenestrations. The minaret has a cone-shaped top above the rotunda. A multi-tiered stalactite is used to crown the entire structure and to join the body with the lantern.

4.2.12. Ghazni minaret in Afghanistan

The minaret was built in (1098-1115 AD). It has a circular stone base. Its body consists of two shafts. The lower storey is unusual cross section in the form of star-shaped polygons formed by setting two equal and concentric squares at 45° and about 21 meters height (Figure 4.45)[68]. Above this portion was a cylindrical shaft which was destroyed by the earthquake in 1902 AD. The total height of the minaret is 42 meters. (Creswell, 1926c, p. 290; Scerrato, 1976, p. 65). No image of the complete minaret exists. Figure 4.45 (left) is the situation before the earthquake, the middle image is the current situation, and the right drawing is the intermediate state assumed by the author.

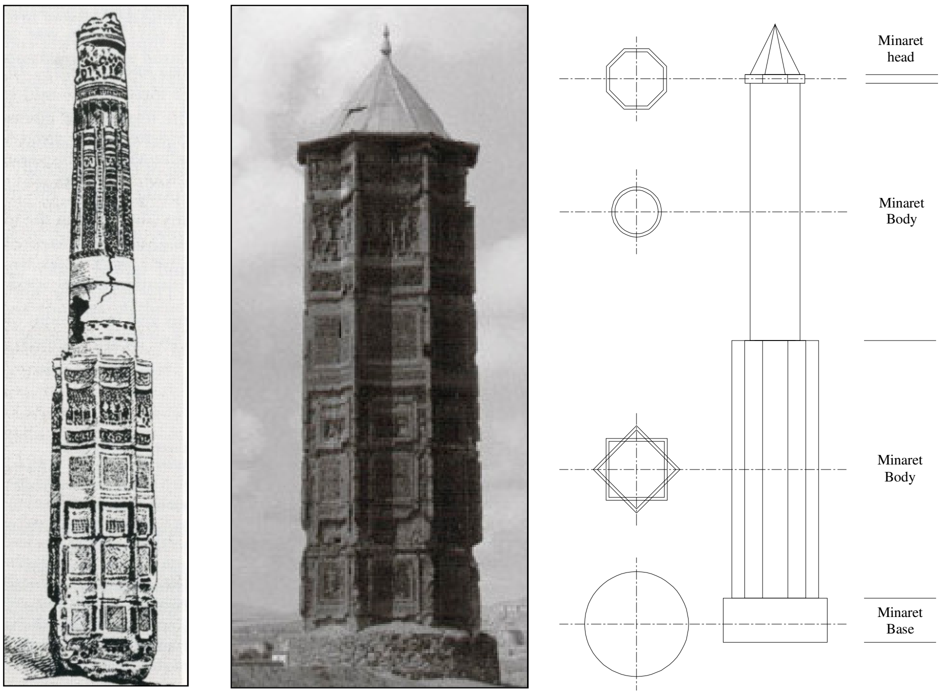


Figure 4.45: Ghazni minaret of Mas'ud III

[68] Figure 4.45 left: remains of the original state of minaret from (J. Bloom, 1989, p. 158), Figure 4.45 middle: the current state of minaret from http://www.archnet.org/library/images/one-image-large.jsp?location_id=12583&image_id=128308, Figure 4.45 right: analytical drawing for the proposed complete minaret has been done by the author.

4.3. Specification of shape grammars for hybrid minarets

This section concerns the definition of shape grammars for hybrid minaret designs. The general procedures presented in the previous chapter for the analysis and synthesis phases need to be dedicated to minarets. Writing rules of shape grammars for the hybrid minarets requires firstly the definition of the original rules within sub-class rules sets. In addition, the new hybrid rules can be added to increase the number of possible hybrid designs and to improve their mixed and individuality characters. On the other hand, detailed definitions of state labels and markers are identified to constrain rule selection and application to derive feasible hybrid minarets. Accordingly, the definition of both original rules and hybrid rules of shape grammars for hybrid minarets including their markers and state labels are elaborated in the next paragraphs.

4.3.1. Original rules of shape grammars for hybrid minarets

The study adopts parallel grammars which consist of a parametric shape grammar, a user guide grammar and an evaluation grammar. The twelve antecedents are analyzed by means of labelled 2D parametric shape grammars to represent their top and front views. The original rules of shape grammars for hybrid minarets are defined in six sub-class rules sets. In addition, specific state labels and markers are defined within the original rules to restrict the resultant designs within the feasible hybrid minarets.

4.3.1.1. Sub-class rules sets of shape grammars for hybrid minarets

The original rules are defined in six subclass rules sets to represent the different configurations of the main components of minarets: a base, a body, a joint, a balcony, a lantern and lastly a head. Each rule has a serial symbol composed of (OR + the sequence number of a rule in the sub-class rules set + a letter indicates the type of sub-class rules set). For example, the rule symbol (OR3a) refers to the third original rule of the minaret bases. The number of original rules in each sub-class rules set is varied as shown in table 4.1.

Table 4.1: Sub-class sets of minaret original rules

Name of sub-class set of original rules	Symbol of the original rules
Minaret bases	OR (1-7)a
Minaret bodies	OR (1-14)b
Minaret joints	OR(1-15)c
Minaret balconies	OR(1-6)d
Minaret lanterns	OR(1-4)e
Minaret heads	OR(1-8)f

The initial shape of this grammar consists of centre lines to represent the coordinate system of minaret top and front views, and the state label *n* which represents all the antecedents in the corpus (Figure 4.46).

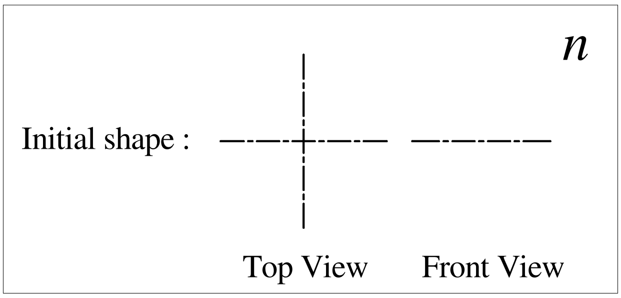


Figure 4.46: Initial shape in the grammar of minarets

The initial shape exists in the LHS of all original rules of the sub-class rules set of minaret bases, whereas the letter n is replaced by state labels to represent the specific antecedents of the current rule (Figure 4.47).

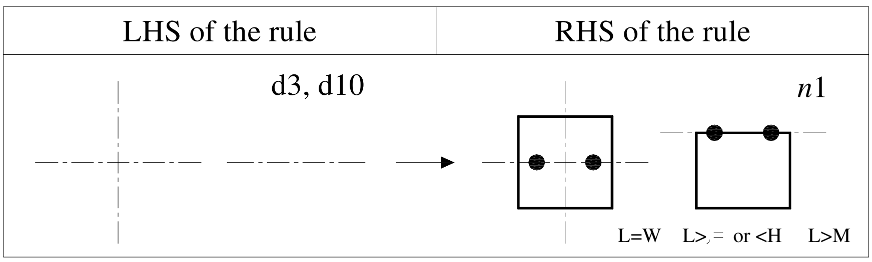


Figure 4.47: Example from sub-class rules set of minaret bases

The state labels of antecedents in the corpus are defined in Table 4.2.

Table 4.2: Labels of the corpus of antecedents

Name of antecedent	Label of antecedent
Sidi 'Uqba minaret	d1
Khanqah of Faraj minaret	d2
Ni’an inaret	d3
Minaret of Jesus	d4
Anna Minaret	d5
Yivli minaret	d6
Blue mosque minaret	d7
Qubbat Talha minaret	d8
Wazir Khan minaret	d9
Abu el-Hajaj minaret	d10
Kalyan minaret	d11
Ghazani minaret	d12

The LHS of all rules in other sub-class rules sets, such as bodies, joints, balconies, lanterns and heads, consists of the following matching conditions (Figure 4.48):

- The centre lines to represent the coordinate system in both the plan and the elevation. This simple and unified representation of the shape in the LHS of all rules in the sub-class rule set facilitates the free choice of rules and then the mixture of rules from varied antecedents.
- The markers to represent the current sub-class rule set; and, if applicable, the functional and formal constraints regarding this rule. Only the LHS of rules in the sub-class rule set of minaret bases have no markers because these rules are applied once at the beginning of the application.
- The constant state labels to represent the antecedents of the current rule, whereas each antecedent has a label as clarified in Table 4.2.

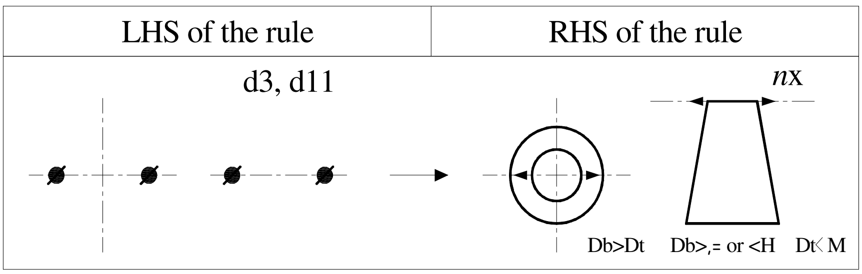


Figure 4.48: Example from sub-class rule set of minaret bodies

The matching conditions in the LHS of rules make it easy to apply rules belonging to varied antecedents on the generated hybrid minaret. On the other hand, the RHS of rules consists of the following elements (Figures 4.47 and 4.48):

- The centre lines to represent the coordinate system of both the plan and the elevation. They are elevated to be located at the top surfaces of the plan and elevation; except the rules of the sub-class rule set of minaret head (Figure 4.49) in which the centre lines remain in the bottom surface as the end of the grammar.
- The varied configurations of each component in each sub-class rule set. However, the RHS of one of the rules in the sub-class set of minaret bases has only the centre lines, the markers and the variable state labels because some of the antecedents in the corpus have no base.
- Parameters of dimensional proportions.
- The variable state labels to represent the possible antecedents of the next rule.
- The markers to represent the subsequent sub-class rule set with possible functional and formal constraints regarding this rule. They are placed on the centre lines in the top surfaces of plan and elevation except the RHS of all rules in the sub-class set of minaret heads (Figure 4.49) which are devoid of markers because they are applied once at the final stage to end the design derivation.

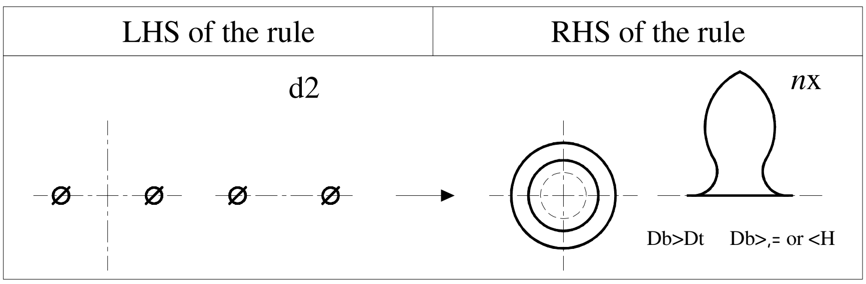


Figure 4.49: Example from sub-class rule set of minaret heads

4.3.1.2. Parametric rules of shape grammars for hybrid minarets

As concluded in the previous section, there is no canon to govern the respected proportion of minaret components (Hillenbrand, 1994). The minaret can be embodied in any dimensions. Therefore, the dimensional proportions in shape grammars for hybrid minaret design are parameterised. Shapes can be parametric in that the geometric scale of the shape can be varied. Parameterisation of shapes can increase the scope of shape grammars enormously and model shapes that could not otherwise be realistically modelled (Cagan & Mitchell, 1994, p. 175). Parametric grammar is flexible in shape recognition and able to generate more design variants than a basic grammar (Chen, 2005, pp. 56, 78). In this case, the individuality character of the generated hybrid designs is enhanced.

Parameters of minarets are variables that define the dimensional proportions of each shape. They are used to restrict the configuration within the same topology irrespective of definite scale values. For example, the proportion of the length, width and height can be represented as either equality (=) or strict inequality (< or >), whereas (L) represents a shape length, (W) represents a shape width, (H) represents a shape height, (D) represents a shape diameter, (Db) represents a base diameter, (Dt) represents a top diameter, and (M) represents a distance between markers, as shown in Figure 4.50.

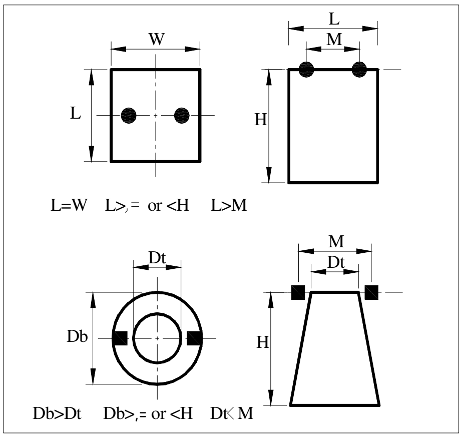


Figure 4.50: The parameters of shapes in minaret rules

4.3.1.3. Markers of shape grammars for hybrid minarets

Markers play an important role in deriving valid designs. Main markers and secondary markers are used in the shape grammar for hybrid minaret designs to ensure that each component is generated and placed in a reasonable relationship with other components.

Main markers symbolize the functional type of each component such as a body (●), a joint (◀), a balcony (■), a lantern (□) and a head (○). The main marker in the LHS of each rule symbolizes the sub-class rule set to which the rule belongs. On the other hand, the main marker in the RHS of each rule symbolizes the next minaret component which is inherited from the rule antecedent(s) to direct the user to the following sub-class rule set. For example, the rule in Figure 4.51 has the body marker (●) in its LHS therefore it belongs to the body sub-class rule set. At the same time, the following rule should belong to the sub-class rule set of minaret balcony because its RHS marker is (■).

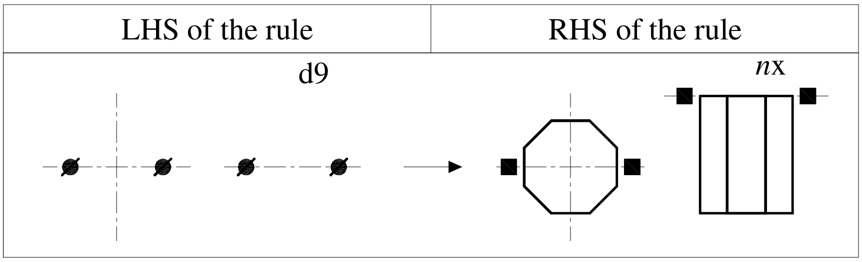


Figure 4.51: Main markers in the LHS and RHS of the rule

The main markers in the RHS of a rule determine the position of added component which is either surrounding or surrounded by the perimeter of existing component. They also keep the valid sequence of minaret components which starts, in most cases, with a minaret base and finishes, in all cases, with a minaret head. The role of these markers is also to prevent the wrong positioning of components, such as adding the body above the lantern or adding the lantern or head above the base directly.

Secondary markers, on the other hand, are part of the main markers which aim to limit the rule application within specific functional and formal constraints derived from the antecedents. The constraints using secondary markers have two tasks: the first concerns the functional sequence of the minaret components while the second concerns the formal sequence of shapes of minaret components. An example of the functional constraints is the secondary markers of joint and balcony which prevent forbidden cases such as adding either a joint followed by a lantern or a balcony followed by a lantern above the minaret base directly or adding either a joint followed by a body or a balcony followed by a body above the lantern.

Formal constraints of the secondary markers keep the proper containment relations between the boundaries of minaret components as found in the antecedents. If the minaret components share the same boundary diameter, then the added shapes must be inscribed in the boundary of the component underneath it, except the lobular and stellar shapes that can be circumscribed by octagonal and circular shapes. Accepted relations between shapes that share the same diameter are clarified in Figure 4.52. For example, it is possible to add an octagon or circle (dashed line) above a square (continuous line), to add a circle (dashed line) above an octagon (continuous line) and lastly to add the octagon (dashed line) above a stellar (continuous line). However, other cases are avoided such as adding a square shape above an octagon or a circle, or adding an octagon above a circle. The reason is although they share the same diameter, even so the perimeter of a square is longer than the circumference of an octagon or a circle, and the circumference of an octagon is longer than the circumference of a circle.

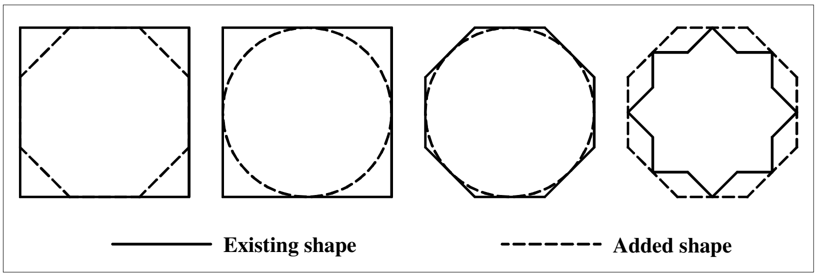


Figure 4.52: Constraints on the sequence of shapes of minaret components

Detailed descriptions of the functional and formal constraints on the sequence of minaret components using main and secondary markers in shape grammars of hybrid minaret designs are listed in the following paragraphs.

4.3.1.3.1. Body markers

Body markers in the RHS of rules direct the user to the sub-class rule set of minaret bodies. They constrain the location and the formal characteristics of the overlaid components in case of having the same diameter, length or width. They include the main marker of body (●) which refer to the possibility of adding any type of body shapes such as a square, an octagon, a circle, a polygon, etc. This marker includes secondary markers with extra constraints, such as adding bodies with an octagonal, a circular, a lobular or a stellar boundary (✖), and more specific secondary markers of adding only a circular, a lobular or a stellar body shape (⊘). Accordingly, the relations between body markers are as follows:

● ⊃ ✖, ● ⊃ ⊘, ✖ ⊃ ⊘

4.3.1.3.2. Joint markers

In shape grammar for minaret designs, different minaret components are connected using joints such as base, body, balcony, lantern and head. The joint markers in the RHS of any rule direct the user to the sub-class rule set of minaret joints. They specify the location where the joint can be added and constrain the sequence of next components. The main marker of joints (◄) allows the user to add any component above the joint. The secondary markers of the joint have functional constraints on the type of component to be added above the joint. For example, the secondary joint marker (•◄) allows adding a body or a balcony followed by a body above the joint, whereas the secondary joint marker (+◄) allows only adding a head or a balcony followed by a lantern or a head above the joint. In this case, the secondary markers prevent forbidden cases in minaret designs such as adding a body above a lantern or adding a lantern or a head above a base directly before adding a body. The relations between the joint markers are as follows:

◄ ⊃ •◄, ◄ ⊃ +◄

4.3.1.3.3. Balcony markers

The balcony markers in the RHS of rules direct the user to select rules from the sub-class rule set of minaret balconies. In all cases, they constrain the location of the added balcony, and in some case, they restrict the formal characteristic of applicable balcony and the types of components which follow it. The main markers (■) allows the user to choose any shape of balcony which can be followed by any component. The secondary markers of the balcony restrict the shape of the balcony boundary, the function of the component following the balcony, or both. Examples of the formal constraints on balcony shape are the balcony secondary marker (⌧) which allows the user to add balconies having an octagonal or a circular boundary, and the balcony secondary marker ( ) which restricts the choice of the balcony's shape to a circular boundary only.

The other balcony secondary markers have constraints on the function of next component, such as the secondary marker ( ) which represents a balcony followed by a body, and the secondary marker ( ) which represents a balcony followed by a lantern or a head. In addition, a combination of the formal and functional constraints of the balcony secondary markers can be found, such as the secondary markers ( ) and ( ) which restrict the choice of the balcony to the ones having either octagonal or circular shapes and followed by a body component in the case of the former and a lantern or a head in the case of the latter. Additionally, the balcony secondary markers ( ) and ( ) constrain the boundary of the balcony within the circular shape followed by a body in the case of the former and a lantern or a head in the case of the latter. The relations between the balcony markers are:

4.3.1.3.4. Lantern and head markers

In all rules, these markers constrain the location of the component to be applied and, in some cases, they restrict their formal characteristics. The boundaries of the lantern and the head in the corpus of twelve minarets are octagonal, polygonal, or circular. Therefore, the main lantern marker (□) adds any lantern shape marker, while the formal constraint of secondary lantern marker (⍁) adds only the circular lantern. Also the main head marker (○) adds any head shape, while the head secondary marker (∅) adds only a circular boundary shape. Both secondary markers of a lantern and a head are part of the main markers of a lantern and a head.

□ ⊃ ⍁, ○ ⊃ ∅

Table 4.3 lists the markers in shape grammars for hybrid minaret designs.

Table 4.3: Functions of markers in shape grammars for hybrid minaret designs

	Markers	Functional and formal constraints
Body Markers	●	Adding any body shape
	⊗	Adding octagonal, circulrar, lobular or stellar shapes
	⊘	Adding circulrar, lobular or stellar shapes
Joint Markers	◀	Adding any component above joint
	•◀	Adding a body or a balcony followed by a body
	+◀	Adding a head or a balcony followed by a lantern or a head
Balcony Markers	■	Adding any balcony shape
	⊠	Adding an octagonal balcony shape
	⧄	Adding a circular balcony shape
	•■	Adding a balcony followed by a body
	o■	Adding a balcony followed by a lantern or a head
Lantern Markers	□	Adding any lantern shape
	⧄	Adding a circular lantern shape
Head Markers	○	Adding any head shape
	∅	Adding a circular head shape

4.3.1.4. State labels of shape grammars for hybrid minarets

In shape grammars for hybrid minaret designs, there are 12 antecedents in the corpus *n*; each one has a symbol, d1 to d12, as defined in Table 4.2. Therefore *n* is:

n = {d1, d2, d3, ..., d12}

The state label *n* is attached to the initial shape which can be replaced by any label or subset of labels of the 12 antecedents in the corpus. For example, the third rule in the base sub-class rules set is derived from the antecedents d3 and d10, therefore the label *n* of the initial shape in the LHS of (OR3a) rule is replaced by d3, d10; as shown in Figure 4.53.

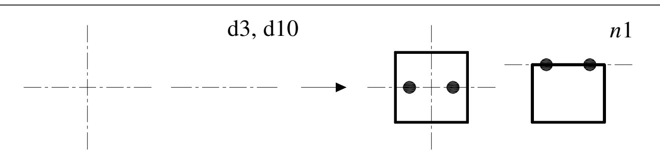
Rule no.	Original rule of sub-class rule set of minaret bases	The user guide grammar for hybrid design
OR 3a		$n1 = \{n \backslash d3, d10\}$ $n1$ = {d1, d2, d4, d5, d6, d7, d8, d9, d11, d12}

Figure 4.53: Example of a rule from the base sub-class rules set

The state label $n1$ is attached to the RHS of all rules in the subclass rule set of minaret bases. The value of $n1$ is defined by the user guide grammar as the whole twelve minarets in the corpus (n) minus the antecedents whose labels are in the LHS of the current rule (Figure 4.53).

The variable state label nx is attached to the RHS of all rules of the other sub-class rule sets of minaret bodies, joints, balconies, lanterns and heads. The variable x is an ascending value which represents the sequence of the rule in the application and varies from 2 to y. The variable y represents the total number of rules needed to derive the minaret design which varies in the corpus of antecedents from 3 rules to 12 rules. Accordingly, the use of the variable state label (nx) maintains the variations in both the total number of minaret components and their sequences in the analyzed antecedents. The values of variable nx are specified by the user guide grammar for hybrid design by excluding the LHS labels of a current rule from the set of state labels of previous rule (nx-1), as shown in Figure 4.54.

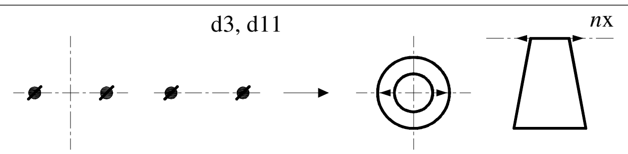
Rule no.	Original rule of sub-class rule set of minaret bodies	The user guide grammar for hybrid design
OR 5b		$nx = \{n(x\text{-}1)\backslash d3, d11\}$

Figure 4.54: Example of a rule from the body sub-class rules set

To ensure the adding of the minaret head is the last stage of the grammar where x=y, the symbol (′) is added as a constraint to the state labels in the LHS of all rules that have the head markers on their RHS, as shown in Figure 4.55. These rules should be selected at the penultimate stage of rule application. Therefore, at grammar runtime, the symbol (′) should be added to the set of nx=$n(y – 2)$ in the user guide grammar of the rule that precedes the penultimate rule.

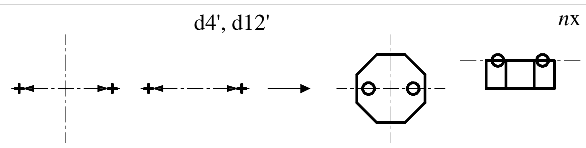
Rule no.	Original rule of sub-class rule set of minaret joints	The user guide grammar for hybrid design
OR 8c		$nx = \{n(x\text{-}1)\backslash d4, d12\}$

Figure 4.55: Adding the symbol (') to LHS rules that add head markers

The penultimate rule adds the head markers to the generated design in order to add the head component to the derived shape by the last rule. The state label at the RHS of the last head rule (*nx* = *ny*) is defined in the user guide grammar as zero. The only rule in the grammar that has a state label (0) in its LHS is the termination rule (Figure 4.56) which deletes the centre lines from top and front views of the generated design. For the full list of original rules of shape grammars for hybrid minaret designs, refer to appendix A-1.

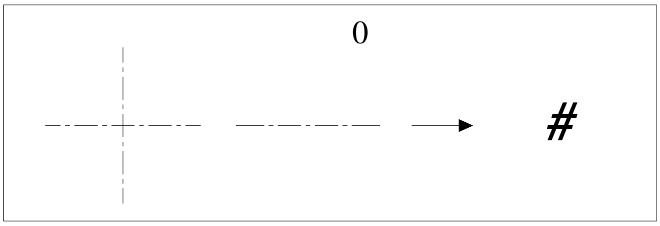


Figure 4.56: The termination rule

4.3.2. Hybrid rules (HR) of shape grammars for hybrid minarets

In shape grammars for hybrid minaret designs, the hybrid rules are isolated from the original rules and organised in sub-class rule sets of minaret bases, bodies, joints, balconies, lanterns and heads. The hybrid rules are derived by merging two original rules belong to the same sub-class rule set. Each rule has a symbol composed of the letters (HR), the rule number in the sub-class rules set and lastly the letter which is a symbol of the sub-class. The hybrid rule with the symbol (HR 3b) is the third rule in the sub-class set of bodies' hybrid rules. The number of hybrid rules in each sub-class rules set is defined in table 4.4.

Table 4.4: Sub-class sets of minaret hybrid rules

Name of sub-class set of hybrid rules	Symbol of the hybrid rules
Minaret bases	HR(1-12)a
Minaret bodies	HR(1-23)b
Minaret joints	HR(1-22)c
Minaret balconies	HR (1-13)d
Minaret lanterns	HR(1-12)e
Minaret heads	HR(1-8)f

The state labels in each hybrid rule are the sum of antecedents of its parents. In this study, the new geometry of a hybrid rule is limited to combining parts of the formal characteristics of both parents' rules. The markers which determine the sequence of the hybrid rule matches one of the parents' markers or both of them if they have the same sequence. Table 4.5 shows the different possibilities of hybrid rules by merging two original rules (A) and (B) in shape grammar for hybrid minaret designs.

Table 4.5: Different possibilities of hybrid rules in shape grammars for hybrid designs

	A & B Geometry	A & B Sequence	A & other sub-class rules Geometry	B & other sub-class rules Geometry	Hybrid rule Geometry	Hybrid rule Sequence
Hybrid rules with new geometry	Different	Different	Different	Different	A+B	A or B
	Different	Similar	Different	Different	A+B	A=B
	Different	Similar	Similar	Different	A+B	A=B
	Different	Different	Similar	Different	A+B	A
	Different	Different	Different	Similar	A+B	B
	Different	Similar	Similar	Similar	A+B	A=B

Two approaches are used to derive the geometry of the hybrid rules. The first approach generates hybrid rules using an interpolation process, while the second approach generates the hybrid rules using an extrapolation process. The term interpolation is "a method of estimating values that lie between two known values", while the extrapolation is a method of estimating values that lie outside a known range from values within a known range" (Terzidis, 2003, pp. 58, 60). The interpolation is used to derive symmetrical hybrid rules, while the extrapolation is used to derive asymmetrical hybrid rules.

4.3.2.1. Symmetrical hybrid rules using interpolation

In shape grammars for hybrid minaret designs, the primary activities of interpolation are both combining the shapes of the parents' rules and maintaining their original characteristics. The formal conventions of the minarets in the corpus, such as the symmetry, are retained. The shape and the spatial relation in the hybrid rule are introduced between the shapes and spatial relations of its parent's original rules, as shown in Figure 4.57.

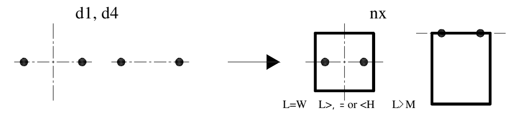
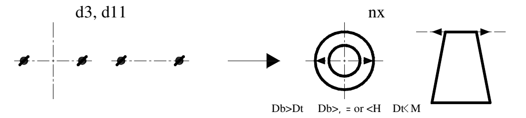
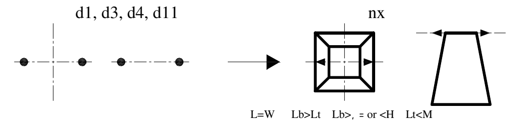
Rule no.	Subclass rule set of minaret body (Hybrid rules)	User guide grammar for hybid design	Evaluation of rules
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.5
OR 5b		nx={n(x-1)\d3, d11}	Rule Prevalence = 0.166 Geometrical difference = 0.75 Sequential difference = 0.333
HR 1b		nx={n(x-1)\d1, d3, d4, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.333

Figure 4.57: Hybrid rule derived using interpolation

4.3.2.2. Asymmetrical hybrid rules using extrapolation

Extrapolation looks beyond the formal conventions of the minarets in the corpus as a means of discovering new forms. The hybrid rule in this approach seeks the shape that falls outside the standards in which the rules of parents are known, such as asymmetrical shape (Figure 4.58).

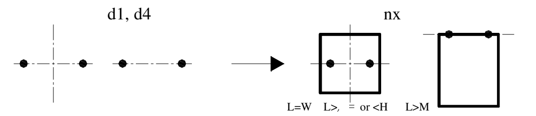
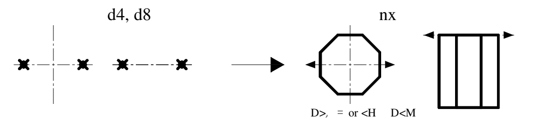
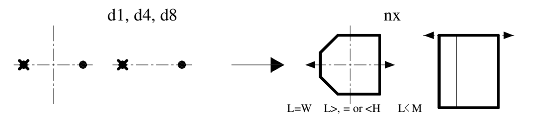
Rule no.	Subclass rule set of minaret body (Hybrid rules)	User guide grammar for hybid design	Evaluation of rules
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.5
OR 3b		nx={n(x-1)\d4, d8}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333
HR 7b		nx={n(x-1)\d1, d4, d8}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.333

Figure 4.58: Hybrid rule derived using extrapolation

4.3.2.3. Evaluation of hybrid rules

In the evaluation of the hybrid rule, the rule prevalence value of the hybrid rules is the ratio of the antecedents of both parents' rules to the total number of antecedents in the corpus. The rule geometrical difference value of the hybrid rule having a new geometry is the maximum value (1.0), while the sequential difference value of the hybrid rule keeps the same value of one of its parent which has the same sequence (Figures 4.57 and 4.58).

The derivations of hybrid rules from original rules are listed in appendix section A-2-1. In addition, the complete lists of hybrid rules of the shape grammar for hybrid minaret designs are provided in appendix section A-2-2.

4.4. Chapter summary

The first section of this chapter presented a review of minaret designs in Islamic architecture. Models of minarets belonging to a variety of periods and regions are syntactically analysed and the constraints and variations in traditional minaret designs are defined. Twelve heterogeneous minarets were introduced to be the sample of shape grammar for hybrid minaret design.

The second section of this chapter identified the detailed descriptions of original and hybrid rules in shape grammars for hybrid component-based design to suit the sample of minarets under investigation. The subclass rule sets were defined and both the state labels and markers were decided. In addition, methods of deriving hybrid rules from minarets' original rules were established.

5. THE IMPLEMENTATION AND RESULTS

In chapter four, a sample of twelve traditional minarets was identified to represent the heterogeneous corpus and to be the subject of the implementation. In addition, a framework of shape grammar for hybrid design was refined to cope with hybrid minarets in particular.

In this chapter, the research hypotheses are put forward in section (5.1). Section (5.2) presents three stages of implementation, each one with its results. Section (5.2.1) shows the derivation of four examples of a hybrid minaret using original rules and hybrid rules to validate the method of shape grammars for hybrid designs. The second stage, in section (5.2.2), validates the innovation measures of hybrid designs by testing their ability to distinguish between copies of existing designs and hybrid designs. Lastly, the third stage, in section (5.2.3), identifies the factors that the grammar user can take into account to boost the innovation values of a hybrid design. A summary of this chapter is presented in section (5.3).

5.1. Research hypotheses

Chapter three (section 3.4) put forward three research objectives. Firstly, to verify that the proposed method of shape grammars is able to generate hybrid designs from the corpus of existing heterogeneous designs. The second objective was to validate the proposed measures of innovation in hybrid designs. Finally, the third objective was to identify the indicators for grammar user to derive hybrid designs with high innovation values.

To achieve these objectives, the following hypotheses need to be proven.

- The proposed method of shape grammar for hybrid design is capable of generating hybrid designs and not simply reproducing existing designs.

- The proposed innovation measures of the mixture and individuality in the generated hybrid design have higher values in hybrid designs than copies of antecedents. In addition, hybrid designs composed of hybrid rules have higher innovation values than hybrid designs composed of original rules.
- That the independent variables which the user can control such as the number of rules used to derive the hybrid design and the rule evaluation factors, especially the rule prevalence values, can be the indicators of innovation measures in the generated hybrid design.

5.2. Stages of implementation

To achieve the research objectives and to examine the hypotheses, three stages of implementation were prepared as follows:

- Stage 1: examples of the derivation of hybrid minarets using original rules and hybrid rules separately are clarified.
- Stage 2: The original rules of shape grammars are used to derive copies of the existing minarets with their innovation measures which are compared with the innovation values of both hybrid designs composed of original rules and hybrid rules separately
- Stage 3: The effects of the number of rules and the rule evaluation factors on the innovation measures were extracted from the examples in stage 2. In addition, further investigation on some factors were done using 10 pairs of hybrid minarets, each one composed of 6 and 10 original rules and hybrid rules separately.

5.2.1. Stage 1: Examples of hybrid minarets

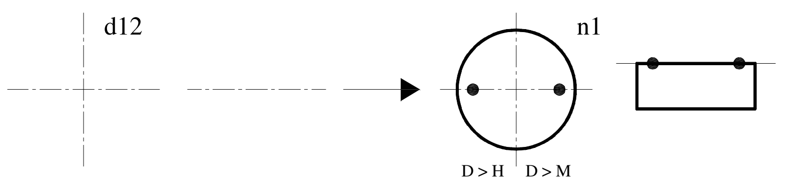
The method of shape grammars for hybrid design is verified by applying the minaret grammar to derive examples of hybrid minarets using the original rules and the hybrid rules separately. In fact, deriving hybrid minaret by mixing both original and hybrid rules is possible, even so the study applied them separately to identify the difference between them. The presented examples are derived using (7 or 8) rules as the average number of rules in the corpus of antecedents which varies between (3–12) rules. The method has proved its ability to generate hybrid designs and to avoid the derivation of copies of existing designs.

5.2.1.1. Examples of hybrid minaret using original rules

The study applied shape grammars to derive hybrid minarets composed of original rules. The complete lists of original rules of the shape grammar for hybrid minarets are provided in appendix section A-1. Two examples were derived using seven and eight original rules and are described in the following paragraphs.

5.2.1.1.1. First example: hybrid design composed of 7 original rules

The first rule in shape grammar of hybrid minaret should belong to the sub-class rules set of minaret bases which has the initial shape in the LHS of all rules. The rules in this sub-class add either a base with markers or only markers if the antecedents are without bases. In this example, the first rule initiates the generation by adding a circular base to the composition (Figure 5.1).

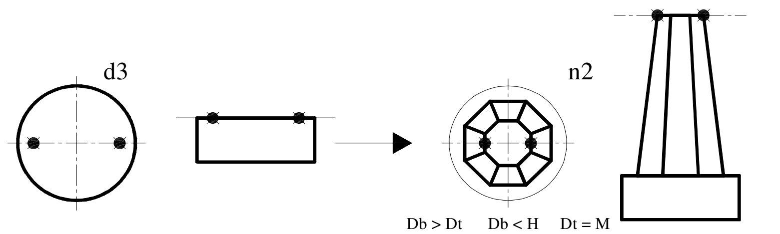
Rule no.1	Example 1: Hybrid design derivation - 7 Original rules
OR 7a	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n1 = {n\d12} n1 = {d1, d2, d3, d4, d5, d6, d7, d8, d9, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5

Figure 5.1: Adding the minaret circular base in the first rule

The rule (OR7a) belongs to the antecedent (d12) in the corpus. Therefore, the variable state label (*n1*) in the RHS of this rule is defined in the user guide grammar for hybrid design as it represents the set of whole antecedents in the corpus (*n*) minus the antecedent (d12). In this case, the user guide for hybrid designs controls the selection of the next rule to belong to varied antecedents. The evaluation of the rule has default values on which the grammar user can depend in selecting the rule. On the other hand, the evaluation of the generated design displays at grammar runtime the innovation values at each stage of the design derivation.

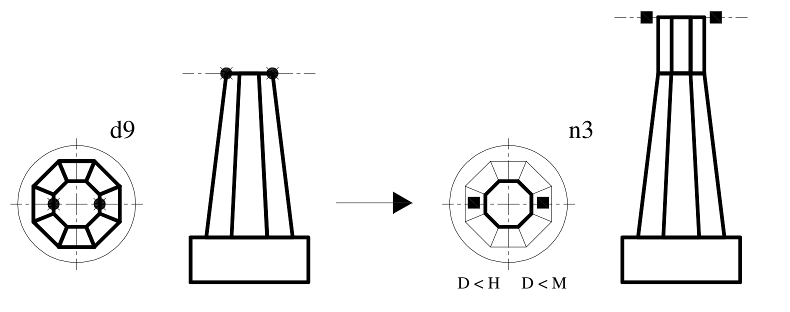
The markers in the RHS of the first rule are the body main markers which allow the user to add a body component without any restriction on the shape of its boundary. Therefore, the second rule to apply belongs to the sub-class rule set of minaret bodies and has body secondary markers in its LHS as part of body main markers to add a tapering octagonal body above the circular base (Figure 5.2). The rule (OR4b) was derived from the antecedent (d3) in the corpus. Thus, the user guide for hybrid designs defines the variable state labels (*n2*) as the set of the previous rule (*n1*) minus the antecedent (d3). The evaluation of the grammar calculated the innovation measures of the generated hybrid design at this stage.

Rule no.2	Example 1: Hybrid design derivation - 7 Original rules
OR 4b	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n2 = {n1\d3} n2 = {d1, d2, d4, d5, d6, d7, d8, d9, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.0 Matching degree = 0.5 Geometrical difference = 0.916 Sequential difference = 0.5

Figure 5.2: Adding the minaret body above the base in the second rule

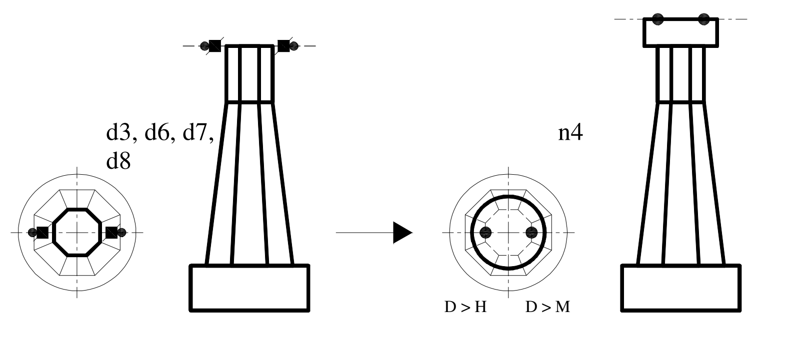
The markers in the RHS of the second rule are the body secondary markers which allow adding the second body with an octagonal, a circular, a lobular or a stellar boundary shape. According to that, the third rule in the grammar (OR12b) adds an octagonal body derived from the antecedent (d9) (Figure 5.3). The variable state label (*n3*) is defined in the user guide grammars for hybrid designs as the set of (*n2*) minus the antecedent (d9). The evaluation of the generated design shows the innovation measures of hybridity at this stage of the derivation.

Rule no.3	Example 1: Hybrid design derivation - 7 Original rules
OR 12b	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n3 = {n2\d9} n3 = {d1, d2, d4, d5, d6, d7, d8, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.916	Design diversity = 0.25 Design abundance = 1.0 Matching degree = 0.333 Geometrical difference = 0.805 Sequential difference = 0.638

Figure 5.3: Adding the second minaret body above the first body in the third rule

The markers in the RHS of the third rule are the main markers of the balcony. These markers admit the addition of a balcony which has any shape and can be followed by any component. Therefore, the fourth rule has balcony secondary markers in its LHS, as part of the balcony main markers, to add the circular balcony followed by a body (Figure 5.4).

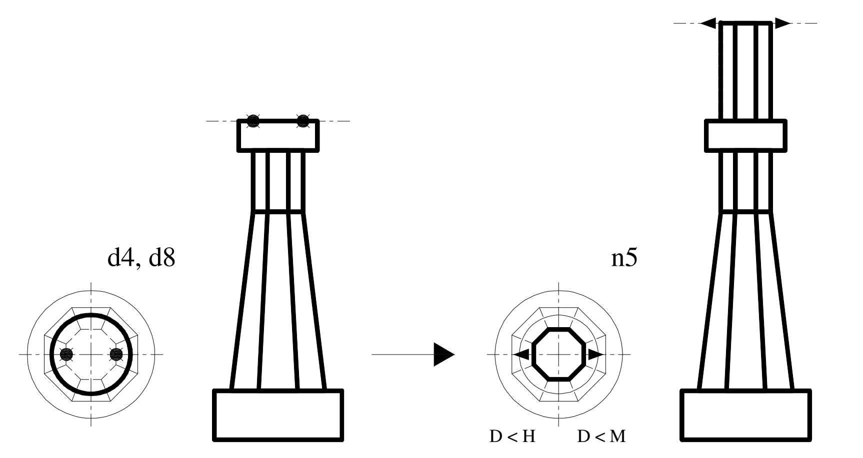
Rule no.4	Example 1: Hybrid design derivation - 7 Original rules
OR 4d	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n4 = {n3\d3, d6, d7, d8} n4 = {d1, d2, d4, d5, d10, d11}	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 1.75 Matching degree = 0.5 Geometrical difference = 0.749 Sequential difference = 0.604

Figure 5.4: Adding the balcony above the second body in the fourth rule

The rule (OR4d) was derived from multi antecedents (d3, d6, d7 and d8) in the corpus. The antecedent (d3) does not belong to the set (*n3*) which defines the variable state labels of the previous rule; even so the rule is eligible for selection and application. In fact, the rules with multi-state labels can be chosen if there is only one state label belong to the set (*nx*) of the previous rule. This rule has high rule prevalence value and boosts the innovation measures of diversity and abundance. On the contrary, the use of two rules derived from the same antecedent (d3) increases the matching degree as an inverse measure of individuality.

The markers in the RHS of the fourth rule are the main markers of the body. Therefore the fifth rule has body secondary markers, as part of the body main markers, to add the octagonal body above the balcony (Figure 5.5).

Rule no.5	Example: Hybrid design derivation - 7 Original rules
OR 6b	

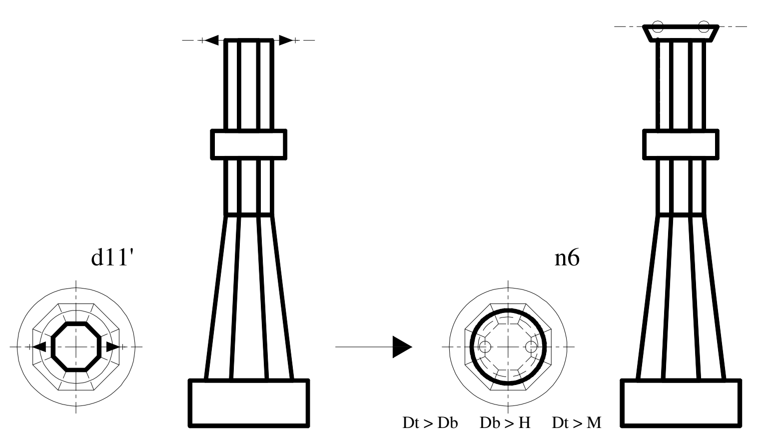
The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n5 = {n4\d4, d8} n5 = {d1', d2', d5', d10', d11'}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.583 Design abundance = 1.8 Matching degree = 0.4 Geometrical difference = 0.716 Sequential difference = 0.549

Figure 5.5: Adding the body above the balcony in the fifth rule

The rule (OR6b) was derived from two antecedents (d4 and d8) whereas only (d4) belongs to the set that defines the variable state label (*n4*) of the previous rule. In the RHS of this rule the variable state label (*n5*) is ($n(y - 2)$), whereas ($y = 7$) is the total number of rules to derive the hybrid design in this example. Therefore, the symbol (′)

is added to all state labels in the set of (*n5*) to constrain the choice of the next rule and to ensure adding the head markers at the RHS of the next rule. The evaluation of the grammar shows update values of innovation measures of the generated design at this stage of derivation.

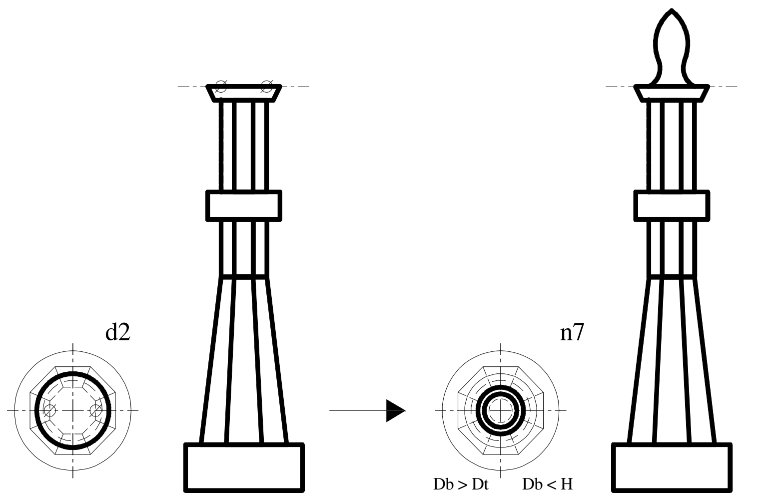
The markers in the RHS of the fifth rule are the main markers of joints which allow the user to add a joint followed by any component. The next rule therefore has joint secondary markers which add the joint component followed by the head (Figure 5.6). The rule (OR15c) was derived from the antecedent (d11) which belongs to the set of variable state labels (*n5*) of the previous rule. Thus, the user guide grammar defines the values of the variable state label (*n6*) by excluding the antecedent (d11) from the set (*n5*).

Rule no.6	**Example 1: Hybrid design derivation - 7 Original rules**
OR 15c	

The user guide grammar for hybid designs	**Evaluation of the rule in the grammar**	**Evaluation of the design in the grammar**
n6 = {n5\d11} n6 = {d1, d2, d5, d10}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.666	Design diversity = 0.666 Design abundance = 1.666 Matching degree = 0.333 Geometrical difference = 0.694 Sequential difference = 0.569

Figure 5.6: Adding the joint above the body in the sixth rule

The head secondary markers in the RHS of the penultimate rule ensure adding a head with a circular base to the generated minaret at the final stage. The seventh rule is the final rule in the derivation of the minaret which ends the configuration by adding the head with a circular base at the top of the minaret (Figure 5.7). The rule (OR2f) was derived from the antecedent (d2) which belongs to the set of variable state labels (*n6*) of the previous rule. The variable state label of the final rule (*ny* = *n7*) is defined in the user guide grammars as (0).

Rule no.7	Example 1: Hybrid design derivation - 7 Original rules
OR 2f	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n7 = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.0	Design diversity = 0.75 Design abundance = 1.571 Matching degree = 0.285 Geometrical difference = 0.725 Sequential difference = 0.487

Figure 5.7: Adding the head above the joint in the seventh rule

The final measures of the innovation in the whole generated design have been calculated at this stage. The values are as follows:

- Diversity value is 0.75, whereas the maximum diversity in all hybrid design is (1.0) which results from selecting rules belonging to all antecedents in the corpus; while the minimum diversity in hybrid designs derived by original rules is (0.083) which results from choosing all original rules belonging to the same one antecedent in the corpus.
- Abundance value is 1.571, whereas the minimum abundance in hybrid designs derived by original rules is (1.0) which results from selecting all original rules which are derived from only one antecedent in the corpus.

- Matching value is 0.285 to both (d3 and d8) antecedents, whereas the minimum matching value in case of hybrid designs composed of seven original rules is (0.142) which results from antecedents in the corpus having only one or no rule in the generated design, and the maximum matching value in hybrid designs composed of original rules is (1.0) which results from one common antecedent among all applied rules.
- Geometrical difference value is 0.725, whereas the maximum geometrical difference in all hybrid designs composed of original rules is (0.916) which results from all applied rules having one antecedent and having no similar geometry among the other antecedents in the corpus. On the other hand, the minimum geometrical difference in hybrid designs composed of original rules is (0) which results from each applied rule having similar geometries to all antecedents in the corpus.
- Sequential difference value is 0.487, whereas the maximum sequential difference in all hybrid designs composed of original rules is (0.916) which results from applied rules having one antecedent and having no similar sequence among the other antecedent in the corpus. On the other hand, the minimum sequential difference in all hybrid designs is (0) which results from each applied rule having a similar sequence to all antecedents in the corpus.

The only rule which has a state label (0) in the LHS is the termination rule which deletes the centre lines from the top and front views of the minaret (Figure 5.8).

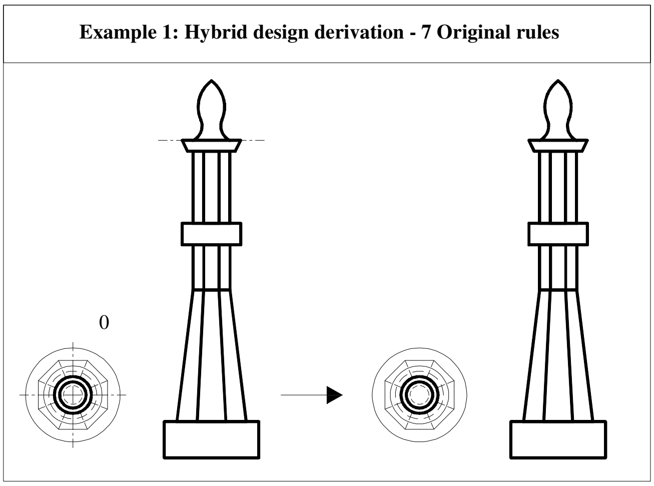
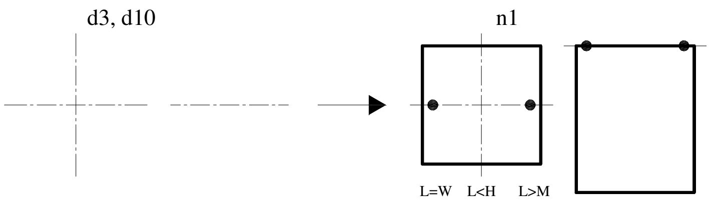


Figure 5.8: Termination rule

5.2.1.1.2. Second example: hybrid design composed of 8 original rules

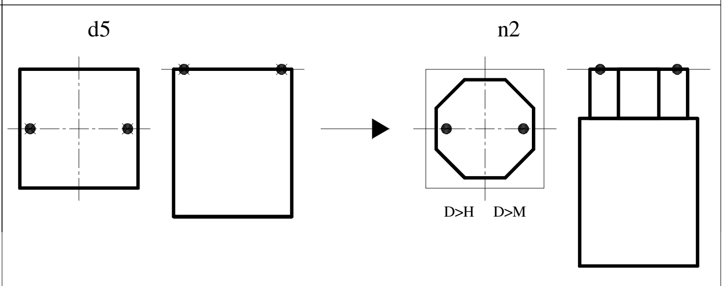
In the second example, the first rule in the derivation process belongs to the sub-class rule set of minaret bases and adds a cubical base (Figure 5.9). The original rule (OR3a) belongs to the antecedents (d3 and d10) which are excluded from the set that defines the variable state label (*n1*). Rules with multi-antecedents boost both diversity and abundance values. The markers in the RHS of this rule are the body main markers which allow the user to add a body with any boundary shape.

Rule no.1	Example 2: Hybrid design derivation - 8 Original rules
OR 3a	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n1 = {n\d3, d10} n1 = {d1, d2, d4, d5, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.583 Sequential difference = 0.5

Figure 5.9: Adding the minaret base at the beginning of the application

The second rule has body secondary markers in its LHS, as part of the body main markers, to add a short octagonal body above the base (Figure 5.10).

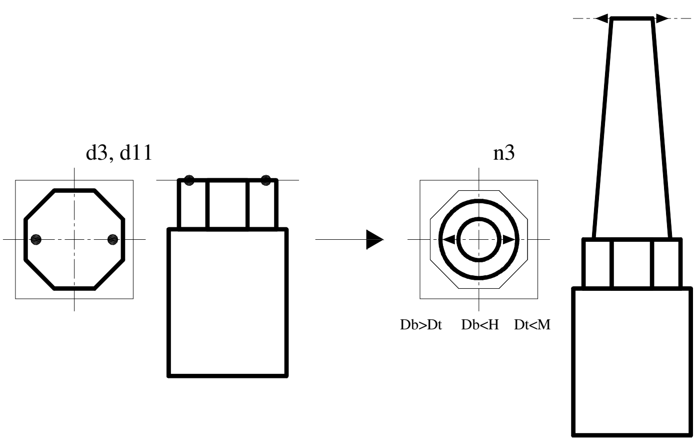
Rule no.2	Example 2: Hybrid design derivation - 8 Original rules
OR 7b	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n2 = {n1\d5} n2 = {d1, d2, d4, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 1.5 Matching degree = 0.5 Geometrical difference = 0.583 Sequential difference = 0.5

Figure 5.10: Adding the minaret first body above the base in the second rule

This rule was derived from the antecedent (d5), therefore the user guide grammar for hybrid design defines the variable state label (*n2*) by removing the antecedent (d5) from the set of variable state label (*n1*). The markers in the RHS of this rule are the body main markers which allow adding the body with any boundary shape by the third rule. The evaluation of the grammar shows the innovation values at this stage.

The Third rule in the grammar has body secondary markers in its LHS which are part of the body main markers in the RHS of the previous rule. It adds a tapering cylindrical shaft above the octagonal one (Figure 5.11). The rule (OR5B) belongs to multi antecedents (d3 and d11) whereas one antecedent (d11) belongs to the set (*n2*) which defines the variable state label of the previous rule. Therefore, the user guide grammar defines the variable state label (*n3*) by excluding the antecedent (d11) from the values of the variable state label (*n2*). The rule has joint main markers in its RHS. Therefore the next rule in the generation process belongs to the sub-class rule set of minaret joints.

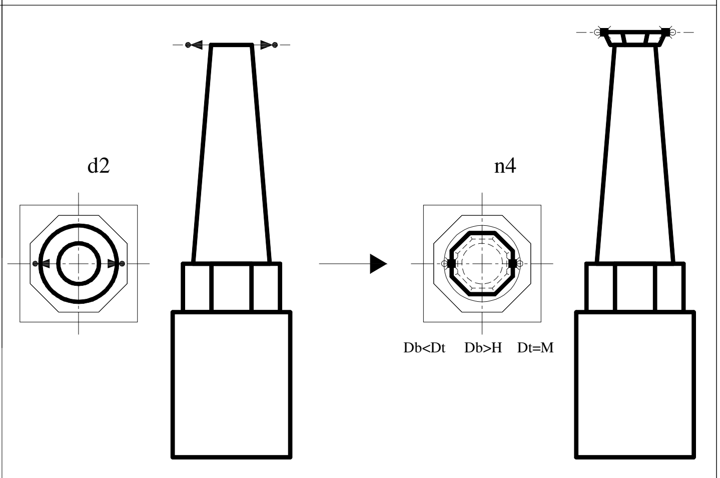
Rule no.3	Example 2: Hybrid design derivation - 8 Original rules
OR 5b	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n3 = {n2\d3, d11} n3 = {d1, d2, d4, d6, d7, d8, d9, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.75 Sequential difference = 0.333	Design diversity = 0.333 Design abundance = 1.666 Matching degree = 0.666 Geometrical difference = 0.638 Sequential difference = 0.444

Figure 5.11: Adding the minaret second body above the first body in the third rule

The fourth rule in the grammar has joint secondary markers in its LHS which restrict the following components to a head or balcony followed by a lantern or a head.

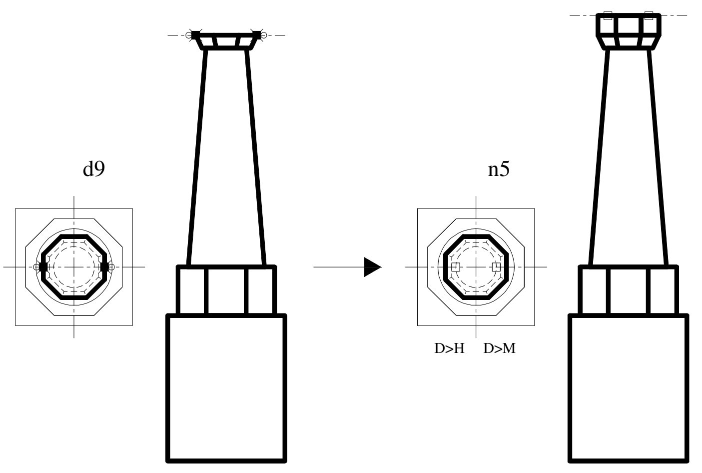
Therefore, this rule adds the joint component which is followed by a balcony above the body (Figure 5.12). The markers in the RHS of this rule are the secondary markers of balcony which restrict the next choice of balcony rule to ones that have an octagonal or a circular boundary shape and followed by a lantern or a head. The rule (OR7c) was derived from the antecedent (d2). The user guide grammar for hybrid design defines the variable state label (*n4*) by excluding the antecedent (d2) from the set (*n3*) which defines the variable state label of the previous rule. The innovation measures at this stage of the derivation are calculated and displayed at the evaluation of the generated design.

Rule no.4	**Example 2: Hybrid design derivation - 8 Original rules**
OR 7c	

The user guide grammar for hybid designs	**Evaluation of the rule in the grammar**	**Evaluation of the design in the grammar**
n4 = {n3\d2} n4 = {d1, d4, d6, d7, d8, d9, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.416 Design abundance = 1.5 Matching degree = 0.5 Geometrical difference = 0.687 Sequential difference = 0.458

Figure 5.12: Adding the minaret joint above the second body in the fourth rule

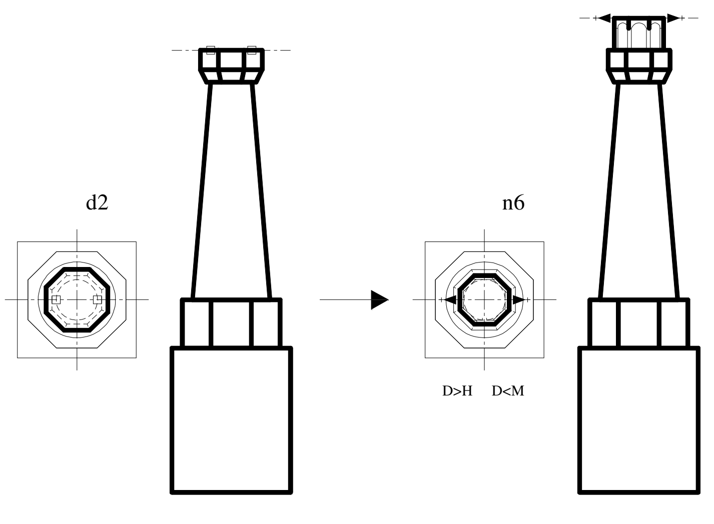
The fifth rule adds the octagonal balcony above the joint (Figure 5.13). The rule was derived from the antecedent (d9). The user guide grammar defines the set of variable state label (*n5*) by excluding the antecedent (d9) from the values of variable state label of the previous rule (*n4*). The markers in the RHS of this rule are lantern main markers. Therefore the next rule should be chosen from the sub-class rule set of minaret lanterns. However, there is no match between the values of (*n5*) and the LHS state labels of the lantern sub-class rule set. Therefore, the user guide grammar considers the value of (*n5*) as Ø and replaces the variable state label (*n5*) with the alternative variable state label (*n5**). The values of (*n5**) are the set of the variable (*m*) minus the antecedent (d9), whereas (*m*) is the antecedents in the corpus having less rules in the previous steps of derivation.

Rule no.5	Example 2: Hybrid design derivation - 8 Original rules
OR 6d	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n5 = {n4\d9} n5 = {d1, d4, d6, d7, d8, d12} n5 = {Ø} n5 = n5* n5* = {m\d9} n5* = {d2, d5, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.75 Sequential difference = 0.833	Design diversity = 0.5 Design abundance = 1.4 Matching degree = 0.4 Geometrical difference = 0.699 Sequential difference = 0.533

Figure 5.13: Adding the minaret balcony above the joint in the fifth rule

The sixth rule adds the lantern above the balcony (Figure 5.14). The rule was derived from the antecedent (d2) which does not belong to the values of the set (*n5*). Therefore, the diversity value at this stage of derivation is the same as the previous stage. The variable state label in the LHS of this rule is (*n6*) which is normally defined in the user guide grammar for hybrid design as the value of the initial variable (*n5*) minus the antecedent (d2). The sequence of this rule in the derivation is (y – 2), therefore the symbol (′) is added to the values of (*n6*) to constrain the choice of next rule to only the rules that have head markers in their RHS. The markers in the RHS of this rule are the secondary markers of a joint (+◄) which allows adding a head above the joint or adding a balcony followed by a lantern or a head above the joint.

Rule no.6	Example 2: Hybrid design derivation - 8 Original rules
OR 1e	

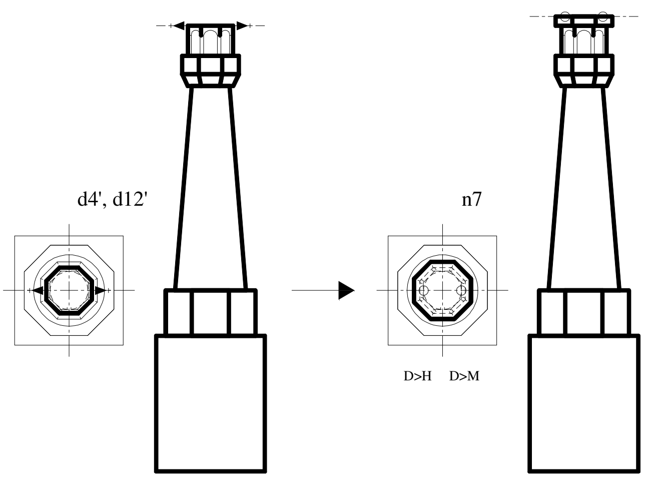
The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n6 = {n5\d2} n6 = {d1', d4', d6', d7', d8', d12'}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.75	Design diversity = 0.5 Design abundance = 1.333 Matching degree = 0.333 Geometrical difference = 0.735 Sequential difference = 0.569

Figure 5.14: Adding the minaret lantern above the balcony in the sixth rule

The seventh rule adds the octagonal joint above the lantern (Figure 5.15). The rule was derived from multi-antecedents (d4 and d12) which both of them belong to the set that defines the variable state label (*n6*). This rule enhances the diversity and abundance values of the generated design at this stage. The user guide grammar for hybrid design defines the variable state label (*n7*) as the values of (*n6*) minus the antecedents (d4 and d12). The markers in the RHS of the penultimate rule are the head main markers which direct the grammar user to add the head with any base shape at the final stage of minaret derivation. Therefore, the final rule in the derivation adds the conical head with polygonal base above the joint (Figure 5.16). The variable state label (*n8*), at the RHS of this rule, is defined by the user guide grammar for hybrid design as zero.

Rule no.7	Example 2: Hybrid design derivation - 8 Original rules

OR 9c

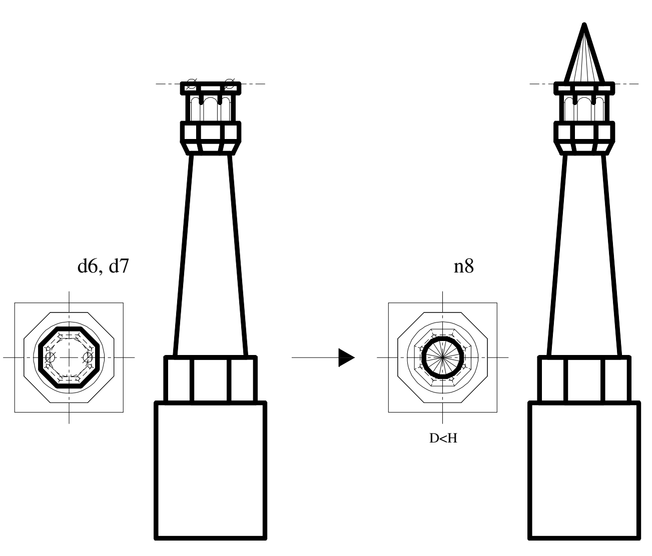


The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n7 = {n6\d4, d12} n7 = {d1, d6, d7, d8}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.666	Design diversity = 0.666 Design abundance = 1.428 Matching degree = 0.285 Geometrical difference = 0.749 Sequential difference = 0.583

Figure 5.15: Adding the minaret joint above the lantern in the seventh rule

Rule no.8	Example 2: Hybrid design derivation - 8 Original rules

OR 6f



The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n8 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.0	Design diversity = 0.833 Design abundance = 1.5 Matching degree = 0.25 Geometrical difference = 0.759 Sequential difference = 0.510

Figure 5.16: Adding the minaret head above the joint at the last stage of design generation

The evaluation of the generated design presents the innovation values of the final hybrid minaret. The values are as follows:

- Diversity value is 0.833, whereas the maximum diversity in all hybrid design is (1) which results from selecting rules belong to all antecedents in the corpus; and the minimum diversity of hybrid designs composed of original rules is 0.083 which results from choosing all original rules which belongs to the same one antecedent in the corpus.
- Abundance value is 1.5. The minimum abundance in all hybrid designs composed of original rules is (1) which results from selecting all original rules derived from only one antecedent in the corpus.
- Matching value is 0.25 to both (d2 and d3) antecedents. The minimum matching value in case of hybrid designs composed of eight original rules is (0.125) which results from antecedents having one or none rule in the generated design, and the maximum matching value is (1.0) which results from one common antecedent among all applied rules.
- Geometrical difference value is 0.759, whereas the maximum geometrical difference in all hybrid designs composed of original rules is (0.916) which results from applied rules belonging to one antecedent and having no similar geometries among the other antecedents in the corpus. The minimum geometrical difference in all hybrid designs composed of original rules is (0) which results from applying rules having similar geometries to all the antecedents in the corpus.
- Sequential difference value is 0.51, whereas the maximum sequential difference in all hybrid designs is (0.916) which results from applying original rules belonging to one antecedent and having no similar sequence among the other antecedents in the corpus. The minimum sequential difference in all hybrid designs is (0) which results from applying rules having a similar sequence to all the antecedents in the corpus.

The termination rule, which has the state label (0) in the LHS, is applied to delete the centre lines from the configuration (Figure 5.17).

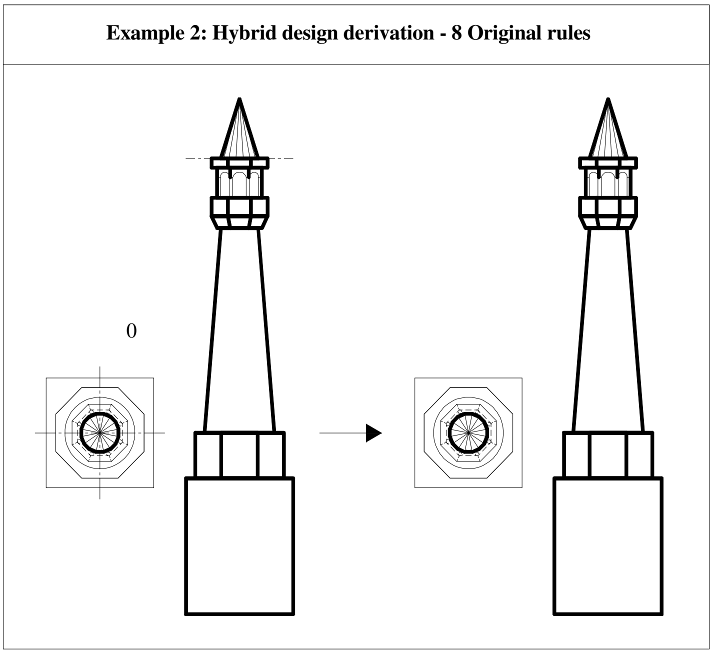


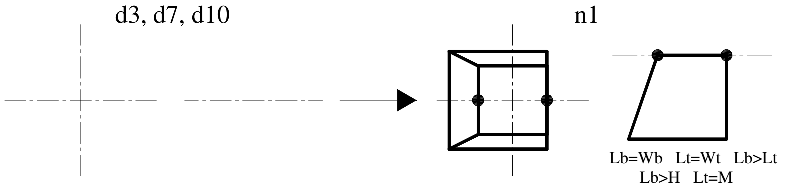
Figure 5.17: The termination rule

5.2.1.2. Examples of hybrid minaret using hybrid rules

The hybrid rule is derived by merging two original rules (see appendix A-2-1). In shape grammars for hybrid minarets, the original rules and hybrid rules are organised separately from each other (see appendix A-1 and A-2-2). In this section, the study shows two hybrid designs derived using seven and eight hybrid rules belonging to the sub-class rules sets of minaret hybrid rules. Both symmetrical and asymmetrical hybrid rules are used in the derivation.

5.2.1.2.1. First example: hybrid design composed of 7 original rules

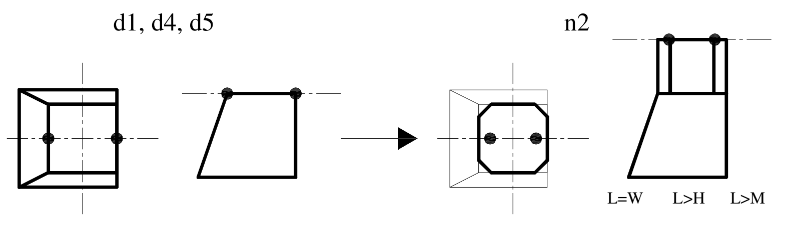
The first rule in the derivation belongs to the sub-class of hybrid rules of minaret bases (Figure 5.18). The rule was derived by merging the base configurations of two original rules which belong to three antecedents (d3, d7 and d10). The user guide grammar for hybrid designs defines the variable state label (*n1*) as the set of (*n*) minus the antecedents (d3, d7 and d10). The markers in the RHS of the first rule are the main markers of the body which allows the user to add a body with any boundary shape. Therefore, the second rule belongs to the sub-class set of bodies' hybrid rules and adds a short body above the base.

Rule no.1	Example 1: Hybrid design derivation - 7 Hybrid rules
HR 12a	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n1 = {n\d3, d7, d10} n1 = {d1, d2, d4, d5, d6, d8, d9, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversit y= 0.25 Design abundanc e= 3.0 Matching degre e= 0.5 Geometrical difference = 1.0 Sequential difference = 0.5

Figure 5.18: Adding the minaret base at the first stage of hybrid minaret generation

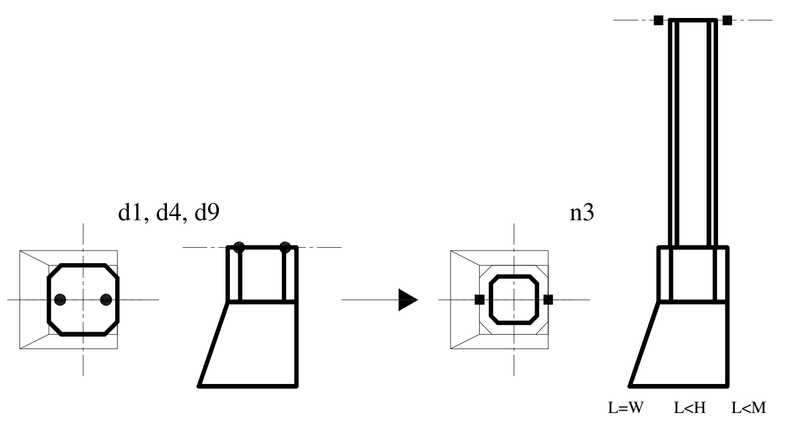
The second rule was derived by combining the characteristics of two original rules of sub-class rule set of minaret bodies (Figure 5.19). The variable state label (*n2*) was defined in the user guide grammar by excluding the antecedents (d1, d4, and d5) from the set (*n1*) which defines the variable state label of the previous rule. The markers in the RHS of this rule are also body main markers which guide the next selection to the same sub-class rule set of minaret bodies. The evaluation of the grammar shows the innovation measures of the rules and generated designs at each step of the derivation.

Rule no.2	Example 1: Hybrid design derivation - 7 Hybrid rules
HR 16b	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n2 = {n1\d1, d4, d5} n2 = {d2, d6, d8, d9, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 3.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5

Figure 5.19: Adding the minaret body above the base in the second rule

The third rule in the grammar was selected from the sub-class rule set of minaret bodies which adds the second body above the first one (Figure 5.20). The hybrid rule has three antecedents, only one of them belongs to the set (*n2*) which defines the variable state label of the previous rule. The markers in the RHS of this rule are the main markers of the balcony which allows adding a balcony without any constraints on its format and its order.

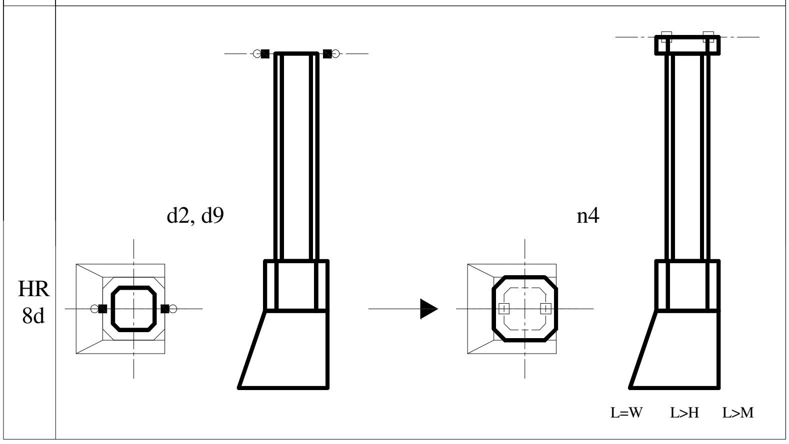
Rule no.3	**Example 1: Hybrid design derivation - 7 Hybrid rules**
HR 15b	

The user guide grammar for hybid designs	**Evaluation of the rule in the grammar**	**Evaluation of the design in the grammar**
n3 = {n2\ d1, d4, d9} n3 = {d2, d6, d8, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.583 Design abundance = 3.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.638

Figure 5.20: Adding the minaret second body in the third rule

The fourth rule was chosen from the sub-class of hybrid rules set of minaret balconies to add the balcony followed by a lantern (Figure 5.21). The hybrid rule was derived by merging two original rules extracted from the antecedents (d2 and d9). The antecedent (d2) belongs to the set (*n3*) which defines the variable state label of the previous rule. The markers in the RHS of this rule are the lantern main markers which allow adding a lantern without any constraint on its boundary shape. The evaluation of the grammar displays the innovation measures at this stage.

Rule no.4	Example 1: Hybrid design derivation - 7 Hybrid rules

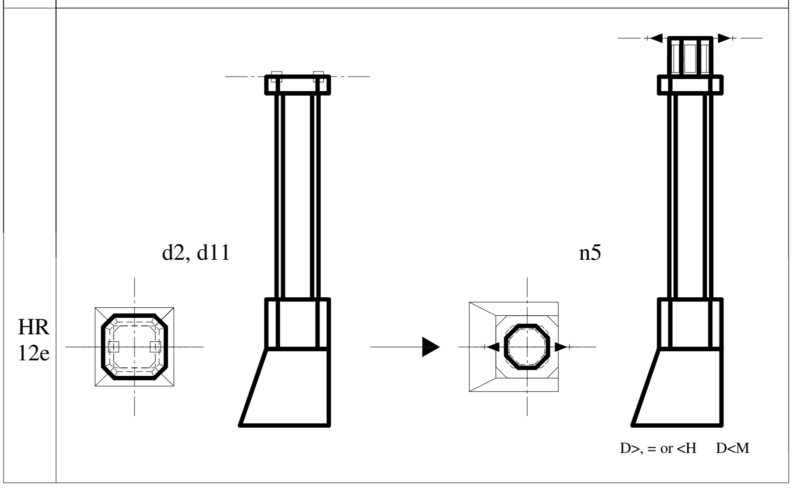


The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n4 = {n3\d2, d9} n4 = {d6, d8, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.666 Design abundance = 2.75 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.687

Figure 5.21: Adding the minaret balcony above the second body in the fourth rule

The fifth rule is one of the hybrid rules in the lantern sub-class rule set. It adds an octagonal lantern followed by a joint (Figure 5.22).

Rule no.5	Example 1: Hybrid design derivation - 7 Hybrid rules



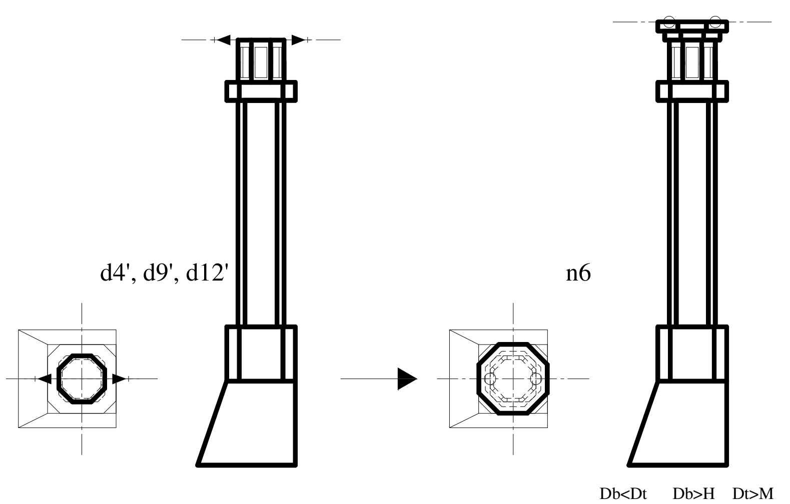
The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n5 = {n4\d2, d11} n5 = {d6', d8', d12'}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.75	Design diversity = 0.75 Design abundance = 2.6 Matching degree = 0.2 Geometrical difference = 1.0 Sequential difference = 0.699

Figure 5.22: Adding the minaret lantern above the balcony in the fifth rule

The rule has two antecedents (d2 and d11) whereas (d11) belongs to the set (*n4*). The total number of rules in this example is seven; therefore the sequence of the fifth rule is ($y – 2$) which requires adding the symbol (′) to the set (*n5*) of variable state label in the user guide grammar. In this case, the choice of the sixth rule is constrained with rules that add head markers to the generated minaret.

The sixth rule is one of the hybrid rules in the sub-class hybrid rules set of minaret joints. It adds the joint component followed by a head (Figure 5.23). The hybrid rule has three antecedents; one of them (d12) belongs to the values of the variable state label (*n5*) of the previous rule. The user guide grammar defines the values for the variable state label (*n6*) as the possible antecedents for the next rule. The markers in the RHS of this rule are the head markers which guide the grammar user toward the subclass hybrid rules set of minaret head. The evaluation of the grammar displays the innovation measures of the generated design which are calculated and prompted at grammar runtime.

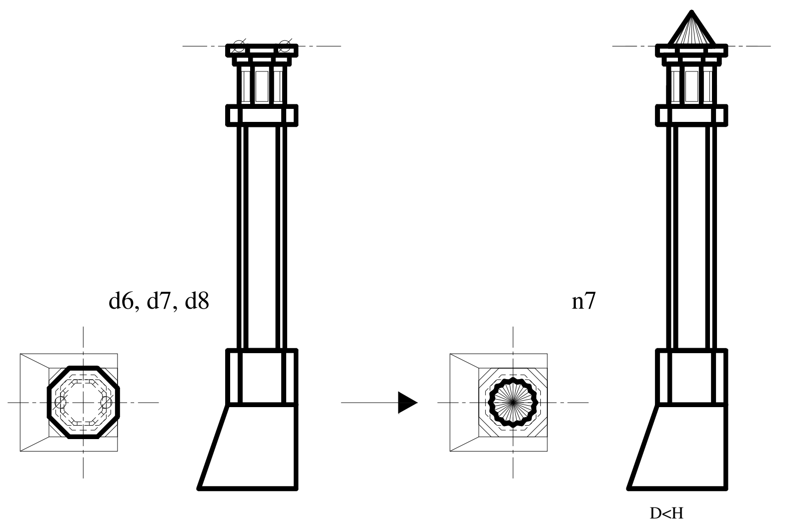
Rule no.6	Example 1: Hybrid design derivation - 7 Hybrid rules
HR 14c	



The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n6 = {n5\d4, d9, d12} n6 = {d6, d8}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.666	Design diversity = 0.833 Design abundance = 2.666 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.694

Figure 5.23: Adding the minaret joint above the lantern in the sixth rule

The final rule adds the head component above the joint (Figure 5.24). The rule has three antecedents (d6, d7 and d8); two of them belong to the set (*n6*) that define the variable state label of the previous rule. In this step, all antecedents in the corpus have hybrid rules in the generated design. Therefore, the value of the diversity at the final stage is (1.0).

Rule no.7	Example 1: Hybrid design derivation - 7 Hybrid rules
HR 8f	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n7 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 2.714 Matching degree = 0.214 Geometrical difference = 1.0 Sequential difference = 0.595

Figure 5.24: Adding the minaret head above the joint at the final stage of derivation

The evaluation of the generated design presents the innovation values of the final hybrid design composed of seven hybrid rules. The values are as follows:

- Diversity value is 1.0 which is the maximum diversity in all hybrid design. It results from selecting rules which belong to all antecedents in the corpus. The minimum diversity in hybrid designs composed of hybrid rules is almost (0.166) which results from choosing hybrid rules derived by merging two original rules extracted from the same two antecedent in the corpus.

- Abundance value is 2.714, whereas the minimum abundance in hybrid designs composed of hybrid rules is almost (2). The minimum abundance results from selecting all hybrid rules derived by merging two original rules which belongs to two antecedents in the corpus.
- Matching value is 0.214 to both (d4 and d9) antecedents, whereas the minimum matching value in case of hybrid designs composed of seven hybrid rules is (0.142) which results from antecedents having at least two hybrid rules in the generated design. The maximum matching value in hybrid designs composed of hybrid rules is (0.5) which results from applying hybrid rules having one common antecedent among all of them. The reason why the maximum matching is (0.5) in case of hybrid design composed of hybrid rules and (1.0) in case of hybrid designs composed of original rules is attributed to the fact that each hybrid rule partially matches the antecedent in contrast to the original rule which completely matches the antecedent. Therefore, the calculation of matching degree of the hybrid rule is multiplied by (0.5).
- Geometrical difference value is 1.0 which is the standard value in hybrid minarets composed of hybrid rules. In shape grammars for hybrid minarets, all hybrid rules have new geometries derived by merging their parents' geometries. Therefore, the new geometry of the hybrid rule differs than all antecedents in the corpus.
- Sequential difference value is 0.595; whereas the maximum sequential difference in all hybrid designs composed of hybrid rules is (0.916) resulted from the application of hybrid rules having the sequence of one of their parent rules which belongs to one antecedent and has no similar sequence among other antecedents in the corpus. On the other hand, the minimum sequential difference in all hybrid designs composed of hybrid rules is (0), which results from the application of hybrid rules having the sequence of one of their parent rules which has a similar sequence to all antecedents in the corpus.

The termination rule, which has the state label (0) in its LHS, is applied to delete the centre lines from the final top and front views of minaret (Figure 5.25).

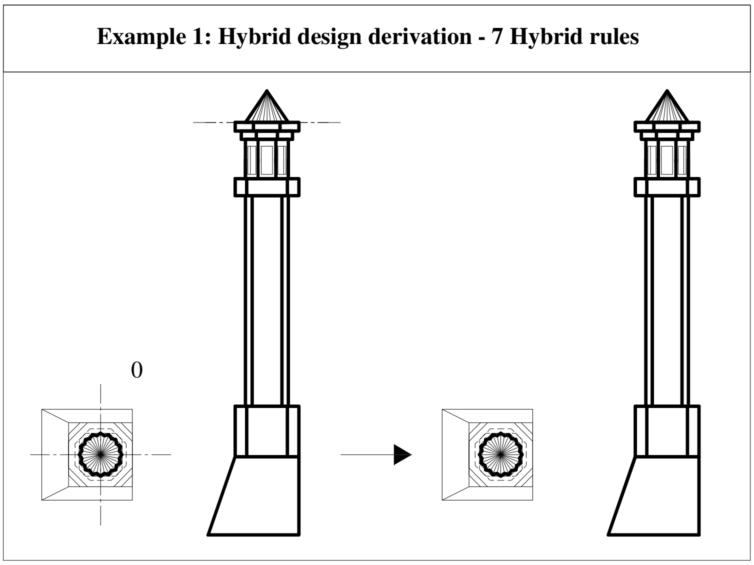
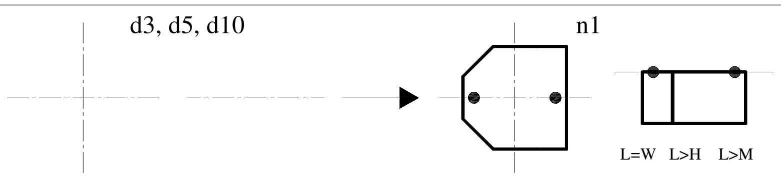


Figure 5.25: Termination rule

5.2.1.2.2. Second example: hybrid design composed of 8 original rules

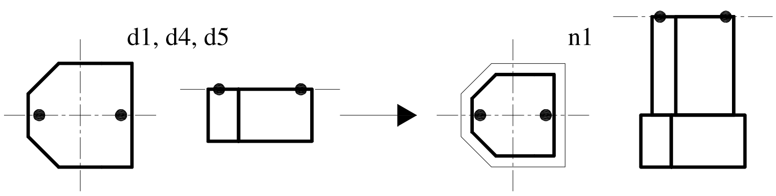
The second example was composed of 8 hybrid rules. The first hybrid rule initiates the generation by adding the base from the sub-class hybrid rules set (Figure 5.26). This rule was derived by merging two original rules belonging to three antecedents (d3, d5 and d10) which are excluded from the set that defines the variable state label (*n1*). The markers in the RHS of this rule are body main markers which guide the selection of the next rule to the sub-class hybrid rules set of minaret bodies.

Rule no.1	Example 2: Hybrid design derivation - 8 Hybrid rules
HR 7a	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n1 = {n\d3, d5, d10} n1 = {d1, d2, d4, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5

Figure 5.26: Adding the minaret base at the first stage of hybrid minaret derivation

The second rule adds the first body above the base (Figure 5.27). The rule has three antecedents (d1, d4 and d5), two of them belong to the set (*n1*) which defines the variable state label of the previous rule. The markers of the RHS of this rule are the body main markers which allow the user to add a body without any restriction on its shape. The evaluation of the generated design calculates at grammar runtime the innovation measures at this stage of derivation.

Rule no.2	Example 2: Hybrid design derivation - 8 Hybrid rules
	

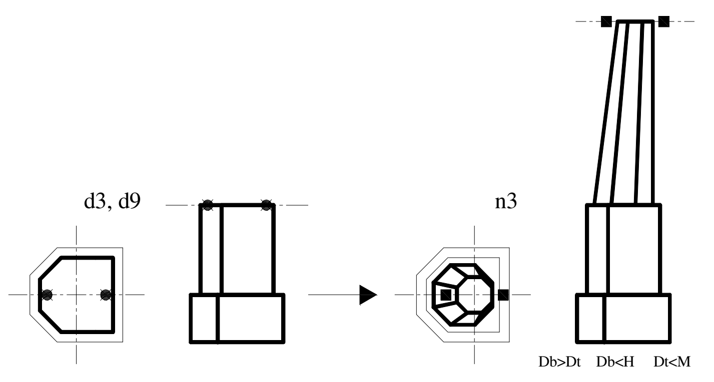
The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n2 = {n1\d1, d4, d5} n2 = {d2, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.416 Design abundance = 3.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5

Figure 5.27: Adding the minaret body above the base in the second rule

The third rule adds the minaret second body above the first body (Figure 5.28). The rule has two antecedents; one of them belongs to the set (*n2*) which defines the variable state label of the previous rule. The markers in the RHS of this rule are the main markers of the balcony which allows adding a balcony having any boundary shape and followed by any minaret component. The evaluation of the hybrid design displays at grammar runtime the innovation values at this stage of design derivation.

Rule no.3	Example 2: Hybrid design derivation - 8 Hybrid rules

HR 21b



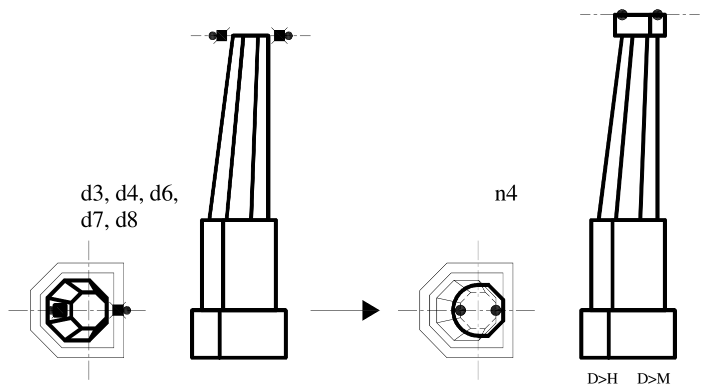
The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n3 = {n2\d3, d9} n3 = {d2, d6, d7, d8, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.5 Design abundance = 2.666 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.638

Figure 5.28: Adding the minaret second body above the first body in the third rule

The fourth rule adds the balcony above the body (Figure 5.29). The rule has multi-antecedents where three of them belong to the set (*n3*) which defines the variable state label of the previous rule. The markers in the RHS of this rule are the body main markers which direct the next selection to the sub-class rule set of minaret bodies.

Rule no.4	Example 2: Hybrid design derivation - 8 Hybrid rules

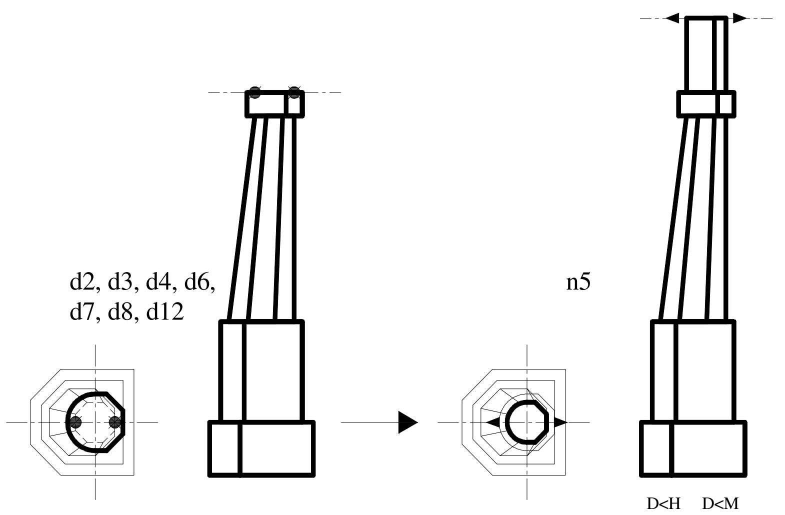
HR 12d



The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n4 = {n3\d3, d4, d6, d7, d8} n4 = {d2, d11, d12}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.75 Design abundance = 3.25 Matching degree = 0.375 Geometrical difference = 1.0 Sequential difference = 0.604

Figure 5.29: Adding the minaret balcony above the second body in the fourth rule

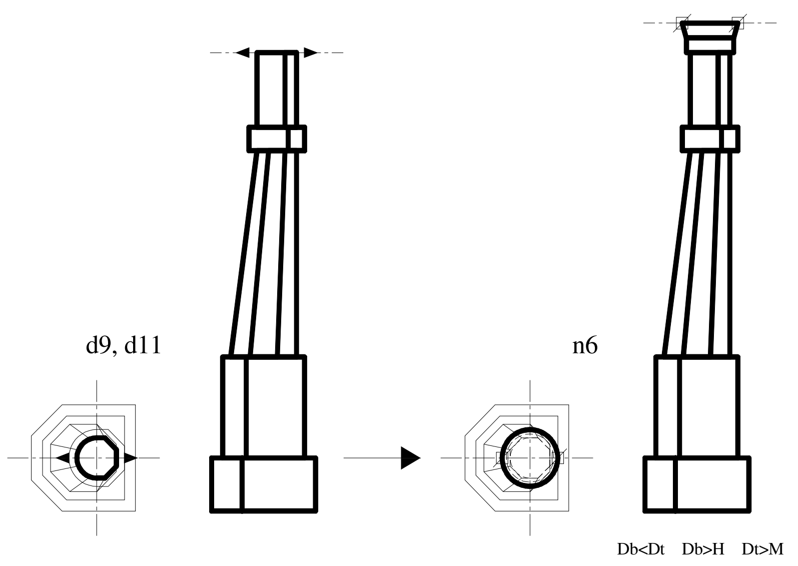
The fifth rule adds the third body above the balcony (Figure 5.30). The rule has seven antecedents; two of them belong to the set (*n4*) which defines the variable state label of the previous rule. The markers in the RHS of this rule are the main markers of the joint which allow adding a joint followed by any component.

Rule no.5	Example 2: Hybrid design derivation - 8 Hybrid rules
HR 13b	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n5 = {n4\d2, d3, d4, d6, d7, d8, d12} n5 = {d11}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.916 Design abundance = 4.0 Matching degree = 0.4 Geometrical difference = 1.0 Sequential difference = 0.549

Figure 5.30: Adding the minaret balcony above the second body in the fifth rule

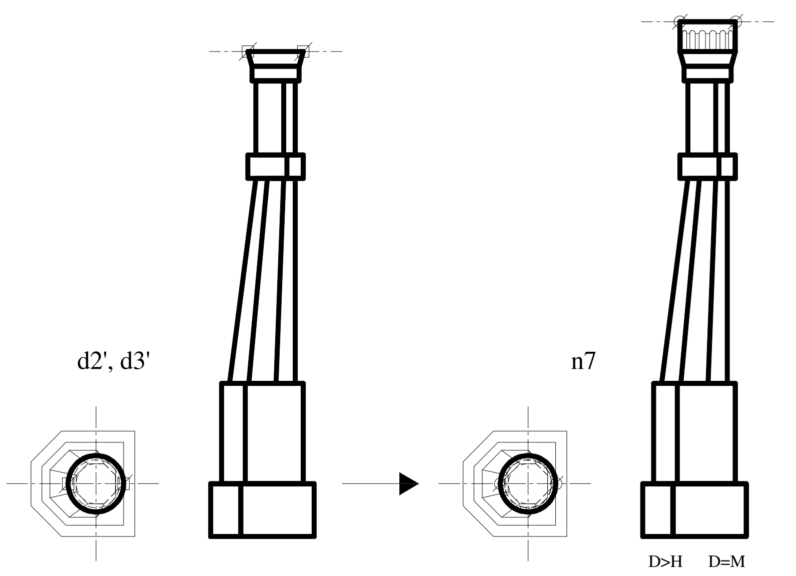
The sixth rule adds a joint followed by a lantern above the third body (Figure 5.31). The rule was derived from two antecedents (d9 and d11); one of them belongs to the set of (*n5*) which consists of one antecedent (d11). Therefore, the variable state label (*n6*) is defined as (Ø) because all antecedents have been exhausted in the previously applied rules. The user guide grammar replaces the variable (*n6*) with the alternative variable (*n6**). The user guide grammar defines the variable state label (*n6**) as it represents the set (*m*) of antecedents which have the less number of applied rules in the previous steps of the shape generation minus the antecedents of the current rule. The diversity value of the generated design at this stage is (1.0) as the maximum diversity which remains the same in the following steps of rule applications. In addition the sequence of this rule is (y – 2) whereas (y=8) is the total number of rules in this example. Therefore, the symbol (′) was added to the set of ($n6^*$) to constrain the choice of the next rule within the rules that have the head markers at their RHS. The markers in the RHS of this rule are the lantern secondary markers which allow adding the circular lantern only.

Rule no.6	Example 2: Hybrid design derivation - 8 Hybrid rules
HR 17c	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n6 = {n5\d9, d11} n6 = {Ø} n6 = n6* n6* = {m\d9, d11} n6* = {d1', d2', d5', d10', d12'}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 1.0 Design abundance = 3.666 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.596

Figure 5.31: Adding the minaret joint above the second body in the sixth rule

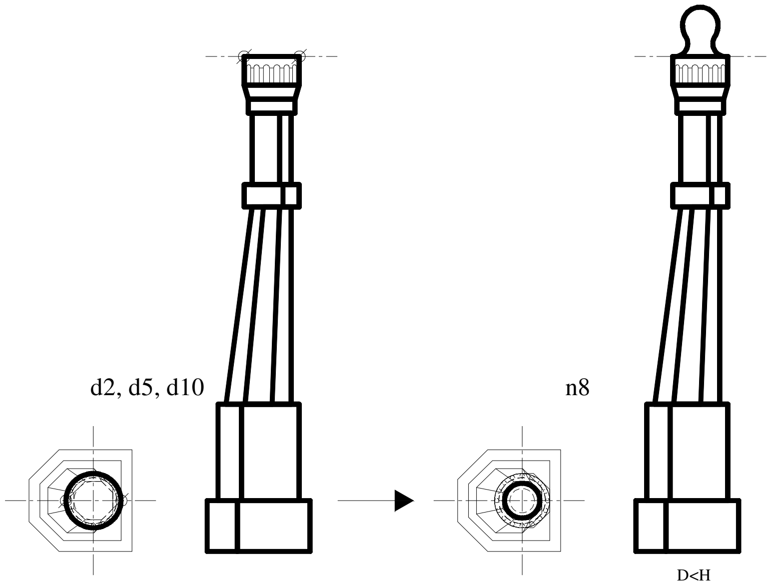
The seventh rule adds the circular lantern above the joint (Figure 5.32). The user guide grammar defines the variable state label of this rule ($n7$) depending on the empty set of ($n6$). Therefore, the user guide grammar define ($n7$) as (Ø) and replaces it with the variable ($n7^*$) which represent the set (m) of antecedents which have the less number of applied rules in the design derivation minus the antecedents (d2 and d3) of the current rule. The markers in the RHS of the penultimate rule are the head secondary markers which add the head having a circular base at the final stage of design generation.

Rule no.7	Example 2: Hybrid design derivation - 8 Hybrid rules
HR 6f	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n7 = {n6\d2, d3} n7 = {Ø} n7 = n7* n7* = {m\d2, d3} n7* = {d1, d5, d10, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 1.0 Design abundance = 3.428 Matching degree = 0.357 Geometrical difference = 1.0 Sequential difference = 0.642

Figure 5.32: Adding the minaret lantern above the joint in the seventh rule

The last rule adds the head component above the lantern (Figure 5.33). The rule has a variable state label ($ny = n8$) which is defined in the user guide grammar as (0). At this stage the derivation of minaret top and front views are completed. The evaluation of the grammar calculates the final innovation measures at this stage.

Rule no.8	Example 2: Hybrid design derivation - 8 Hybrid rules
HR 2f	

The user guide grammar for hybid designs	Evaluation of the rule in the grammar	Evaluation of the design in the grammar
n8 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 3.375 Matching degree = 0.312 Geometrical difference = 1.0 Sequential difference = 0.561

Figure 5.33: Adding the minaret head above the lantern at the final stage of derivation

The evaluation of the generated design presents the innovation values of the final hybrid design composed of eight hybrid rules. The values are as follows:

- Diversity value is 1.0 which is the maximum diversity in all hybrid design. It results from selecting rules belong to all antecedents in the corpus; and the minimum diversity in hybrid designs composed of hybrid rules is almost 0.166 which results from choosing hybrid rules derived by merging two original rules belonging to the same two antecedents in the corpus.
- Abundance value is 3.375, whereas the minimum abundance in hybrid designs composed of hybrid rules is almost (2). The minimum abundance results almost from selecting all hybrid rules derived from two original rules belonging to two antecedents in the corpus.

- Matching value is 0.312 to the antecedent (d3), whereas the minimum matching value in case of hybrid designs composed of eight hybrid rules is (0.125) which results from antecedents having only two hybrid rules in the generated design. The maximum matching value in hybrid designs composed of hybrid rules is (0.5) which results from applying hybrid rules having a one common antecedent in the corpus. The reason why the maximum matching is (0.5) in case of hybrid designs derived by hybrid rules and (1.0) in case of hybrid designs derived by original rules is attributed to the fact that each hybrid rule is partially matches the antecedent in contrast to the original rule which completely matches the antecedent. Therefore, the calculation of matching degree of the hybrid rule is multiplied by (0.5).
- Geometrical difference value = 1.0, is the standard value in hybrid minarets composed of hybrid rules. In shape grammars for hybrid minarets, all hybrid rules have new geometry derived from merging their parents' geometries. Therefore, the new geometry of the hybrid rule has the maximum geometrical difference than all antecedents in the corpus.
- Sequential difference value is 0.561, whereas the maximum sequential difference in all hybrid designs composed of hybrid rules is (0.916) which results from using hybrid rules having the sequence of one of their parent rules which belongs to one antecedent and have no similar sequence among the other antecedents in the corpus. On the other hand, the minimum sequential difference in all hybrid designs composed of hybrid rules is (0), which results from the application of hybrid rules having the sequence of one of their parent rules which has a similar sequence to all antecedents in the corpus.

The termination rule, which has the state label (0) in its LHS, is applied to delete the centre lines from the final top and front views of minaret (Figure 5.34).

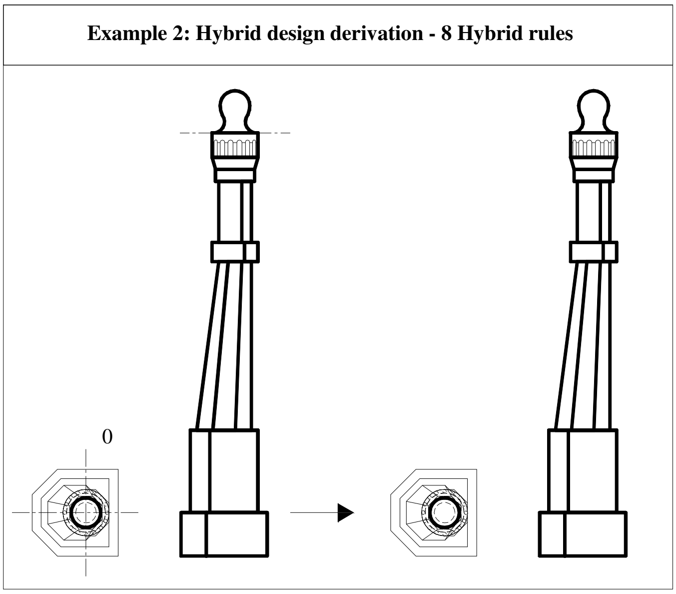


Figure 5.34: Termination rule

5.2.1.3. Stage 1: Main findings

The implementation of shape grammars for hybrid designs in stage 1 shows successive examples of deriving hybrid minarets. Four examples are derived using (7 or 8) original rules (ORs) and hybrid rules (HRs) separately. These numbers of rules are the average in the corpus of antecedents which varies between (3-12) rules. The innovation measures of these examples show varied values of diversity, abundance and geometrical difference between hybrid designs composed of original and hybrid rules (Table 5.1).

Table 5.1: Innovation measures of hybrid minaret examples

	Hybrid minaret using ORs		Hybrid minaret using HRs	
Innovation measures	**7 original rules**	**8 original rules**	**7 original rules**	**8 original rules**
Diversity	0.75	0.833	1.0	1.0
Abundance	1.571	1.5	2.714	3.375
Matching degree	0.285	0.25	0.214	0.312
Geometrical difference	0.725	0.759	1.0	1.0
Sequential difference	0.485	0.510	0.595	0.561

There are no significant differences in both matching degree and sequential difference values between hybrid designs composed of original and hybrid rules. In three examples, the higher values of matching were to the same antecedent (d3) as shown in Figure 4.35. In spite of the hybrid design composed of hybrid rules has high matching

(0.312) to the antecedent d3, even so the maximum geometrical difference value (1.0) boosts its individuality.

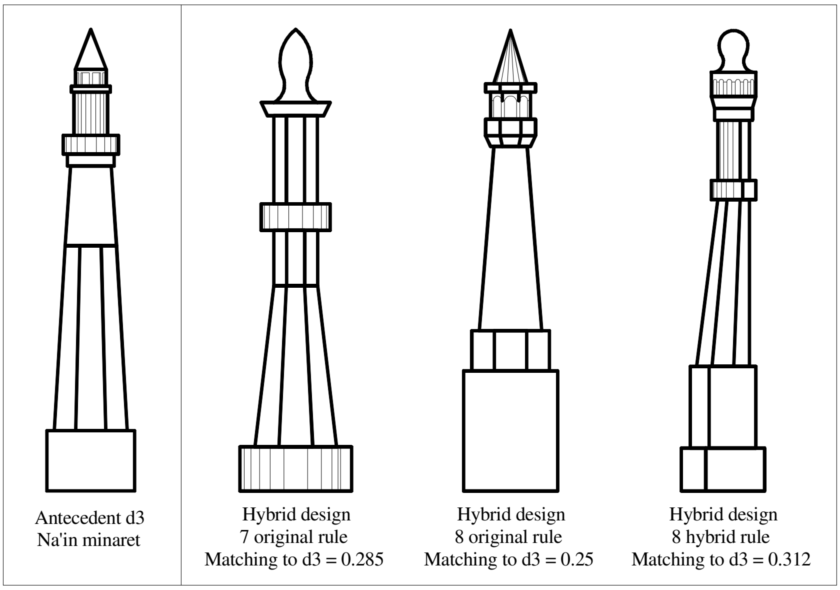


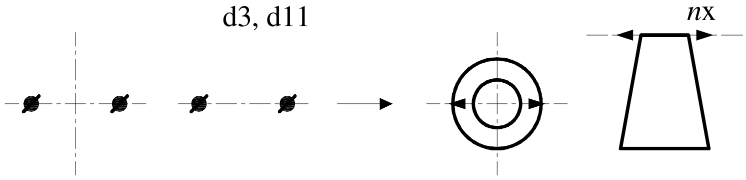
Figure 5.35: Matching degree of three examples of hybrid minaret to the antecedent (d3)

However, the validity of the innovation measures of hybrid designs is questionable and needs investigation for verification. Therefore, the second stage aims to verify the innovation measures of hybrid designs.

5.2.2. Stage 2: Innovation measures of hybrid designs versus copies of existing designs

The second stage investigates the validity of the proposed innovation measures of hybrid design by applying them to both non-hybrid and hybrid designs. The ability of these measures to distinguish non-hybrid designs from hybrid ones is the proof of their reliability. Therefore, the study compares the values of proposed innovation measures of copies of existing designs with hybrid designs composed of original rules and hybrid rules separately.

To derive a copy of antecedent, the user guide grammar for hybrid designs is replaced by the user guide grammar for copies of existing designs. The latter guide defines the variable state label in the RHS of each rule as identical to one of the antecedents in the LHS of the same rule. The variable state labels (*n1* & *nx*) in the RHS of the applied rules are replaced by the antecedent to be copied (Figure 5.36). For detailed derivations of copies of existing designs, please refer to appendix section B-1.

Rule no.	**Original rule of sub-class rule set of minaret bodies**
OR 5b	

The user guide grammar for hybrid designs	**The user guide grammar for copies of existing designs**
*n*x = {*n*(*x*-1)\d3, d11}	either *n*x = d3 If the generated copy is d3 or *n*x = d11 If the generated copy is d11

Figure 5.36: The user guide for copies of the antecedents

The number of rules in the corpus of twelve antecedents varies among (3, 5, 6, 7, 9, 10, 11 and 12) rules. Fair comparison demands that the innovation values of each copy of existing design is compared with two corresponding hybrid designs derived from the same number of original rules and hybrid rules. Innovation measures are calculated for twelve copies of antecedents (Figure 5.37) and twenty four hybrid designs: twelve of them are composed of original rules (ORs) (Figure 5.38), and the other are composed of hybrid rules (HRs) (Figure 5.39). For detailed derivations of the 24 hybrid minarets, please refer to appendix section B-2 for hybrid minarets composed of (ORs), and section B-3 for hybrid minarets composed of (HRs).

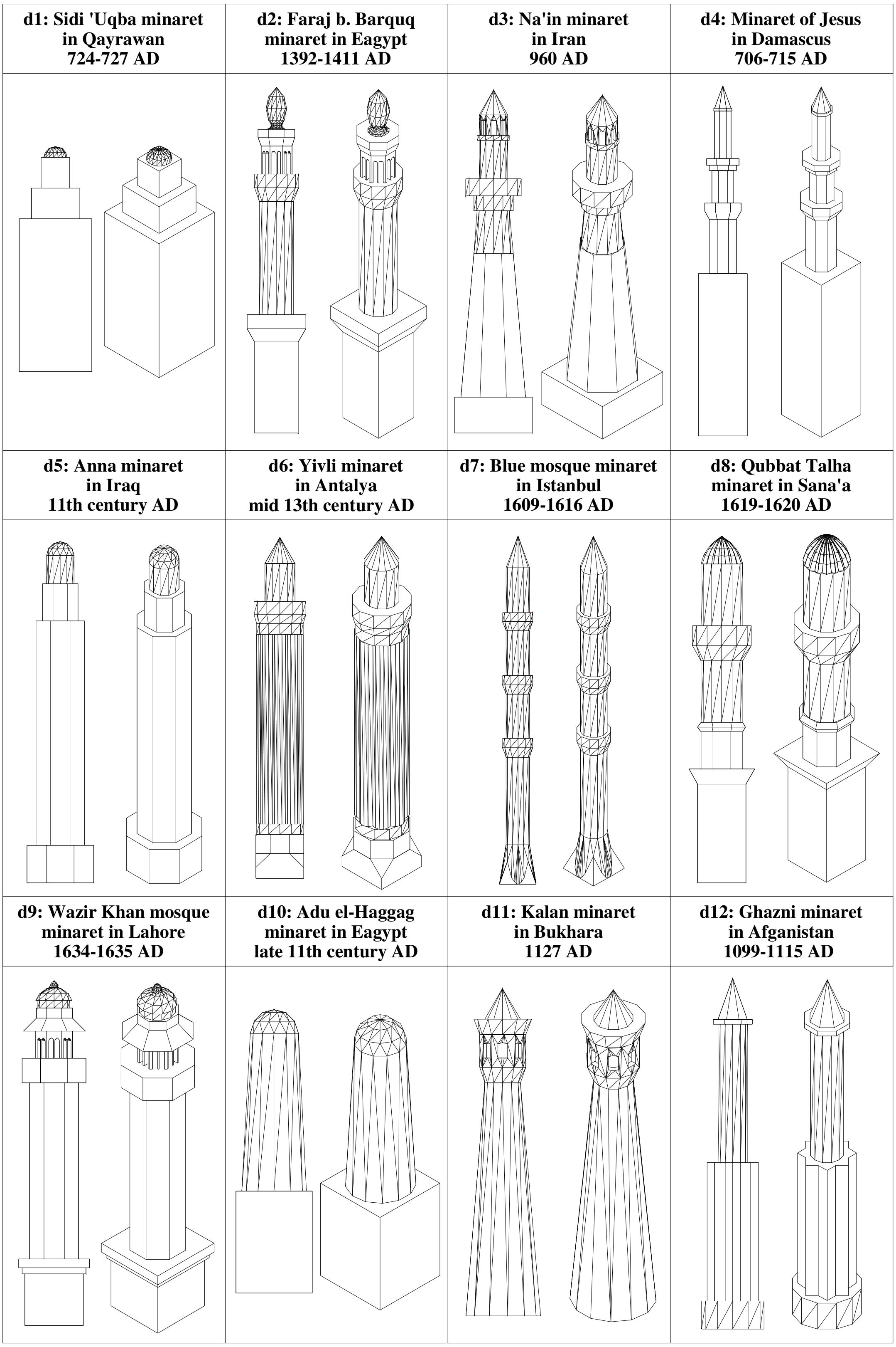


Figure 5.37: Copies of existing designs

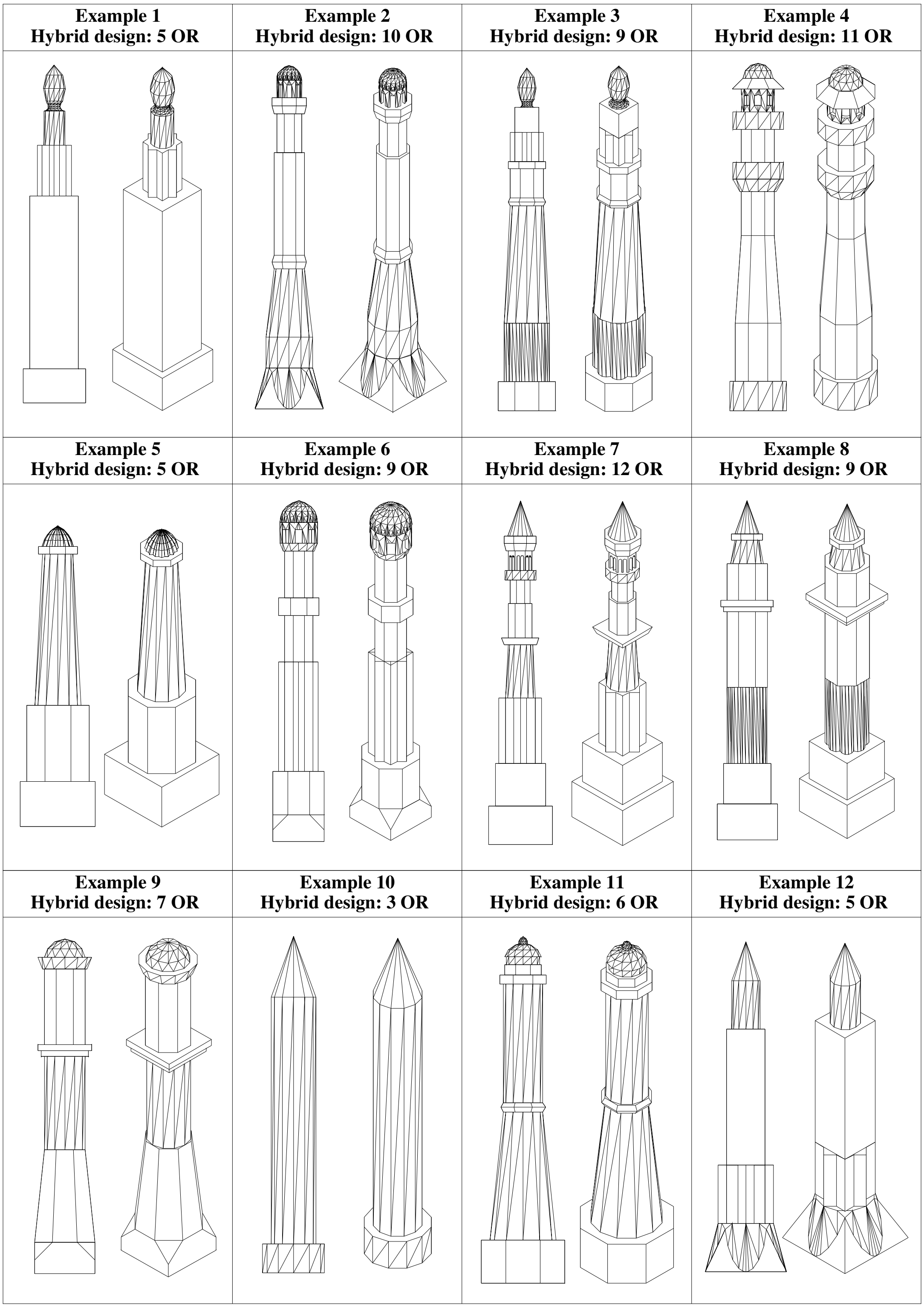


Figure 5.38: Hybrid designs composed of original rules

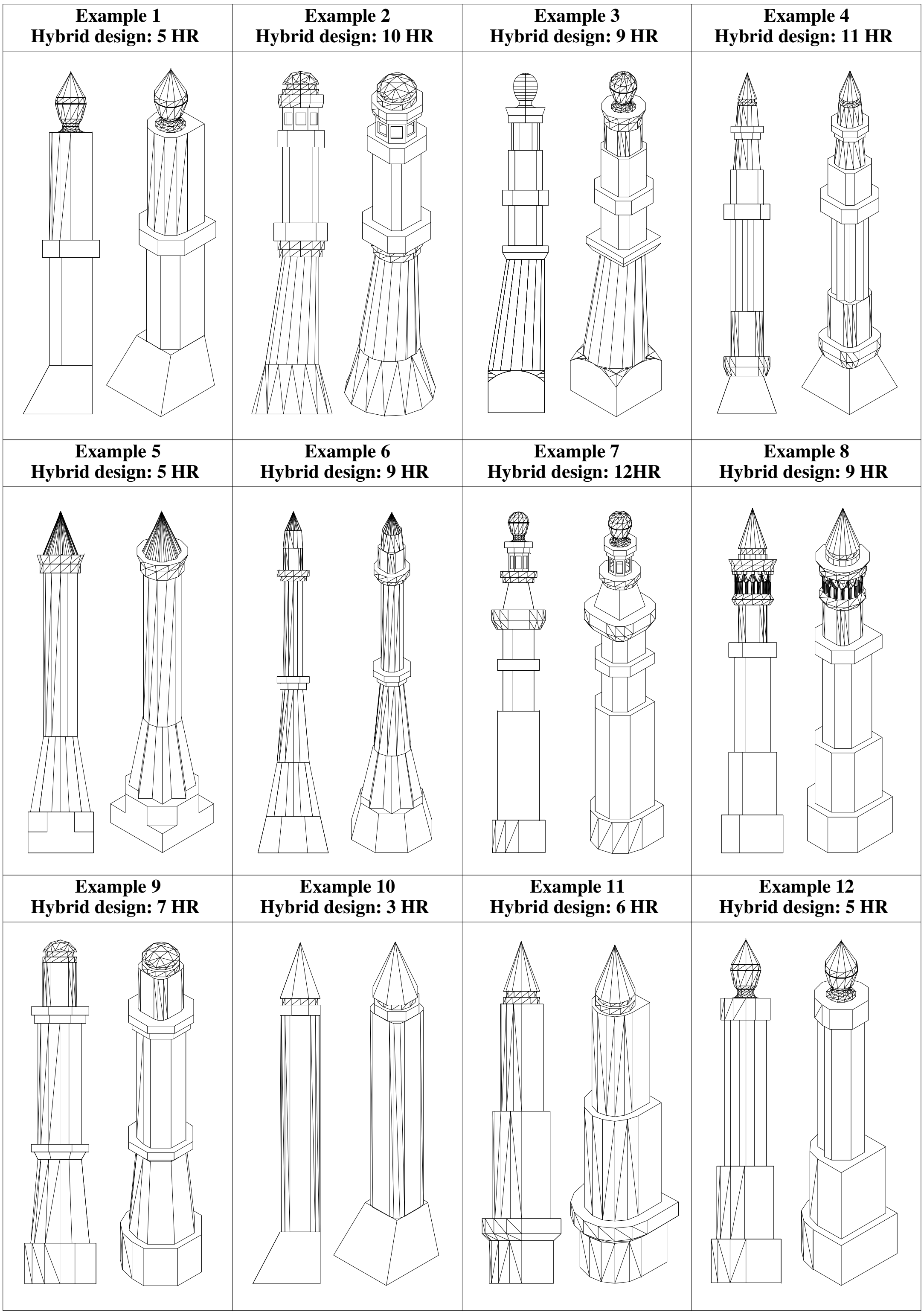


Figure 5.39: Hybrid designs composed of hybrid rules

5.2.2.1. Results of innovation measures

Innovation metrics measure the mixed character of the generated design via diversity and abundance on one hand, and the individuality of the generated design via matching degree, geometrical difference and sequential difference on the other hand. To avoid the effect of different numbers of rules on the innovation measures, the results of each innovation measure in each copy of antecedent is compared with the results of the same measure of two hybrid minarets composed of the same number of original rules and hybrid rules separately. Table 5.2 shows the results of the innovation measures in copies of existing minarets, hybrid minarets composed of original rules, and hybrid minarets composed of hybrid rules.

Table 5.2: Results of innovation measures of copies of existing designs and hybrid designs

Innovation measures of copies of existing designs												
State labels	**d1**	**d2**	**d3**	**d4**	**d5**	**d6**	**d7**	**d8**	**d9**	**d10**	**d11**	**d12**
No. of ORs	**5**	**10**	**9**	**11**	**5**	**9**	**12**	**9**	**7**	**3**	**6**	**5**
Diversity	0.25	0.583	0.666	0.416	0.416	0.583	0.583	0.75	0.25	0.25	0.333	0.583
Abundance	1.8	1.7	2.222	1.727	1.8	2.444	3.75	2.777	1.428	1.666	1.666	2.4
Matching	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Geo. diff	0.833	0.732	0.762	0.741	0.666	0.61	0.575	0.647	0.797	0.722	0.735	0.782
Seq. diff.	0.466	0.557	0.489	0.446	0.4	0.425	0.423	0.509	0.69	0.333	0.555	0.399
Innovation measures of hybrid designs derived by original rules												
No. of ORs	**5**	**10**	**9**	**11**	**5**	**9**	**12**	**9**	**7**	**3**	**6**	**5**
Diversity	0.833	0.833	0.833	0.916	0.583	0.833	1.0	0.916	0.916	0.583	0.833	0.75
Abundance	2.0	1.2	1.111	1.636	1.6	1.444	1.333	1.555	2.0	2.333	2.166	2.0
Matching	0.2	0.2	0.111	0.272	0.4	0.222	0.25	0.333	0.285	0.333	0.333	0.4
Geo. diff.	0.732	0.757	0.832	0.726	0.732	0.731	0.749	0.758	0.737	0.721	0.791	0.782
Seq. diff.	0.4	0.524	0.509	0.545	0.399	0.555	0.589	0.462	0.452	0.333	0.444	0.4
Innovation measures of hybrid designs derived by hybrid rules												
No. of HRs	**5**	**10**	**9**	**11**	**5**	**9**	**12**	**9**	**7**	**3**	**6**	**5**
Diversity	0.916	1.0	1.0	1.0	0.916	1.0	1.0	1.0	1.0	0.833	0.916	0.916
Abundance	3.2	2.9	3.555	3.0	3.2	2.888	3.0	3.111	3.428	3.666	4.0	3.2
Matching	0.2	0.3	0.222	0.181	0.3	0.277	0.208	0.222	0.214	0.333	0.333	0.3
Geo. diff.	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Seq. diff.	0.499	0.574	0.509	0.537	0.399	0.453	0.527	0.555	0.487	0.361	0.472	0.566

In the next paragraphs, the results are analysed to verify the measures ability to recognise the differences between copies of antecedents and hybrid designs on one hand, and between hybrid designs composed of original rules and ones composed of hybrid rules on the other hand.

5.2.2.1.1. Diversity values

The diversity results are presented in Figure 5.40. They show the different values of variety in the mixture of copies of antecedents and both hybrid designs. In all the examined cases, the hybrid designs have higher diversity values than the copies of antecedents. Additionally, the hybrid designs composed of hybrid rules have higher diversity values than hybrid designs composed of original rules.

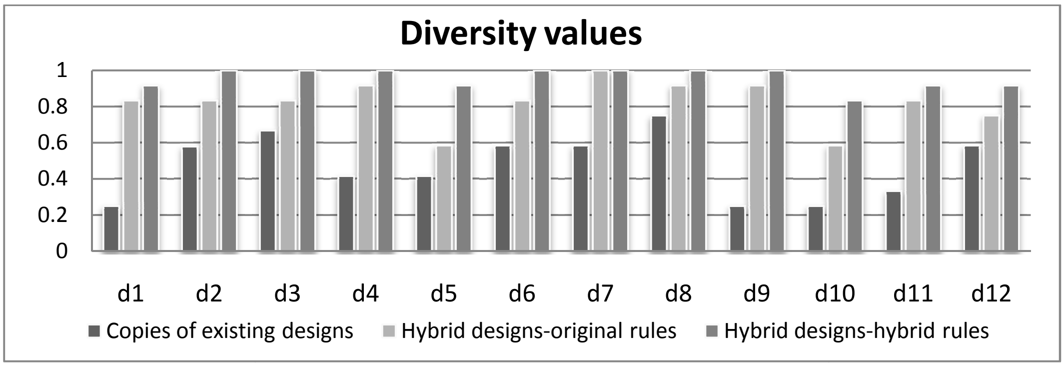


Figure 5.40: Comparing diversity values of copies of antecedents with hybrid designs

5.2.2.1.2. Abundance values

The results of abundance values in Figure 5.41 reflect the density of mixture in copies of existing designs and hybrid designs. The Graph shows that in 11 of 12 cases the abundance values of hybrid designs derived by hybrid rules are higher than the abundance values of copies of antecedents. Furthermore, in all cases, the abundance values of hybrid designs composed of hybrid rules are higher than the abundance values of hybrid designs composed of original rules.

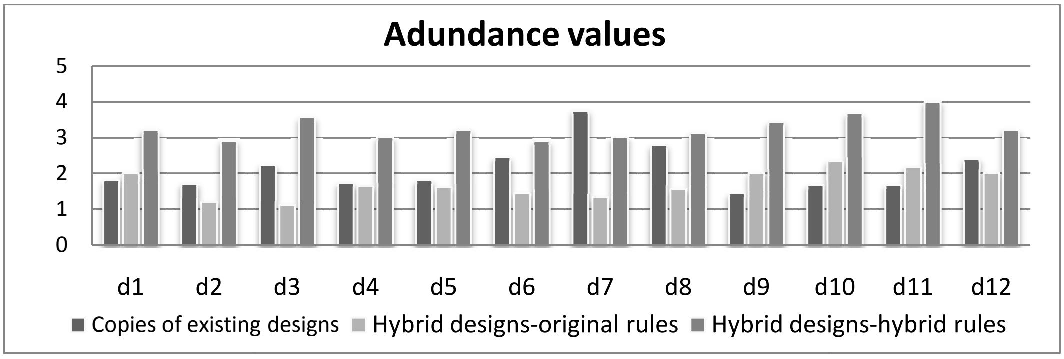


Figure 5.41: Comparing abundance values of copies of antecedents with hybrid designs

However, the abundance values of 7 existing designs are higher than the abundance values of hybrid designs derived by original rules. The reason can be attributed to the rule prevalence values which enhance the abundance results. In the grammar, the original rules with high rule prevalence value (derived from more than 3 designs) are only 5.555%, original rules with medium rule prevalence values (derived from 2 and 3 designs) are 22.222%, and original rules with low rule prevalence values (derived from one design) are 72.222% (Table 5.3). In fact, the high abundance of existing designs such as d7 results from the repetition of a rule with high rule prevalence value in contrast to a hybrid design in which the user guide grammar prevents any rule repetition, except the cases in which the set of (*nx*) in the user guide grammar is Ø. However, the high percentage of original rules with low rule prevalence value (72.222%) which were derived from one antecedent in the corpus, confirms the heterogeneous features of the antecedents.

Table 5.3: Percentage of rule prevalence values in original and hybrid rules

	Original rules	**Hybrid rules**
High rule prevalence (4-8) state labels	5.555%	18.888%
Medium rule prevalence (2-3) state labels	22.222%	74.444%
Low rule prevalence (1) state label	72.222%	6.666%

5.2.2.1.3. Matching degree

The results of matching degrees are an inverse measure of the individuality in the generated design. Figure 5.42 shows that the copies of existing designs have minimum individuality resulted from their full match to the antecedents in the corpus. On the other hand, the results of hybrid designs varied between the maximum individuality in case of the minimum matching 0.111 and the minimum individuality in case of the maximum matching 0.4. In 6 cases, hybrid designs derived from hybrid rules have higher individuality; their matching values are less than the matching values of hybrid designs derived by original rules. In three cases, the matching values of both hybrid designs derived by original and hybrid rules are identical.

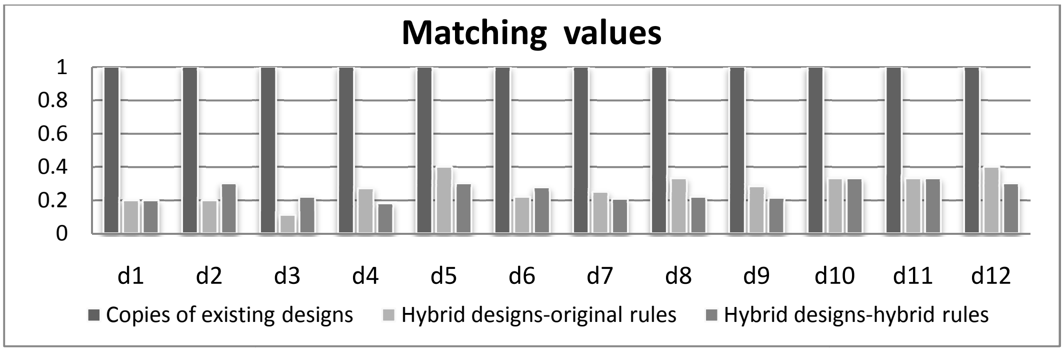


Figure 5.42: Comparing matching values of copies of existing designs with hybrid designs

5.2.2.1.4. Geometrical difference values

The results of geometrical difference values in Figure 5.43 reflect the individuality of the generated design in terms of their rule formats. They shows that hybrid designs derived from hybrid rules have maximum geometrical difference values as a result of all hybrid rules having new geometries emerged by combining features from two original rules. On the other hand, the geometrical difference values of 7 hybrid designs derived from original rules are higher than the values of copies of existing designs. In two cases, the geometrical difference values of copies of existing designs and hybrid designs composed of original rules are the same.

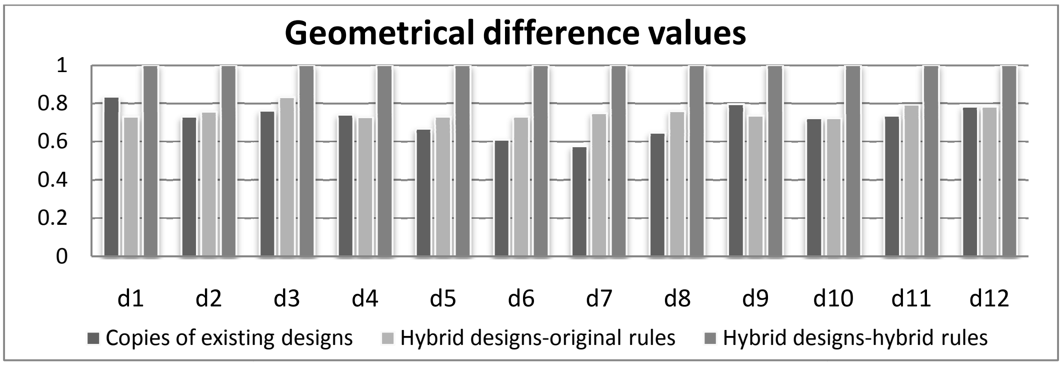


Figure 5.43: Comparing geometrical difference values of copies of existing designs and hybrid designs

5.2.2.1.5. Sequential difference values

The results of sequential difference values in Figure 5.44 reflect the individuality of the generated design in terms of the rule orders. They show that there is no remarkable difference in sequential difference values among exiting and hybrid designs. Nevertheless, in 9 of 12 cases, the hybrid designs composed of the hybrid rules have higher sequential difference values than the existing design. In addition, the sequential difference values of 7 hybrid designs consisting of hybrid rules are higher than the hybrid designs consisting of original rules.

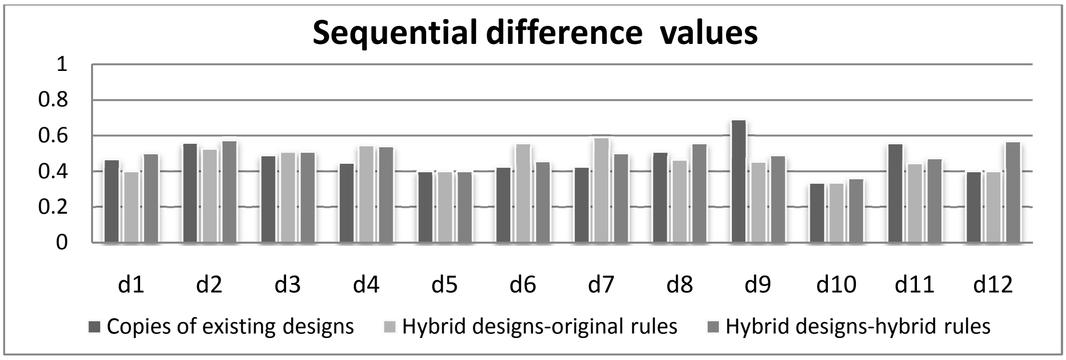


Figure 5.44: Comparing sequential difference values of copies of existing designs and hybrid designs

5.2.2.2. Stage 2: Main findings

Comparing the innovation measures of the mixed character and the individuality of copies of existing designs and hybrid designs composed of original and hybrid rules reveals the following matters:

- There are differences between copies of existing designs and hybrid designs. The metrics of the mixed characters defined in terms of diversity and abundance are mostly higher in hybrid designs than copies of antecedents. However, the high abundance values of copies of antecedents compared to hybrid designs are negative abundance. The reason is the high density of copies of antecedents is accompanied with low diversity values contrary to positive abundance of hybrid designs in which the high density is accompanied with high diversity values. In addition, the hybrid designs in general have higher individuality described in terms of the matching degree, the geometrical difference and the sequential difference.
- There are differences between hybrid designs composed of original rules and hybrid designs composed of hybrid rules. In all cases, the mixed characters of hybrid designs composed of hybrid rules have higher diversity and abundance, except one case in which the diversity is the same. In addition, in all cases, the hybrid designs composed of hybrid rules have higher

individuality in their rule format as a result of having the maximum geometrical difference values. As well, in most cases, the individuality of hybrid designs composed of hybrid rules has less matching and higher sequential difference than hybrid designs composed of original rules.

In conclusion, these results prove that the proposed assessment method of the innovation in hybrid designs is able to distinguish copies of existing designs from hybrid design, as well as hybrid designs composed of original rules from hybrid designs composed of hybrid rules. According to that, the validity of the innovation assessment of hybrid designs is verified. In the next section, the research investigates the predictable factors of high innovation in a hybrid design.

5.2.3. Stage 3: Indicators of high innovative hybrid designs

In shape grammars, the application of rules to generate a design space requires a search technique for a desired solution. In this study, the desired solution is the more innovative hybrid design. The search process includes many options that the user may not gain without the aid of the grammar, thus paving the way for possible innovative design.

In shape grammars for hybrid designs, the search for innovative hybrid design can be done twice giving the grammar user the potential to explore and compare many alternative rules during the generation process firstly and to explore a large number of alternative designs after the generation process secondly. In the former, the grammar user depends on comparing the feedback signals of high innovative hybrid design. While in the latter, the results of the innovation measures of hybrid designs can be compared to automatically guide the user search for the best configuration from the population of generated solutions. This stage of implementation concerns the first selection and aims to identify the factors that the grammar user can take into account to direct the generated hybrid design to have high innovation values.

Being a mixture and having individuality are the main characteristics of innovation in hybrid designs measured by diversity and abundance for the former, matching degree, geometrical difference and sequential difference for the latter. They are dependent variables affected by the independent variables which the grammar user can control in the derivation process such as: the number of rules (NR) used to derive a hybrid design, and the user feedback signals defined in terms of rule evaluation factors. The latter are default values added to each rule in the grammar to represent the followings:

- The rule prevalence value (RPV) is an indicator of the mixed character in a generated design.
- The rule geometrical difference value (RGDV) is an indicator of the individuality in a rule format.
- The rule sequential difference value (RSDV) is an indicator of the individuality in a rule order.

5.2.3.1. Factors affecting innovation: Initial hypotheses

The initial expectations of the effects of the independent variables (the number of rules (NR) and the user feedback signals including rule prevalence value (RPV), rule geometrical difference value (RGDV) and rule sequential difference value (RSDV)) on the dependent variables (diversity, abundance, matching degree, design geometrical difference value and design sequential difference value) are summarised as follows:

- A high number of rules (NR) may have a positive effect on the diversity which measures the variety of the design mixture.
- The different number of rules (NR) may not affect other innovation measures such as abundance, matching degree, geometrical difference value and sequential difference value.
- A high average of rule prevalence values (RPV) may affect positively the diversity which measures the variety of the design mixture.
- A high average of rule prevalence values (RPV) has a positive effect on the abundance which measures the density of the design mixture.
- A high average of rule prevalence values (RPV) may affect negatively the individuality by increasing the matching degree.

- A high average of rule prevalence values (RPV) may affect negatively the individuality in the original rule format by decreasing the geometrical difference value of the generated design.
- A high average of rule prevalence values (RPV) may affect negatively the individuality in the rule order by decreasing the sequential difference value of the generated design.
- A high average of rule geometrical difference values (RGDV) has definitely a positive effect on the individuality in rule format by increasing the geometrical difference value of the generated design.
- The average of rule geometrical difference values (RGDV) does not affect directly other innovation measures such as diversity, abundance, matching degree, and sequential difference value.
- A high average of rule sequential difference values (RSDV) has definitely a positive effect on the individuality in rule order by increasing the sequential difference value of the generated design.
- The average of rule sequential difference values (RSDV) does not affect directly other innovation measures such as diversity, abundance, matching degree, and geometrical difference value.

The assumptions regarding the effects of rule numbers (NR) and the average rule prevalence values (RPV) on the innovation measures need to be verified. On the other hand, the effects of rule geometrical difference value (RGDV) and rule sequential difference value (RSDV) on the innovation measures seems to be inevitable. Therefore, the study in the next paragraphs concentrates on examining the relations between the independent variables represented by the number of rules (NR) and the average rule prevalence values (RPV) and the dependent variables represented by the innovation measures.

5.2.3.2. The effects of the NR and RPV on the innovation measures – phase 1

To verify the assumptions above, the relation between the independent variables and the dependent variables in the innovation assessment results of the generated designs of stage 2 of implementation is investigated. The effects of the number of rules (NR) and average rule prevalence value (RPV) on the values of diversity, abundance, matching degree, geometrical difference and sequential difference are calculated using correlation coefficients.

A correlation coefficient is an indicator of a predictive relationship between two variables. It is a statistical technique that can show whether and how strongly one variable can be influenced by changes in another variable. It is used to calculate the effect of the changes in the number of rules (NR) and the average rule prevalence values (RPV) on each of the dependent variables in 12 copies of existing designs (CED), 12 hybrid designs derived by original rules (HD-OR), and 12 hybrid designs derived by hybrid rules (HD-HR) generated in the previous stage. The results are presented in Table 5.4 which shows predictive relations such as:

- A correlation coefficient of (1) means that the two variables are perfectly correlated: If one increase so does the other.
- A correlation coefficient of (-1) means that the two variables are perfectly inversely correlated: If one increases the other decreases.
- A correlation coefficient of (0) means that the two variables are not related.
- A correlation coefficient close to (0) means that the relation between the two variables is not certain to be useful.
- The study explained the other values of correlation coefficient as follows:
- A correlation coefficient from (0.7 to 1.0) means that the relation between the two variables is direct strong. While, a correlation coefficient varying from (–1.0 to –0.7) means that the relation between the two variables is inverse strong.
- A correlation coefficient from (0.4 to 0.7) means that the relation between the two variables is direct moderate. While a correlation coefficient from (–0.7 to –0.4) means that the relation between the two variables is inverse moderate.

- A correlation coefficient from (0.4 to 0.0) means that the relation between the two variables is direct weak. While a correlation coefficient varying from (0 to –0.4) means that the relation between the two variables is inverse weak.

Table 5.4: Correlation coefficient values between innovation measures and each of NR and RPV in copies of existing designs CED and hybrid designs HD using OR and HR

		Innovation measures				
		Diversity	Abundance	Matching	Geo. difference	Seq. difference
CED	NR	+0.589	+0.519	0.0	–0.47	+0.232
	RPV	+0.662	+0.997	0.0	–0.72	–0.353
HD-OR	NR	+0.786	–0.781	–0.5	+0.114	+0.96
	RPV	–0.363	+0.996	+0.592	–0.299	–0.77
HD-HR	NR	+0.887	–0.562	–0.565	0.0	+0.551
	RPV	–0.649	+0.919	+0.418	0.0	–0.553

The results show that the number of rules (NR) has a strong direct relation with diversity values in both hybrid designs composed of original rules and hybrid rules. They are compatible with the research assumption regarding these variables. At the same time, the number of rules (NR) has a moderate direct relation with diversity values in copies of antecedents which reflect some difference between copies of existing designs and hybrid designs.

The relations between the number of rules (NR) and the abundance values show a moderate direct relation in case of copies of existing designs, while the relations are strong inverse and moderate inverse in case of hybrid designs derived by original rules and hybrid rules respectively. The relation between the number of rules (NR) and the matching degree values show that there is no relation in case of copies of existing designs, while the relations are moderate inverse in case of hybrid designs composed of original rules and hybrid rules. The relation between the number of rules (NR) and the geometrical difference values show a moderate inverse relation in case of copies of antecedents and a weak direct relation in case of hybrid design composed of original rules, while there is no relation in case of hybrid designs composed of hybrid rules. Lastly, the relations between the number of rules (NR) and the sequential difference values show a weak direct relation in case of copies of antecedents and a strong direct relation and a moderate direct relation in case of hybrid designs composed of original rules and hybrid rules correspondingly.

However, the study assumed that there are no relations between the number of rules (NR) and the innovation measures such as abundance, matching degree, geometrical difference and sequential difference. Therefore, the results above need more investigation for verification.

On the other hand, the relations between the average rule prevalence values (RPV) and the diversity values reveal a moderate direct relation in case of copies of existing designs, and weak inverse and moderate inverse relations in case of hybrid designs composed of original rules and hybrid rules respectively. The results here contradict the research assumption that there is a direct relation between the rule prevalence values (RPV) and the diversity values which require more investigation. The relations between the average rule prevalence values (RPV) and the abundance values show strong direct relations in both copies of antecedents and hybrid designs composed of both original rules and hybrid rules. The results here are compatible with the research assumption regarding these variables. The relations between the average rule prevalence values (RPV) and the matching degree values show that there is no relation in case of the copies of antecedents because the whole copies of antecedents have the maximum matching value (1) irrespective of their rule prevalence values (RPV), while the relations are moderate direct in both hybrid designs derived by original rules and hybrid rules. The direct relations between the rule prevalence values (RPV) and the matching degree values are expected in the research assumption in the previous section.

The relations between the average rule prevalence values (RPV) and the geometrical difference values show a strong inverse relation in case of copies of existing designs, a weak inverse relation in case of hybrid designs derived by original rules, and no relation in case of hybrid designs derived by hybrid rules because the whole hybrid rules have the maximum geometrical difference value (1) irrespective of their rule prevalence values (RPV). Lastly, the relations between the average rule prevalence values (RPV) and the sequential difference values reveal a weak inverse relation in case of copies of antecedents, a strong inverse relation in case of hybrid designs composed of original rules, and a moderate inverse in case of hybrid designs composed of hybrid rules.

The correlation results between the independent and dependent variables confirm the differences between existing and hybrid designs. However, some results of hybrid designs are contrary to expectation and need more investigation, such as the inverse relation between the number of rules and the matching degree, the inverse relation between the rule prevalence values (RPV) and the diversity values. Accordingly, the research needs to verify the effects of each predictor variable: the number of rules (NR) and the rule prevalence value (RPV) on the innovation measures of the mixture and individuality in hybrid designs. Both predictor variables can direct the grammar user to generate hybrid designs with high innovation. However, the required investigation should take into consideration to isolate, as much as possible, the effect of each factor on the other factor.

5.2.3.3. The effects of the NR and RPV on the innovation measures – phase 2

To reduce the mutual influence of the number of rules (NR) and the average rule prevalence values (RPV) on each other, this study derives 10 pairs of hybrid designs, 5 of them using original rules (ORs) and the others using hybrid rules (HRs). Each pair consists of hybrid designs derived by 6 and 10 original or hybrid rules. The number of rules 6 and 10 were chosen to represent the varied numbers of rules in the corpus which are (3, 5, 6, 7, 9, 10, 11 and 12). The rules in each pair have the same (4-5) rules to diminish the effects of varied rule prevalence values (RPV) on the innovation measures. On one hand, the comparison between designs of 6 and 10 rules in each pair reveals the impacts of the different number of rules (NR) on the innovation measures. On the other hand, comparison between hybrid designs composed of the same number of rules (6 or 10) separately, explains the effects of various rule prevalence values (RPV) on the innovation measures. For detailed derivations of the 10 pairs of hybrid minarets, please refer to appendix section C-1 for the 5 pairs of hybrid minarets, each composed of (6 and 10) original rules (ORs), and section C-2 for the other 5 pairs of hybrid minarets, each composed of (6 and 10) hybrid rules (HRs).

5.2.3.3.1. The effect of NR on the innovation measures

The results of diversity values of hybrid minarets derived by 6 and 10 original rules are clarified in Figure 5.45. In 4 of 5 pairs, the diversity of hybrid designs derived by

10 ORs are higher than the diversity values of hybrid designs derived by 6 ORs. Only in one case, the diversity values of both of them are the same.

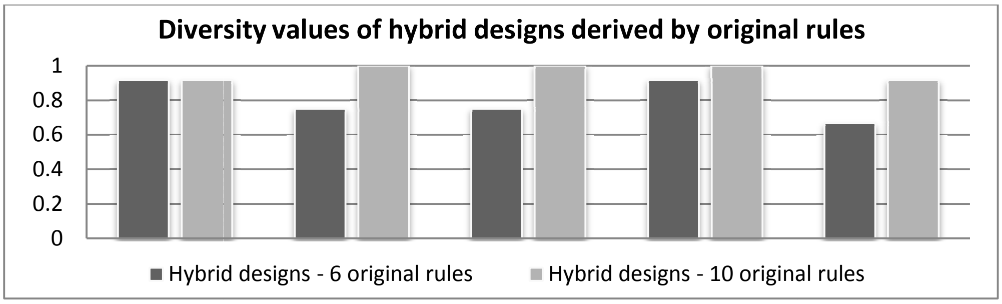


Figure 5.45: Diversity values of hybrid designs derived by 6 and 10 original rules

On the other hand, in 3 of 5 pairs, the diversity values of hybrid designs derived by 10 hybrid rules are higher than the ones derived by 6 hybrid rules, as shown in Figure 5.46. In two pairs the diversity results are the same.

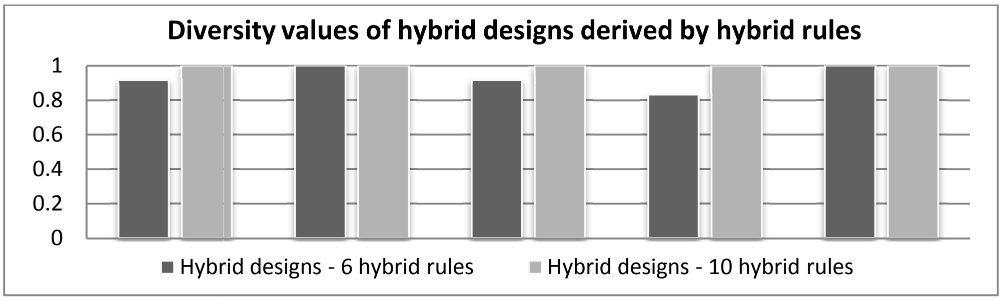


Figure 5.46: Diversity values of hybrid designs derived by 6 and 10 hybrid rules

In both graphs, the diversity values of hybrid designs composed of 10 rules are either the same or higher than the diversity values of hybrid designs composed of 6 rules. The results above are compatible with the strong direct relation, concluded in Table 5.4, between the number of rules (NR) and the diversity values. Also, in all cases, the diversity values of hybrid designs composed of hybrid rules are the same or higher than the diversity values of hybrid designs composed of the same number of original rules

The results of the effect of different number of rules on abundance values of hybrid minarets derived by 6 and 10 original rules are shown in Figure 5.47. In all 5 pairs, the abundance values of hybrid designs derived by 6 original rules are higher than the abundance values of hybrid designs derived by 10 original rules.

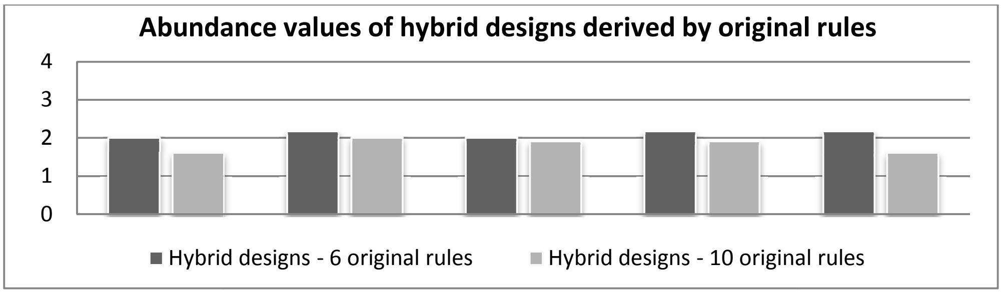


Figure 5.47: Abundance values of hybrid designs derived by 6 and 10 original rules

On the other hand, in Figure 5.48, three of the five cases show that the abundance values of hybrid designs composed of 6 hybrid rules are higher than the abundance values of hybrid designs composed of 10 hybrid rules. In only two pairs, the abundance values of hybrid designs composed of 10 hybrid rules are slightly higher than those composed of 6 hybrid rules.

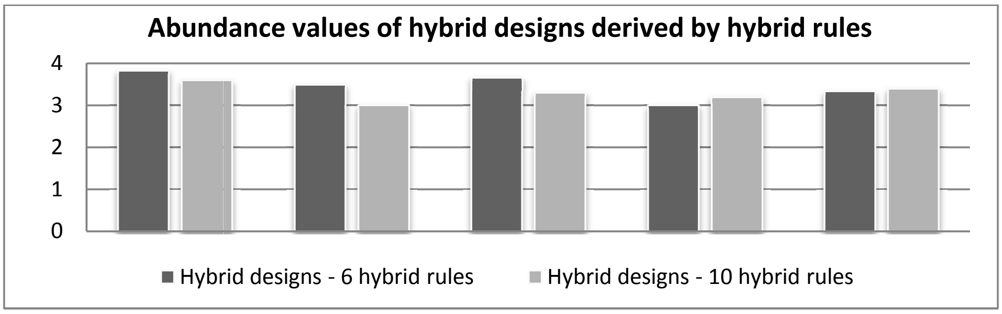


Figure 5.48: Abundance values of hybrid designs derived by 6 and 10 hybrid rules

Accordingly, the abundance values of hybrid designs composed of 6 rules are mostly higher than hybrid designs composed of 10 rules. These results are compatible with the previous results in table 5.4 which show the strong inverse relation for hybrid designs composed of original rules and the moderate inverse relation for hybrid designs composed of hybrid rules. In addition, the abundance values of hybrid designs composed of hybrid rules are higher than the abundance values of hybrid designs composed of the same number of original rules.

As in the abundance results, matching values of hybrid designs composed of six rules are mostly higher than hybrid designs composed of 10 rules, as shown in Figures 5.49 and 5.50. All hybrid designs derived by 6 original rules have matching values higher than hybrid designs composed of 10 original rules. In addition, 4 of 5 cases of hybrid designs derived by 6 hybrid rules have matching values higher than the ones of 10 hybrid rules. These results are compatible with the inverse relation between the number of rules (NR) and the matching degree values concluded in Table 5.4.

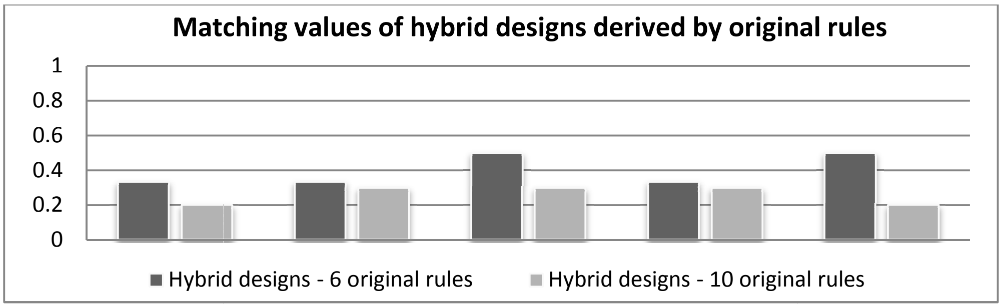


Figure 5.49: Matching values of hybrid designs derived by 6 and 10 original rules

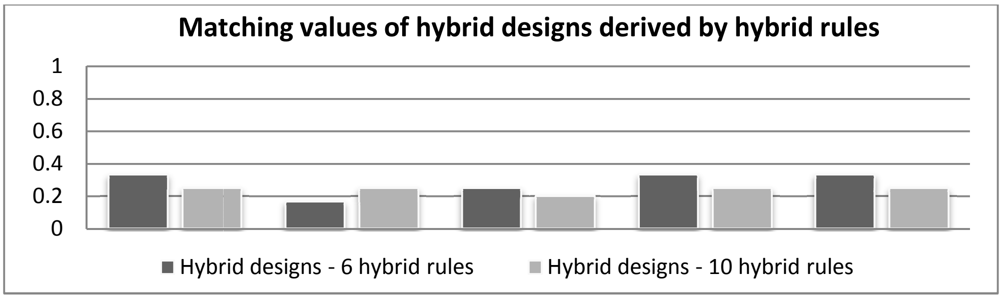


Figure 5.50: Matching values of hybrid designs derived by 6 and 10 hybrid rules

On the other hand, the average of matching values for hybrid designs composed of 6 and 10 original rules are higher than the ones of hybrid designs composed of 6 and 10 hybrid rules.

In 4 of 5 cases, the geometrical difference values of hybrid designs derived by 10 original rules are slightly higher than the values of hybrid designs derived by 6 original rules, as shown in Figure 5.51. This result is compatible with the direct relation concluded in Table 5.4.

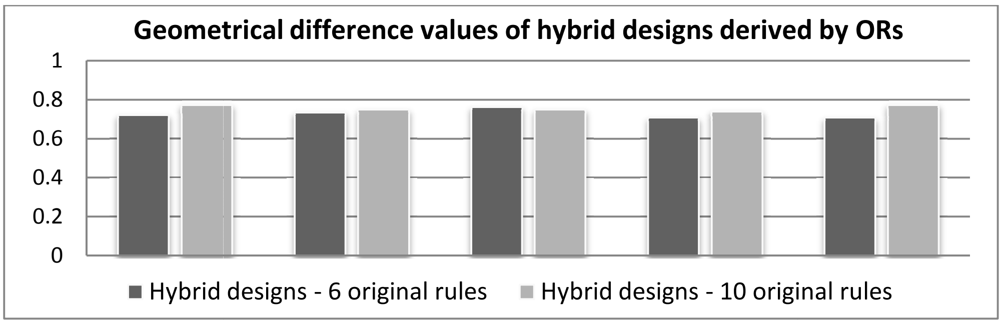


Figure 5.51: Geometrical difference values of hybrid designs derived by 6 and 10 original rules

On the other hand, in Figure 5.52, all hybrid designs derived by hybrid rules have the maximum geometrical difference values (1.0). Therefore, there is no relation between the different number of rules and the geometrical difference values in case of hybrid designs composed of hybrid rules.

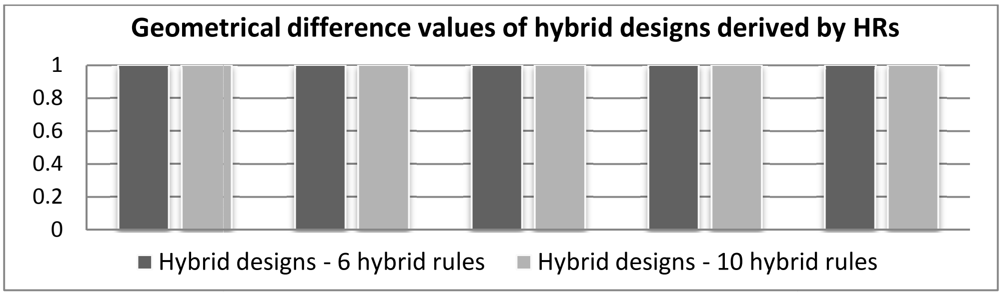


Figure 5.52: Geometrical difference values of hybrid designs derived by 6 and 10 hybrid rules

Lastly, in all cases the sequential difference values of hybrid designs composed of 10 rules are higher than the ones composed of 6 rules, as shown in Figures 5.53 and 5.54. These results are compatible with the direct relation concluded in Table 5.4

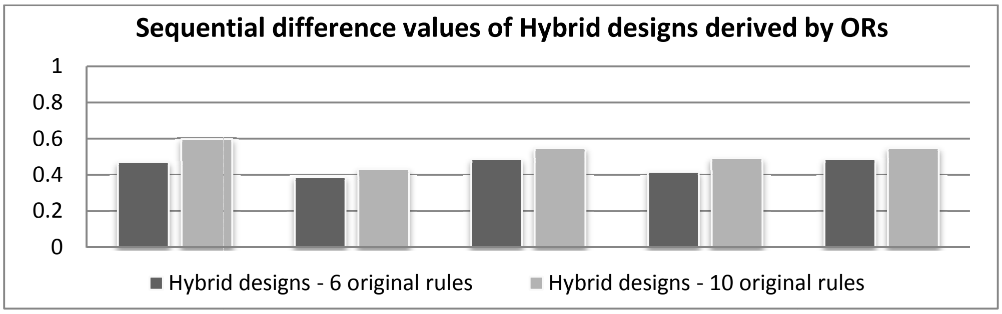


Figure 5.53: Sequential difference values of hybrid designs derived by 6 and 10 original rules

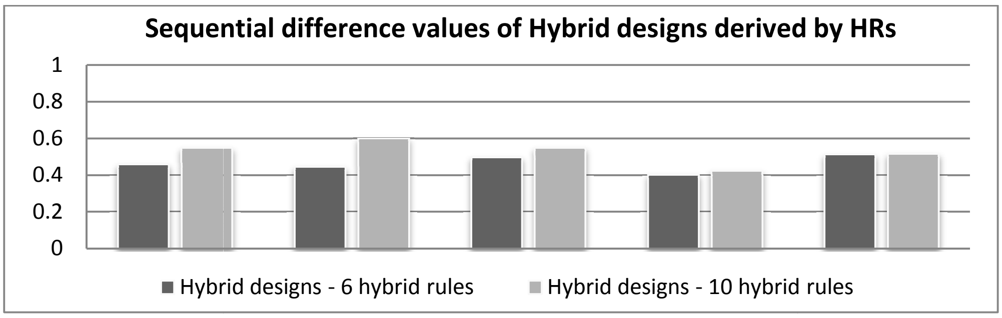


Figure 5.54: Sequential difference values of hybrid designs derived by 6 and 10 hybrid rules

5.2.3.3.2. The effect of rule prevalence values (RPV) on the innovation measures

In Table 5.5, the correlation coefficients are used to show the effects of the average rule prevalence values (RPV) on the innovation measures of hybrid designs (HD) derived by 6 and 10 original rules (OR) and hybrid rules (HR).

Table 5.5: Correlation coefficient between the rule prevalence values and the innovation measures of 10 pairs of hybrid designs using 6 and 10 original and hybrid rules

	Innovation measures				
	Diversity	**Abundance**	**Matching**	**Geo. difference**	**Seq. difference**
HD – 6 ORs	-0.273	+1.0	-0.166	-0.6	-0.664
HD – 10 ORs	+0.744	+0.986	+0.927	-0.863	-0.851
HD – 6 HRs	+0.196	+0.962	-0.038	0.0	+0.316
HD – 10 HRs	0.0	+0.919	+0.418	0.0	-0.148

The effects of the average rule prevalence values (RPV) on the diversity values show unexpected random relations. In both hybrid designs composed of 6 original rules (OR) and 6 hybrid rules (HR), the rule prevalence values (RPV) and diversity values show weak relations. These results are compatible with the weak relation between the rule prevalence values (RPV) and diversity values in hybrid designs composed of original rules as concluded in Table 5.4. The rule prevalence values (RPV) and diversity values show a strong direct relation in case of hybrid designs composed of 10 original rules. However, there is no relation between the rule prevalence values (RPV) and the diversity values in case of hybrid designs composed of 10 hybrid rules because all diversity values are 1.0.

The results above confirm that the rule prevalence value (RPV) on its own cannot be a feedback signal of diversity value. The reason can be attributed to the fact that the applied rules with high rule prevalence value, which hold multi-state labels in their LHS, have common antecedents among them. For example, an applied rule with a rule prevalence value (RPV=0.333) derived from four antecedents in the corpus may have only one antecedent belong to the set of variable state label $n(x\text{-}1)$ of the previous rule. This rule boosts the diversity value in the same way as the applied rule derived from one antecedent does. According to that, there is a need to develop a diversity indicator to reflect the expected raising of the diversity value. The indicator of diversity (ID) value is equal or less than the rule prevalence value (RPV $\geq$ ID). It is calculated to represent the ratio of the number of antecedents of the current rule that exist in the set of $n(x\text{-}1)$ of the previous rule to the total number of antecedents in the corpus. However, if the set of $n(x\text{-}1)$ is Ø, it is replaced by the sets $\{n(x\text{-}1)^*$ or $n(x\text{-}1)^{**}\}$ whereas the diversity reaches or does not reach the maximum value one; in these cases the value of the diversity indicator (ID) at this stage is 0/12=0.0.

Unlike the rule prevalence value which is a default value added to each rule in the grammar, the indicator of diversity (ID) is an updated value added automatically at grammar runtime to only the eligible rules for selection which have the possible matching conditions in their LHS. The value of the indicator of diversity (ID) is calculated and attached to the rule evaluation values as a feedback signal to enable the grammar user to choose the suitable rule. Its value changes for the same rule at each stage of design derivation depending on the sequence of rules in the application which determines the set of $n(x\text{-}1)$.

At the same time, the relations between the average rule prevalence values (RPV) and the matching values are also unexpected. The relations between them are weak in hybrid designs composed of 6 original rules, hybrid designs composed of 6 hybrid rules and hybrid designs composed of 10 hybrid rules. On the other hand, the relation between them is a strong direct in hybrid designs composed of 10 original rules. These results reveal that the rule prevalence value is also not sufficient to indicate the matching value.

According to that, an indicator of matching degree (IM) is needed to be in the same vein as the indicator of diversity (ID). Its value is calculated automatically, at grammar runtime, as the ratio of the number of antecedents in the LHS of the current rule which does not exist in the set of the possible antecedents defined by the variable state label $\{n(x\text{-}1)$, $n(x\text{-}1)^*$ and $n(x\text{-}1)^{**}\}$ of the previous rule to the total number of antecedents in the corpus. In all cases, the value of the indicator of matching degree (IM) is less than the rule prevalence value (RPV > IM). If all the antecedents in the LHS exist in the set of the possible antecedents of the previous rule $\{n(x\text{-}1)$, $n(x\text{-}1)^*$ and $n(x\text{-}1)^{**}\}$ then, the value of the indicator of matching degree is 0/12=0.0.

The indicators of diversity (ID) and matching degrees (IM) are clarified in the following examples:

If the rule is derived from the antecedents: {d3, d4 and d9}, then the rule prevalence value is (RPV=0.25).

If the set of the variable $n(x\text{-}1)$ was defined in the user guide grammar of the previous rule as: {d1, d3, d6, d9, d11}

Then, the indicator of diversity (ID) is 2/12=0.166, to represent the ratio of the two antecedents {d3, d9} which exist in the set $n(x\text{-}1)$ to total number of the antecedents in the corpus, and the indicator of matching (IM) is 1/12=0.083, to represent the ratio of the one antecedent {d4} which does not exist in the set of $n(x\text{-}1)$ to total number of the antecedents in the corpus.

RPV = 0.25 — Default value

ID = 0.166
IM = 0.083 — Updated values

The indicator of diversity (ID) of any other rule has the same rule prevalence value (RPV=0.25) but different antecedents {d1, d2, d10} is 1/12=0.083 because only one antecedent (d1) exists in the set $n(x\text{-}1)$ of the last rule, while the indicator of matching degree (IM) is 2/12=0.166 because two antecedents (d2 and d10) does not exist in the set $n(x\text{-}1)$ of the previous rule.

In table 5.5, the relations between the rule prevalence values (RPV) and the abundance values are strong direct in all hybrid designs composed of 6 and 10 original rules and hybrid rules. These results are compatible with the relations concluded in table 5.4.

The relations between the rule prevalence values (RPV) and each of the geometrical difference values and sequential difference values are moderate inverse in case of hybrid designs composed of 6 original rules and strong inverse in case of hybrid designs composed of 10 original rules (Table 5.5). On the other hand, there is no relation between the rule prevalence values (RPV) and the geometrical difference values of hybrid designs derived by hybrid rules because all hybrid rules have the maximum geometrical difference value (1.0). However, the relations between the rule prevalence values (RPV) and the sequential difference values are weak in case of the hybrid designs composed of hybrid rules. In spite the fact that the rule geometrical and sequential difference values are the main feedback signals of the design geometrical and sequential difference values, even so in some cases, the rule prevalence values of original rules can indicate the design geometrical and sequential difference values with high certainty. For example, original rules with high rule prevalence values have low geometrical and sequential difference values. On the other hand, the low certainty results from the original rules with low rule prevalence values which have high, medium or low geometrical and sequential difference values (Table 5.6).

Table 5.6: The relations in original rules between the rule prevalence values and the geometrical or sequential difference values

		Geometrical or sequential difference value of original rules		
		High	**Medium**	**Low**
Rule Prevalence value of original rules	**High**			●
	Medium		●	●
	Low	●	●	●

5.2.3.4. Stage 3: Main findings

The search technique for the innovative hybrid design gives the grammar user the potential to explore many alternative rules during the generation process. The study considered that the number of rules (NR) and the rule evaluation factors have significant effects on the innovation measures. However, the effects of some rule evaluation factors such as rule geometrical difference values and rule sequential difference values are definitely positive and limited to some innovation measures such as design geometrical difference and design sequential difference respectively. Therefore, the study examined only the effects of the number of rules (NR) and the average rule prevalence values (RPV) on the innovation measures of hybrid designs within two phases. In the first phase, the correlation coefficients between the independent variables and the innovation measures are extracted from the copies of existing designs and hybrid designs generated in the previous stage. The results on one hand enhanced the differences between the copies of antecedents and hybrid designs; and on the other hand showed predicted and in some cases unexpected effects of the independent variables on the innovation measures. The second phase aims were to verify the first phase result and to neutralize the effects of the number of rules (NR) and the average rule prevalence values (RPV) on each other.

Based on the results of two phases, the relations between the number of rules (NR) and the innovation measures reveal that the high number of rules (NR) enhances the diversity of the mixture in hybrid designs composed of original rules (ORs) and hybrid rules (HRs). In contrast, the high number of rules (NR) may affect negatively the abundance value as the measure of the density of hybrid design mixture. The reason can be attributed to the fact that the grammar is derived from the corpus of heterogeneous antecedents, therefore it has a limited number of original and hybrid rules with high rule prevalence value (RPV): 5.555% of ORs and 18.888% of HRs (Table 5.3). In addition, repeating a rule with high rule prevalence value (RPV) is an exceptional case in shape grammar for hybrid design. Accordingly, the percentage of rules with high rule prevalence value (RPV) is more likely higher in designs having few rules than many rules. Furthermore, the high number of rules (NR) strengthens the design individuality by decreasing the matching degree. It can be justified, to some extent, for the same reason mentioned above where the low number of rules has greater chances to choose rules with high rule prevalence values (RPV) which in turn are most likely to increase the generated design matching degree. Lastly, the number of rules (NR) has a direct relation with both design geometrical and sequential difference values except hybrid designs composed of hybrid rules in which the number of rules does not affect the design geometrical difference values.

The relations between the average rule prevalence values (RPV) and the innovation measures reveal that high rule prevalence values (RPV) boost the abundance which measures the density in hybrid design mixture. In contrast, the correlation between rule prevalence values (RPV) and both diversity and matching degree values reveal uncertainty relations. The reason why the applied rules with high RPV may not enhance the diversity values can be attributed to the fact that the applied rules have some common antecedents in their state labels. Additionally, this reason is more likely to raise the matching value. These results revealed the need to add other feedback signals to represent the automated indicator of diversity (ID) and matching degree (IM). Diversity and matching indicators are two updated values added at grammar runtime to rule evaluation values of each eligible rule for selection. Diversity indicator (ID) calculates the ratio of rule antecedents that exist in the set $n(x\text{-}1)$ of the previous rule to the total number of antecedents in the corpus. While, matching indicator calculates the ratio of rule antecedents that do not exist in the set $n(x\text{-}1)$ of the previous rule to the total number of antecedents in the corpus. Lastly, the high rule prevalence values (RPV) have inverse relation with both design geometrical and sequential differences.

Built on the results above, a key for rule selection is summarized in Figure 5.55 which can be used to direct the grammar user to a range of possible innovation measures. The figure presents the effects of the number of rules, the rule prevalence values (RPV) - defined in terms of the incidence of rule antecedents in the set $n(x\text{-}1)$ of previous rule - on the innovation measures: diversity, abundance matching degree, geometrical difference and sequential difference. The number of rules (NR) is constrained to be less than the total number of antecedents in the corpus (ND): (NR<ND) and grouped to (NR<0.5ND), (NR=0.5ND) or (NR>0.5D). The varied rule prevalence values (RPV) are classified to high, medium or low which in turn are classified according to the incidence of rule antecedents in the set of $n(x\text{-}1)$, such as all antecedents, some antecedents or one antecedent.

Figure 5.55 helps the grammar user to decide the number of rules and to realize the effects of the independent variables on the diversity, abundance, matching degree, geometrical difference and sequential difference. The grammar user can follow the tactics to derive hybrid designs with high innovation values and can distinguish the rules (original or hybrid) that meet the required level of innovation measures (high, medium or low).

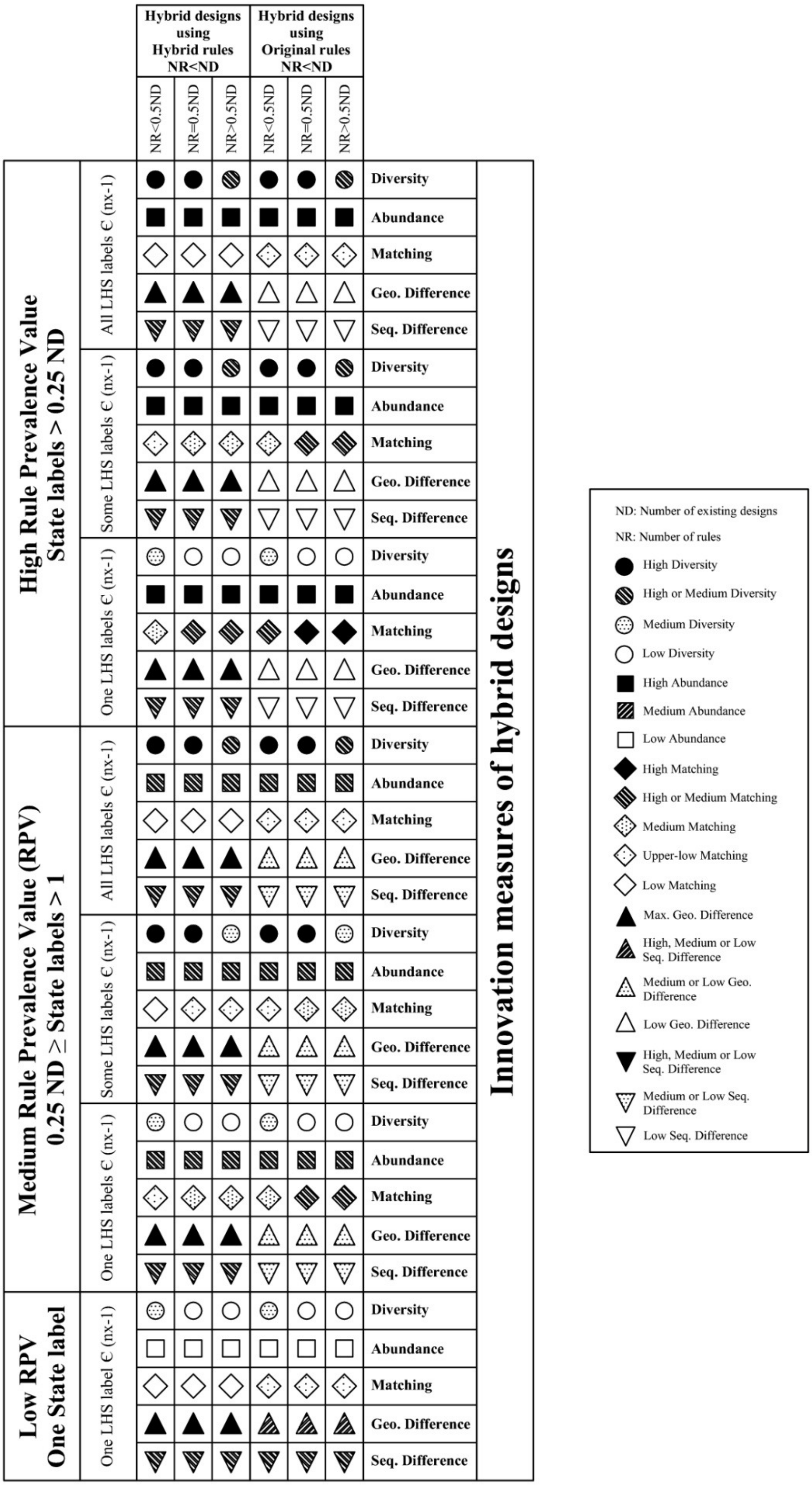


Figure 5.55: The effects of RPV and NR on the innovation measures of hybrid designs

5.3. Chapter summary

The study has sought to achieve three objectives; therefore, the implementations of shape grammars for hybrid minaret designs are presented in three stages. The first stage show successive examples of hybrid minarets derived using (7 and 8) original rules and hybrid rules. The second stage removed the doubts about the validity of innovation measures in a generated hybrid design. The ability of these measures to distinguish the mixed character and individuality between copies of existing designs and hybrid designs was the test of their reliability. This was done by comparing the innovation metrics of copies of antecedents with hybrid designs composed of original and hybrid rules separately. The results show significant differences regarding diversity, matching, and geometrical difference especially.

The third stage highlighted the signals of grammar user feedback that contribute efficiently in deriving hybrid design with high innovation values. The study analysed the relations between the independent variables: the number of rules (NR) used to derive a design, and the average rule prevalence value (RPV); and the dependent variables of innovation metrics: diversity, abundance, matching degree, geometrical difference and sequential difference in the generated designs from the second stage. The results revealed disagreement with the research hypotheses in some cases. Therefore, further investigation was needed to verify these results and to neutralize the effects of number of rules (NR) and rule prevalence values (RPV) on each other. This is done by comparing hybrid designs composed of 6 and 10 original and hybrid rules separately. The results of this phase underlined the need to add automatic indicators to rule default evaluation system as mechanisms for predictability of both diversity and matching measures.

6. DISCUSSION AND CONCLUSION

The research reported in this thesis explores the use of shape grammars to derive hybrid component-based designs from a corpus of heterogeneous antecedents. The points of strength and weakness in this method are identified in section 6.1 of this chapter. The contribution of the study is highlighted in section 6.2. Lastly, recommendations for further research in this topic are discussed in section 6.3.

6.1. Shape grammar for hybrid designs: strength and weakness

The implementation of shape grammars for hybrid minarets in the previous chapter reveals the validity of the method. However, there are sides of strength and weakness in this method which are reviewed in the following paragraphs.

6.1.1. Strength aspects in shape grammar for hybrid designs

Shape grammar for hybrid design have been developed in chapter three, refined in chapters four, and implemented to derive hybrid minaret designs in chapter five. This section highlights areas of strength in this method. Some of these aspects are attributed to shape grammar method in general and are enhanced in the developed method at hand whilst others are related specifically to this method.

The main aspects of strength are:

- Generating designs using shape grammars are time and effort saving compared to traditional design methods.
- Shape grammars for hybrid design are a bottom-up component-based modelling approach which has the advantage of the ease of use by grammar users who may know nothing about shape grammars.

- Because of the heterogeneity features of the existing designs in the corpus, this method is able to generate a large number of hybrid designs from a limited number of antecedents.
- The results of implementing a shape grammar for hybrid minarets prove the possibility of generating a large variety of new hybrid designs. They contradicts the views that shape grammars are useful in exploring a large number of similar variations and are less useful for the new modelling of existing building (Huang et al., 2009).
- In spite of this, there is a set of pre-determined steps in this method that should be followed to produce a hybrid design, even so there is no prior knowledge about the final configuration to be derived using this process. This fact complies with the unexpected and unpredictable characters associated with the process of generating innovative design using original shape grammar[69] whereas surprise is a "strength of grammars because it opens up new, unimagined design possibilities" (Knight, 1998).
- Shape grammars for hybrid designs boost the two aspects of human innovative design behaviour: exploration and adaptation. It is suitable for computationally modelling the open-ended nature of design by incorporating and adapting knowledge drawn from past experiences of valuable precedents. The search process for a desired solution gives the grammar user the potential to explore and compare a large number of alternative designs including many options that the user may not gain without the aid of the grammar, thus paving the way for highly innovative design. In addition, hybrid adaptation in shape grammar provides the rules with a new order and a new format in which the generated hybrid design is incrementally evolved toward a new state of solution.
- There are two levels of exploration in shape grammars for hybrid designs: internal and external. The internal level happens during the derivation processes whereas the grammar user has alternative rules to choose from at

[69] The author of original shape grammars creates grammar rules from scratch.

each stage of application. The external level, on the other hand, happens after the completion of many hybrid designs to find the best configuration from the population of generated solutions.

- In this method, the selection processes by grammar users can be innovation measures-guided or self-guided. They allow the user to browse through applicable rules or generated designs and explicitly select the desired one. In the former, the grammar user participates in the filtering process using the evaluation schemes (rule evaluation and grammar evaluation) to filter the choices of rules during the generation process and the choices of final designs after completing the generation process. However, the user dependence on the innovation measures alone in the choice of applicable rules or final designs may not lead to satisfactory results. Therefore, the grammar user can use his/her aesthetic preference to guide the selection process.
- The differences between original rules as keeping the design constraints of their antecedents and hybrid rules as breaking these restrictions enhance the variety in the generated hybrid designs which can be derived from original rules only, mixture of original rules and hybrid rules, or hybrid rules only.
- The ability to innovate in any design lies in the ability to generate diverse alternatives that break away from the norms and the governing constraints. The use of hybrid adaptation techniques in shape grammars provides opportunities for the emergence of unexpected or unpredictable designs by combining original rules to generate hybrid rules. The blended shapes and spatial relations in a hybrid rule inherit features from its input original rules leading to emergent features of its own by merging characteristics of their antecedents.
- The innovation measures in shape grammar for hybrid design are internal metrics which determine how well the mixed character and individuality in the generated hybrid designs. Contrary to external metrics such as golden ratio... etc., the internal metrics here depend on the grammar structure and

give the grammar indicators such as (NR, RPV, RGDV, RSDV) a larger role in generating innovative hybrid designs.

- The benefits arising from measuring the innovation in hybrid designs can be attributed to creating a platform for final designs comparison and selection, and creating feedback signals for generated designs improvement.
- To the best of the author's knowledge, shape grammar for hybrid designs is the only grammar that makes a link between rules in the grammar and their antecedents in the corpus. The antecedent role in deriving a hybrid design is not anonymous but can be traced easily. This feature is useful if the user wants to select rules belonging to specific precedents in the corpus as sources of hybridisation. Furthermore, it enables the customization of the whole corpus of antecedents to be sub-corpus. The computer implementation can deal with this matter easily. For example, the rules that have the LHS constant state labels of the customized sub-corpus are active, the others are inactive. Additionally, the default innovation measures of the active rules are automatically recalculated to satisfy the new number of antecedents in the sub-corpus and the new number of rules in each sub-class rules set.

6.1.2. Weakness aspects in shape grammar for hybrid designs

The weakness points in shape grammars for hybrid designs relate in some cases to shape grammars method in general, while in others are attributed to the method itself. The main weakness aspects are:

- Shape grammars for hybrid design are component-based generative design process which analyse precedents to a set of components and use them to build new hybrid designs. However, this simple component-based approach may not suit alone many classes of architectural and product designs. In fact, this method is more likely to satisfy specific type of simple formal artefacts composed of clear components with juxtaposed and containment relations whereas these components are the basis for grouping rules in sub-class rule sets.

- The user dependence on the innovation measures to choose a rule or a final design may not lead him/her to satisfactory results. Despite of the design configurations being obtainable with rules derived from existing designs with aesthetic, the grammar alone cannot guarantee aesthetic results of the generated design from the user point of view (Huang et al., 2009). The reason can be attributed to the fact that considering aesthetic design principle by shape grammar to yield pleasing results limits the variety of the generated designs.
- Combing original rules to derive hybrid rules may lead to nonsense configurations. According to Knight, "understanding the predictabilities of grammars is central to the successful design of shape grammars in creative design application" (1998, p. 499). Therefore, the grammar author should be aware of all the possibilities of deriving and applying the hybrid rules.
- The grammar based computational design is accused of restraining the creativity of the grammar users who lack "the creative control, responsibility and challenges they enjoy in traditional design" (Knight, 1998, p. 500). The user role is limited to select applicable rules according to their innovation indicators or his/her personal desire and to decide their parameters which are more likely to be boring mechanical tasks.
- The grammar author cannot deal manually with a large number of existing designs in the corpus whereas the increase in the number of antecedents leads to a direct increase in the number of rules as a result of the heterogeneous characters of antecedents.
- Although writing rules in sub-class rule sets makes expanding the grammar an easy matter if new antecedents are added to the corpus at any time; even so the default values of rule evaluation (RPV, RGDV and RSDV) in all grammar rules need to be updated. They should consider the increase in the total number of antecedents in the corpus on one hand, and the effects of new

rules on both geometrical and sequential differences values of other rules on the other hand. These changes are time consuming in case of manual handling of grammar, contrary to computer implementation which can deal with them easily and quickly.

6.2. Contribution of the research

The main research contribution can be attributed to the use of hybrid adaptation in shape grammars and the assessment of the generated hybrid design. Unlike previous works, which focused on transformation techniques to derive new designs from the corpus of antecedents, this research concentrated on hybridisation techniques irrespective of other adaptation techniques. The contribution of this thesis is summarised in the following paragraphs.

6.2.1. Shape grammars as both interpolation and extrapolation tools

Previous works on analytical shape grammars enabled the grammar user not only to understand and generate designs in the original style but to generate new designs in an extension of the style[70]. However, except Orsbone et al (2006) on vehicle designs, shape grammars were applied on a corpus of homogeneous antecedents. The new designs generated by transforming the original language such as Bosnian house grammar (Colakoglu, 2001, 2005), Yingzao Fashi grammar (A. Li, 2001), Siza houses grammar (Duarte, 2005) are interpolations of the building type in which the new designs data is constructed within the range of a set of antecedents' data.

In this study, the primary contribution has been to propose an approach that enables the incorporation of shape grammar method into hybrid adaptation techniques. Shape grammars for hybrid design create new and different designs from the corpus of heterogeneous antecedents using interpolation and extrapolation. In the former process, the hybrid design is derived by original rules only. While in the latter, the

[70] Http://www.mit.edu/~tknight/IJDC/frameset_history_analysis.html

hybrid design is derived by hybrid rules only or a combination between original rules and hybrid rules in which the data of a new hybrid design is constructed outside the range of a set of antecedents' data.

6.2.2. Shape grammars as multi-guides tool for grammar user

The user interaction in shape grammar practices can be classified into three roles (Chase, 2002). In the first role, the user has a full control on rule selection and grammar development. In the second role, the grammar user has a partial control on some aspects of rule selection. While in the third one the grammar user has no control and the designs are generated without user intervention.

In shape grammar for hybrid designs, the partial control of grammar user is directed via parallel grammars composed of three descriptive grammars. Firstly, it is the user guide grammar for hybrid design which restricts the choice of the next rule within a set of predefined antecedents. Secondly, it is the rule evaluation metrics of the innovation in hybrid designs which have default values (RPV, RGDV and RSDV) and automated values (ID and IM) to give grammar user feedback signals before choosing the rule and to provide ground for comparison between eligible rules for selection. The last descriptive grammar is the evaluation metrics of the innovation in the generated hybrid designs which have automated values (diversity, abundance, matching degree, geometrical difference and sequential difference) to give grammar user feedback after applying the rule. Through the generation process, these measures are the feedback loop that can be the input in the next rule selection. After the completion of derivations, the final measures provide a ground for comparison between final designs.

6.2.3. Shape grammars as an assessment tool

In previous shape grammar practices, the assessment tools are derived from other domains such as the aesthetics in case of golden ratio (Shea, 1997), or economics in the case of costing (Agarwal et al., 1999). In this study, the assessment method is built in to the grammar rules to make the shape grammar not only an analytical and generative tool but also an assessment tool (Figure 6.1).

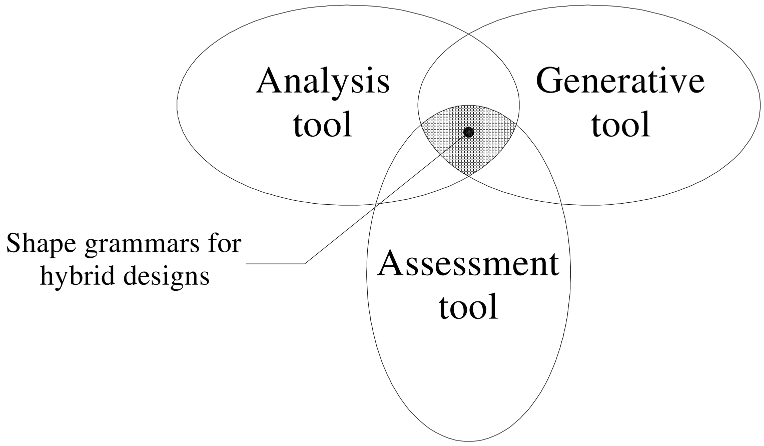


Figure 6.1: Tasks of shape grammar for hybrid designs

6.3. Recommendations for future research

This study identified an approach for deriving hybrid designs using shape grammar. A range of issues have emerged in relation to the proposed approach which requires further investigation.

6.3.1. Further applications of the method

This study implemented shape grammars for hybrid designs on a corpus of heterogeneous traditional minarets. More applications on different architectural artefacts, engineering designs and product designs are required to enhance the proposed approach.

6.3.2. Enrichment of the method

The central part of shape grammar for hybrid designs involves capturing the component-based design information and representing it via grammar rules. Therefore, there is a need to improve this method to suit other architectural products in which component-based analysis alone is not sufficient.

6.3.3. Application in other domains

It is possible to identify hybrid adaptation technique in other design computing domains such as case-based designs. This will offer a platform to compare shape grammar method with case-based method.

6.3.4. Computer implementation

Shape grammars for hybrid minaret design were conducted manually. A computer implementation is useful to perform direct housekeeping work such as parametric shape representation and computation of user guide grammar, innovation indicators and innovation measures. According to that, creating a user interface will facilitate and save time for the grammar user compared with the hand-based method.

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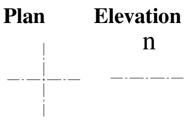
Appendix

Appendix–A: Rules of shape grammars for hybrid minarets

A-1 Original rules of shape grammar for hybrid minarets

Initial Shape:



n={d1, d2, d3, d4, d5, d6, d7, d7, d8, d9, d10, d11, d12}

nx: is a variable state label to be defined in the user guide grammar

x: is an ascending integer variable that represents stages of adding minaret components. x={2, ..., y},
y: is the total number of rules needed to generate the minaret. In the corpus of antecedents y={3, ..., 12}

m: is a set of designs that have the least number of applied rules in the perevios steps of design derivation

Parameters of shapes are
L: length, W: width, Lb: base length, Lt: top length, H: hieght, D: diameter, Db: base diameter, Dt: top diameter, M: distance betweem markers

Markers:

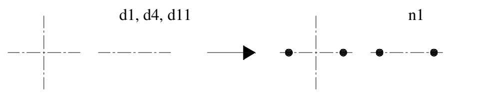
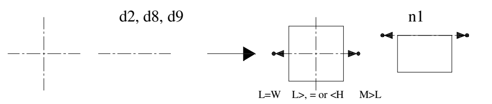
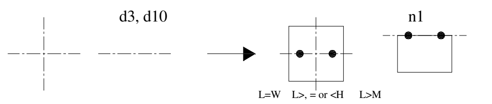
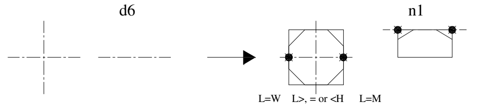
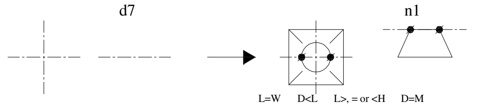
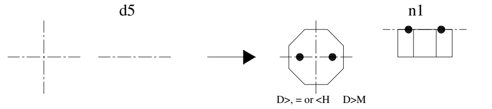
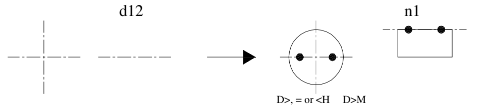
1-Body markers:
- Adding any body shape
- Adding an octagonal, a circular, a lobular or a stellar boby shape
- Adding a circular, a lobular or a stellar boby shape

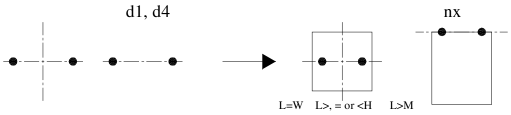
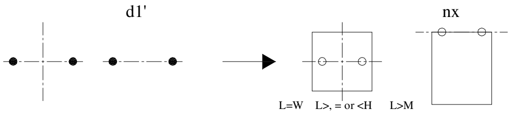
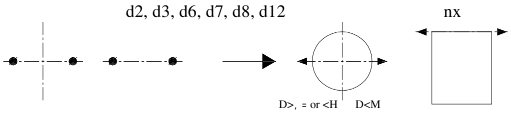
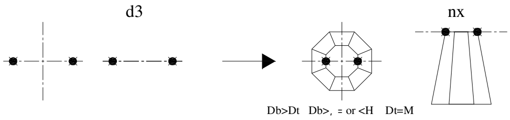
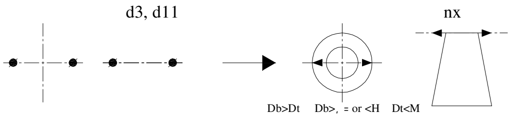
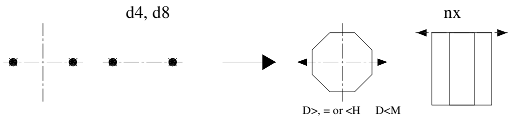
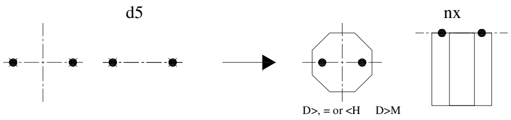
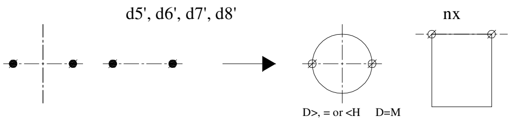
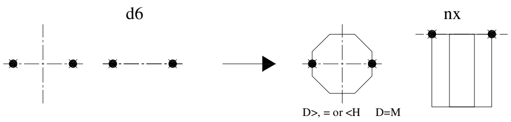
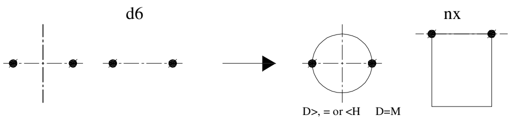
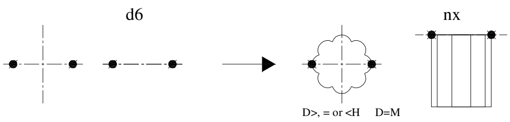
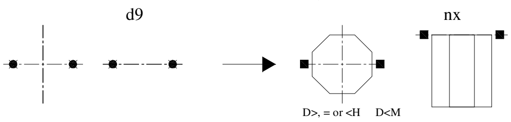
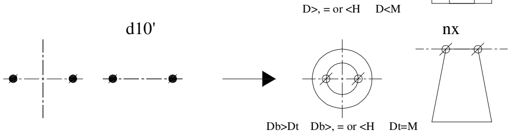
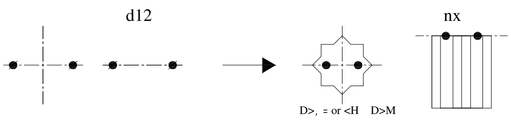
2-Joint markers:
- Adding any component above the joint
- Adding a body or a balcony followed by a body
- Adding a head or a balcony followed by a lantern or a head above the joint

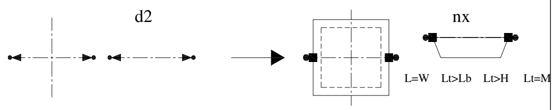
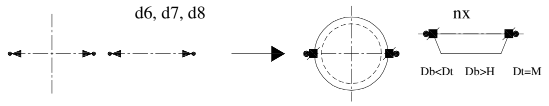
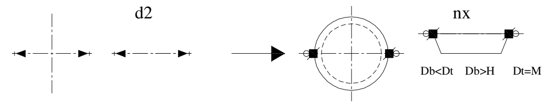
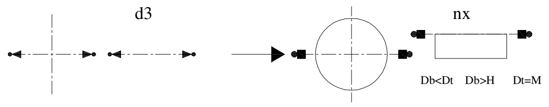
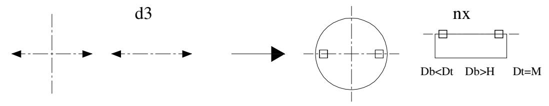
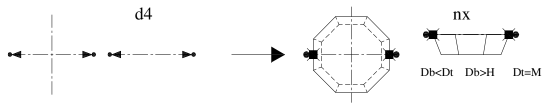
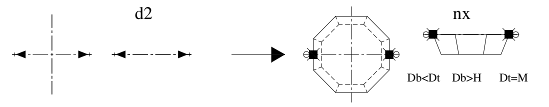
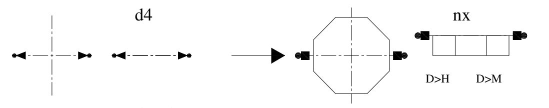
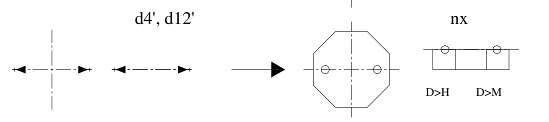
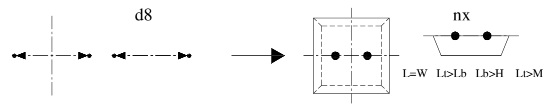
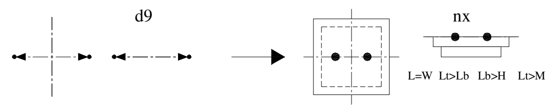
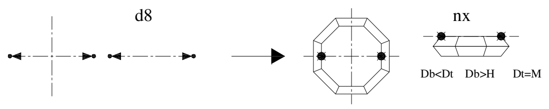
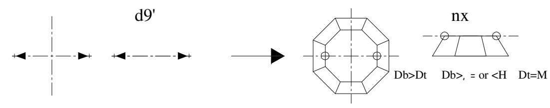
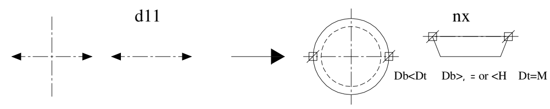
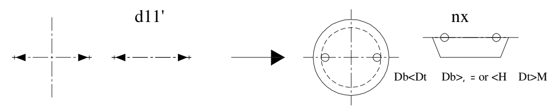
3-Balcony markers:
- Adding any balcony shape
- Adding an octagonal or a circular balcony shape
- Adding a circular balcony shape
- Adding a balcony followed by a body
- Adding a balcony followed by a lantern or a head

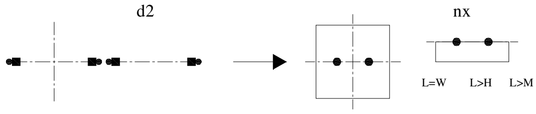
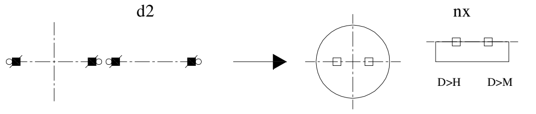
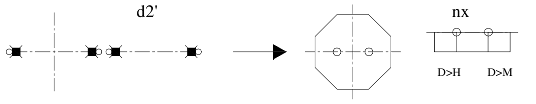
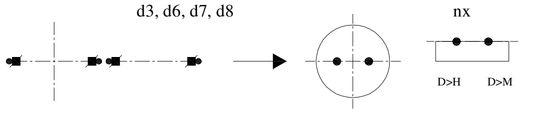
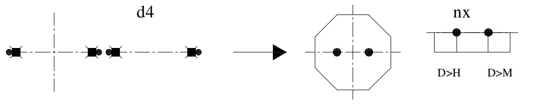
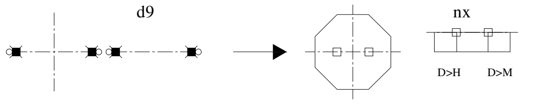
4-Lantern markers:
- Adding any lantern shape
- Adding a circular lantern shape

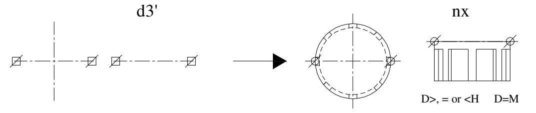
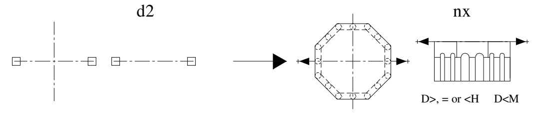
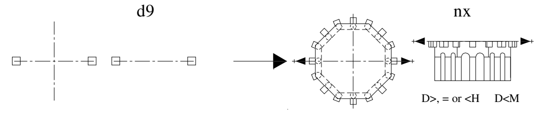
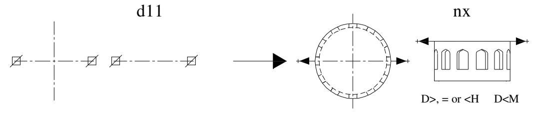
5-Head markers:
- Adding any head shape
- Adding a circular head shape

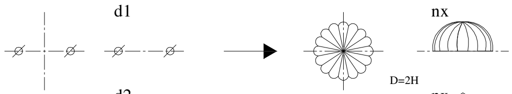
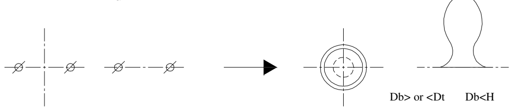
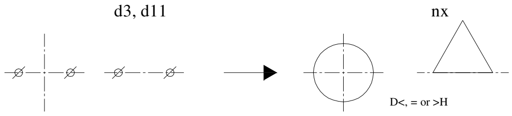
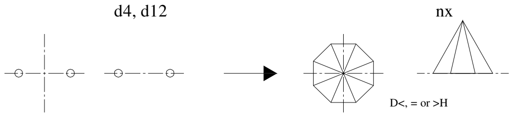
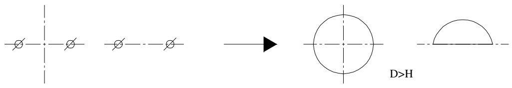
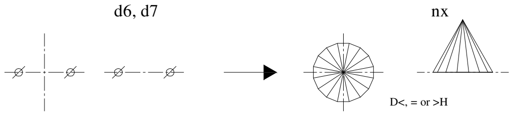
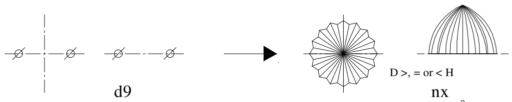
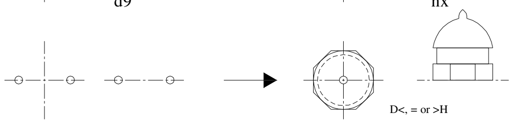
Rule no.	Subclass rule set of minaret bases (Original rules)	User guide grammar for hybid design	Evaluation of the original rules	Evaluation of the generated design
OR 1a		n1={n\d1, d4, d11}	Rule Prevalence=0.25 Geometrical difference=0.75 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2a		n1={n\d2, d8, d9}	Rule Prevalence=0.25 Geometrical difference=0.583 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3a		n1={n\d3, d10}	Rule Prevalence=0.166 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4a		n1={n\d6}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5a		n1={n\d7}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6a		n1={n\d5}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 7a		n1={n\d12}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

Rule no.	Subclass rule set of minaret bodies (Original rules)	User guide grammar for hybid design	Evaluation of the original rules	Evaluation of the generated design
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2b		nx={n(x-1)\d1}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3b		nx={n(x-1)\d2, d3, d6, d7, d8, d12}	Rule Prevalence=0.5 Geometrical difference=0.416 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4b		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5b		nx={n(x-1)\d3, d11}	Rule Prevalence=0.166 Geometrical difference=0.75 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6b		nx={n(x-1)\d4, d8}	Rule Prevalence=0.166 Geometrical difference=0.583 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 7b		nx={n(x-1)\d5}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 8b		nx={n(x-1)\d5, d6, d7, d8}	Rule Prevalence=0.333 Geometrical difference=0.416 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 9b		nx={n(x-1)\d6}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 10b		nx={n(x-1)\d6}	Rule Prevalence=0.083 Geometrical difference=0.416 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 11b		nx={n(x-1)\d6}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 12b		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 13b		nx={n(x-1)\d10}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 14b		nx={n(x-1)\d12}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

Rule no.	Subclass rule set of minaret joints (Original rules)	User guide grammar for hybid design	Evaluation of the original rules	Evaluation of the generated design
OR 1c		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2c		nx={n(x-1)\d6, d7, d8}	Rule Prevalence=0.25 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3c		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4c		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5c		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6c		nx={n(x-1)\d4}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 7c		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 8c		nx={n(x-1)\d4}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 9c		nx={n(x-1)\d4, d12}	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 10c		nx={n(x-1)\d8}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 11c		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 12c		nx={n(x-1)\d8}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 13c		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 14c		nx={n(x-1)\d11}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 15c		nx={n(x-1)\d11}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

Rule no.	Subclass rule set of minaret balconies (Original rules)	User guide grammar for hybid design	Evaluation of the original rules	Evaluation of the generated design
OR 1d	d2 nx  L=W L>H L>M	nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2d	d2 nx  D>H D>M	nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3d	d2' nx  D>H D>M	nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4d	d3, d6, d7, d8 nx  D>H D>M	nx={n(x-1)\d3, d6, d7, d8}	Rule Prevalence=0.333 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5d	d4 nx  D>H D>M	nx={n(x-1)\d4}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6d	d9 nx  D>H D>M	nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

Rule no.	Subclass rule set of minaret Lanterns (Original rules)	User guide grammar for hybid design	Evaluation of the original rules	Evaluation of the generated design
OR 1e	d3' nx  D>, = or <H D=M	nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2e	d2 nx  D>, = or <H D<M	nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3e	d9 nx  D>, = or <H D<M	nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4e	d11 nx  D>, = or <H D<M	nx={n(x-1)\d11}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

Rule no.	Subclass rule set of minaret heads (Hybrid rules)	User guide grammar for hybid design	Evaluation of the original rules	Evaluation of the generated design
OR 1f	d1 nx  D=2H	nx=0	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2f	d2 nx  Db> or <Dt Db<H	nx=0	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3f	d3, d11 nx  D<, = or >H	nx=0	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4f	d4, d12 nx  D<, = or >H	nx=0	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5f	d5, d10 nx  D>H	nx=0	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6f	d6, d7 nx  D<, = or >H	nx=0	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 7f	d8 nx  D >, = or < H	nx=0	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 8f	d9 nx  D<, = or >H	nx=0	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

0

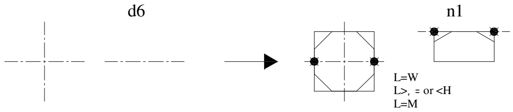
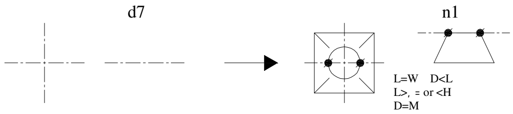
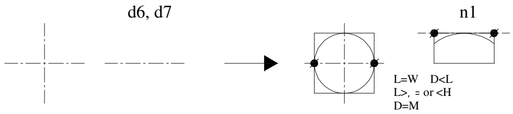
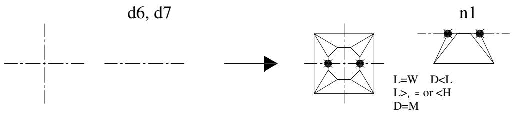
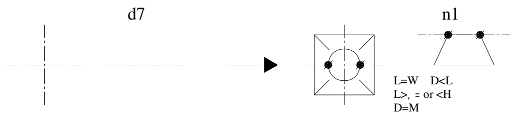
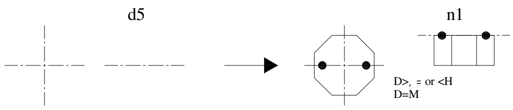
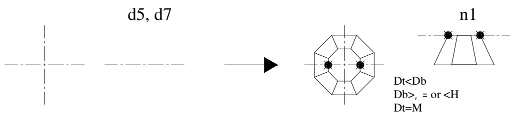
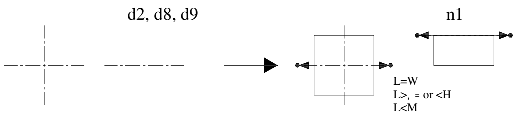
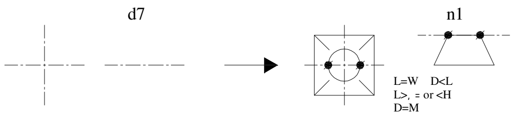
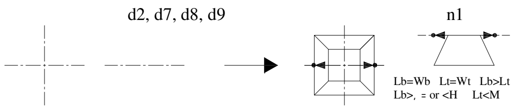
Termination rule

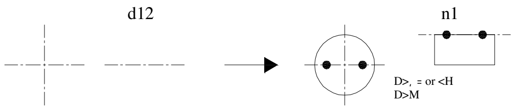
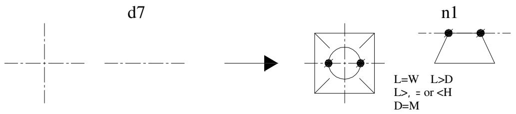
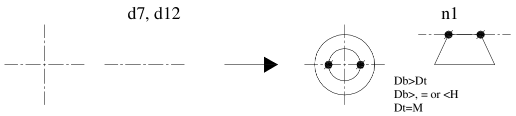
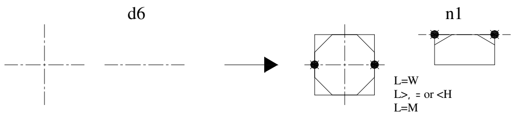
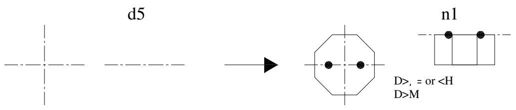
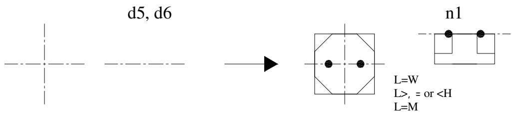
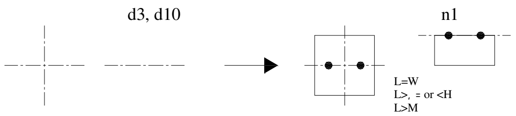
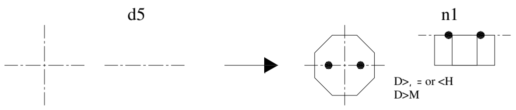
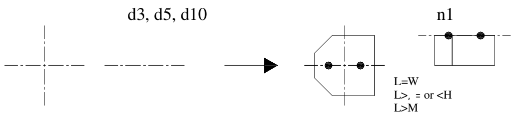
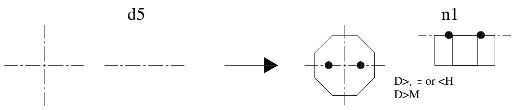
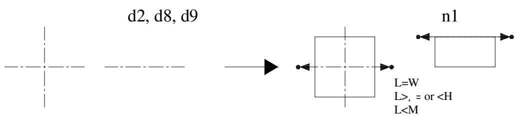
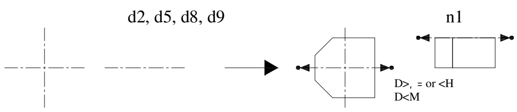
A-2 Hybrid rules of shape grammar for hybrid minarets

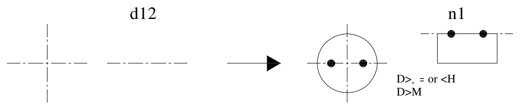
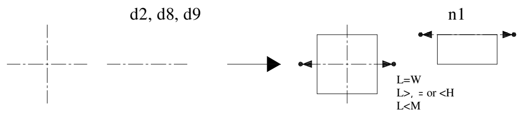
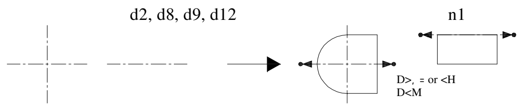
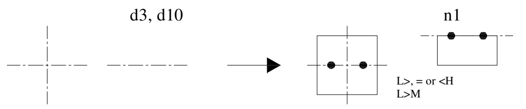
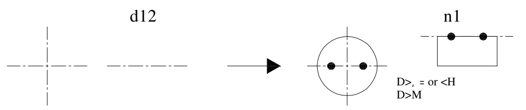
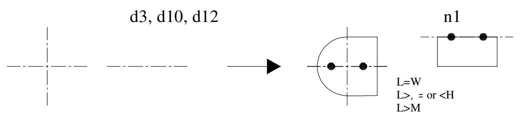
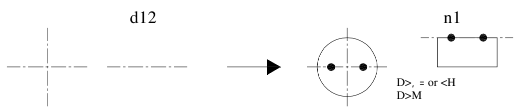
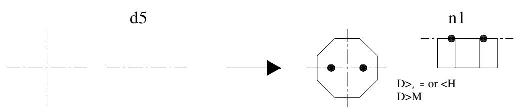
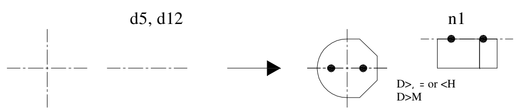
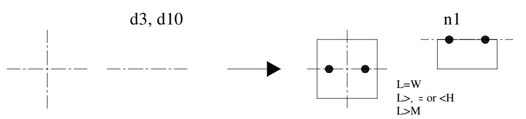
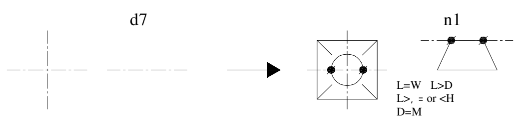
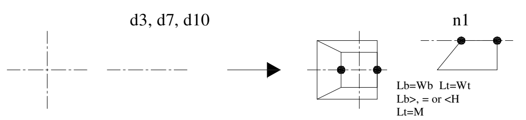
A-2-1 Derivation of hybrid rules

Merging original rules to derive hybrid rules for each component are presented as follows.

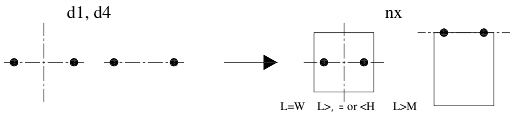
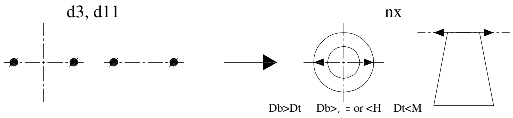
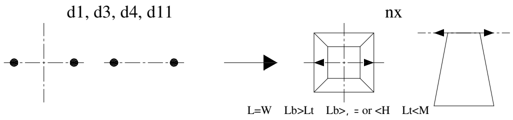
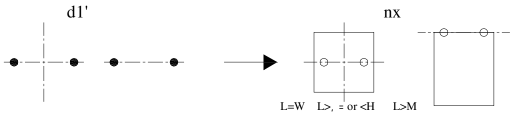
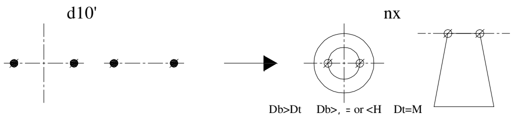
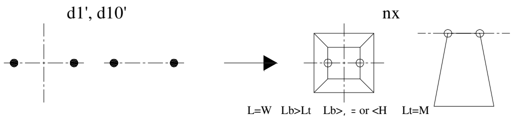
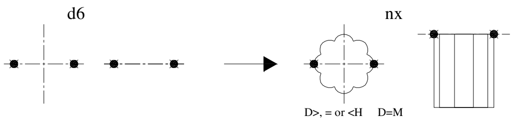
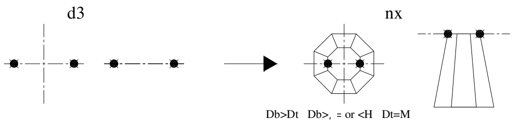
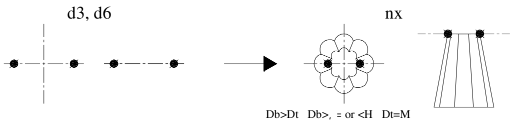
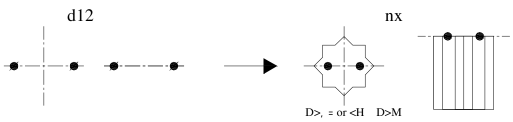
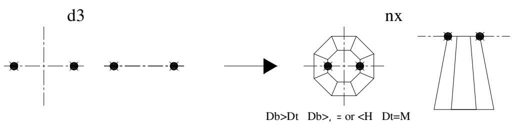
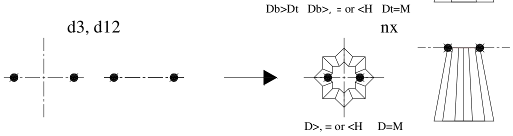
Minaret base

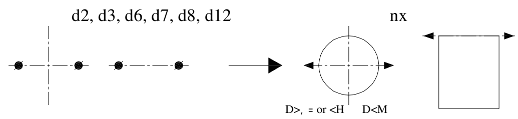
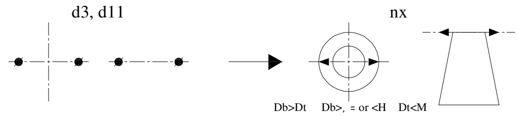
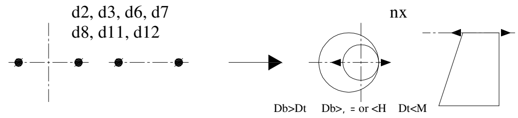
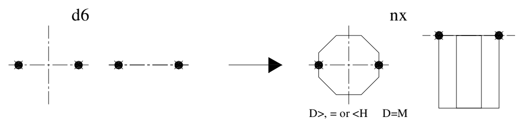
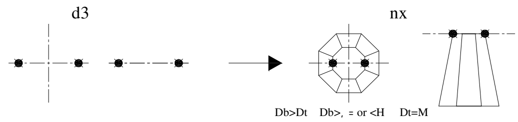
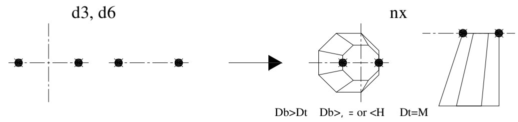
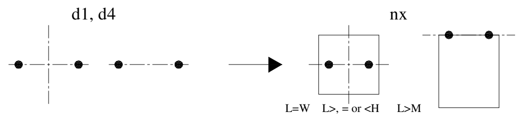
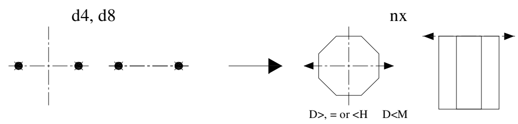
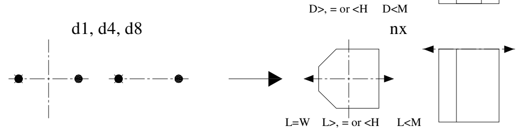
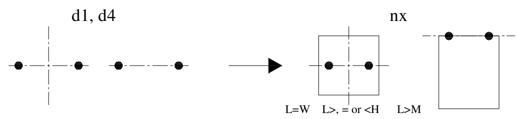
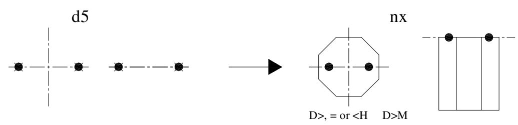
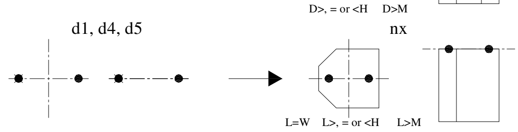
Rule no.	Dreivation of hybrid rules of minaret base	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 4a		n1={n\d6}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5a		n1={n\d7}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 1a		n1={n\d6, d7}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 2a		n1={n\d6, d7}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5a		n1={n\d7}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6a		n1={n\d5}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 3a		n1={n\d5, d7}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2a		n1={n\d2, d8, d9}	Rule Prevalence=0.25 Geometrical difference=0.583 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4a		n1={n\d7}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 4a		n1={n\d2, d7, d8, d9}	Rule Prevalence=0.33 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

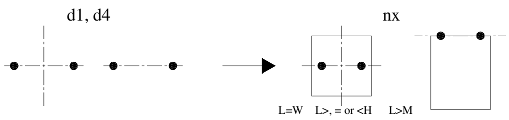
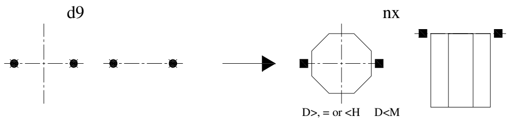
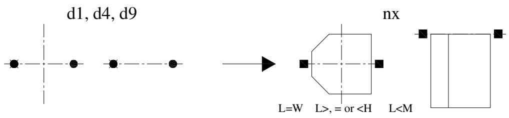
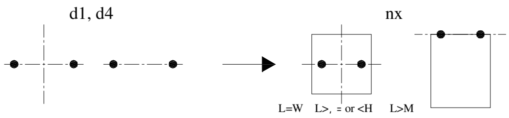
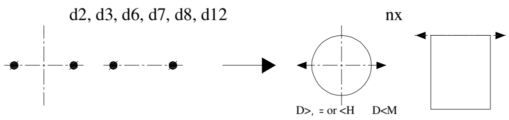
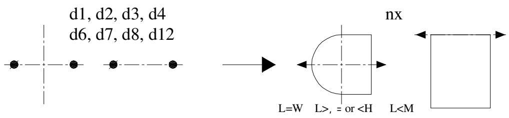
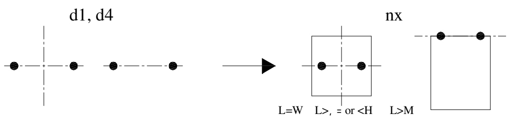
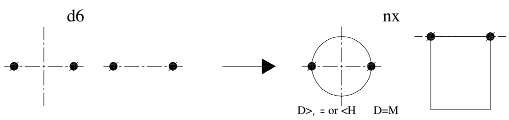
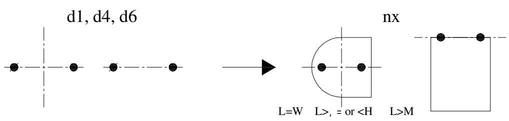
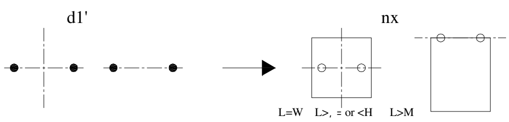
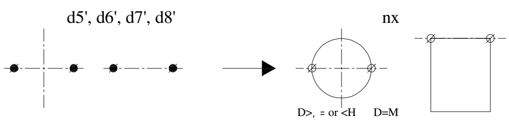
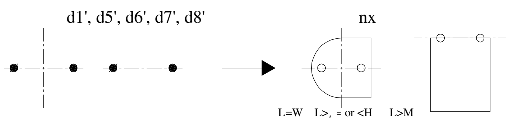
Rule no.	Dreivation of hybrid rules of minaret base	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 7a		n1={n\d12}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5a		n1={n\d7}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 5a		n1={n\d7, d12}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4a		n1={n\d6}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6a		n1={n\d5}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 6a		n1={n\d5, d6}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3a		n1={n\d3, d10}	Rule Prevalence=0.166 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6a		n1={n\d5}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 7a		n1={n\d3, d5, d10}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6a		n1={n\d5}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2a		n1={n\d2, d8, d9}	Rule Prevalence=0.25 Geometrical difference=0.583 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 8a		n1={n\d2, d5, d8, d9}	Rule Prevalence=0.333 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

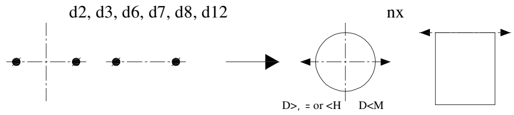
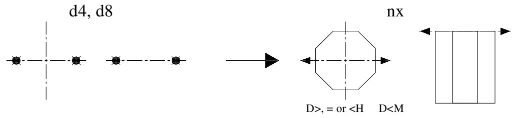
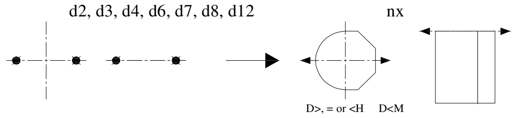
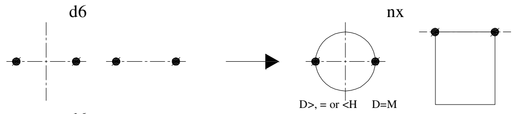
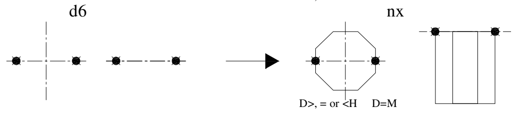
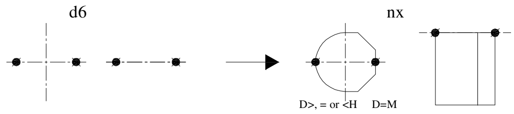
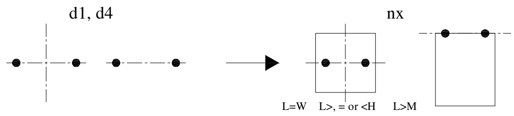
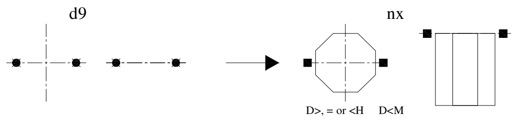
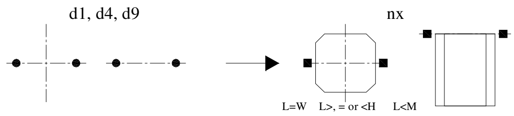
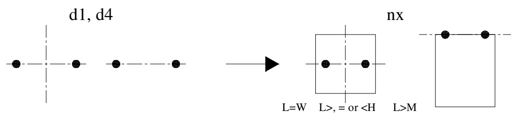
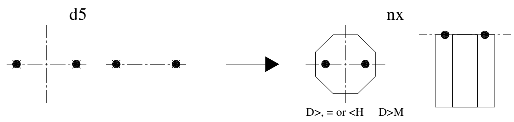
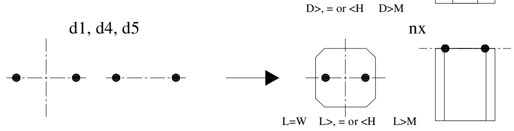
Rule no.	Dreivation of hybrid rules of minaret base	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 7a		n1={n\d12}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2a		n1={n\d2, d8, d9}	Rule Prevalence=0.25 Geometrical difference=0.583 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 9a		n1={n\d2, d8, d9, d12}	Rule Prevalence=0.333 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3a		n1={n\d3, d10}	Rule Prevalence=0.166 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 7a		n1={n\d12}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 10a		n1={n\d3, d10, d12}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 7a		n1={n\d12}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6a		n1={n\d5}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 11a		n1={n\d5, d12}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3a		n1={n\d3, d10}	Rule Prevalence=0.166 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5a		n1={n\d7}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 12a		n1={n\d3, d7, d10}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

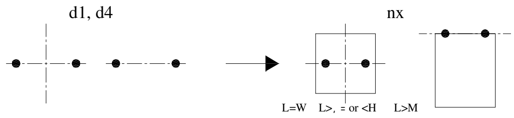
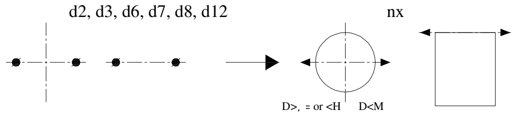
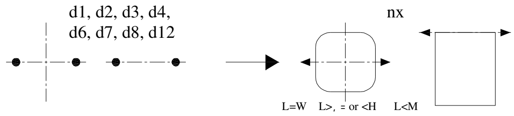
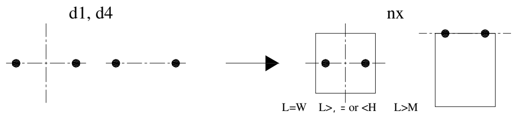
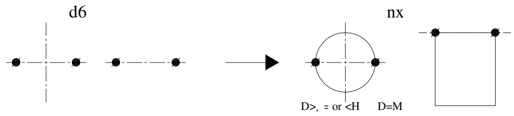
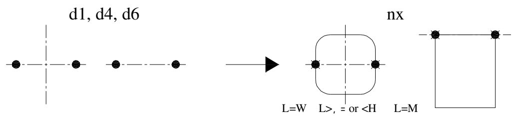
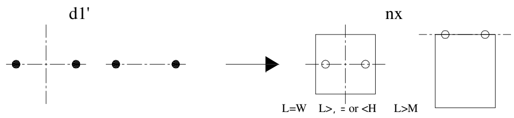
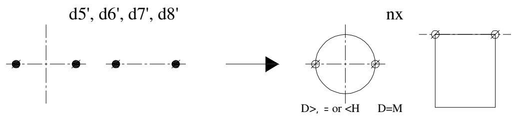
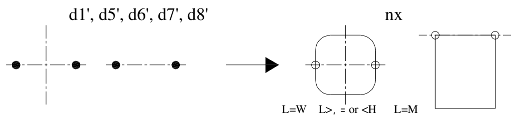
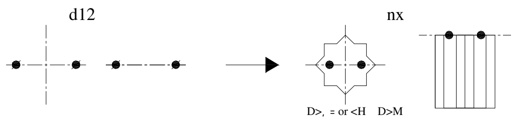
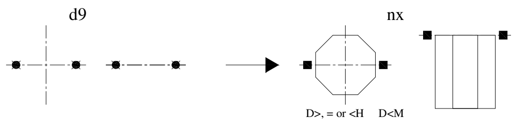
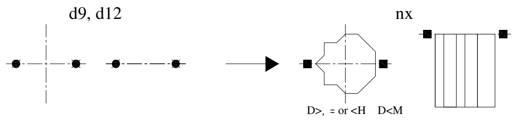
Minaret body

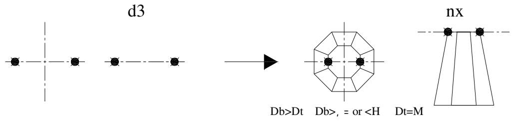
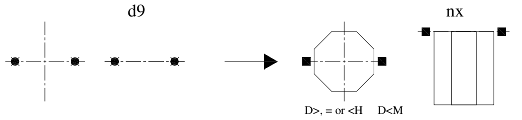
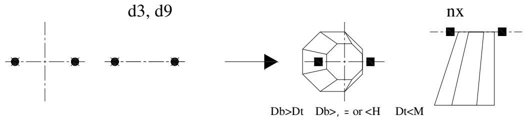
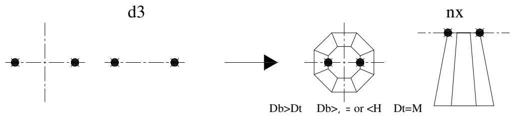
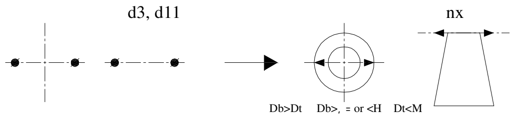
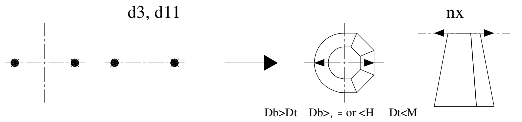
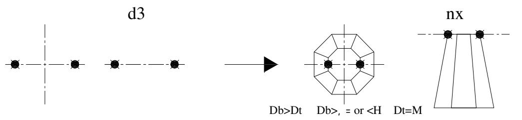
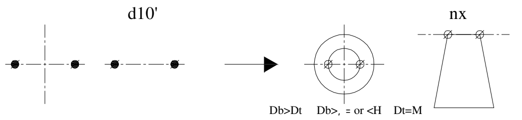
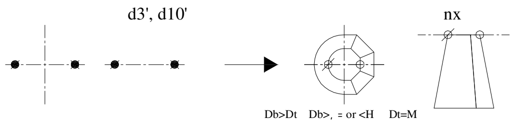
Rule no.	Dreivation of hybrid rules of minaret body	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5b		nx={n(x-1)\d3, d11}	Rule Prevalence=0.166 Geometrical difference=0.75 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 1b		nx={n(x-1)\d1, d3, d4, d11}	Rule Prevalence=0.333 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2b		nx={n(x-1)\d1}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 13b		nx={n(x-1)\d10}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 2b		nx={n(x-1)\d1, d10}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 11b		nx={n(x-1)\d6}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4b		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 3b		nx={n(x-1)\d3, d6}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 14b		nx={n(x-1)\d12}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4b		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 4b		nx={n(x-1)\d3, d12}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

Rule no.	Dreivation of hybrid rules of minaret body	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 3b		nx={n(x-1)\d2, d3, d6, d7, d8, d12}	Rule Prevalence=0.5 Geometrical difference=0.416 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5b		nx={n(x-1)\d3, d11}	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 5b		nx={n(x-1)\d2, d3, d6, d7, d8, d11, d12}	Rule Prevalence=0.583 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 9b		nx={n(x-1)\d6}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4b		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 6b		nx={n(x-1)\d3, d6, d8}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6b		nx={n(x-1)\d4, d8}	Rule Prevalence=0.166 Geometrical difference=0.583 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 7b		nx={n(x-1)\d1, d4, d8}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 7b		nx={n(x-1)\d5}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 8b		nx={n(x-1)\d1, d4, d5}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

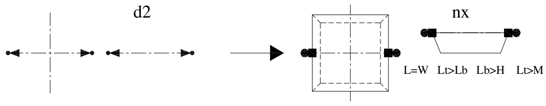
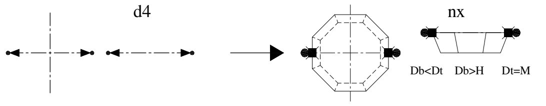
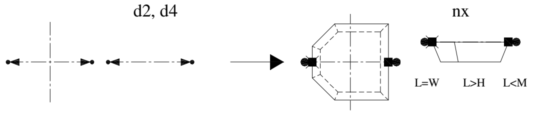
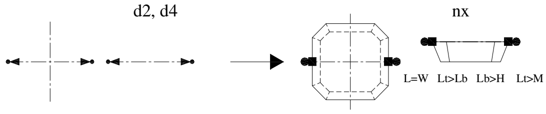
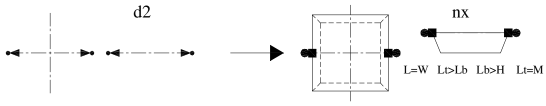
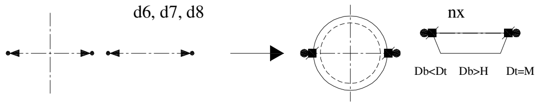
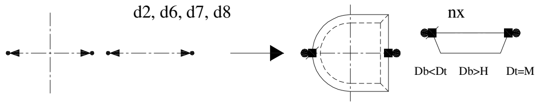
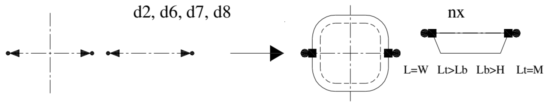
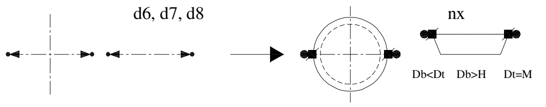
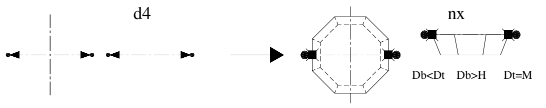
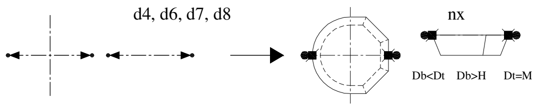
Rule no.	Dreivation of hybrid rules of minaret body	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6b		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 9b		nx={n(x-1)\d1, d4, d9}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3b		nx={n(x-1)\d2, d3, d6, d7, d8, d12}	Rule Prevalence=0.5 Geometrical difference=0.416 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 10b		nx={n(x-1)\d1, d2, d3, d4, d6, d7, d8, d12}	Rule Prevalence=0.666 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3b		nx={n(x-1)\d6}	Rule Prevalence=0.083 Geometrical difference=0.416 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 11b		nx={n(x-1)\d1, d4, d6}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1b		nx={n(x-1)\d1}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.583	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3b		nx={n(x-1)\d5 d6, d7}	Rule Prevalence=0.333 Geometrical difference=0.416 Sequential difference=0.583	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 12b		nx={n(x-1)\d1, d5, d6, d7, d8}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.583	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

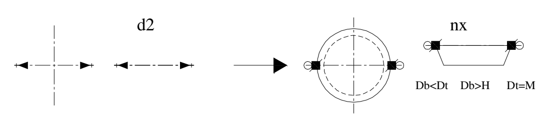
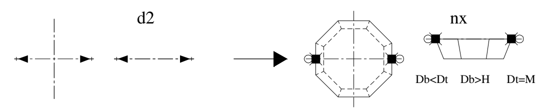
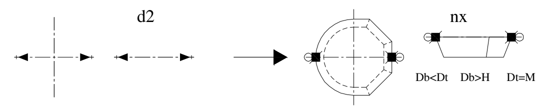
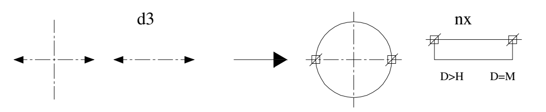
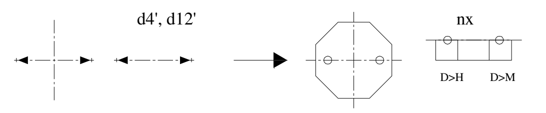
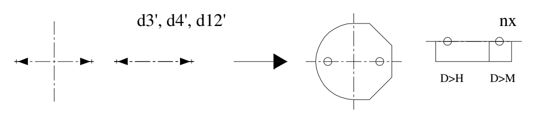
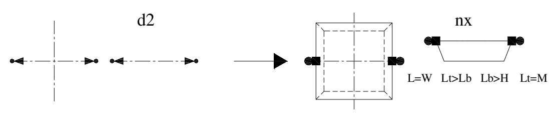
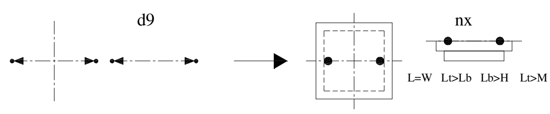
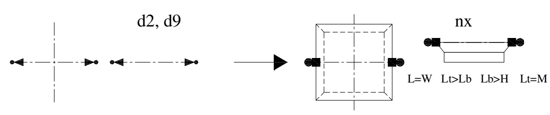
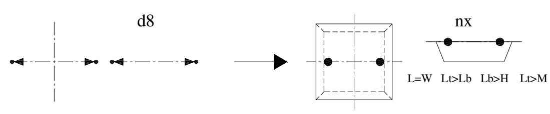
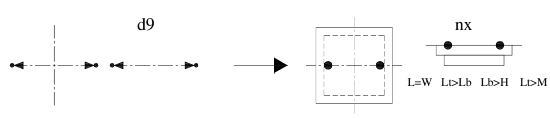
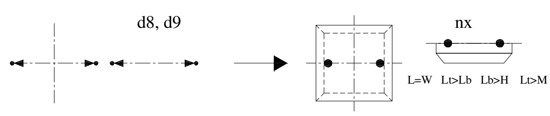
Rule no.	Dreivation of hybrid rules of minaret body	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 3b		nx={n(x-1)\d2, d3, d6, d7, d8, d12}	Rule Prevalence=0.5 Geometrical difference=0.416 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6b		nx={n(x-1)\d4, d8}	Rule Prevalence=0.166 Geometrical difference=0.583 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 13b		nx={n(x-1)\d2, d3, d4, d6, d7, d8, d12}	Rule Prevalence=0.583 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 10b		nx={n(x-1)\d6}	Rule Prevalence=0.083 Geometrical difference=0.416 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 9b		nx={n(x-1)\d6}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 14b		nx={n(x-1)\d6}	Rule Prevalence=0.083 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence=0.166 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 12b		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 15b		nx={n(x-1)\d1, d4, d9}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence=0.166 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 7b		nx={n(x-1)\d5}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 16b		nx={n(x-1)\d1, d4, d5}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

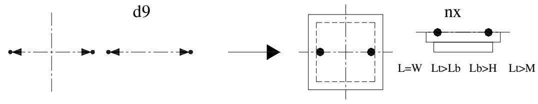
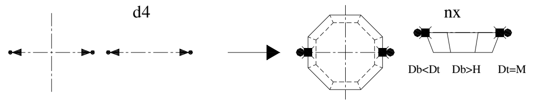
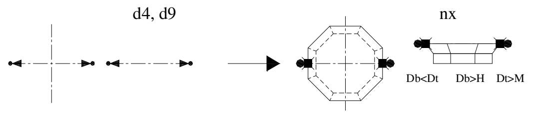
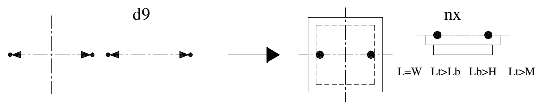
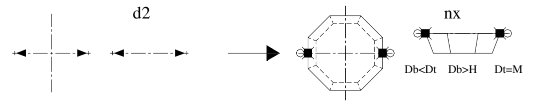
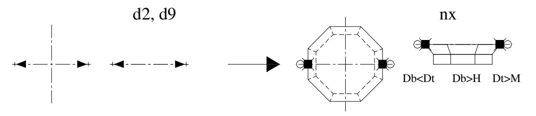
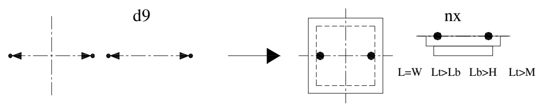
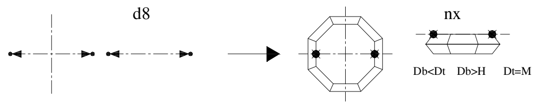
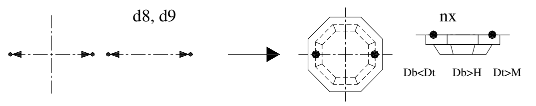
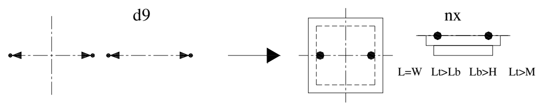
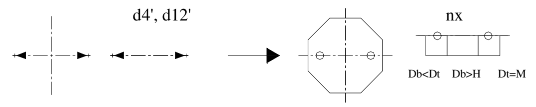
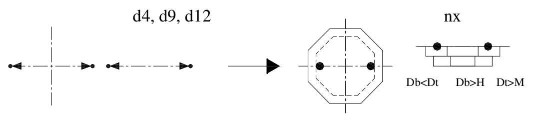
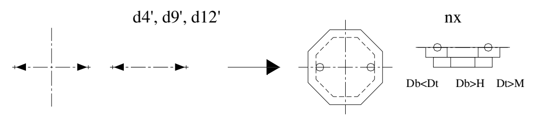
Rule no.	Dreivation of hybrid rules of minaret body	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence=0.166 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3b		nx={n(x-1)\d2, d3, d6, d7, d8, d12}	Rule Prevalence=0.5 Geometrical difference=0.416 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 17b		nx={n(x-1)\d1, d2, d3, d4, d6, d7, d8, d12}	Rule Prevalence=0.666 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1b		nx={n(x-1)\d1, d4}	Rule Prevalence=0.166 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3b		nx={n(x-1)\d6}	Rule Prevalence=0.083 Geometrical difference=0.416 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 18b		nx={n(x-1)\d1, d4, d6}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1b		nx={n(x-1)\d1'}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.583	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3b		nx={n(x-1)\d5, d6, d7, d8}	Rule Prevalence=0.25 Geometrical difference=0.416 Sequential difference=0.583	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 19b		nx={n(x-1)\d1, d5, d6, d7, d8}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.583	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 14b		nx={n(x-1)\d12}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 12b		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 20b		nx={n(x-1)\d9, d12}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

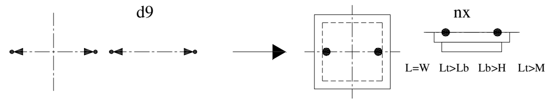
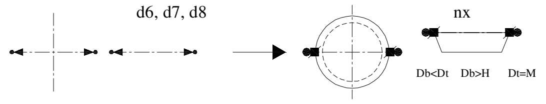
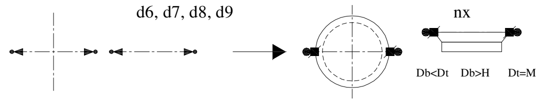
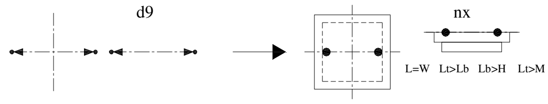
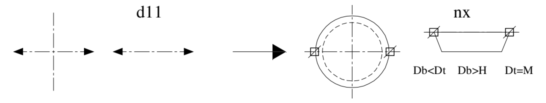
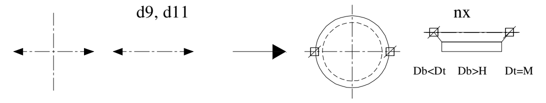
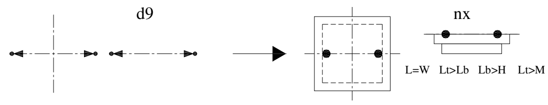
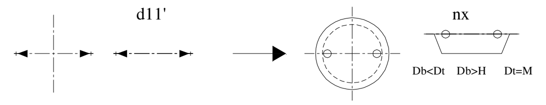
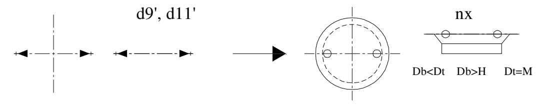
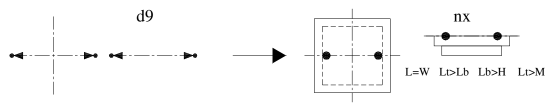
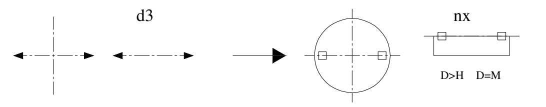
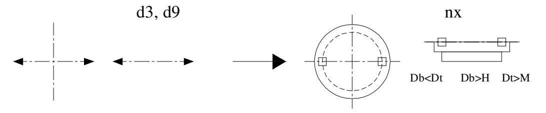
Rule no.	Dreivation of hybrid rules of minaret body	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 4b		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 12b		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 21b		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4b		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5b		nx={n(x-1)\d3, d11}	Rule Prevalence=0.166 Geometrical difference=0.75 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 22b		nx={n(x-1)\d3, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4b		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 13b		nx={n(x-1)\d10}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.583	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 23b		nx={n(x-1)\d3, d10}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.583	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

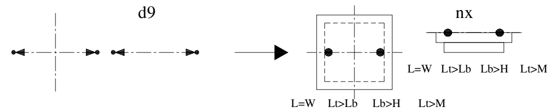
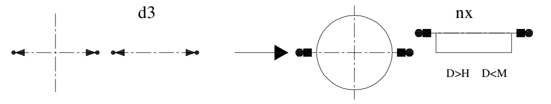
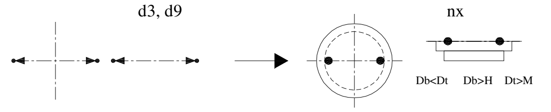
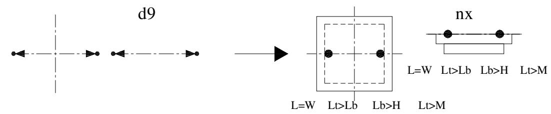
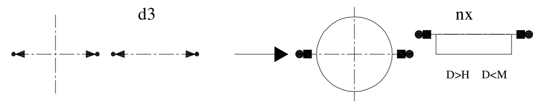
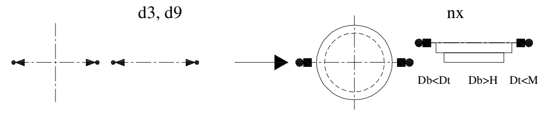
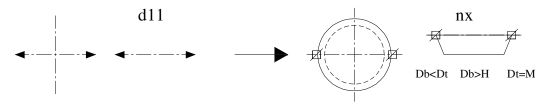
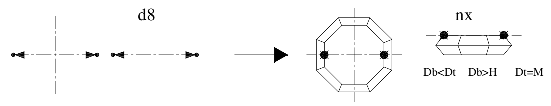
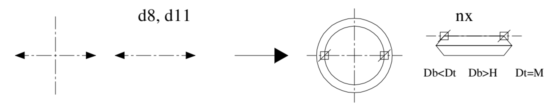
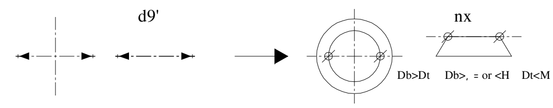
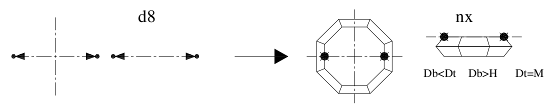
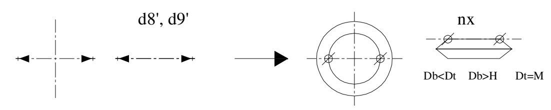
Minaret joint

Rule no.	Dreivation of hybrid rules of minaret joint	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 1c	 d2; nx; L=W Lt>Lb Lb>H Lt>M	nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6c	 d4; nx; Db<Dt Db>H Dt=M	nx={n(x-1)\d2, d4}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 1c	 d2, d4; nx; L=W L>H L<M	nx={n(x-1)\d2, d4}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 2c	 d2, d4; nx; L=W Lt>Lb Lb>H Lt>M	nx={n(x-1)\d2, d4}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1c	 d2; nx; L=W Lt>Lb Lb>H Lt=M	nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2c	 d6, d7, d8; nx; Db<Dt Db>H Dt=M	nx={n(x-1)\d6, d7, d8}	Rule Prevalence=0.25 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 3c	 d2, d6, d7, d8; nx; Db<Dt Db>H Dt=M	nx={n(x-1)\d2, d6, d7, d8}	Rule Prevalence=0.333 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 4c	 d2, d6, d7, d8; nx; L=W Lt>Lb Lb>H Lt=M	nx={n(x-1)\d2, d6, d7, d8}	Rule Prevalence=0.333 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2c	 d6, d7, d8; nx; Db<Dt Db>H Dt=M	nx={n(x-1)\d6, d7, d8}	Rule Prevalence=0.25 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6c	 d4; nx; Db<Dt Db>H Dt=M	nx={n(x-1)\d4}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 5c	 d4, d6, d7, d8; nx; Db<Dt Db>H Dt=M	nx={n(x-1)\d4, d6, d7, d8}	Rule Prevalence=0.333 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

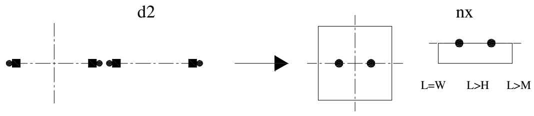
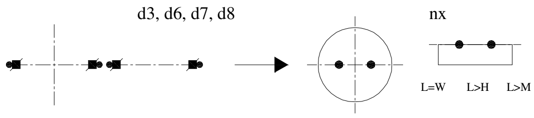
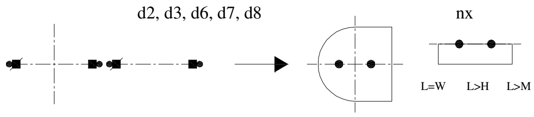
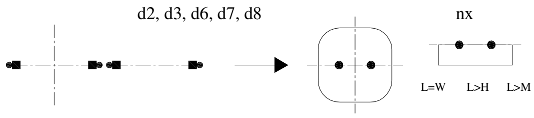
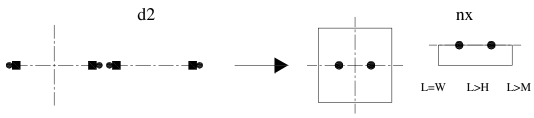
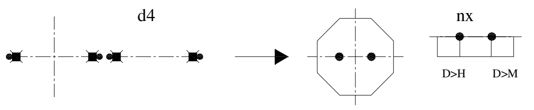
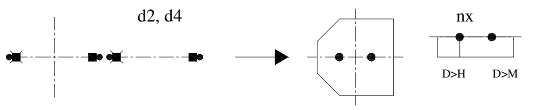
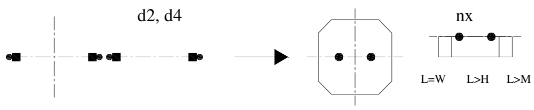
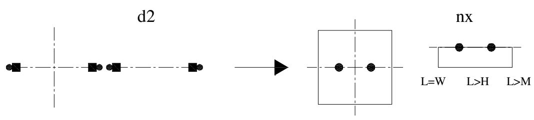
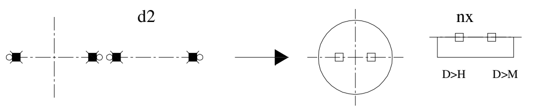
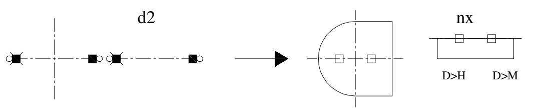
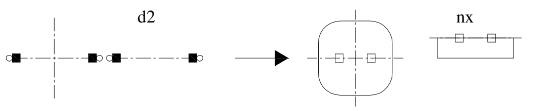
Rule no.	Dreivation of hybrid rules of minaret joint	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 2c		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6c		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 6c		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5c		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 9c		nx={n(x-1)\d4, d12}	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 7c		nx={n(x-1)\d3, d4, d12}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1c		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 11c		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 8c		nx={n(x-1)\d2, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 10c		nx={n(x-1)\d8}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 11c		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 9c		nx={n(x-1)\d8, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

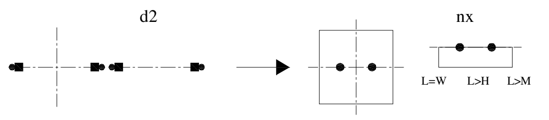
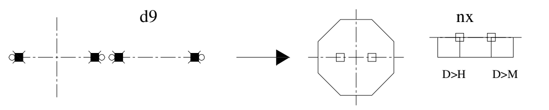
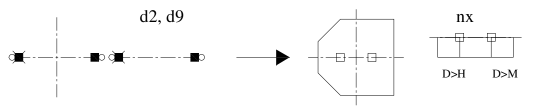
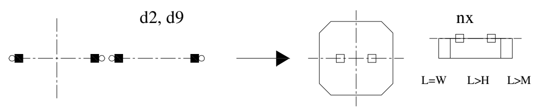
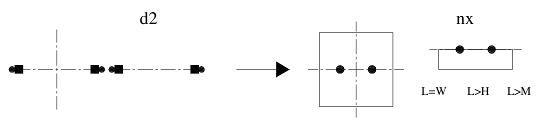
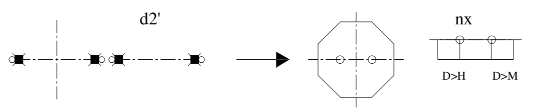
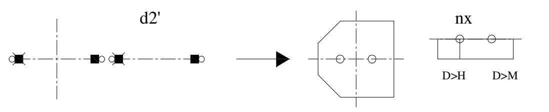
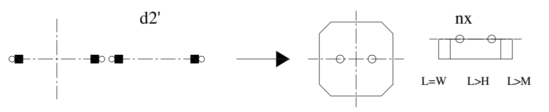
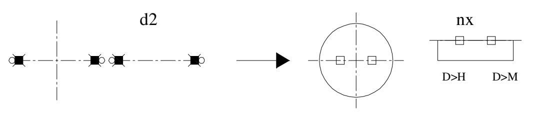
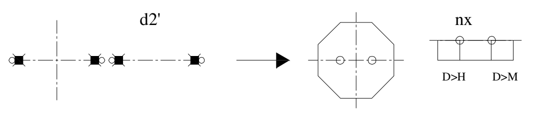
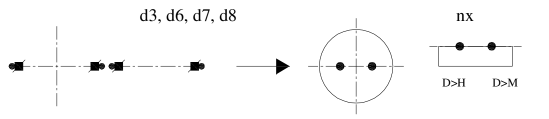
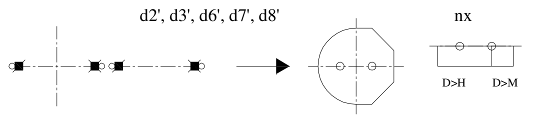
Rule no.	Dreivation of hybrid rules of minaret joint	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 11c		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6c		nx={n(x-1)\d4}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 10c		nx={n(x-1)\d4, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 11c		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 7c		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 11c		nx={n(x-1)\d2, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 11c		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 12c		nx={n(x-1)\d8}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 12c		nx={n(x-1)\d8, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 11c		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 9c		nx={n(x-1)\d4, d12}	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 13c		nx={n(x-1)\d4, d9, d12}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 14c		nx={n(x-1)\d4, d9, d12}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

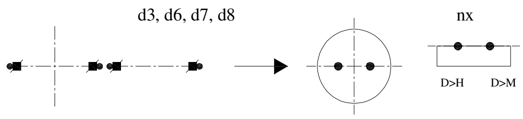
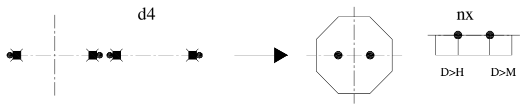
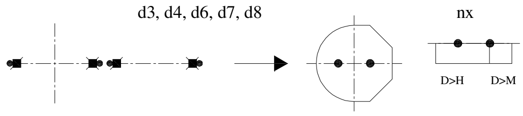
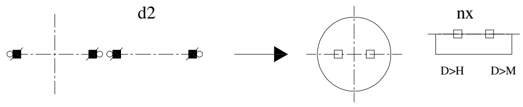
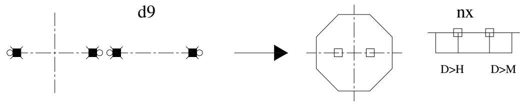
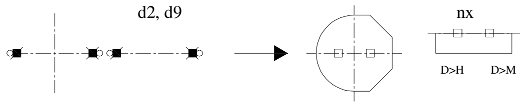
Rule no.	Dreivation of hybrid rules of minaret joint	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 11c	d9  nx L=W Lt>Lb Lb>H Lt>M	nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2c	d6, d7, d8  nx Db<Dt Db>H Dt=M	nx={n(x-1)\d6, d7, d8}	Rule Prevalence=0.25 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 15c	d6, d7, d8, d9  nx Db<Dt Db>H Dt=M	nx={n(x-1)\d6, d7, d8, d9}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 11c	d9  nx L=W Lt>Lb Lb>H Lt>M	nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 14c	d11  nx Db<Dt Db>H Dt=M	nx={n(x-1)\d11}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 16c	d9, d11  nx Db<Dt Db>H Dt=M	nx={n(x-1)\ d9, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 11c	d9  nx L=W Lt>Lb Lb>H Lt>M	nx={n(x-1)\d9'}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 15c	d11'  nx Db<Dt Db>H Dt=M	nx={n(x-1)\d11}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 17c	d9', d11'  nx Db<Dt Db>H Dt=M	nx={n(x-1)\ d9, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 11c	d9  nx L=W Lt>Lb Lb>H Lt>M	nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5c	d3  nx D>H D=M	nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 18c	d3, d9  nx Db<Dt Db>H Dt>M	nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

Rule no.	Dreivation of hybrid rules of minaret joint	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 11c		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4c		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 19c		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 11c		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4c		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 20c		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 14c		nx={n(x-1)\d11}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 12c		nx={n(x-1)\d8}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 21c		nx={n(x-1)\d8, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 13c		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 12c		nx={n(x-1)\d8}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 22c		nx={n(x-1)\d8, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

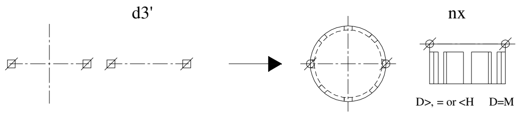
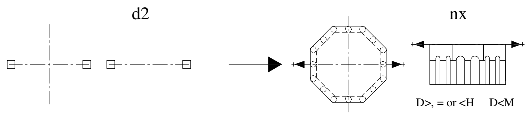
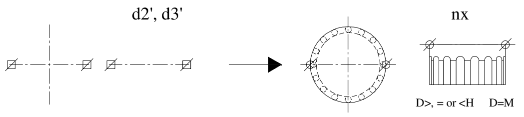
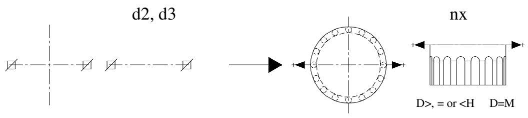
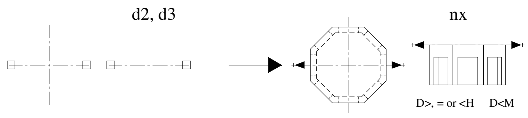
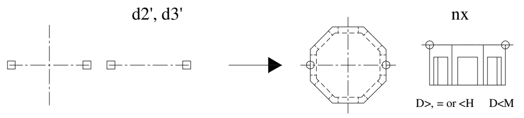
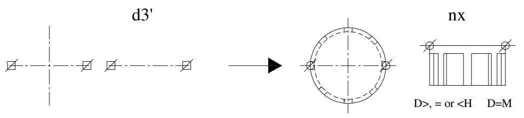
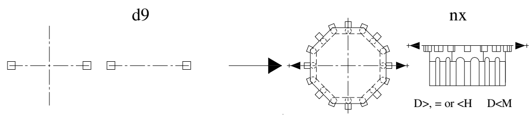
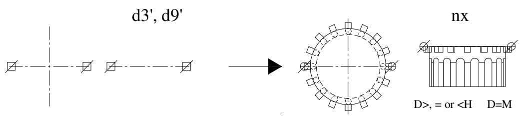
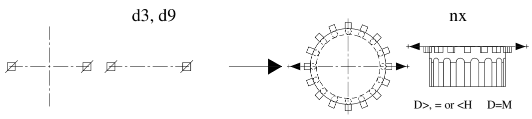
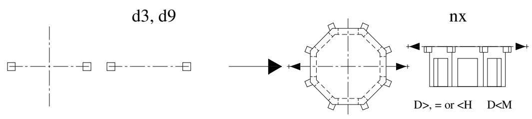
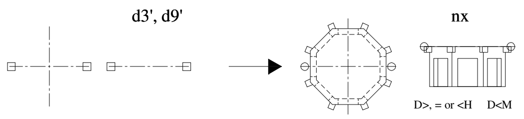
Minaret balcony

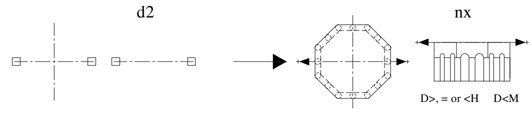
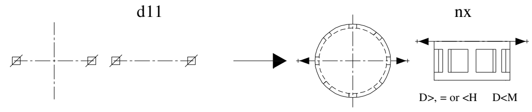
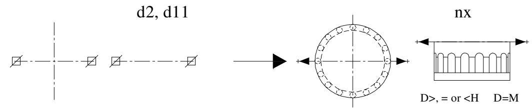
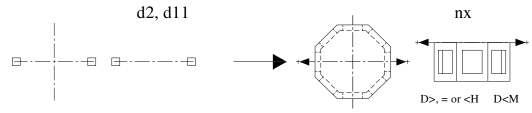
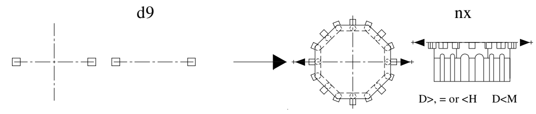
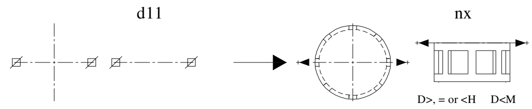
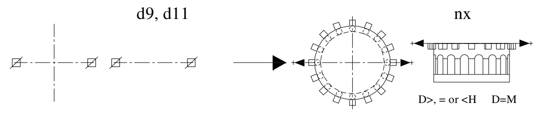
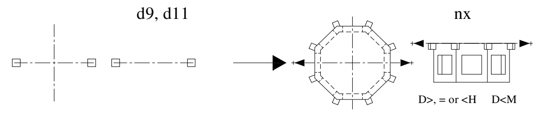
Rule no.	Dreivation of hybrid rules of minaret balcony	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 1d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4d		nx={n(x-1)\d3, d6, d7, d8}	Rule Prevalence=0.333 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 1d		nx={n(x-1)\d2, d3, d6, d7, d8}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 2d		nx={n(x-1)\d2, d3, d6, d7, d8}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5d		nx={n(x-1)\d4}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 3d		nx={n(x-1)\d2, d4}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 4d		nx={n(x-1)\d2, d4}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 5d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degrees= Geometrical difference= Sequential difference=
HR 6d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degrees= Geometrical difference= Sequential difference=

Rule no.	Dreivation of hybrid rules of minaret balcony	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 1d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6d		nx={n(x-1)\d11}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 7d		nx={n(x-1)\d2, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 8d		nx={n(x-1)\d2, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 9d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 10d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2d		nx={n(x-1)\d11}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4d		nx={n(x-1)\d3, d6, d7, d8}	Rule Prevalence=0.333 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 11d		nx={n(x-1)\d2, d3, d6, d7, d8}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

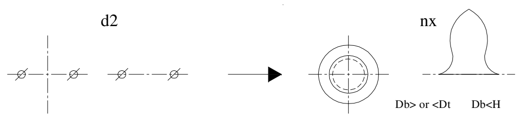
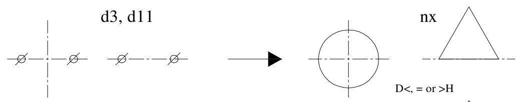
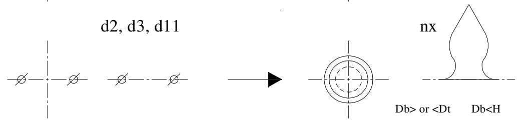
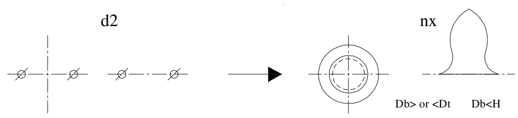
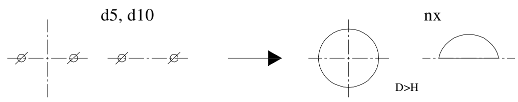
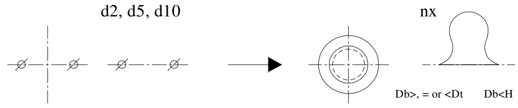
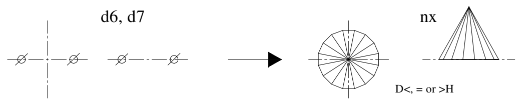
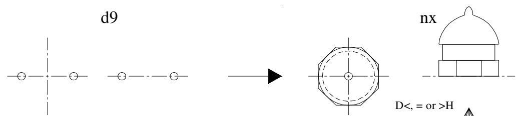
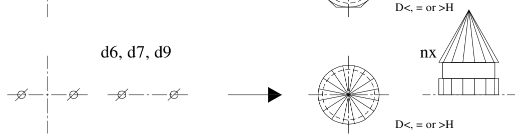
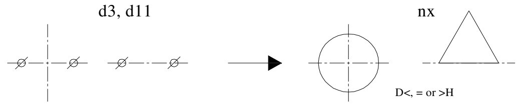
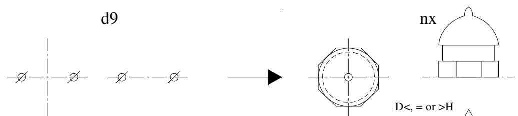
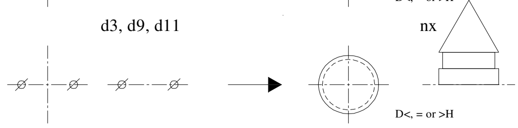
Rule no.	Dreivation of hybrid rules of minaret balcony	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 4d		nx={n(x-1)\d3, d6, d7, d8}	Rule Prevalence=0.333 Geometrical difference=0.583 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5d		nx={n(x-1)\d4}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 12d		nx={n(x-1)\d3, d4, d6, d7, d8}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2d		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6d		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 13d		nx={n(x-1)\d2, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

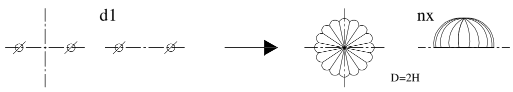
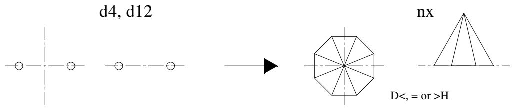
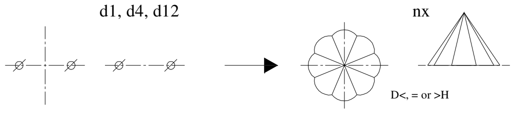
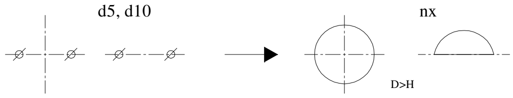
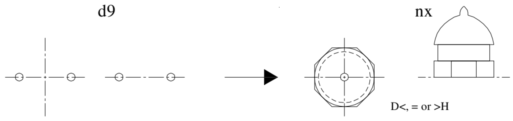
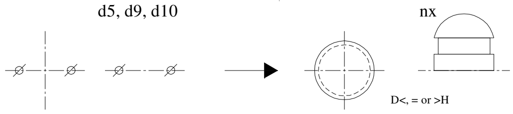
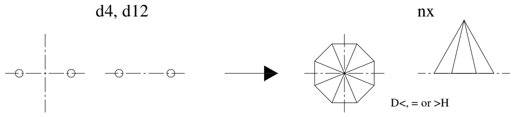
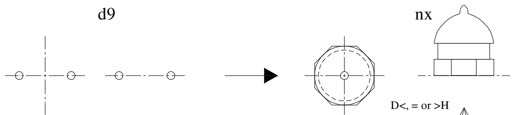
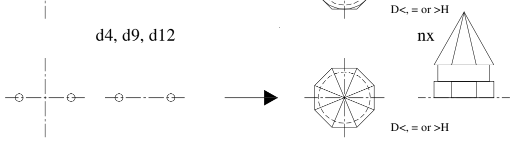
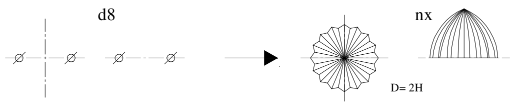
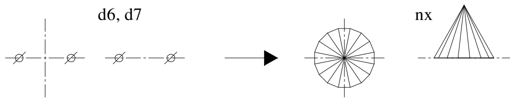
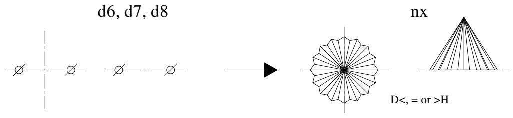
Minaret lantern

Rule no.	Dreivation of hybrid rules of minaret lantern	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 1e		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2e		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 1e		nx={n(x-1)\d2, d3}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 2e		nx={n(x-1)\d2, d3}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 3e		nx={n(x-1)\d2, d3}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 4e		nx={n(x-1)\d2, d3}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 1e		nx={n(x-1)\d3}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3e		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 5e		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 6e		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 7e		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 8e		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

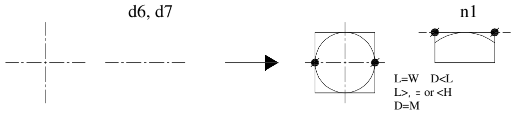
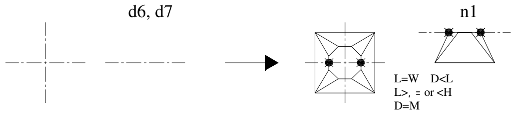
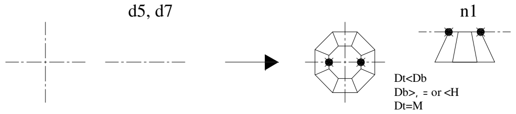
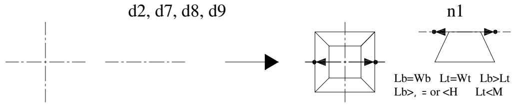
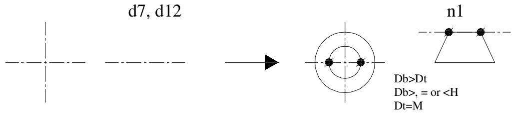
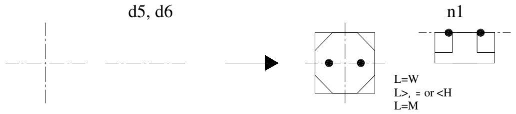
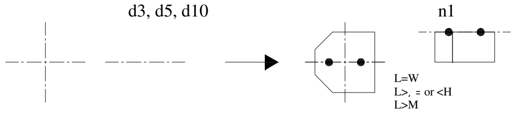
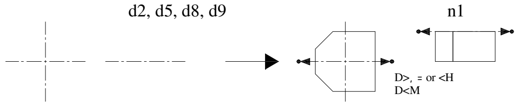
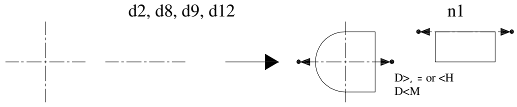
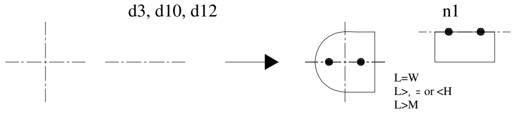
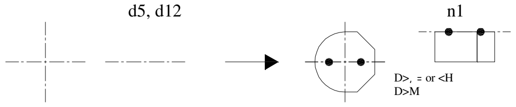
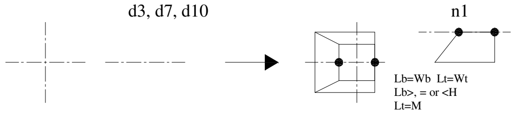
Rule no.	Dreivation of hybrid rules of minaret lantern	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 2e		nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4e		nx={n(x-1)\d11}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 9e		nx={n(x-1)\d2, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 10e		nx={n(x-1)\d2, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3e		nx={n(x-1)\d9}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4e		nx={n(x-1)\d11}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 11e		nx={n(x-1)\d9, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 12e		nx={n(x-1)\d9, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

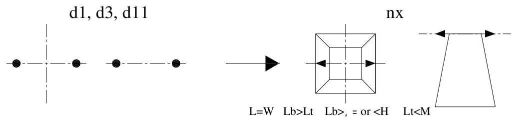
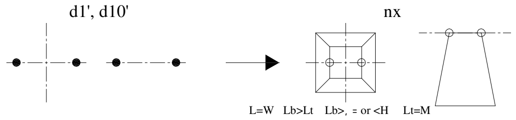
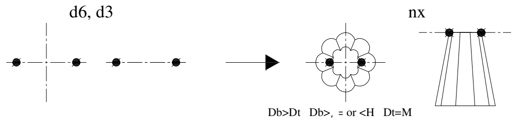
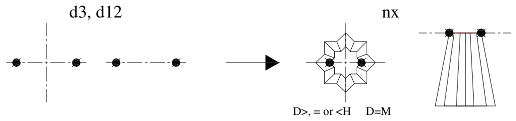
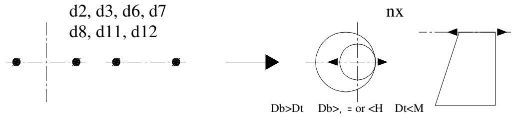
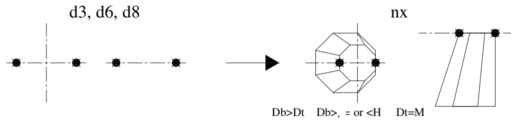
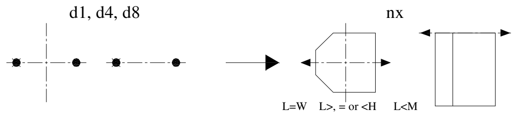
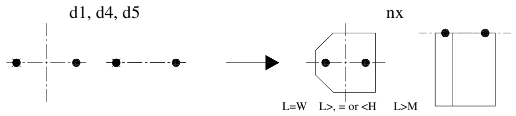
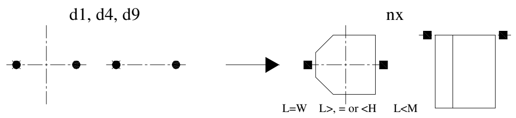
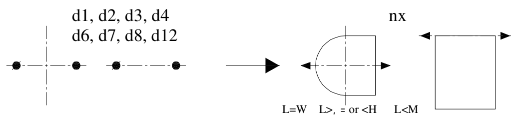
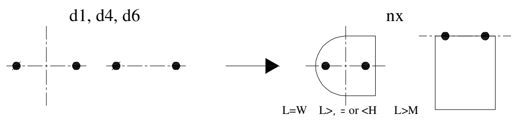
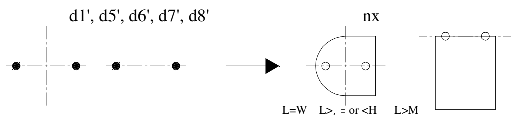
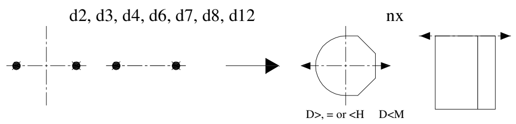
Minaret head

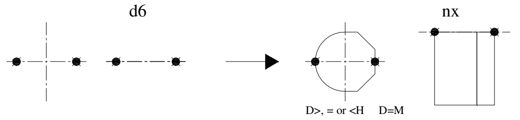
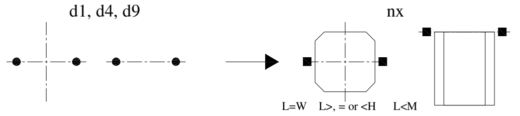
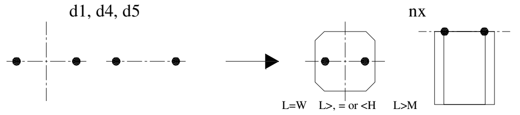
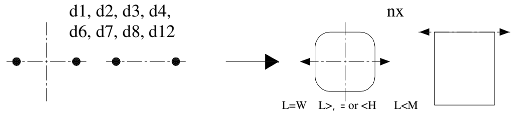
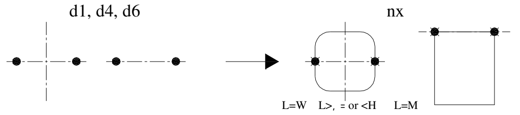
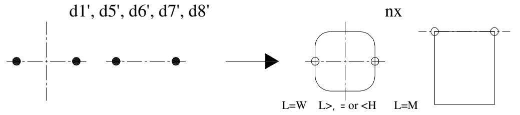
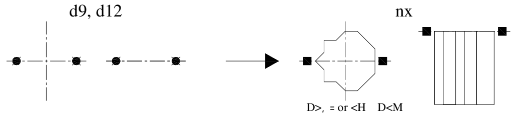
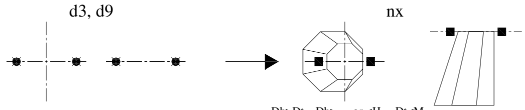
Rule no.	Dreivation of hybrid rules of minaret head	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 2f		nx=0	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3f		nx=0	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 1f		nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 2f		nx=0	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5f		nx=0	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 2f		nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6f		nx=0	Rule Prevalence=1.66 Geometrical difference=0.833 Sequential difference=0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 8f		nx=0	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 3f		nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 3f		nx=0	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 8f		nx=0	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 4f		nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

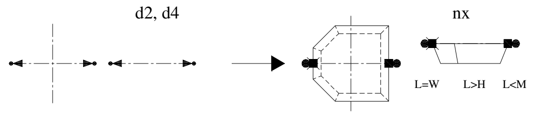
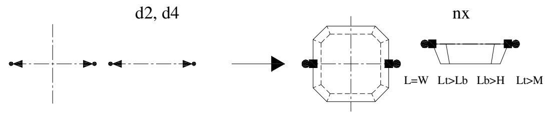
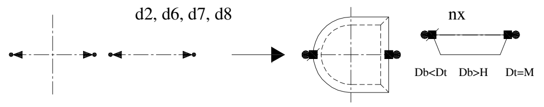
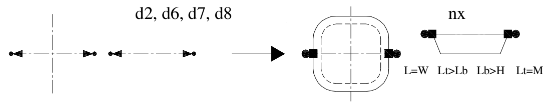
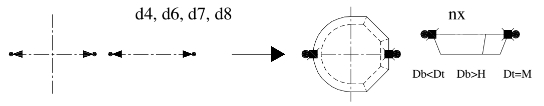
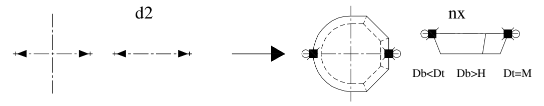
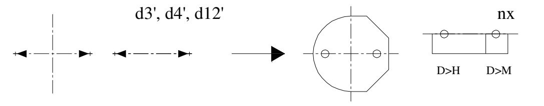
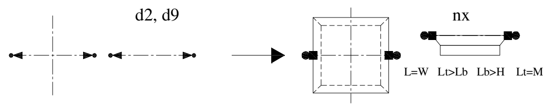
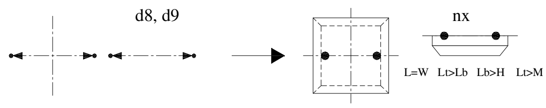
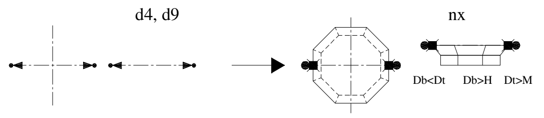
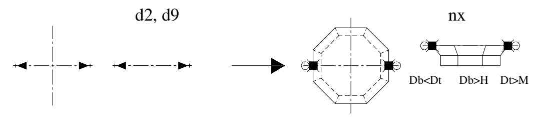
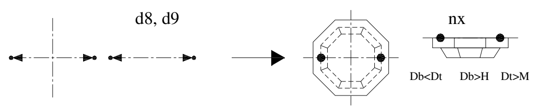
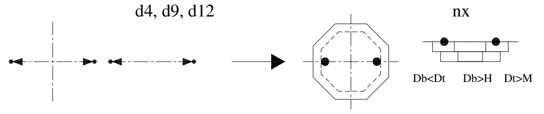
Rule no.	Dreivation of hybrid rules of minaret head	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
OR 1f	d1  nx D=2H	nx=0	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4f	d4, d12  nx D<, = or >H	nx=0	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 5f	d1, d4, d12  nx D<, = or >H	nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 5f	d5, d10  nx D>H	nx=0	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 8f	d9  nx D<, = or >H	nx=0	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 6f	d5, d9, d10  nx D<, = or >H	nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 4f	d4, d12  nx D<, = or >H	nx=0	Rule Prevalence=0.166 Geometrical difference=0.833 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 8f	d9  nx D<, = or >H	nx=0	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 7f	d4, d9, d12  nx D<, = or >H	nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 7f	d8  nx D= 2H	nx=0	Rule Prevalence=0.083 Geometrical difference=0.961 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
OR 6f	d6, d7  nx	nx=0	Rule Prevalence=1.66 Geometrical difference=0.833 Sequential difference=0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 8f	d6, d7, d8  nx D<, = or >H	nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

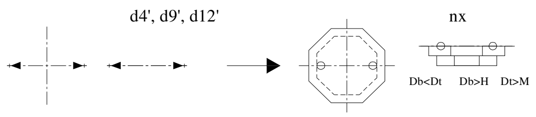
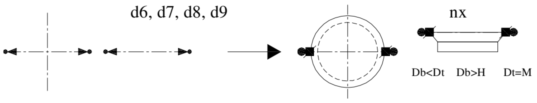
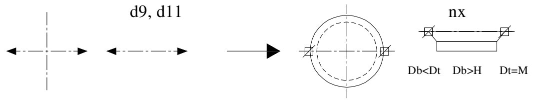
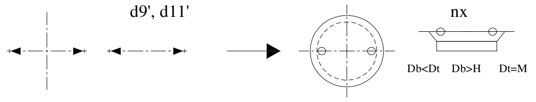
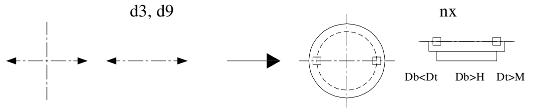
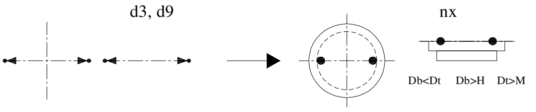
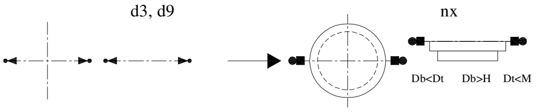
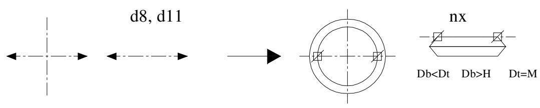
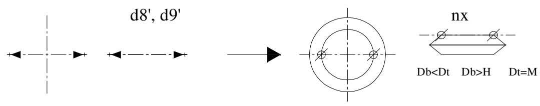
A-2-2 Lists of hybrid rules

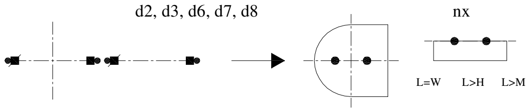
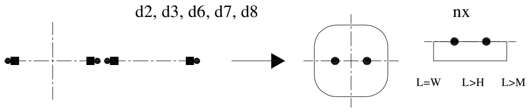
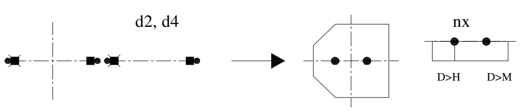
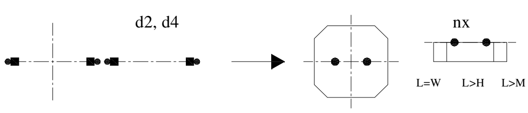
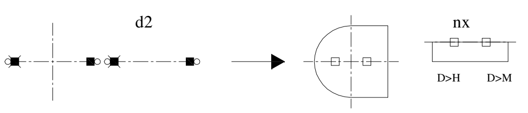
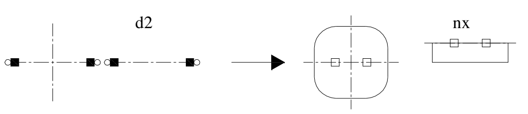
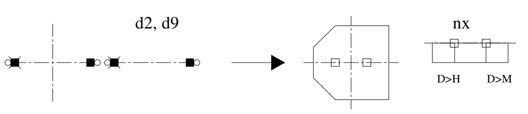
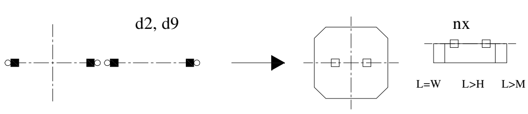
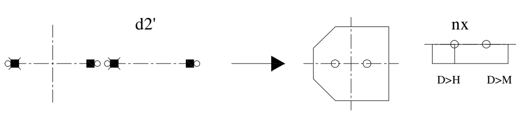
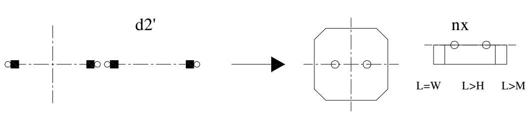
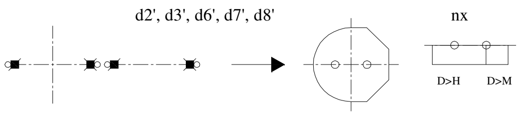
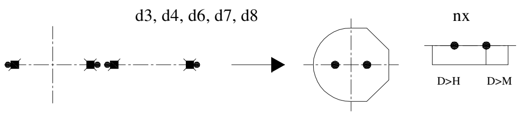
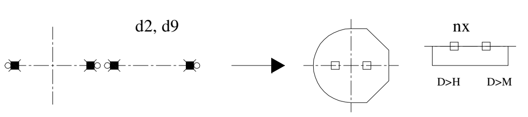
Rule no.	Subclass rule set of minaret base (Hybrid rules)	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
HR 1a	d6, d7 n1 	n1={n\d6, d7}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 2a	d6, d7 n1 	n1={n\d6, d7}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 3a	d5, d7 n1 	n1={n\d5, d7}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 4a	d2, d7, d8, d9 n1 	n1={n\d2, d7, d8, d9}	Rule Prevalence=0.33 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 5a	d7, d12 n1 	n1={n\d7, d12}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 6a	d5, d6 n1 	n1={n\d5, d6}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 7a	d3, d5, d10 n1 	n1={n\d3, d5, d10}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 8a	d2, d5, d8, d9 n1 	n1={n\d2, d5, d8, d9}	Rule Prevalence=0.333 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 9a	d2, d8, d9, d12 n1 	n1={n\d2, d8, d9, d12}	Rule Prevalence=0.333 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 10a	d3, d10, d12 n1 	n1={n\d3, d10, d12}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 11a	d5, d12 n1 	n1={n\d5, d12}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 12a	d3, d7, d10 n1 	n1={n\d3, d7, d10}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

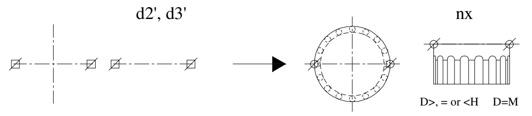
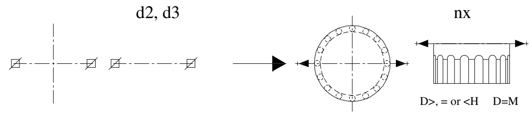
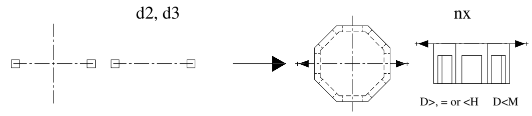
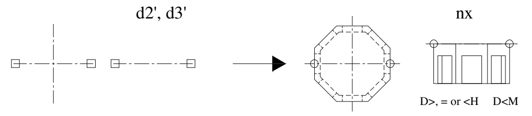
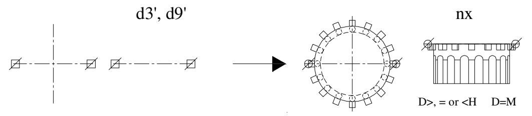
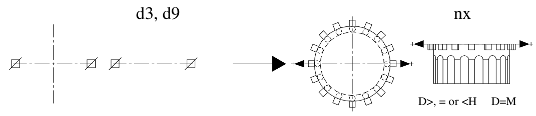
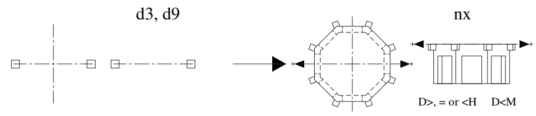
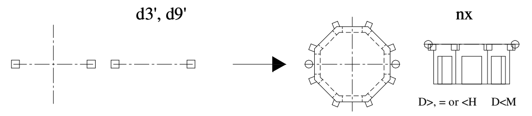
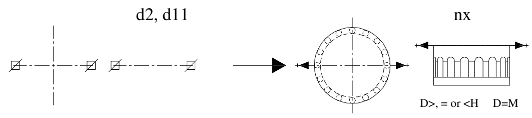
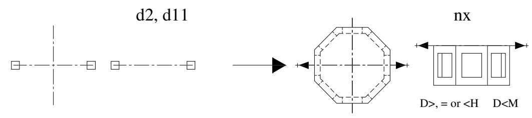
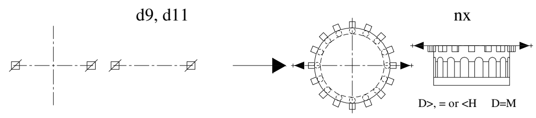
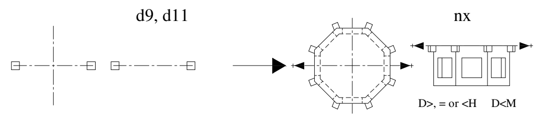
Rule no.	Subclass rule set of minaret body (Hybrid rules)	User guide grammar for hybid design	Evaluation of the hybrid rules	Evaluation of the generated design
HR 1b		nx={n(x-1)\d1, d3, d11}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Design uniqueness= Geometrical difference= Sequential difference=
HR 2b		nx={n(x-1)\d1, d10}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.583	Design diversity= Design abundance= Design uniqueness= Geometrical difference= Sequential difference=
HR 3b		nx={n(x-1)\d6, d3}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Design uniqueness= Geometrical difference= Sequential difference=
HR 4b		nx={n(x-1)\d3, 12}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Design uniqueness= Geometrical difference= Sequential difference=
HR 5b		nx={n(x-1)\d2, d3, d6, d7, d8, d11, d12}	Rule Prevalence=0.583 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Design uniqueness= Geometrical difference= Sequential difference=
HR 6b		nx={n(x-1)\d3, d6, d8}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Design uniqueness= Geometrical difference= Sequential difference=
HR 7b		nx={n(x-1)\d1, d4, d8}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Design uniqueness= Geometrical difference= Sequential difference=
HR 8b		nx={n(x-1)\d1, d4, d5}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 9b		nx={n(x-1)\d1, d4, d9}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 10b		nx={n(x-1)\d1, d2, d3, d4, d6, d7, d8, d12}	Rule Prevalence=0.666 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 11b		nx={n(x-1)\d1, d4, d6}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 12b		nx={n(x-1)\d1, d5, d6, d7, d8}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.583	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 13b		nx={n(x-1)\d2, d3, d4, d6, d7, d8, d12}	Rule Prevalence=0.583 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Design uniqueness= Geometrical difference= Sequential difference=

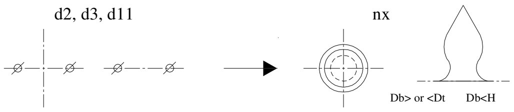
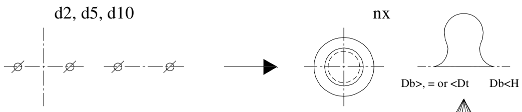
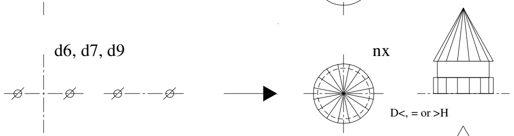
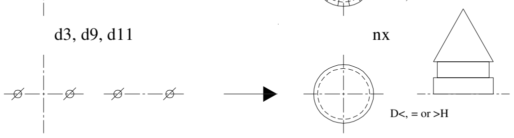
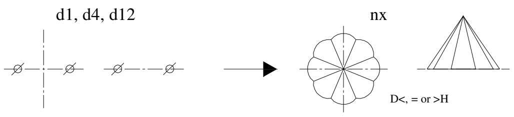
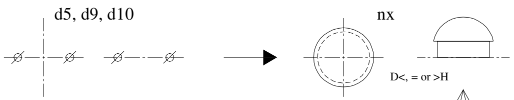
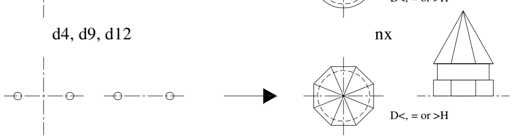
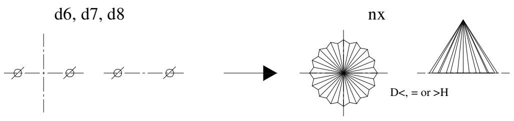
Rule no.	Subclass rule set of minaret body (Hybrid rules)	User guide grammar for hybid design	Evaluation of the hybrid rules	Evaluation of the generated design
HR 14b	d6 nx  D>, = or <H D=M	nx={n(x-1)\d6}	Rule Prevalence=0.083 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Design uniqueness= Geometrical difference= Sequential difference=
HR 15b	d1, d4, d9 nx  L=W L>, = or <H L<M	nx={n(x-1)\d1, d4, d9}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 16b	d1, d4, d5 nx  L=W L>, = or <H L>M	nx={n(x-1)\d1, d4, d5}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 17b	d1, d2, d3, d4, d6, d7, d8, d12 nx  L=W L>, = or <H L<M	nx={n(x-1)\d1, d2, d3, d4, d6, d7, d8, d12}	Rule Prevalence=0.666 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 18b	d1, d4, d6 nx  L=W L>, = or <H L=M	nx={n(x-1)\d1, d4, d6}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 19b	d1', d5', d6', d7', d8' nx  L=W L>, = or <H L=M	nx={n(x-1)\d1, d5, d6, d7, d8}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.583	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 20b	d9, d12 nx  D>, = or <H D<M	nx={n(x-1)\d9, d12}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 21b	d3, d9 nx  Db>Dt Db>, = or <H Dt<M	nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Design uniqueness= Geometrical difference= Sequential difference=
HR 22b	d3, d11 nx  Db>Dt Db>, = or <H Dt<M	nx={n(x-1)\d3, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.333	Design diversity= Design abundance= Design uniqueness= Geometrical difference= Sequential difference=
HR 23b	d3', d10' nx  Db>Dt Db>, = or <H Dt=M	nx={n(x-1)\d3, d10}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.583	Design diversity= Design abundance= Design uniqueness= Geometrical difference= Sequential difference=

Rule no.	Subclass rule set of minaret joints (Hybrid rules)	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
HR 1c	 d2, d4 nx L=W L>H L<M	nx={n(x-1)\d2, d4}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 2c	 d2, d4 nx L=W Lt>Lb Lb>H Lt>M	nx={n(x-1)\d2, d4}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 3c	 d2, d6, d7, d8 nx Db<Dt Db>H Dt=M	nx={n(x-1)\d2, d6, d7, d8}	Rule Prevalence=0.333 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 4c	 d2, d6, d7, d8 nx L=W Lt>Lb Lb>H Lt=M	nx={n(x-1)\d2, d6, d7, d8}	Rule Prevalence=0.333 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 5c	 d4, d6, d7, d8 nx Db<Dt Db>H Dt=M	nx={n(x-1)\d4, d6, d7, d8}	Rule Prevalence=0.333 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 6c	 d2 nx Db<Dt Db>H Dt=M	nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 7c	 d3', d4', d12' nx D>H D>M	nx={n(x-1)\d3, d4, d12}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 8c	 d2, d9 nx L=W Lt>Lb Lb>H Lt=M	nx={n(x-1)\d2, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 9c	 d8, d9 nx L=W Lt>Lb Lb>H Lt>M	nx={n(x-1)\d8, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 10c	 d4, d9 nx Db<Dt Db>H Dt>M	nx={n(x-1)\d4, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 11c	 d2, d9 nx Db<Dt Db>H Dt>M	nx={n(x-1)\d2, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 12c	 d8, d9 nx Db<Dt Db>H Dt>M	nx={n(x-1)\d8, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 13c	 d4, d9, d12 nx Db<Dt Db>H Dt>M	nx={n(x-1)\d4, d9, d12}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

Rule no.	Subclass rule set of minaret joints (Hybrid rules)	User guide grammar for hybid design	Evaluation of the hybrid rules	Evaluation of the generated design
HR 14c		nx={n(x-1)\d4, d9, d12}	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 15c		nx={n(x-1)\d6, d7, d8, d9}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 16c		nx={n(x-1)\ d9, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 17c		nx={n(x-1)\ d9, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 18c		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 19c		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 20c		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 21c		nx={n(x-1)\d8, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 22c		nx={n(x-1)\d8, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.666	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

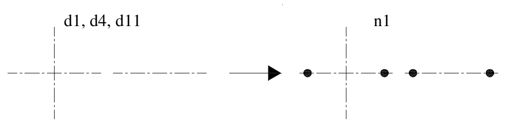
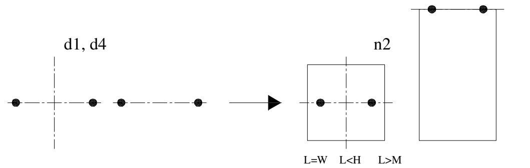
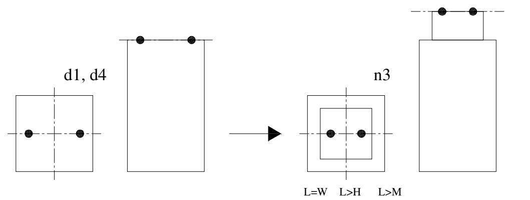
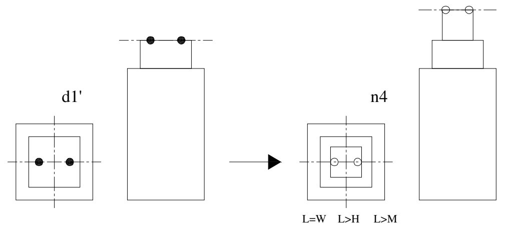
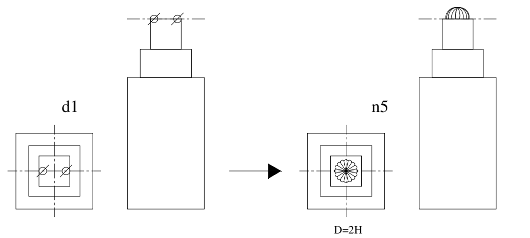
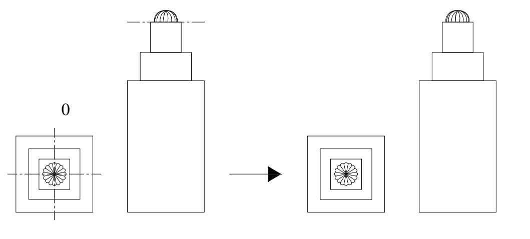
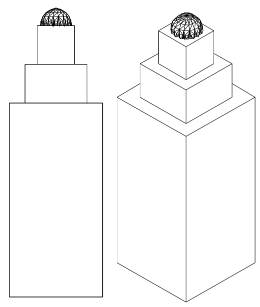
Rule no.	Subclass rule set of minaret balconies (Hybrid rules)	User guide grammar for hybid design	Evaluation of the hybrid rules	Evaluation of the generated design
HR 1d	d2, d3, d6, d7, d8 	nx={n(x-1)\d2, d3, d6, d7, d8}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 2d	d2, d3, d6, d7, d8 	nx={n(x-1)\d2, d3, d6, d7, d8}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 3d	d2, d4 	nx={n(x-1)\d2, d4}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 4d	d2, d4 	nx={n(x-1)\d2, d4}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 5d	d2 	nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degrees= Geometrical difference= Sequential difference=
HR 6d	d2 	nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degrees= Geometrical difference= Sequential difference=
HR 7d	d2, d9 	nx={n(x-1)\d2, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 8d	d2, d9 	nx={n(x-1)\d2, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 9d	d2' 	nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.75 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 10d	d2' 	nx={n(x-1)\d2}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 11d	d2', d3', d6', d7', d8' 	nx={n(x-1)\d2, d3, d6, d7, d8}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 12d	d3, d4, d6, d7, d8 	nx={n(x-1)\d3, d4, d6, d7, d8}	Rule Prevalence=0.416 Geometrical difference=1.0 Sequential difference=0.5	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 13d	d2, d9 	nx={n(x-1)\d2, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.833	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

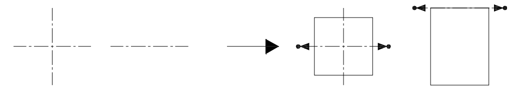
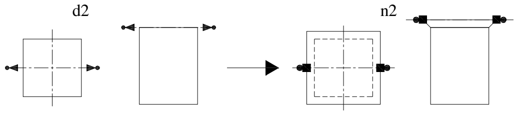
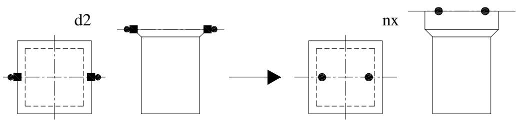
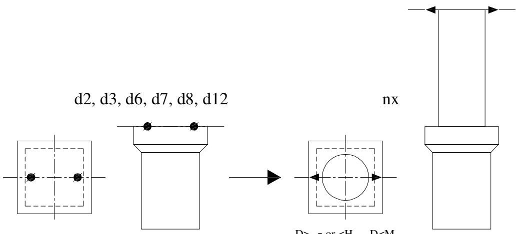
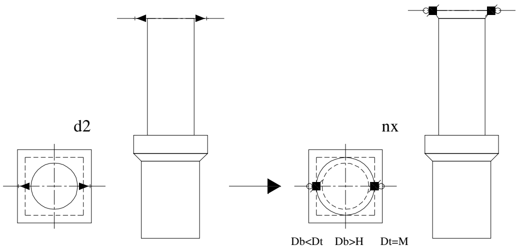
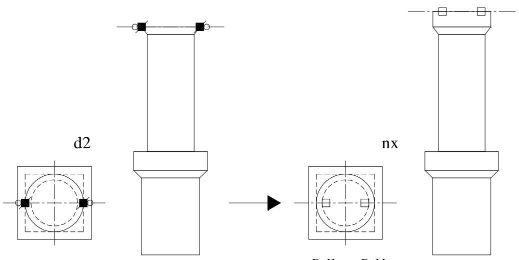
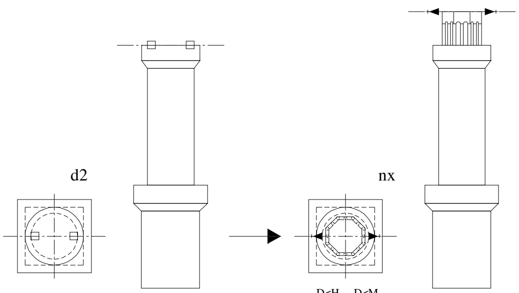
Rule no.	Subclass rule set of minaret lantern (Hybrid rules)	User guide grammar for hybid design	Evaluation of the rules	Evaluation of the generated design
HR 1e		nx={n(x-1)\d2, d3}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 2e		nx={n(x-1)\d2, d3}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 3e		nx={n(x-1)\d2, d3}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 4e		nx={n(x-1)\d2, d3}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 5e		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 6e		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 7e		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 8e		nx={n(x-1)\d3, d9}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.916	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 9e		nx={n(x-1)\d2, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 10e		nx={n(x-1)\d2, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 11e		nx={n(x-1)\d9, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 12e		nx={n(x-1)\d9, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.75	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

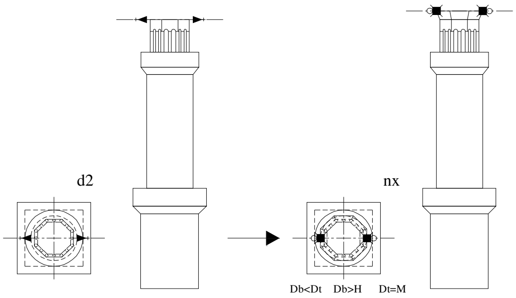
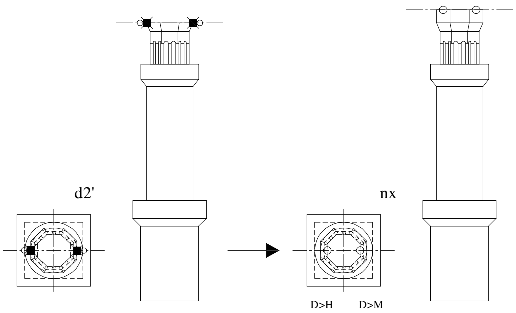
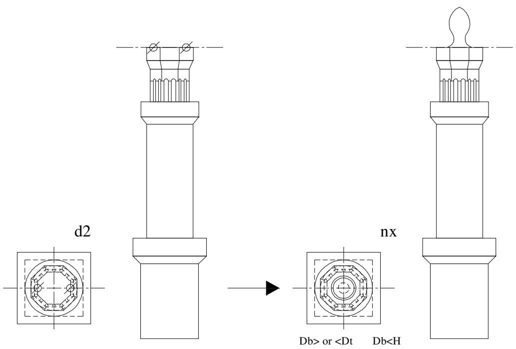
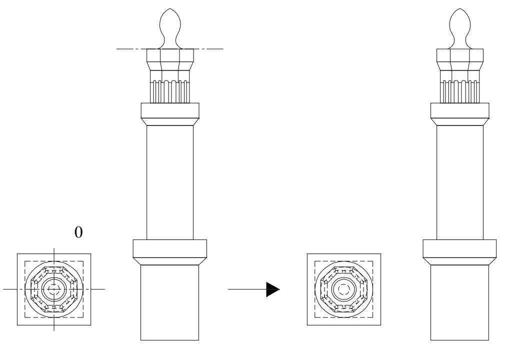
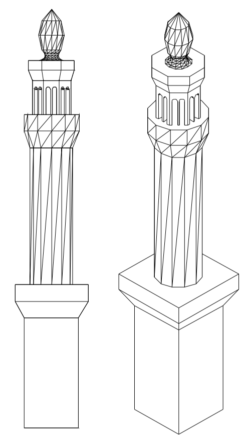
Rule no.	Subclass rule set of minaret head (Hybrid rules)	User guide grammar for hybid design	Evaluation of the hybrid rules	Evaluation of the generated design
HR 1f		nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 2f		nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 3f		nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 4f		nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 5f		nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 6f		nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 7f		nx=0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=
HR 8f		nx=0	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.0	Design diversity= Design abundance= Matching degree= Geometrical difference= Sequential difference=

Appendix–B: Copies of antecedents and examples of hybrid designs

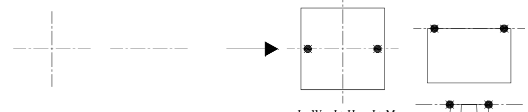
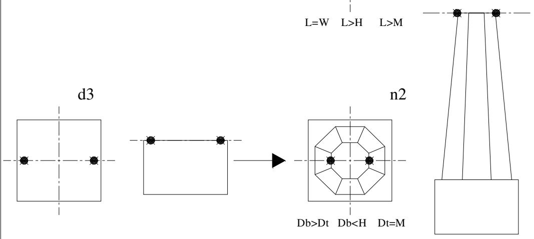
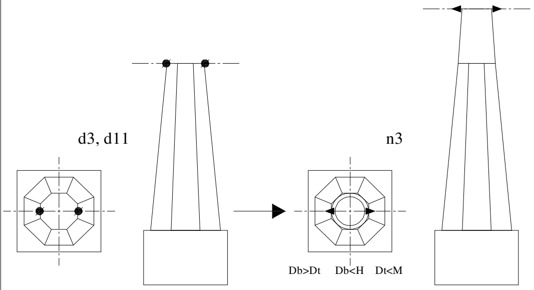
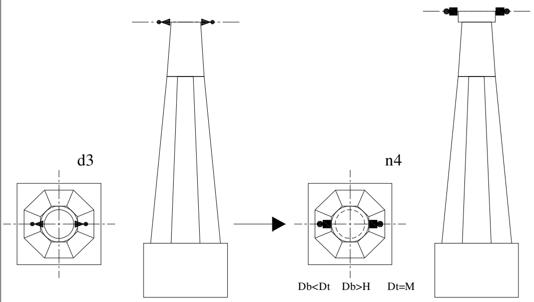
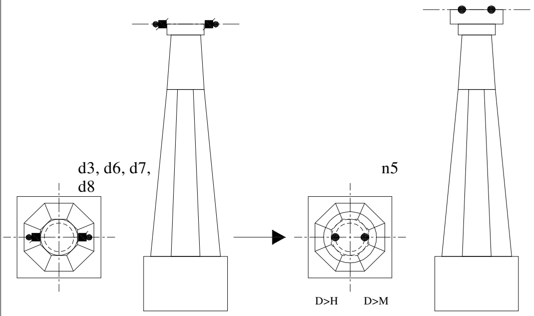
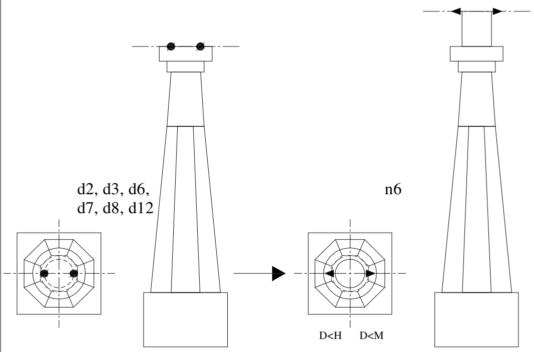
B-1 Copies of existing designs

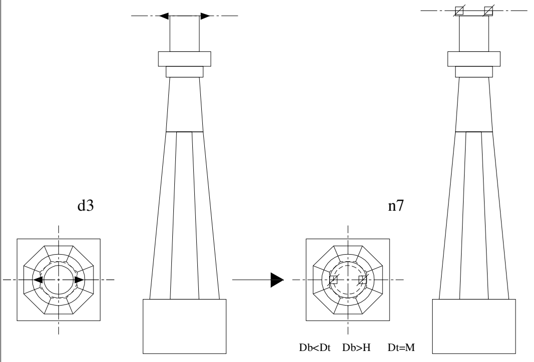
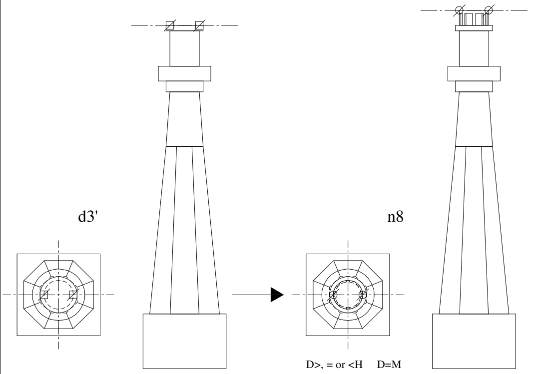
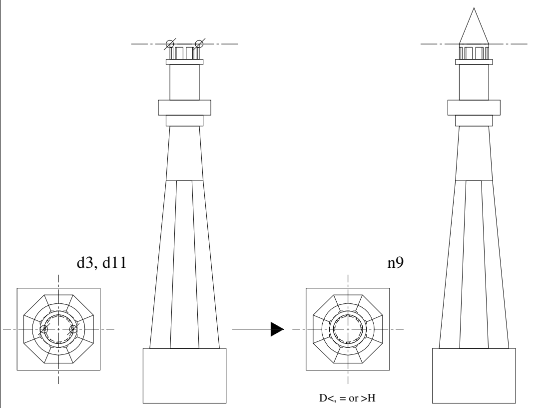
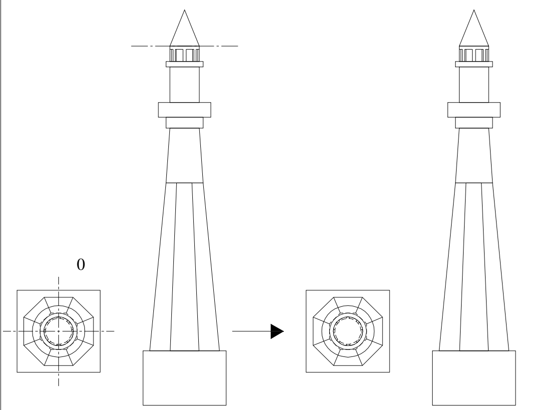
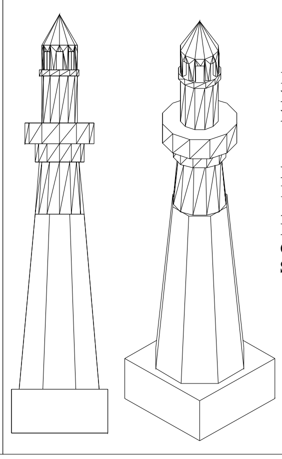
Rule no.	**Original design derivation -** Sidi 'Uqba minaret **(d1) -** 5 Original rules	**User guide grammar for original design**	**Evaluation of the rule**	**Evaluation of the design**
OR 1a		n1 = d1	Rule Prevalence = 0.25 Geometrical difference = 0.75 Sequential difference = 0.75	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 1.0 Geometrical difference = 0.75 Sequential difference = 0.75
OR 1b		n2 = d1	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 2.5 Matching degree = 1.0 Geometrical difference = 0.791 Sequential difference = 0.625
OR 1b		n3 = d1'	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 2.333 Matching degree = 1.0 Geometrical difference = 0.805 Sequential difference = 0.583
OR 2b		n4 = d1	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.812 Sequential difference = 0.583
OR 1f		n5 = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0	Design diversity = 0.25 Design abundance = 1.8 Matching degree = 1.0 Geometrical difference = 0.833 Sequential difference = 0.466
			**Design 1 (d1)** **Sidi 'Uqba minaret** **No. of Original Rules =5** **Evaluation Criteria:** **Diversity= 0.25** **Abundance= 1.8** **Matching= 1.0** **Geometrical diff.= 0.833** **Sequential diff.= 0.466**	

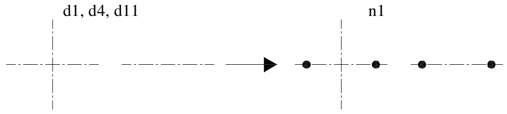
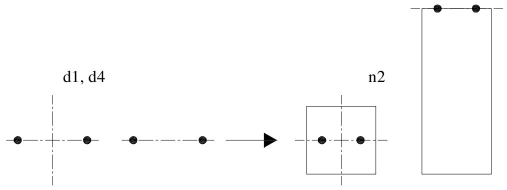
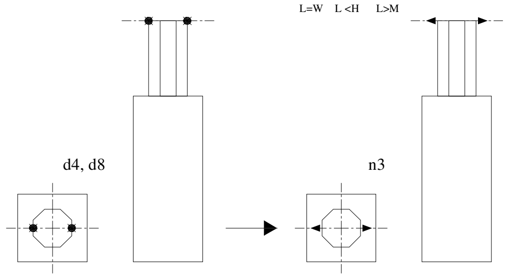
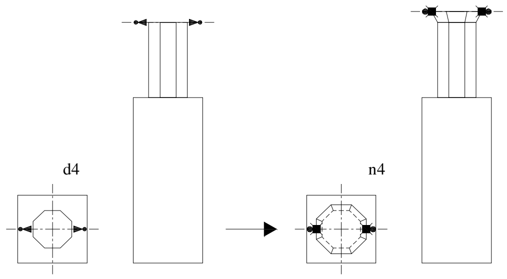
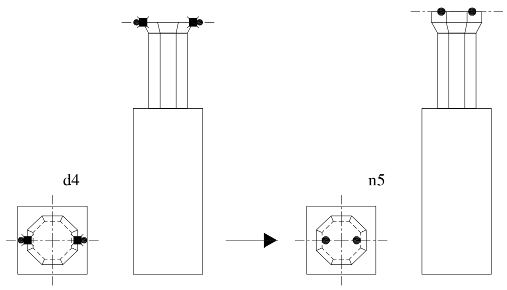
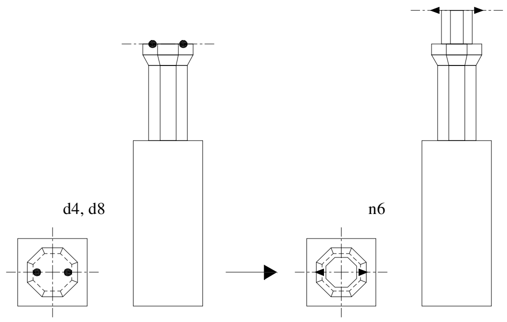
Rule no.	Original design derivation - *Khanqah* of Faraj b. Barquq minaret (d2) - 10 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 2a	d2, d8, d9 n1  L=W L<H M>L	n1 = d2	Rule Prevalence = 0.25 Geometrical difference = 0.583 Sequential difference = 0.75	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 1.0 Geometrical difference = 0.583 Sequential difference = 0.75
OR 1c	d2 n2  L=W Lt>Lb Lt>H Lt=M	n2 = d2	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.708 Sequential difference = 0.625
OR 1d	d2 nx  L=W L>H L>M	n3 = d2	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 1.666 Matching degree = 1.0 Geometrical difference = 0.777 Sequential difference = 0.583
OR 3b	d2, d3, d6, d7, d8, d12 nx  D>, = or <H D<M	n4 = d2	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.583 Design abundance = 2.75 Matching degree = 1.0 Geometrical difference = 0.687 Sequential difference = 0.520
OR 3c	d2 nx  Db<Dt Db>H Dt=M	n5 = d2	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 2.4 Matching degree = 1.0 Geometrical difference = 0.666 Sequential difference = 0.516
OR 2d	d2 nx  D>H D>M	n6 = d2	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.833	Design diversity = 0.583 Design abundance = 2.166 Matching degree = 1.0 Geometrical difference = 0.652 Sequential difference = 0.568
OR 2e	d2 nx  D<H D<M	n7 = d2	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.75	Design diversity = 0.583 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.689 Sequential difference = 0.594

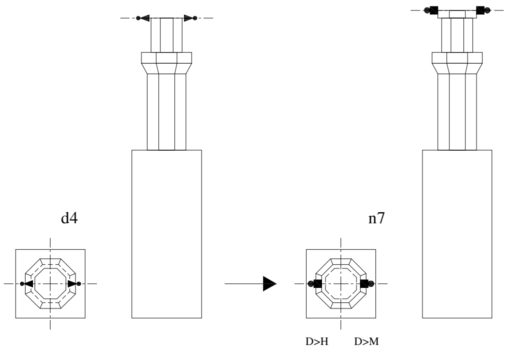
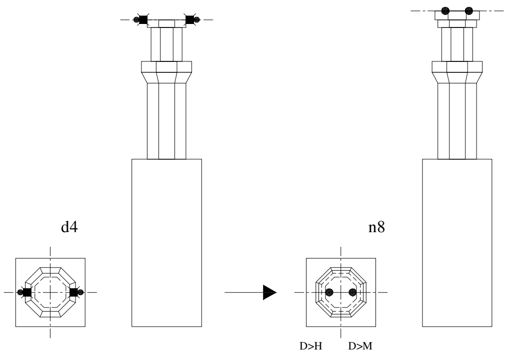
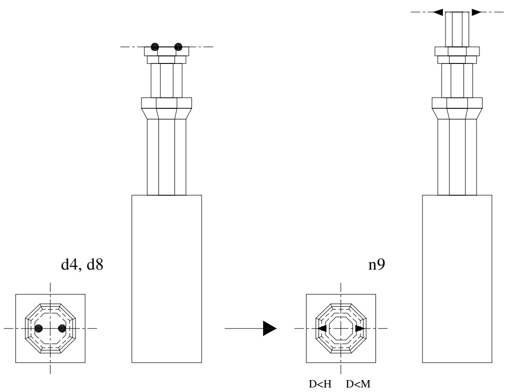
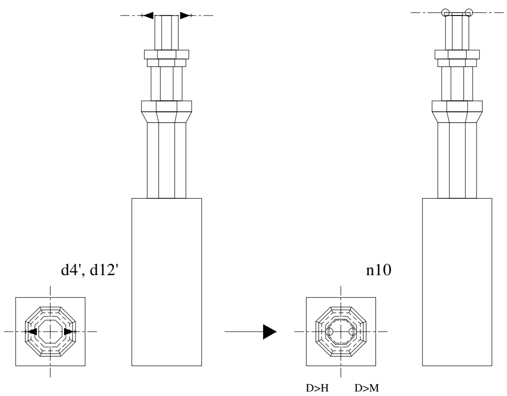
<table>
<tr><th>Rule no.</th><th>Original design derivation - Khanqah of Faraj b. Barquq minaret (d2) - 10 Original rules</th><th>User guide grammar for original design</th><th>Evaluation of the rule</th><th>Evaluation of the design</th></tr>
<tr><td>OR 7c</td><td>
</td><td>n8 = d2'</td><td>Rule Prevalence = 0.083
Geometrical difference = 0.833
Sequential difference = 0.5</td><td>Design diversity = 0.583
Design abundance = 1.875
Matching degree = 1.0
Geometrical difference = 0.707
Sequential difference = 0.582</td></tr>
<tr><td>OR 3d</td><td>
</td><td>n9 = d2</td><td>Rule Prevalence = 0.083
Geometrical difference = 0.75
Sequential difference = 0.916</td><td>Design diversity = 0.583
Design abundance = 1.777
Matching degree = 1.0
Geometrical difference = 0.712
Sequential difference = 0.619</td></tr>
<tr><td>OR 2f</td><td>
</td><td>n10 = 0</td><td>Rule Prevalence = 0.083
Geometrical difference = 0.916
Sequential difference = 0</td><td>Design diversity = 0.583
Design abundance = 1.7
Matching degree = 1.0
Geometrical difference = 0.732
Sequential difference = 0.557</td></tr>
<tr><td></td><td>
</td><td colspan="3">
Design 2 (d2)
Khanqah of Faraj b. Barquq minaret
No. of Original Rules =10

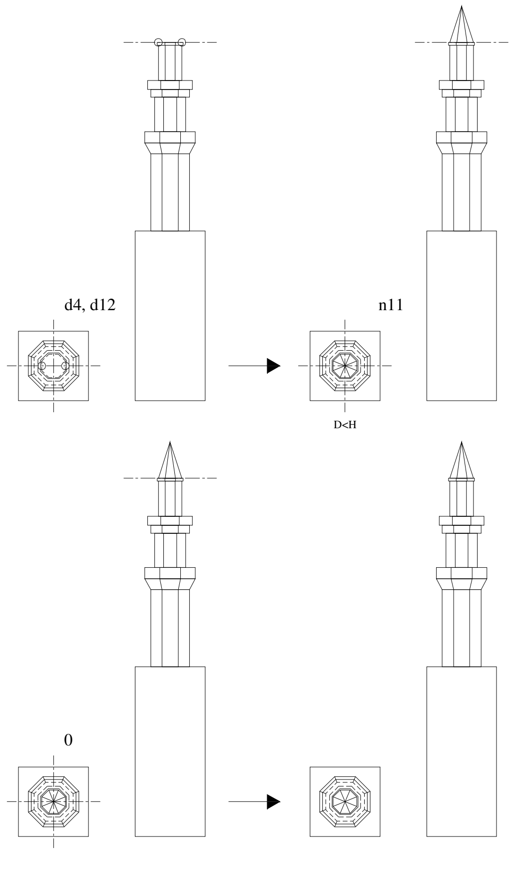
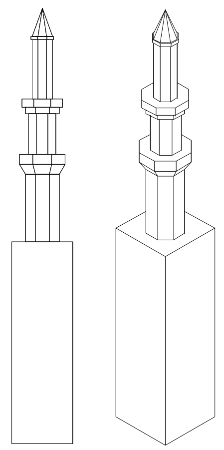
Evaluation Criteria:
Diversity= 0.583
Abundance= 1.7
Matching= 1.0
Geometrical diff.= 0.732
Sequential diff.= 0.557</td></tr>
</table>

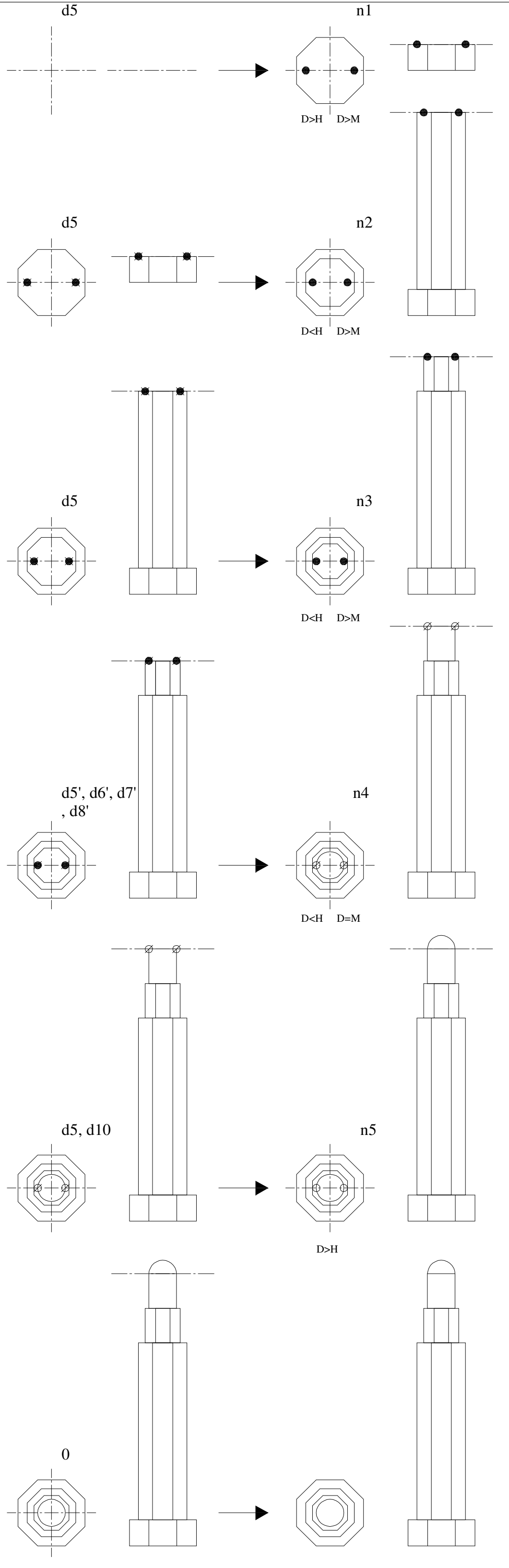
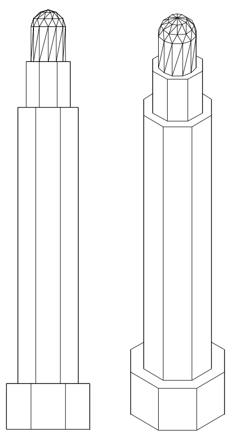
Rule no.	**Original design derivation - Na'in** minaret **(d3) -** 9 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 3a	d3, d10 n1 	n1 = d3	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.583 Sequential difference = 0.5
OR 4b	d3 n2 	n2 = d3	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.5 Matching degree = 1.0 Geometrical difference = 0.749 Sequential difference = 0.5
OR 5b	d3, d11 n3 	n3 = d3	Rule Prevalence = 0.166 Geometrical difference = 0.75 Sequential difference = 0.333	Design diversity = 0.25 Design abundance = 1.666 Matching degree = 1.0 Geometrical difference = 0.749 Sequential difference = 0.444
OR 4c	d3 n4 	n4 = d3	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 1.5 Matching degree = 1.0 Geometrical difference = 0.790 Sequential difference = 0.458
OR 4d	d3, d6, d7, d8 n5 	n5 = d3	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.749 Sequential difference = 0.466
OR 3b	d2, d3, d6, d7, d8, d12 n6 	n6 = d3	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.666 Design abundance = 2.666 Matching degree = 1.0 Geometrical difference = 0.701 Sequential difference = 0.444

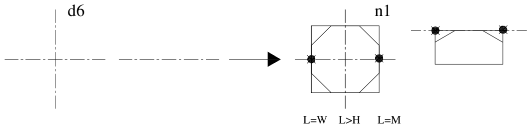
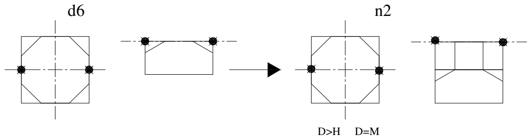
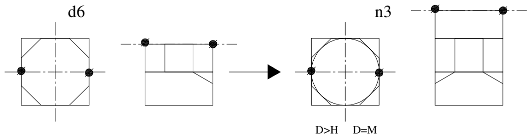
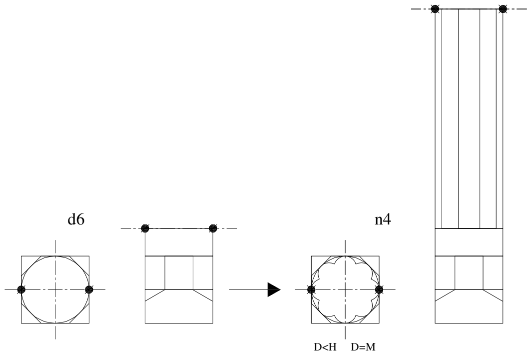
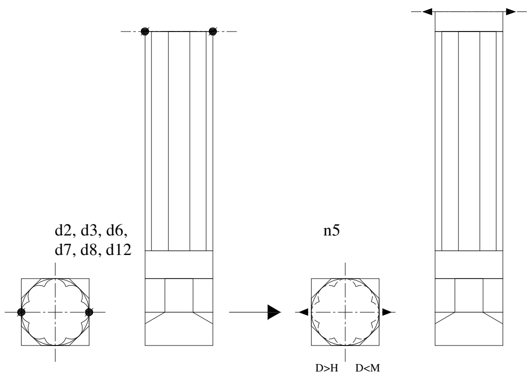
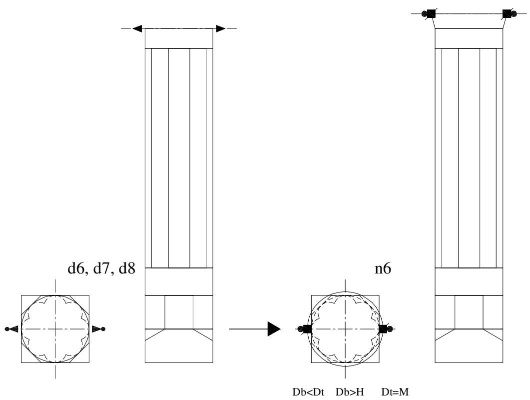
Rule no.	Original design derivation - Na'in minaret (d3) - 9 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 5c		n7 = d3'	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.833	Design diversity = 0.666 Design abundance = 2.428 Matching degree = 1.0 Geometrical difference = 0.731 Sequential difference = 0.499
OR 1e		n8 = d3	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.916	Design diversity = 0.666 Design abundance = 2.25 Matching degree = 1.0 Geometrical difference = 0.754 Sequential difference = 0.551
OR 3f		n9=0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.666 Design abundance = 2.222 Matching degree = 1.0 Geometrical difference = 0.762 Sequential difference = 0.489
			**Design 3 (d3)** **Na'in minaret** **No. of Original Rules =9** **Evaluation Criteria:** **Diversity= 0.666** **Abundance= 2.222** **Matching= 1.0** **Geometrical diff.= 0.762** **Sequential diff.= 0.489**	

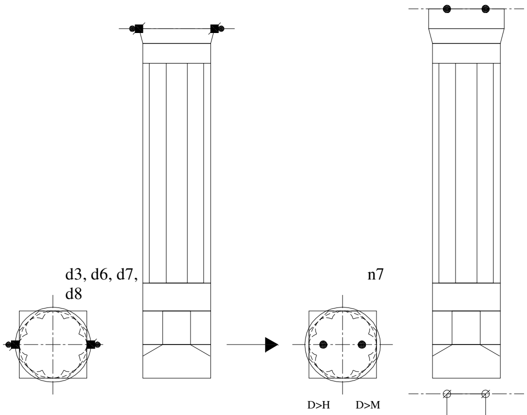
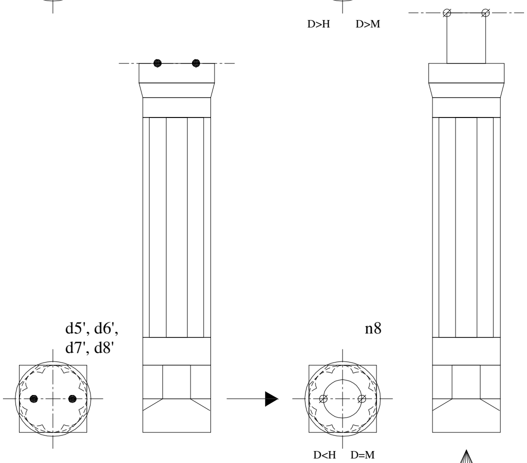
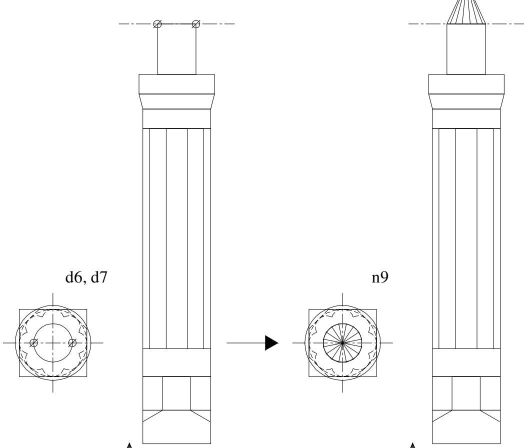
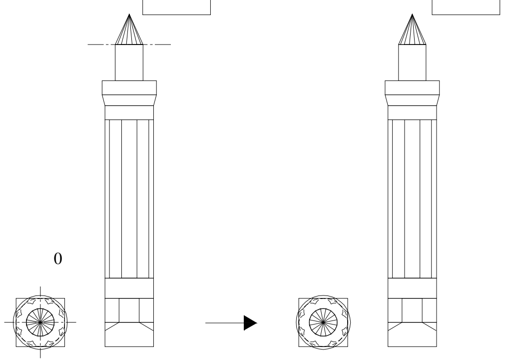
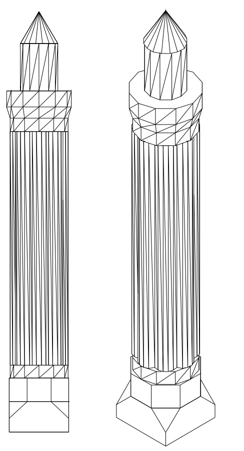
Rule no.	**Original design derivation -** Jesus minaret **(d4) -** 11 Original rules	**User guide grammar for original design**	**Evaluation of the rule**	**Evaluation of the design**
OR 1a	d1, d4, d11 n1 	n1= d4	Rule Prevalence = 0.25 Geometrical difference = 0.75 Sequential difference = 0.75	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 1.0 Geometrical difference = 0.75 Sequential difference = 0.75
OR 1b	d1, d4 n2  L=W L <H L>M	n2 = d4	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 2.5 Matching degree = 1.0 Geometrical difference = 0.791 Sequential difference = 0.625
OR 6b	d4, d8 n3  D<H D<M	n3 = d4	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.333 Design abundance = 2.333 Matching degree = 1.0 Geometrical difference = 0.722 Sequential difference = 0.527
OR 6c	d4 n4  Db<Dt Db>H Dt=M	n4 = d4	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.749 Sequential difference = 0.52
OR 5d	d4 n5  D>H D>M	n5 = d4	Rule Prevalence = 0.083 Geometrical difference = 0.75 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 1.8 Matching degree = 1.0 Geometrical difference = 0.749 Sequential difference = 0.516
OR 6b	d4, d8 n6  D<H D<M	n6 = d4	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.333 Design abundance = 1.833 Matching degree = 1.0 Geometrical difference = 0.722 Sequential difference = 0.486

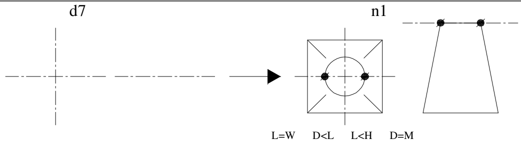
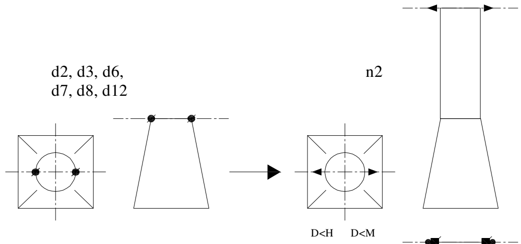
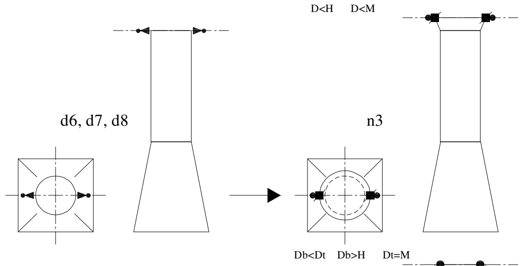
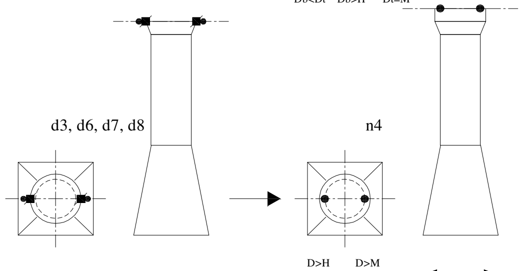
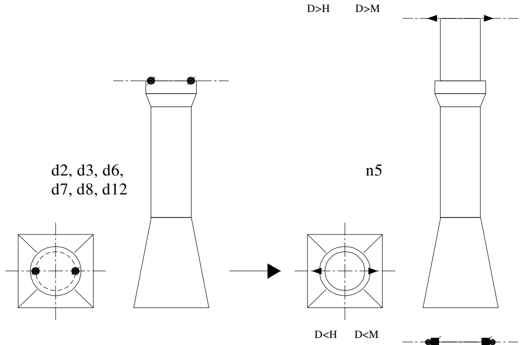
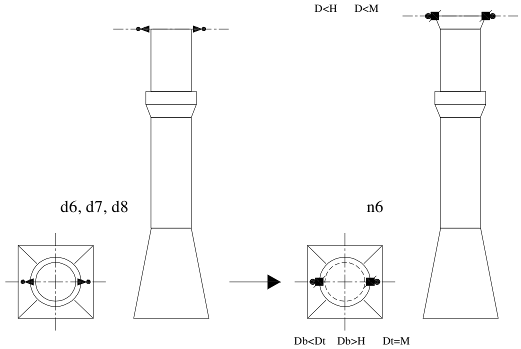
Rule no.	**Original design derivation -** Jesus minaret **(d4) -** 11 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 8c		n7 = d4	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 1.714 Matching degree = 1.0 Geometrical difference = 0.737 Sequential difference = 0.488
OR 5d		n8 = d4	Rule Prevalence = 0.083 Geometrical difference = 0.75 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 1.625 Matching degree = 1.0 Geometrical difference = 0.738 Sequential difference = 0.489
OR 6b		n9 = d4'	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.333 Design abundance = 1.666 Matching degree = 1.0 Geometrical difference = 0.721 Sequential difference = 0.472
OR 9c		n10 = d4	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.666	Design diversity = 0.416 Design abundance = 1.7 Matching degree = 1.0 Geometrical difference = 0.732 Sequential difference = 0.491

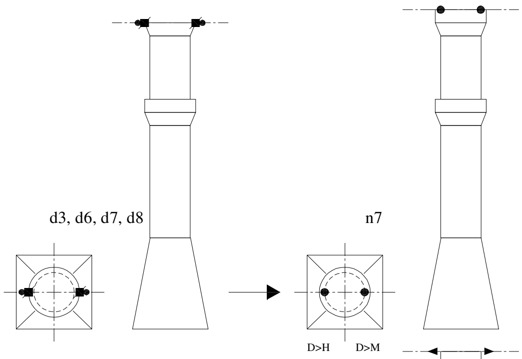
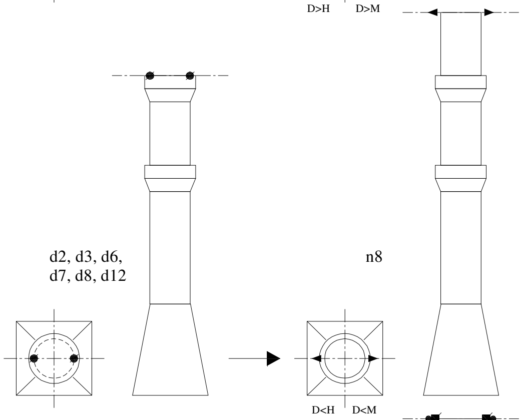
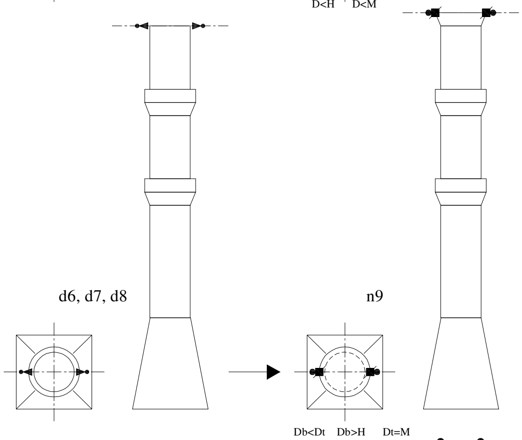
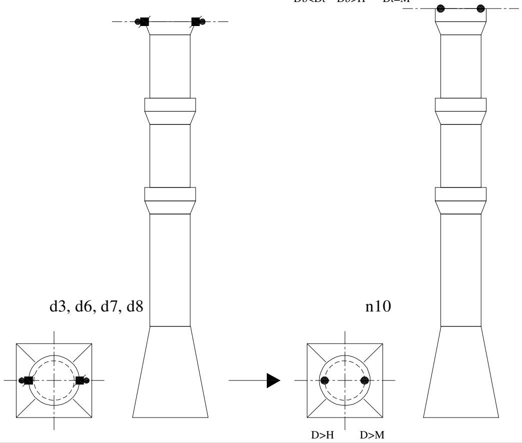
Rule no.	**Original design derivation -** Jesus minaret **(d4) -** 11 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 4f		n11 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.416 Design abundance = 1.727 Matching degree = 1.0 Geometrical difference = 0.741 Sequential difference = 0.446
		 **Design 4 (d4)** **Jesus minaret** **No. of Original Rules =11** **Evaluation Criteria:** **Diversity= 0.416** **Abundance= 1.727** **Matching= 1.0** **Geometrical diff.= 0.741** **Sequential diff.= 0.446**		

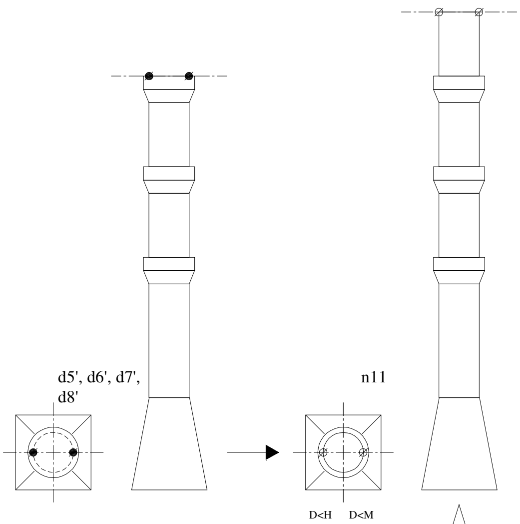
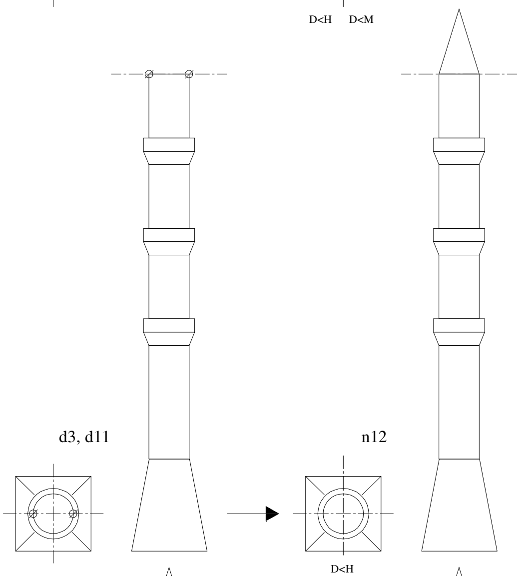
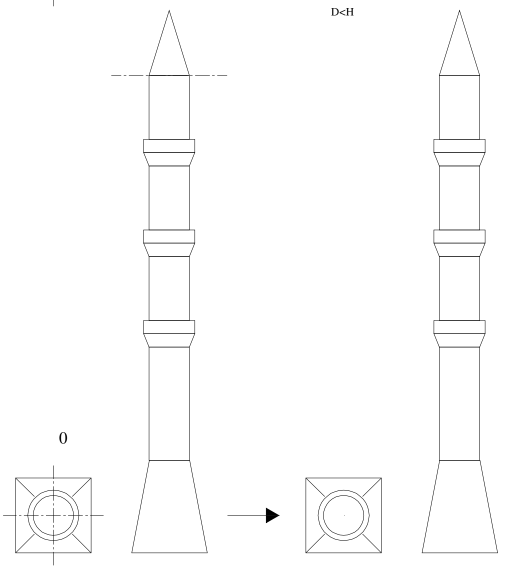
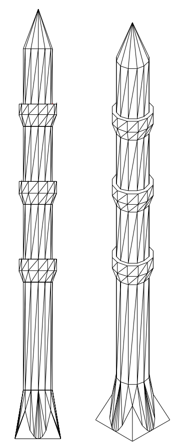
Rule no.	**Original design derivation - Anna** minaret **(d5) -** 5 Original rules	**User guide grammar for original design**	**Evaluation of the rule**	**Evaluation of the design**
OR 6a		n1 = d5	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 7b		n2 = d5	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.749 Sequential difference = 0.5
OR 7b		n3 = d5'	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.694 Sequential difference = 0.5
OR 8b		n4 = d5	Rule Prevalence = 0.333 Geometrical difference = 0.416 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 1.75 Matching degree = 1.0 Geometrical difference = 0.624 Sequential difference = 0.5
OR 5f		n5 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.416 Design abundance = 1.8 Matching degree = 1.0 Geometrical difference = 0.666 Sequential difference = 0.4
			**Design 5 (d5)** **Anna minaret** **No. of Original Rules =5**	**Evaluation Criteria:** **Diversity= 0.416** **Abundance= 1.8** **Matching= 1.0** **Geometrical diff.= 0.666** **Sequential diff.= 0.4**

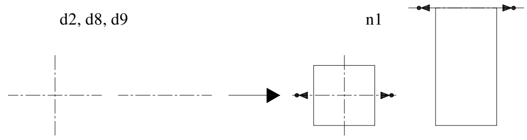
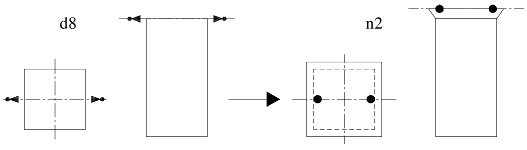
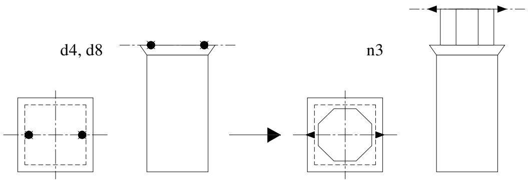
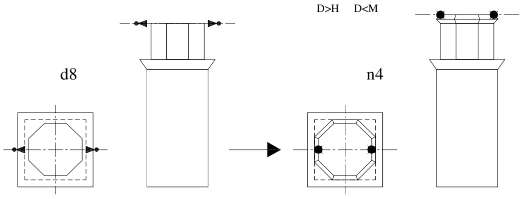
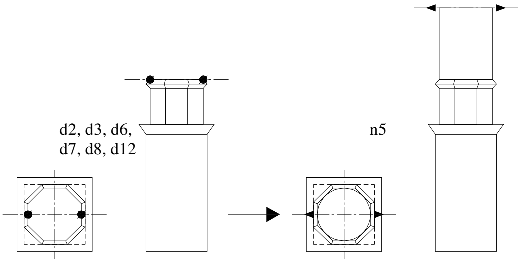
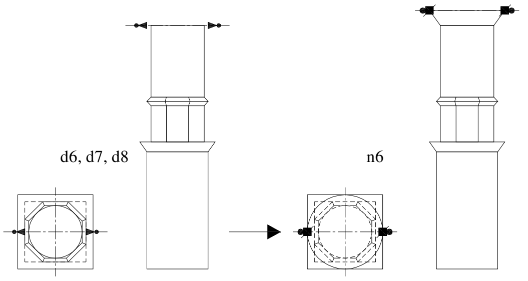
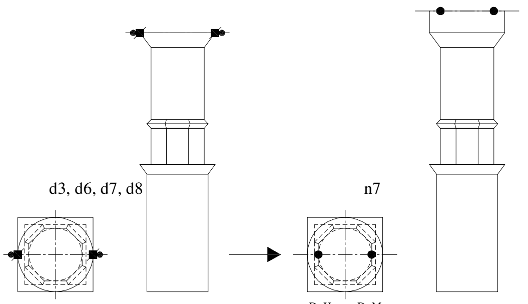
Rule no.	Original design derivation - Yivili minaret **(d6) -** 9 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 4a		n1 = d6	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 9b		n2 = d6	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.749 Sequential difference = 0.5
OR 10b		n3 = d6	Rule Prevalence = 0.083 Geometrical difference = 0.416 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.638 Sequential difference = 0.5
OR 11b		n4 = d6	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.707 Sequential difference = 0.5
OR 3b		n5 = d6	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.5 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.649 Sequential difference = 0.466
OR 2c		n6 = d6	Rule Prevalence = 0.25 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 2.166 Matching degree = 1.0 Geometrical difference = 0.61 Sequential difference = 0.472

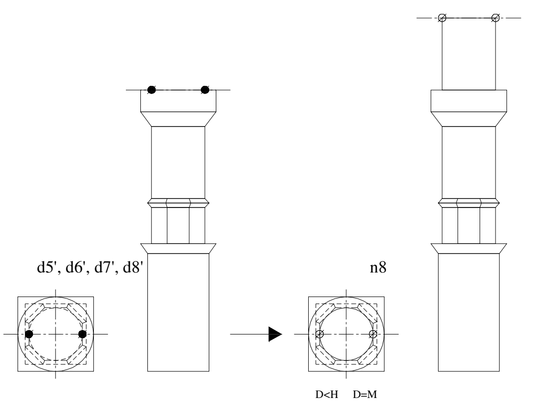
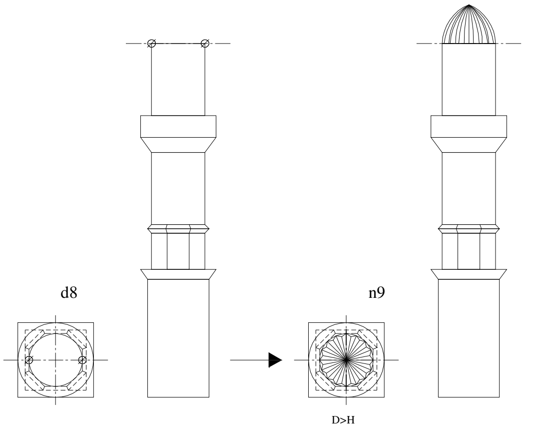
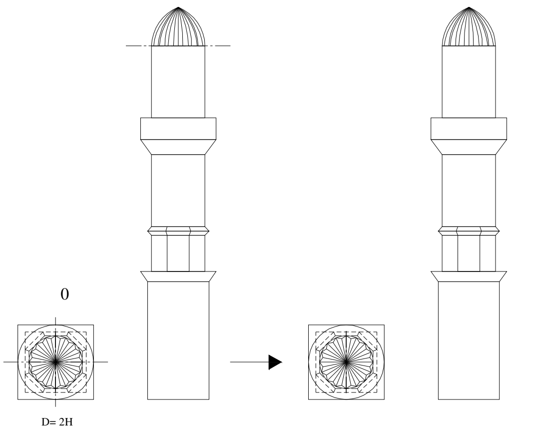
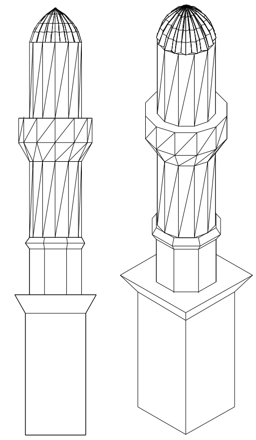
Rule no.	Original design derivation - Yivili minaret (d6) - 9 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 4d		n7 = d6'	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 2.571 Matching degree = 1.0 Geometrical difference = 0.606 Sequential difference = 0.476
OR 8b		n8 = d6	Rule Prevalence = 0.333 Geometrical difference = 0.416 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 2.5 Matching degree = 1.0 Geometrical difference = 0.582 Sequential difference = 0.479
OR 6f		n9 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.583 Design abundance = 2.444 Matching degree = 1.0 Geometrical difference = 0.61 Sequential difference = 0.425
		 **Design 6 (d6)** **Yivili minaret** **No. of Original Rules =9** **Evaluation Criteria:** **Diversity= 0.583** **Abundance= 2.444** **Matching= 1.0** **Geometrical diff.= 0.61** **Sequential diff.= 0.425**		

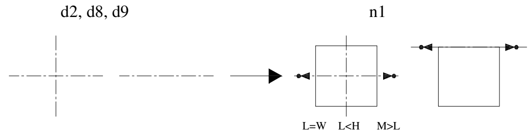
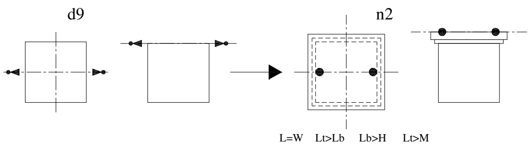
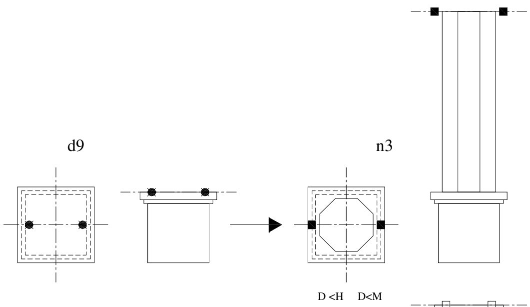
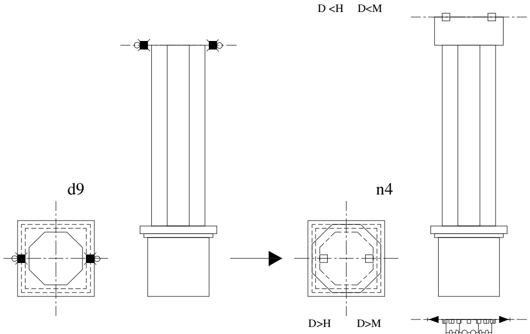
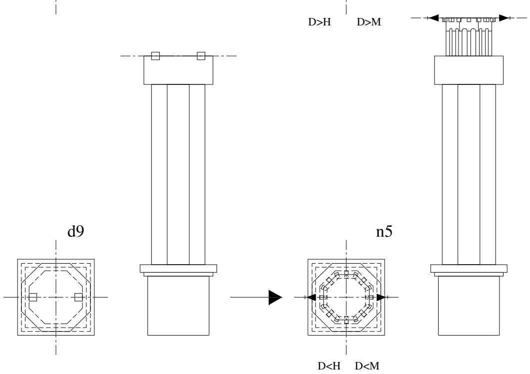
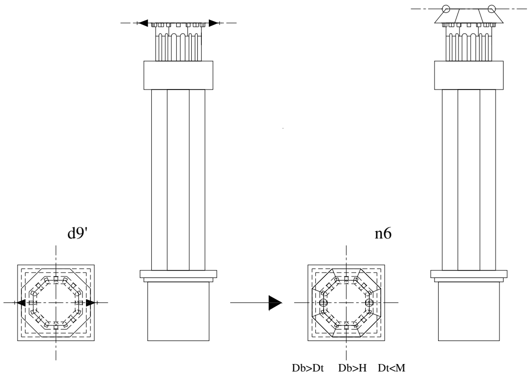
Rule no.	**Original design derivation -** Blue Mosque minaret **(d7) -** 12 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 5a	d7 n1 	n1 = d7	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 3b	d2, d3, d6, d7, d8, d12 n2 	n2 = d7	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.5 Design abundance = 3.5 Matching degree = 1.0 Geometrical difference = 0.666 Sequential difference = 0.416
OR 2c	d6, d7, d8 n3 	n3 = d7	Rule Prevalence = 0.25 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 3.333 Matching degree = 1.0 Geometrical difference = 0.638 Sequential difference = 0.444
OR 4d	d3, d6, d7, d8 n4 	n4 = d7	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 3.5 Matching degree = 1.0 Geometrical difference = 0.624 Sequential difference = 0.458
OR 3b	d2, d3, d6, d7, d8, d12 n5 	n5 = d7	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.5 Design abundance = 4.0 Matching degree = 1.0 Geometrical difference = 0.582 Sequential difference = 0.433
OR 3b	d6, d7, d8 n6 	n6 = d7	Rule Prevalence = 0.25 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 3.833 Matching degree = 1.0 Geometrical difference = 0.582 Sequential difference = 0.444

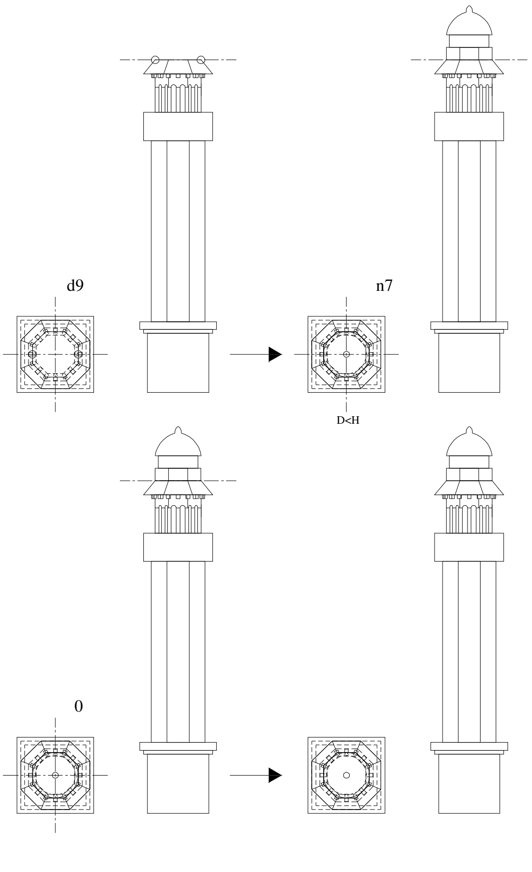
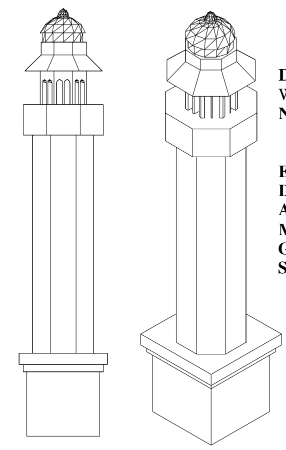
Rule no.	**Original design derivation -** Blue Mosque minaret **(d7) -** 12 Original rules	**User guide grammar for original design**	**Evaluation of the rule**	**Evaluation of the design**
OR 4d		n7 = d7	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 3.857 Matching degree = 1.0 Geometrical difference = 0.582 Sequential difference = 0.452
OR 3b		n8 = d7	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.5 Design abundance = 4.125 Matching degree = 1.0 Geometrical difference = 0.562 Sequential difference = 0.437
OR 2c		n9 = d7	Rule Prevalence = 0.25 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 4.0 Matching degree = 1.0 Geometrical difference = 0.564 Sequential difference = 0.444
OR 4d		n10 = d7'	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance =4.0 Matching degree = 1.0 Geometrical difference = 0.566 Sequential difference = 0.449

Rule no.	**Original design derivation -** Blue Mosque minaret **(d7) -** 12 Original rules	**User guide grammar for original design**	**Evaluation of the rule**	**Evaluation of the design**
OR 8b		n11 = d7	Rule Prevalence = 0.333 Geometrical difference = 0.416 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 3.909 Matching degree = 1.0 Geometrical difference = 0.552 Sequential difference = 0.462
OR 3f		n12 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.583 Design abundance = 3.75 Matching degree = 1.0 Geometrical difference = 0.575 Sequential difference = 0.423
				**Design 7 (d7)** **Blue Mosque minaret** **No. of Original Rules =12** **Evaluation Criteria:** **Diversity= 0.583** **Abundance= 3.75** **Matching= 1.0** **Geometrical diff.= 0.575** **Sequential diff.= 0.423**

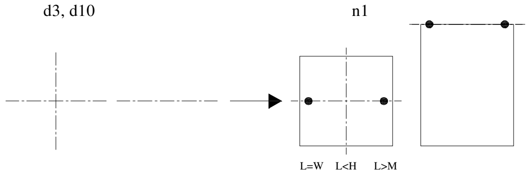
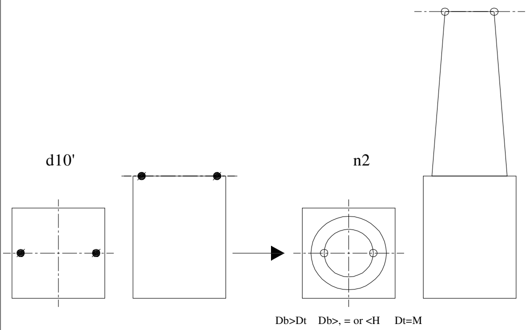
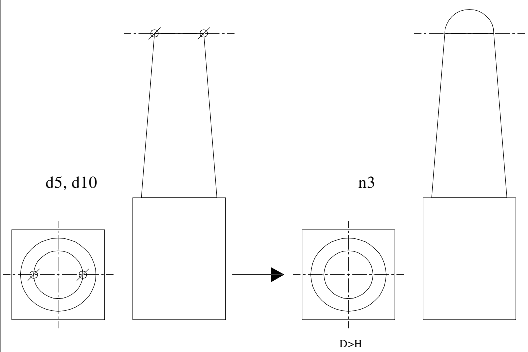
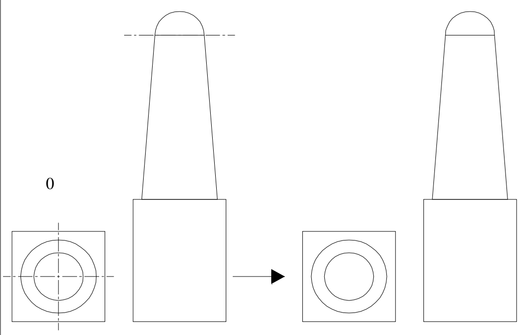
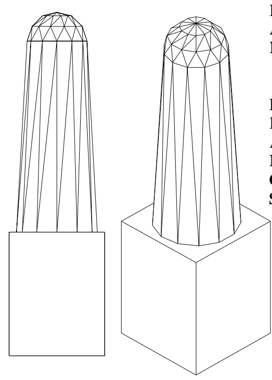
Rule no.	Original design derivation - Qubbat Talha minaret (d8) - 9 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 2a	d2, d8, d9 → n1  L=W L<H M>L	n1 = d8	Rule Prevalence = 0.25 Geometrical difference = 0.583 Sequential difference = 0.75	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 1.0 Geometrical difference = 0.583 Sequential difference = 0.75
OR 10c	d8 → n2  L=W Lt>Lb Lb>H Lt>M	n2 = d8	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.833	Design diversity = 0.25 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.708 Sequential difference = 0.791
OR 6b	d4, d8 → n3  D>H D<M	n3 = d8	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.333 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.666 Sequential difference = 0.638
OR 12c	d8 → n4  Db<Dt Db>H Dt=M	n4 = d8	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.833	Design diversity = 0.333 Design abundance = 1.75 Matching degree = 1.0 Geometrical difference =0.728 Sequential difference = 0.687
OR 3b	d2, d3, d6, d7, d8, d12 → n5  D<H D<M	n5 = d8	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.666 Design abundance = 2.6 Matching degree = 1.0 Geometrical difference = 0.666 Sequential difference = 0.616
OR 2c	d6, d7, d8 → n6  Db<Dt Db>H Dt=M	n6 = d8	Rule Prevalence = 0.25 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 2.666 Matching degree = 1.0 Geometrical difference = 0.652 Sequential difference = 0.597
OR 4d	d3, d6, d7, d8 → n7  D>H D>M	n7 = d8'	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.666 Design abundance =2.857 Matching degree = 1.0 Geometrical difference =0.642 Sequential difference = 0.583

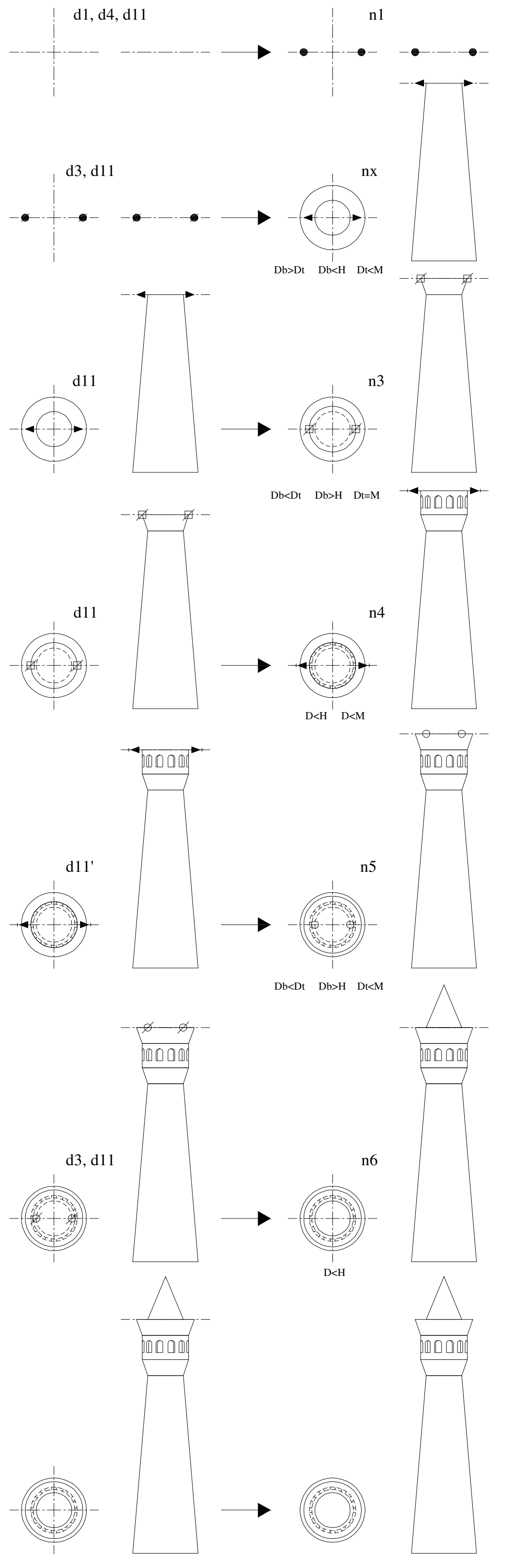
Rule no.	Original design derivation - Qubbat Talha minaret (d8) - 9 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 8b		n8 = d8	Rule Prevalence = 0.333 Geometrical difference = 0.416 Sequential difference = 0.5	Design diversity = 0.75 Design abundance = 3.0 Matching degree = 1.0 Geometrical difference = 0.614 Sequential difference = 0.572
OR 7f		n9 = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0	Design diversity = 0.75 Design abundance = 2.777 Matching degree = 1.0 Geometrical difference = 0.647 Sequential difference = 0.509
				**Design 8 (d8)** **Qubbat Talha minaret** **No. of Original Rules =9** **Evaluation Criteria:** **Diversity= 0.75** **Abundance= 2.777** **Matching= 1.0** **Geometrical diff.= 0.647** **Sequential diff.= 0.509**

Rule no.	Original design derivation - Wazir Khan Mosque minaret (**d**9) - 7 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 2a		n1 = d9	Rule Prevalence = 0.25 Geometrical difference = 0.583 Sequential difference = 0.75	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 1.0 Geometrical difference = 0.583 Sequential difference = 0.75
OR 11c		n2 = d9	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.833	Design diversity = 0.25 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.873 Sequential difference = 0.791
OR 12b		n3 = d9	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.916	Design diversity = 0.25 Design abundance = 1.666 Matching degree = 1.0 Geometrical difference = 0.694 Sequential difference = 0.848
OR 6d		n4 = d9	Rule Prevalence = 0.083 Geometrical difference = 0.75 Sequential difference = 0.833	Design diversity = 0.25 Design abundance = 1.5 Matching degree = 1.0 Geometrical difference = 0.708 Sequential difference = 0.844
OR 3e		n5 = d9'	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.75	Design diversity = 0.25 Design abundance = 1.4 Matching degree = 1.0 Geometrical difference = 0.749 Sequential difference = 0.825
OR 13c		n6 = d9	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.666	Design diversity = 0.25 Design abundance = 1.333 Matching degree = 1.0 Geometrical difference = 0.777 Sequential difference = 0.798

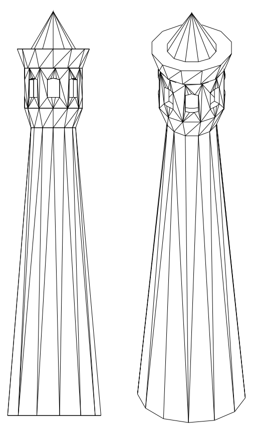
<table>
<tr><th>Rule no.</th><th>Original design derivation - Wazir Khan Mosque minaret
(d9) - 7 Original rules</th><th>User guide grammar for original design</th><th>Evaluation of the rule</th><th>Evaluation of the design</th></tr>
<tr><td rowspan="2">OR
8f</td><td rowspan="2"></td><td>n7 = 0</td><td>Rule Prevalence = 0.083
Geometrical difference = 0.916
Sequential difference = 0</td><td>Design diversity = 0.25
Design abundance = 1.428
Matching degree = 1.0
Geometrical difference = 0.797
Sequential difference = 0.69</td></tr>
<tr><td colspan="3">
Design 9 (d9)
Wazir Khan minaret
No. of Original Rules = 7

Evaluation Criteria:
Diversity= 0.25
Abundance= 1.428
Matching= 1.0
Geometrical diff.= 0.797
Sequential diff.= 0.69</td></tr>
</table>

Rule no.	**Original design derivation -** Abu el-Hajaj minaret **(d10) -** 3 Original rules	**User guide grammar for original design**	**Evaluation of the rule**	**Evaluation of the design**
OR 3a		n1 = d10'	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.583 Sequential difference = 0.5
OR 13b		n2 = d10	Rule Prevalence = 0.083 Geometrical difference = 0.75 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.5 Matching degree = 1.0 Geometrical difference = 0.666 Sequential difference = 0.5
OR 5f		n3 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.25 Design abundance = 1.666 Matching degree = 1.0 Geometrical difference = 0.722 Sequential difference = 0.333
		 **Design 10 (d10)** **Abu el-Hajaj minaret** **No. of Original Rules = 3** **Evaluation Criteria:** **Diversity= 0.25** **Abundance= 1.666** **Matching= 1.0** **Geometrical diff.= 0.722** **Sequential diff.= 0.333**		

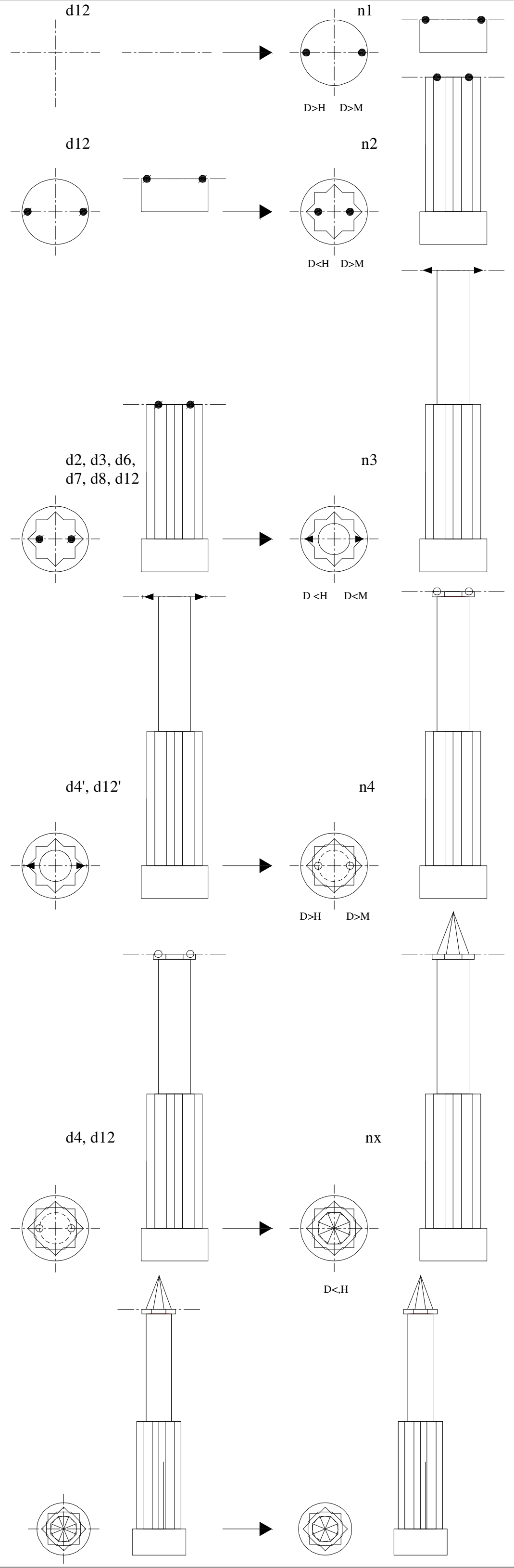


Rule no.	Original design derivation - Kalan minaret (d11) - 6 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 1a		n1 = d11	Rule Prevalence = 0.25 Geometrical difference = 0.75 Sequential difference = 0.75	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 1.0 Geometrical difference = 0.75 Sequential difference = 0.75
OR 5b		n2 = d11	Rule Prevalence = 0.166 Geometrical difference = 0.75 Sequential difference = 0.333	Design diversity = 0.333 Design abundance = 2.5 Matching degree = 1.0 Geometrical difference = 0.75 Sequential difference = 0.541
OR 14c		n3 = d11	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.833	Design diversity = 0.333 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference =0.694 Sequential difference = 0.638
OR 4e		n4 = d11'	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.75	Design diversity = 0.333 Design abundance = 1.75 Matching degree = 1.0 Geometrical difference = 0.749 Sequential difference = 0.666
OR 15c		n5 = d11	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.666	Design diversity = 0.333 Design abundance = 1.6 Matching degree = 1.0 Geometrical difference = 0.716 Sequential difference = 0.666
OR 3f		n6 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.333 Design abundance = 1.666 Matching degree = 1.0 Geometrical difference =0.735 Sequential difference = 0.555

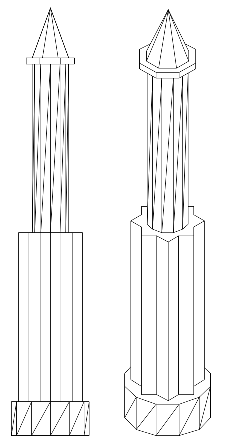


Design 11 (d11)
Kalan minaret
No. of Original Rules = 6

Evaluation Criteria:
Diversity= 0.333
Abundance= 1.666
Matching= 1.0
Geometrical diff.= 0.735
Sequential diff.= 0.555



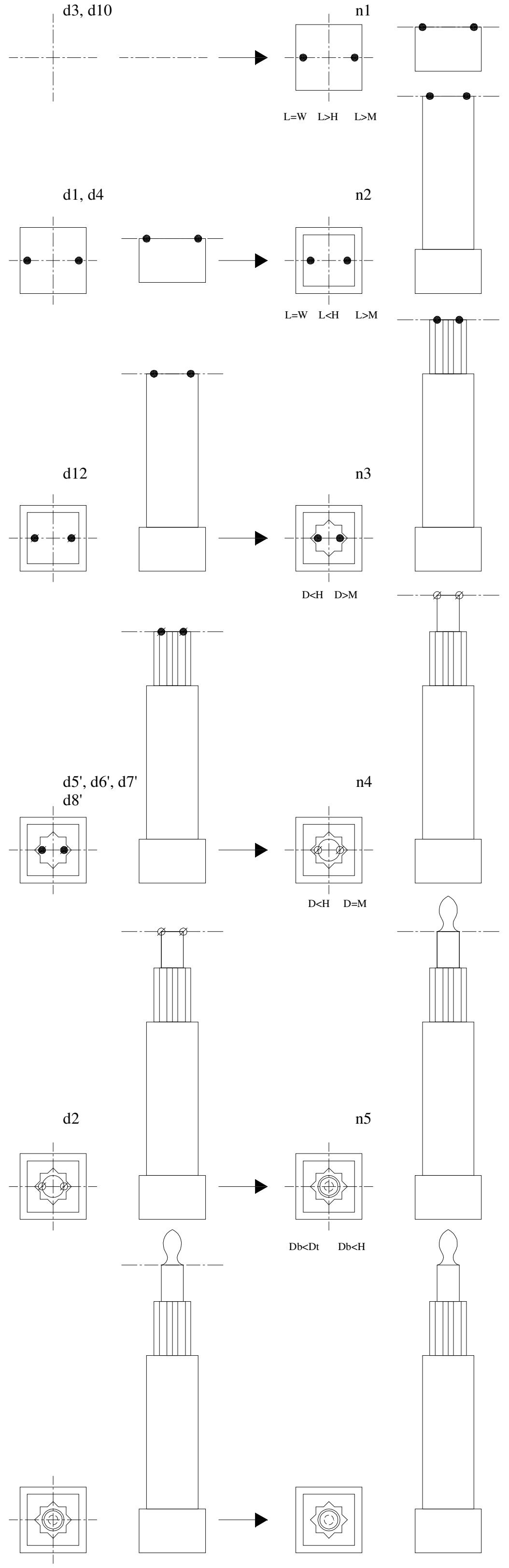
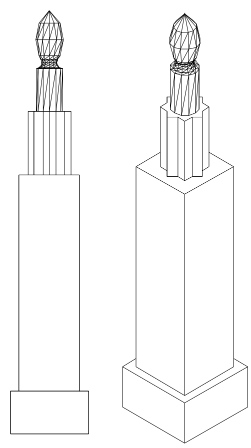
Rule no.	**Original design derivation -** Ghazni minaret **(d12) -** 5 Original rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
OR 7a	d12 → n1 (D>H D>M)	n1 = d12	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 14b	d12 → n2 (D<H D>M)	n2 = d12	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 3b	d2, d3, d6, d7, d8, d12 → n3 (D <H D<M)	n3 = d12	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.5 Design abundance = 2.666 Matching degree = 1.0 Geometrical difference = 0.749 Sequential difference = 0.444
OR 9c	d4', d12' → n4 (D>H D>M)	n4 = d12	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.666	Design diversity = 0.583 Design abundance = 2.5 Matching degree = 1.0 Geometrical difference = 0.77 Sequential difference = 0.499
OR 4f	d4, d12 → nx (D<,H)	n6 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.583 Design abundance = 2.4 Matching degree = 1.0 Geometrical difference = 0.782 Sequential difference = 0.399

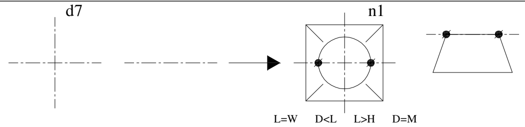
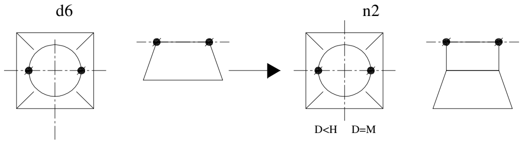
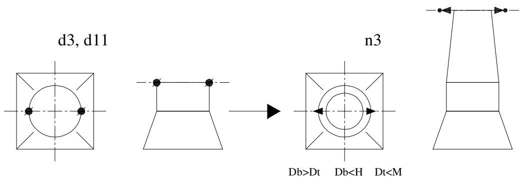
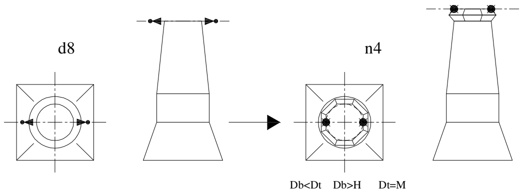
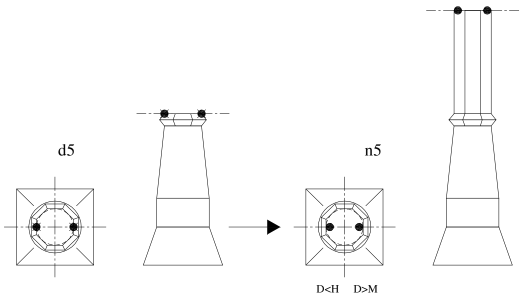
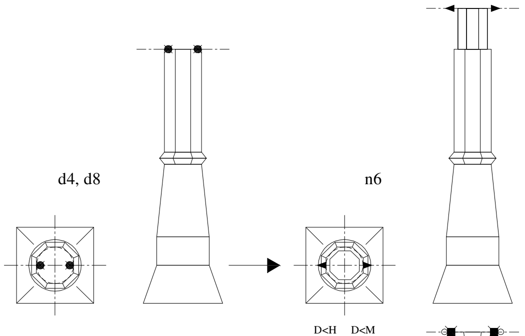
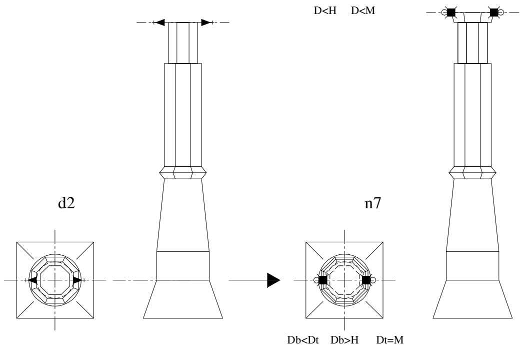


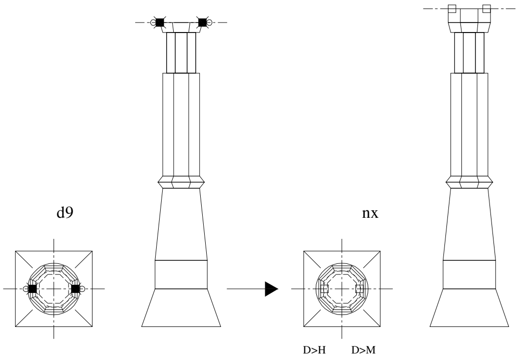
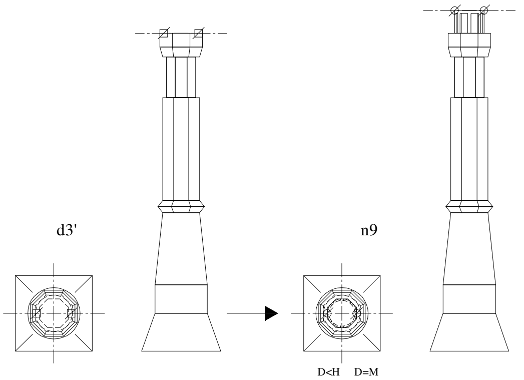
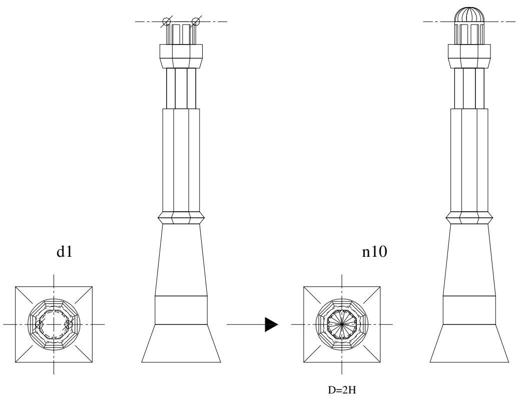
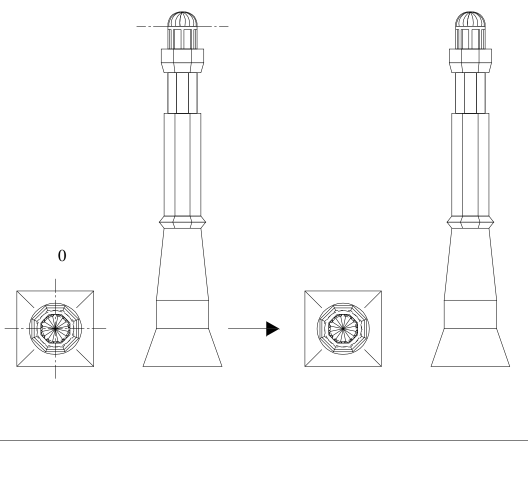
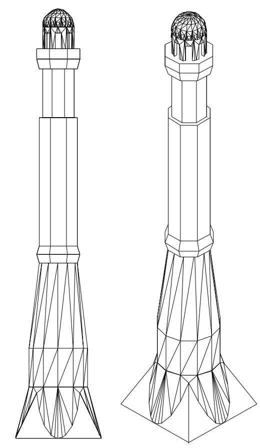
Design 12 (d12)
Ghazni minaret
No. of Original Rules = 5

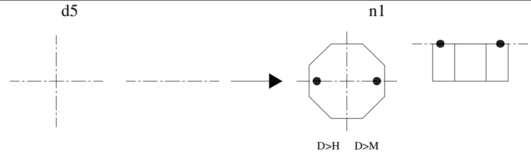
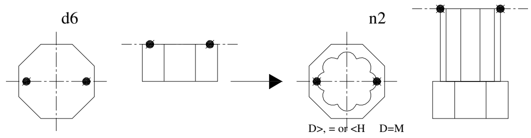
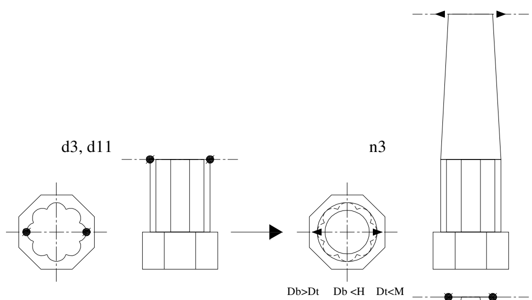
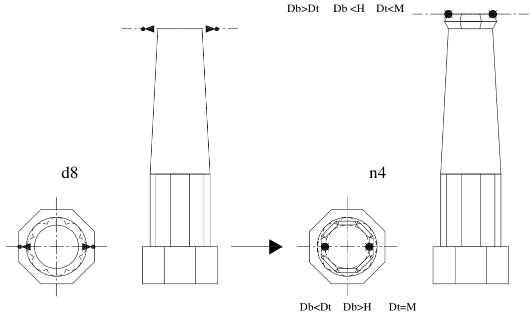
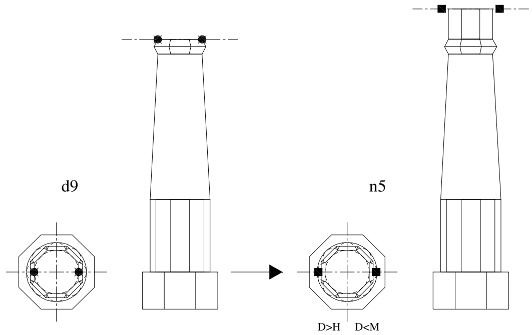
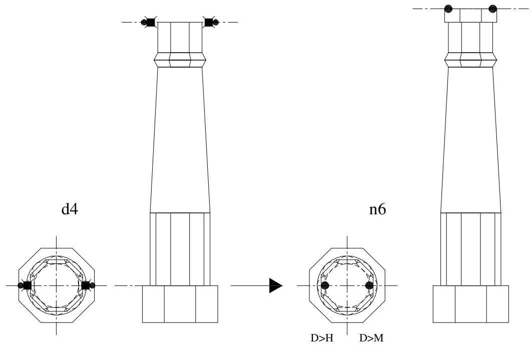
Evaluation Criteria:
Diversity= 0.583
Abundance= 2.4
Matching= 1.0
Geometrical diff.= 0.782
Sequential diff.= 0.399

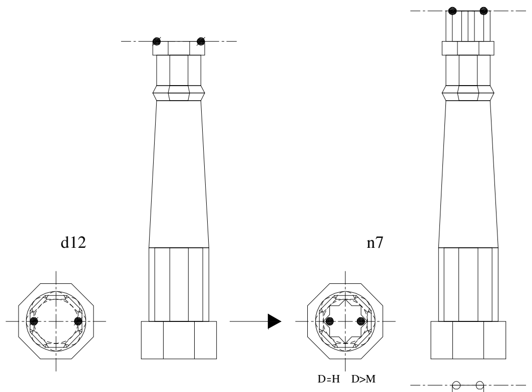
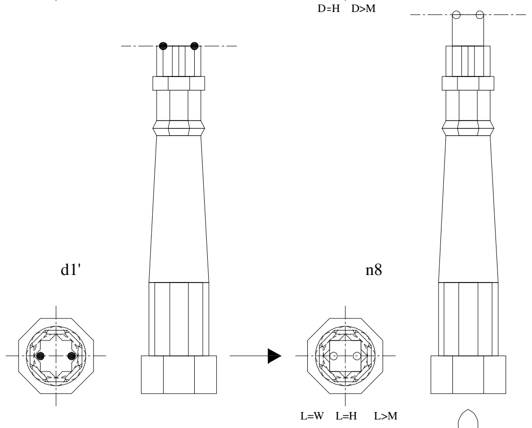
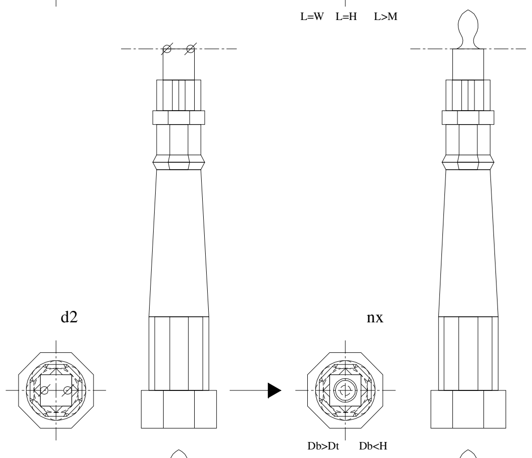
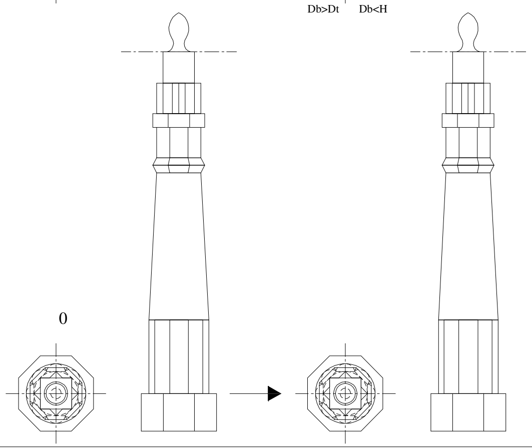
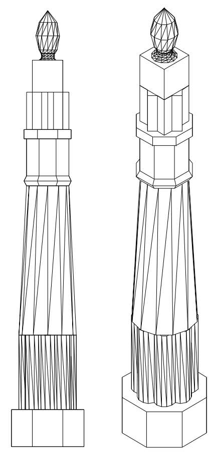
B-2 Hybrid designs composed of original rules

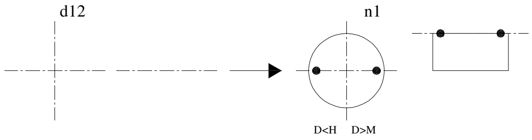
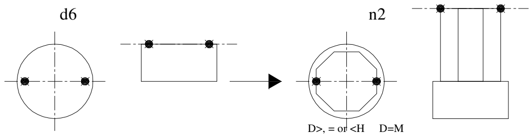
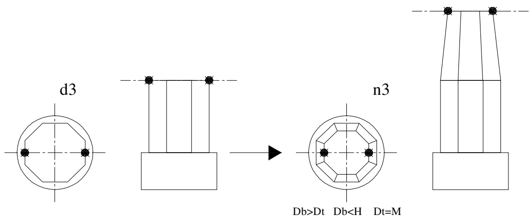
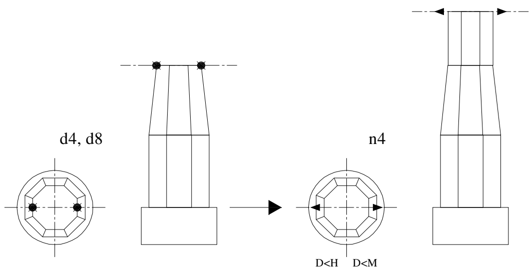
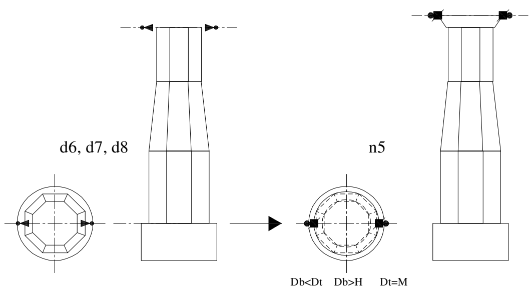
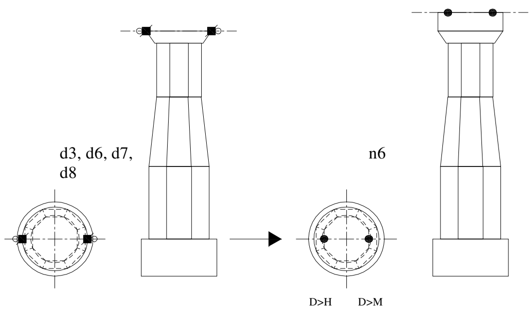
Rule no.	Example 1: Hybrid design derivation - 5 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 3a		n1 = {n\d3, d10} n1 = {d1, d2, d4, d5, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.583 Sequential difference = 0.5
OR 1b		n2 = {n1\d1, d4} n2 = {d2, d5, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.333 Design abundance =2.0 Matching degree = 0.5 Geometrical difference = 0.708 Sequential difference = 0.5
OR 14b		n3 = {n2\d12} n3 = {d2', d5', d6', d7', d8', d9', d11'}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.416 Design abundance = 1.666 Matching degree= 0.333 Geometrical difference = 0.777 Sequential difference = 0.5
OR 8b		n4 = {n(x-1)\d5, d6, d7, d8} n4 = {d2, d9, d11}	Rule Prevalence = 0.333 Geometrical difference = 0.416 Sequential difference = 0.5	Design diversity = 0.75 Design abundance = 2.25 Matching degree = 0.25 Geometrical difference = 0.687 Sequential difference = 0.5
OR 2f		n5 = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0	Design diversity = 0.833 Design abundance = 2.0 Matching degree = 0.2 Geometrical difference = 0.732 Sequential difference = 0.4
			**Example 1** **Hybrid design - OR** **No. of Original Rules =5** **Evaluation Criteria:** **Diversity= 0.833** **Abundance= 2.0** **Matching= 0.2** **Geometrical diff.= 0.732** **Sequential diff.= 0.4**	

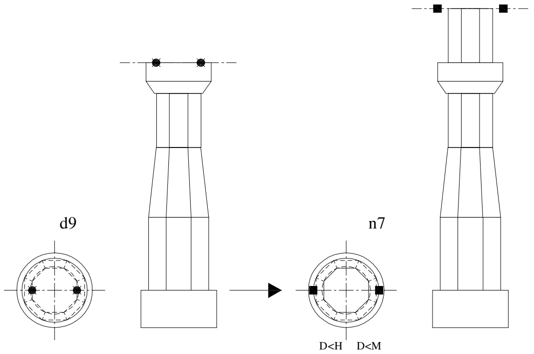
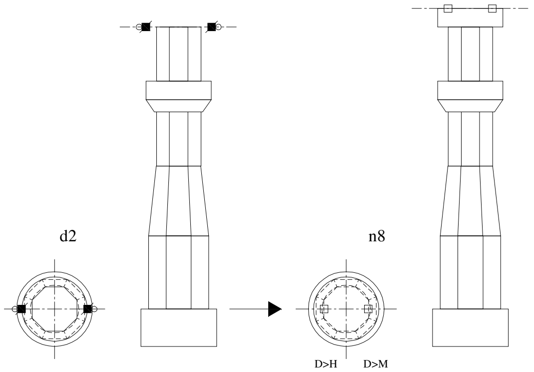
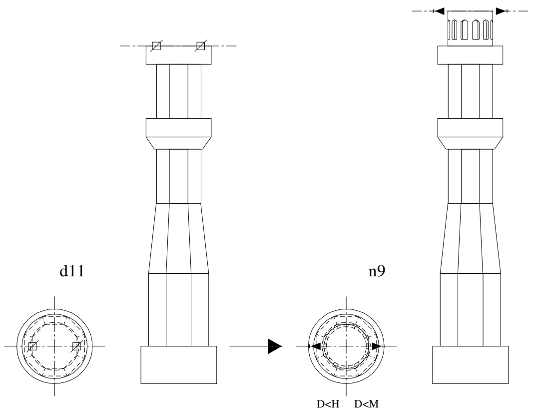
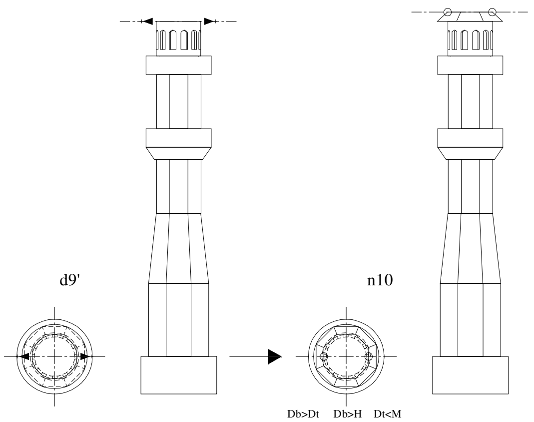
Rule no.	Example 2: Hybrid design derivation - 10 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 5a		n1 = {n\d7} n1 = {d1, d2, d3, d4, d5, d6, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 10b		n2 = {n1\d6} n2 = {d1, d2, d3, d4, d5, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.416 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.0 Matching degree = 0.5 Geometrical difference = 0.666 Sequential difference = 0.5
OR 5b		n3 = {n2\d3, d11} n3 = {d1, d2, d4, d5, d8, d9, d10, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.75 Sequential difference = 0.333	Design diversity = 0.333 Design abundance = 1.333 Matching degree = 0.333 Geometrical difference = 0.694 Sequential difference = 0.444
OR 12c		n4 = {n3\d8} n4 = {d1, d2, d4, d5, d9, d10, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.833	Design diversity = 0.416 Design abundance = 1.25 Matching degree = 0.25 Geometrical difference = 0.749 Sequential difference = 0.541
OR 7b		n5 = {n4\d5} n5 = {d1, d2, d4, d9, d10, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 1.2 Matching degree = 0.2 Geometrical difference = 0.716 Sequential difference = 0.533
OR 6b		n6 = {n5\d4, d8} n6 = {d1, d2, d9, d10, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.583 Design abundance = 1.333 Matching degree = 0.333 Geometrical difference = 0.694 Sequential difference = 0.499
OR 7c		n7 = {n6\d2} n7 = {d1, d9, d10, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 1.285 Matching degree = 0.285 Geometrical difference = 0.713 Sequential difference = 0.499

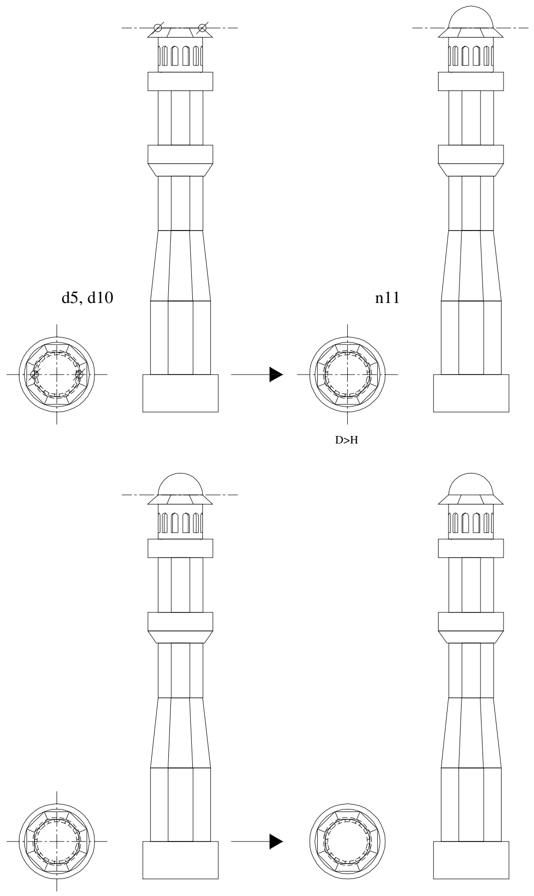
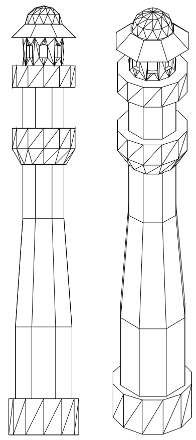
Rule no.	Example 2: Hybrid design derivation - 10 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 6d		n8 = {n7\d9} n8 = {d1', d10', d12'} n8 = Ø n8 = n8* n8* = {m\d9} n8* = {d2', d3', d4', d5', d6', d7', d11'}	Rule Prevalence = 0.083 Geometrical difference = 0.75 Sequential difference = 0.833	Design diversity = 0.75 Design abundance = 1.25 Matching degree = 0.25 Geometrical difference = 0.718 Sequential difference = 0.541
OR 1e		n9 = {n8\d3} n9 = {d1, d10, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.916	Design diversity = 0.75 Design abundance = 1.222 Matching degree = 0.222 Geometrical difference = 0.74 Sequential difference = 0.583
OR 1f		n10 = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0	Design diversity = 0.833 Design abundance = 1.2 Matching degree = 0.2 Geometrical difference = 0.757 Sequential difference = 0.524
			**Example 2** **Hybrid design - OR** **No. of Original Rules = 10** **Evaluation Criteria:** **Diversity= 0.833** **Abundance= 1.2** **Matching= 0.2** **Geometrical diff.= 0.757** **Sequential diff.= 0.524**	

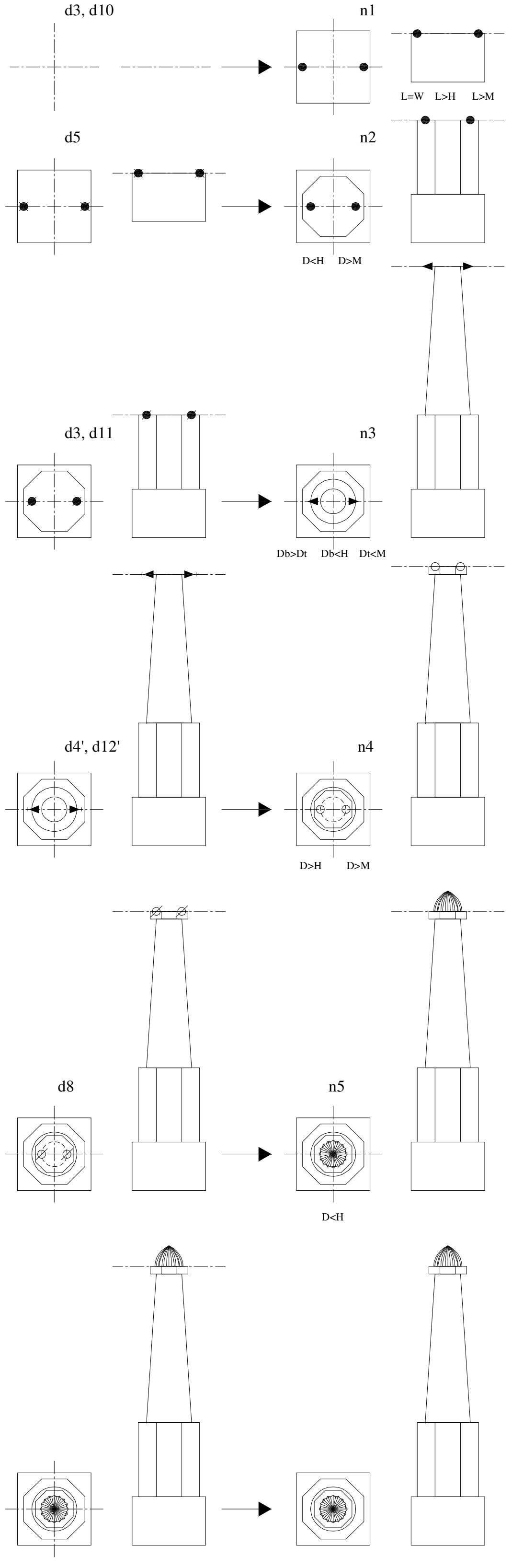
Rule no.	Example 3: Hybrid design derivation - 9 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 6a		n1 = {n\d5} n1 = {d1, d2, d3, d4, d6, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 11b		n2 = {n1\d6} n1 = {d1, d2, d3, d4, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.0 Matching degree = 0.5 Geometrical difference = 0.916 Sequential difference = 0.5
OR 5b		n3 = {n2\d3, d11} n3 = {d1, d2, d4, d7, d8, d9, d10, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.75 Sequential difference = 0.333	Design diversity = 0.333 Design abundance = 1.333 Matching degree = 0.333 Geometrical difference = 0.86 Sequential difference = 0.444
OR 12c		n4 = {n3\d8} n4 = {d1, d2, d4, d7, d9, d10, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.833	Design diversity = 0.416 Design abundance = 1.25 Matching degree = 0.25 Geometrical difference = 0.874 Sequential difference = 0.541
OR 12b		n5 = {n4\d9} n5 = {d1, d2, d4, d7, d10, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.916	Design diversity = 0.5 Design abundance = 1.2 Matching degree = 0.2 Geometrical difference = 0.816 Sequential difference = 0.616
OR 5d		n6 = {n5\d4} n6 = {d1, d2, d7, d10, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.75 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 1.166 Matching degree = 0.166 Geometrical difference = 0.805 Sequential difference = 0.597

Rule no.	Example 3: Hybrid design derivation - 9 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 14b		n7 = {n6\d12} n7 = {d1, d2, d7, d10}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 1.142 Matching degree = 0.142 Geometrical difference = 0.821 Sequential difference = 0.583
OR 2b		n8 = {n7\d1} n8 = {d2, d7, d10}	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.75 Design abundance = 1.125 Matching degree = 0.125 Geometrical difference = 0.822 Sequential difference = 0.572
OR 2f		n9 = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0	Design diversity = 0.833 Design abundance = 1.111 Matching degree = 0.111 Geometrical difference = 0.832 Sequential difference = 0.509
			**Example 3** **Hybrid design - OR** **No. of Original Rules = 9** **Evaluation Criteria:** **Diversity= 0.833** **Abundance= 1.111** **Matching= 0.111** **Geometrical diff.= 0.832** **Sequential diff.= 0.509**	

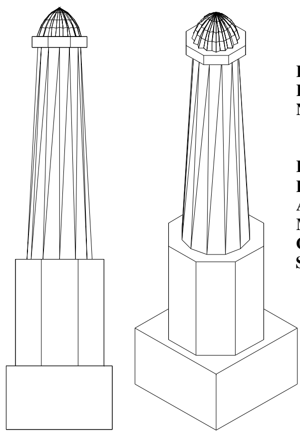
Rule no.	Example 4: Hybrid design derivation - 11 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 7a		n1 = {n\d12} n1 = {d1, d2, d3, d4, d5, d6, d7, d8, d9, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 9b		n2 = {n1\d6} n2 ={d1, d2, d3, d4, d5, d7, d8, d9, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.0 Matching degree = 0.5 Geometrical difference = 0.749 Sequential difference = 0.5
OR 4b		n3 = {n2\d3} n3 = {d1, d2, d4, d5, d7, d8, d9, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 1.0 Matching degree = 0.333 Geometrical difference = 0.805 Sequential difference = 0.5
OR 6b		n4 = {n3\d4, d8} n4 = {d1, d2, d5, d7, d9, d10, d11}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.416 Design abundance = 1.25 Matching degree = 0.25 Geometrical difference = 0.749 Sequential difference = 0.458
OR 2c		n5 = {n4\d6, d7, d8} n5 = {d1, d2, d5, d9, d10, d11} n5 = Ø n5 = n5* n5* = {m\d6, d7, d8} n5* = {d3, d4, d12}	Rule Prevalence = 0.25 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 1.6 Matching degree = 0.4 Geometrical difference = 0.716 Sequential difference = 0.466
OR 4d		n6 = {n5\d3, d6, d7, d8} n6 = {d1, d2, d5, d9, d10, d11}	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference =0.5	Design diversity = 0.5 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 0.694 Sequential difference = 0.472

Rule no.	Example 4: Hybrid design derivation - 11 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 12b		n7 = {n6\d9} n7 = {d1, d2, d5, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.916	Design diversity = 0.583 Design abundance = 1.857 Matching degree = 0.428 Geometrical difference = 0.678 Sequential difference = 0.535
OR 2d		n8 ={n7\d2} n8 ={d1, d5, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.833	Design diversity = 0.666 Design abundance = 1.75 Matching degree = 0.375 Geometrical difference = 0.666 Sequential difference = 0.572
OR 4e		n9 = {n8\d11} n9 = {d1', d5', d10'} n9 = Ø n9 = n9* n9* = {m\d11} n9* = {d2', d4', d9', d12'}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.75	Design diversity = 0.75 Design abundance = 1.666 Matching degree = 0.333 Geometrical difference = 0.694 Sequential difference = 0.592
OR 13c		n10 = {n9\d9} n10 = {d1, d5, d10}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.666	Design diversity = 0.75 Design abundance = 1.6 Matching degree = 0.3 Geometrical difference = 0.716 Sequential difference = 0.599

Rule no.	Example 4: Hybrid design derivation - 11 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 5f		n11 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.916 Design abundance = 1.636 Matching degree = 0.272 Geometrical difference = 0.726 Sequential difference = 0.545
			**Example 4** **Hybrid design - OR** **No. of Original Rules = 11** **Evaluation Criteria:** **Diversity= 0.916** **Abundance= 1.636** **Matching= 0.272** **Geometrical diff.= 0.726** **Sequential diff.= 0.545**	

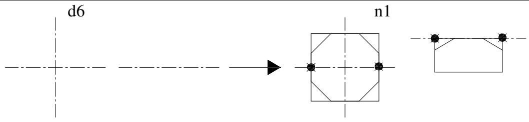
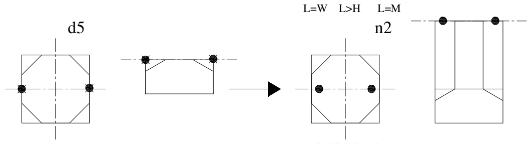
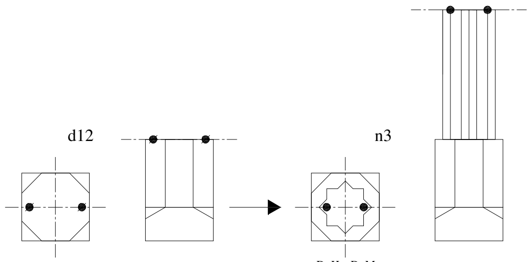
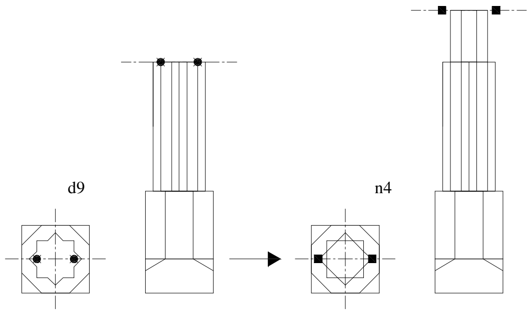
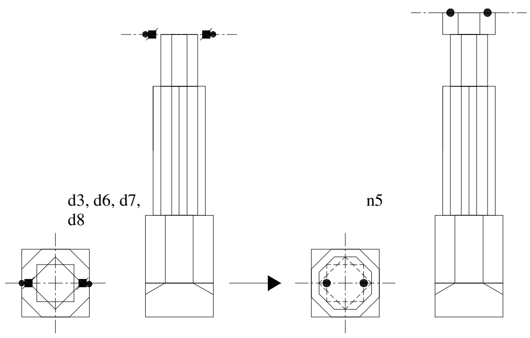
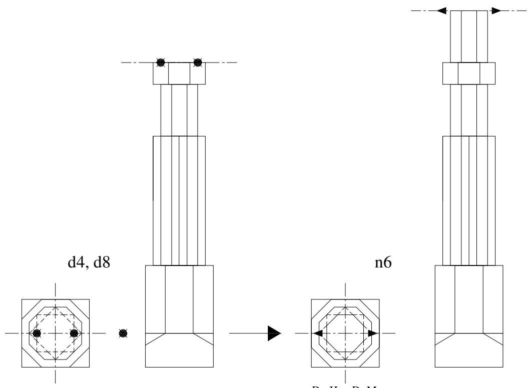


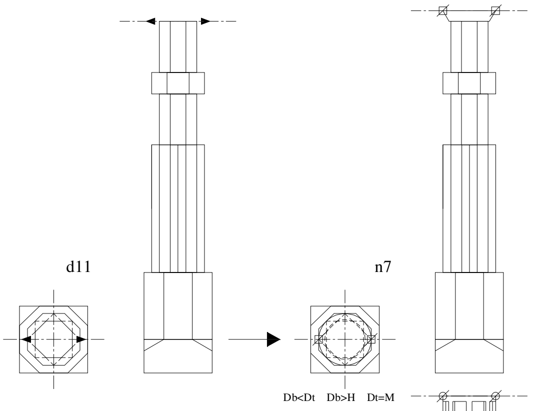
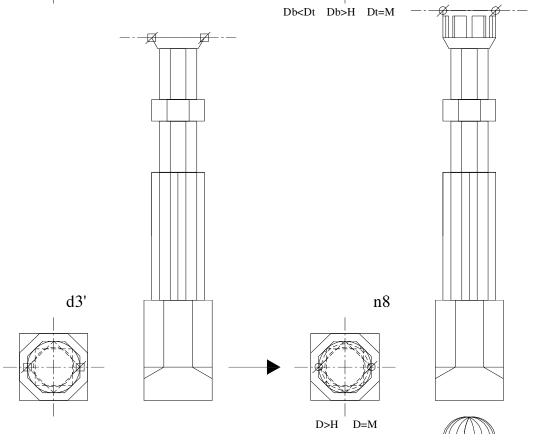
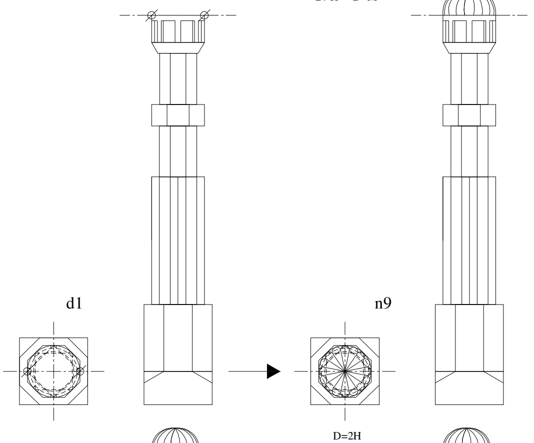
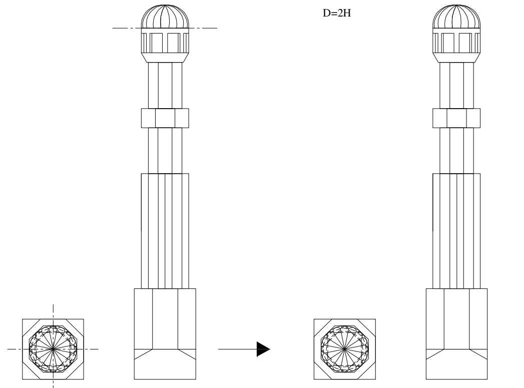
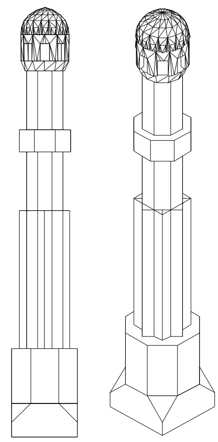
Rule no.	Example 5: Hybrid design derivation - 5 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 3a	d3, d10 → n1	n1 = {n\d3, d10} n1 = {d1, d2, d4, d5, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.583 Sequential difference = 0.5
OR 7b	d5 → n2	n2 = {n1\d5} n2 = {d1, d2, d4, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 1.5 Matching degree = 0.5 Geometrical difference = 0.583 Sequential difference = 0.5
OR 5b	d3, d11 → n3	n3 = {n2\d3, d11} n3 = {d1, d2, d4, d6, d7, d8, d9, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.75 Sequential difference = 0.333	Design diversity = 0.333 Design abundance = 1.666 Matching degree = 0.666 Geometrical difference = 0.638 Sequential difference = 0.444
OR 9c	d4', d12' → n4	n4 = {n3\d4, d12} n4 = {d1, d2, d6, d7, d8, d9}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.666	Design diversity = 0.5 Design abundance = 1.75 Matching degree = 0.5 Geometrical difference = 0.687 Sequential difference = 0.499
OR 6f	d8 → n5	n5 = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0	Design diversity = 0.583 Design abundance = 1.6 Matching degree = 0.4 Geometrical difference = 0.732 Sequential difference = 0.399



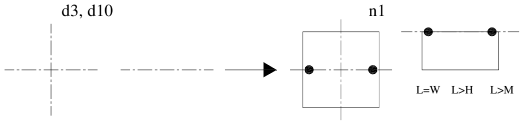
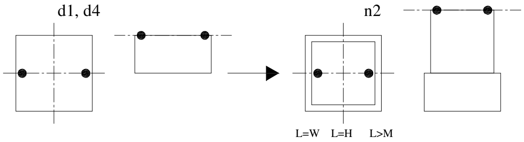
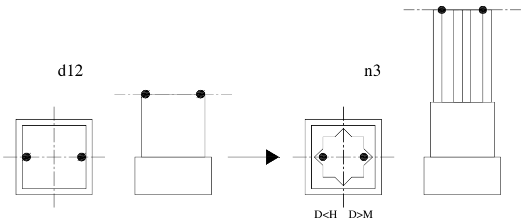
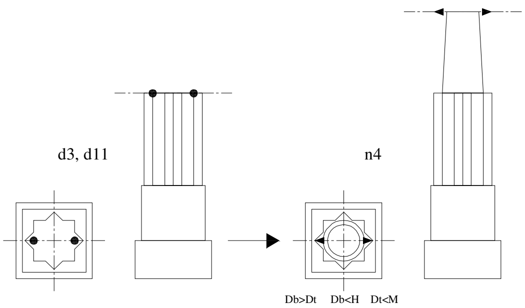
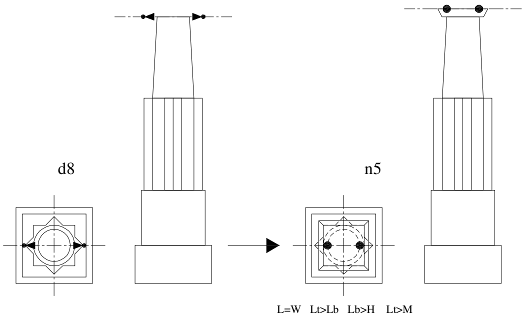
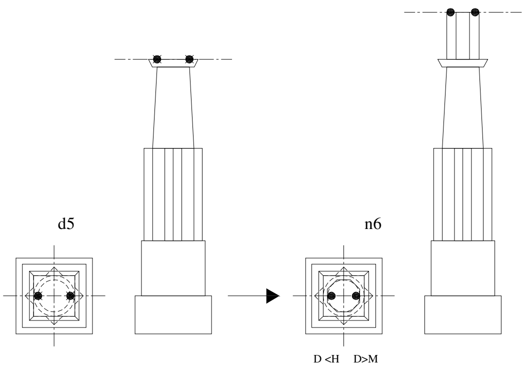
Example 5
Hybrid design - OR
No. of Original Rules = 5

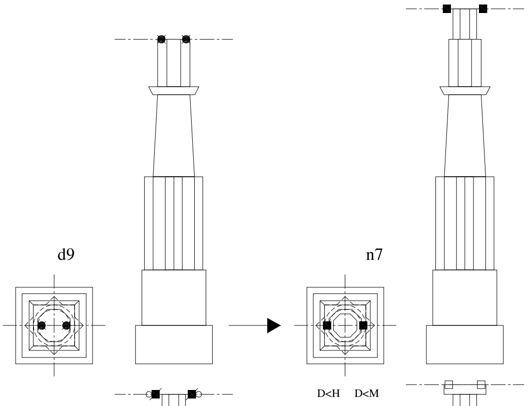
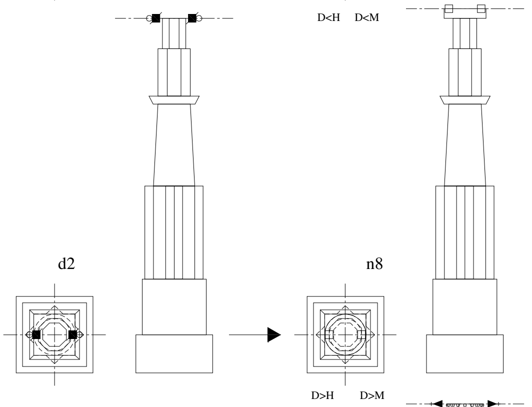
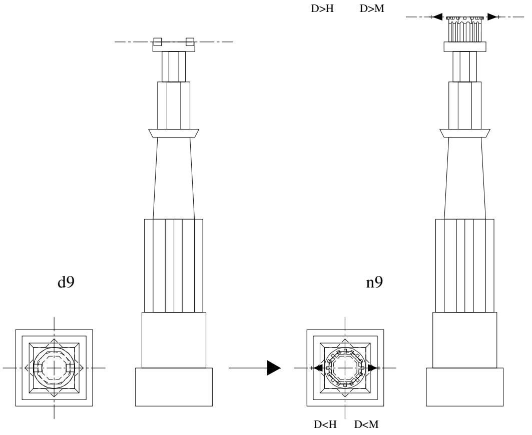
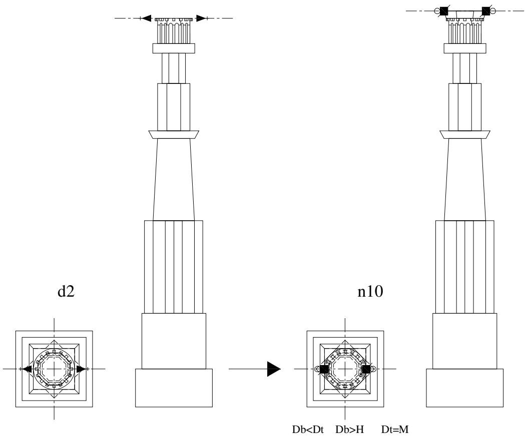
Evaluation Criteria:
Diversity= 0.583
Abundance= 1.6
Matching= 0.4
Geometrical diff.= 0.732
Sequential diff.= 0.399

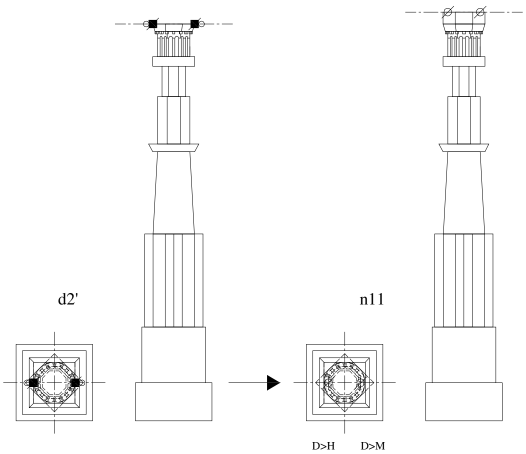
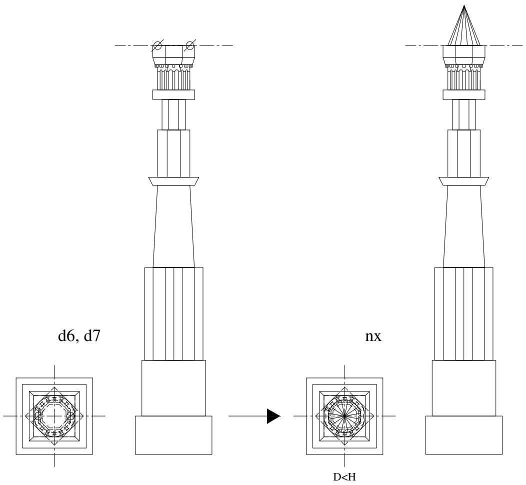
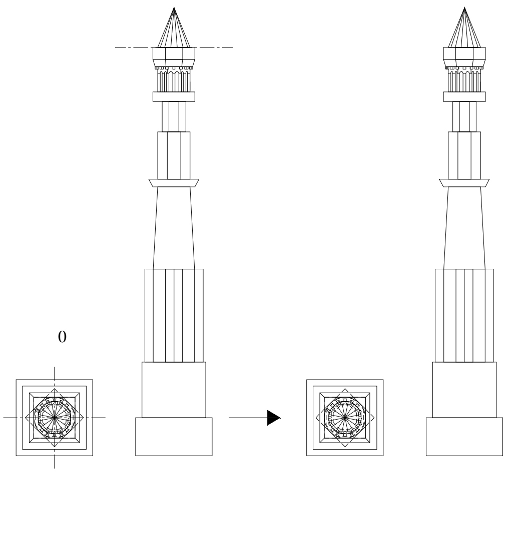
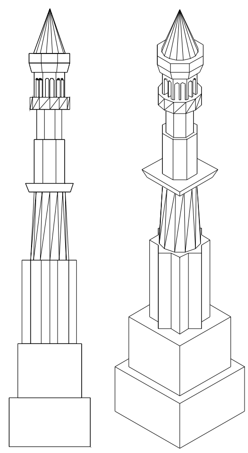
Rule no.	Example 6: Hybrid design derivation - 9 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 4a	 d6 n1 L=W L>H L=M	n1 = {n\d6} n1 = {d1, d2, d3, d4, d5, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 7b	 d5 n2 D<H D>M	n2 = {n1\d5} n2 = {d1, d2, d3, d4, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.0 Matching degree = 0.5 Geometrical difference = 0.749 Sequential difference = 0.5
OR 14b	 d12 n3 D<H D>M	n3 = {n2\d12} n3 = {d1, d2, d3, d4, d7, d8, d9, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 1.0 Matching degree = 0.333 Geometrical difference = 0.805 Sequential difference = 0.5
OR 12b	 d9 n4 D<H D<M	n4 = {n3\d9} n4 = {d1, d2, d3, d4, d7, d8, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.916	Design diversity = 0.333 Design abundance = 1.0 Matching degree = 0.25 Geometrical difference = 0.749 Sequential difference = 0.604
OR 4d	 d3, d6, d7, d8 n5 D>H D>M	n5 = {n4\d3, d6, d7, d8} n5 = {d1, d2, d4, d10, d11}	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 1.6 Matching degree = 0.4 Geometrical difference = 0.716 Sequential difference = 0.583
OR 6b	 d4, d8 n6 D <H D<M	n6 = {n6\d4, d8} n6 = {d1, d2, d10, d11}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.666 Design abundance = 1.666 Matching degree = 0.333 Geometrical difference = 0.694 Sequential difference = 0.541

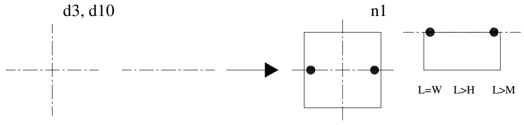
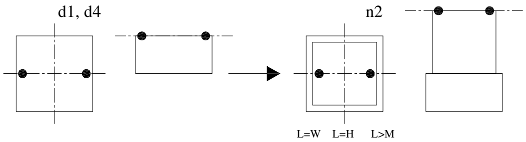
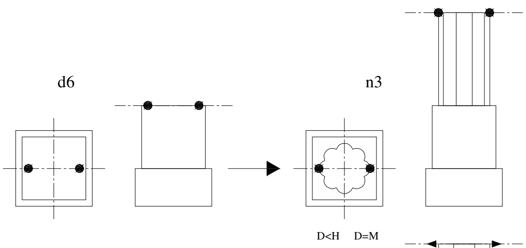
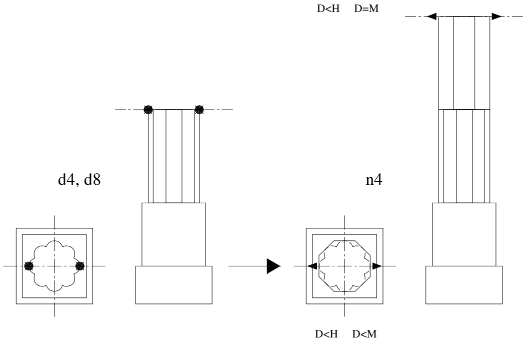
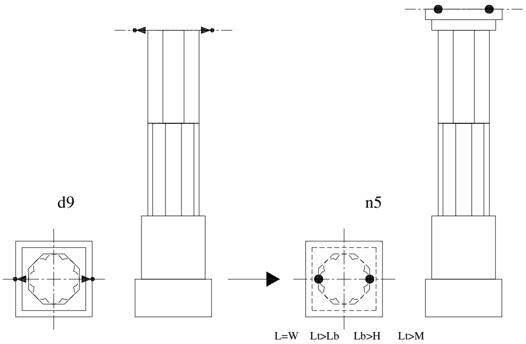
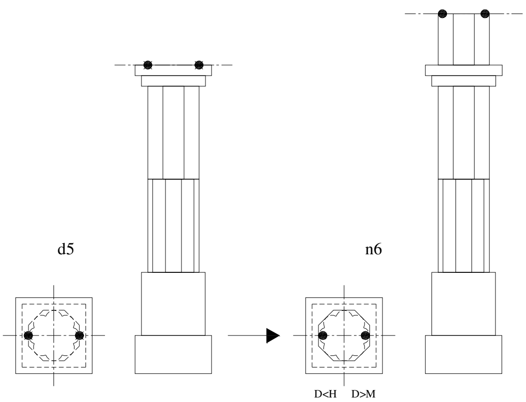
<table>
<tr><th>Rule no.</th><th>Example 6: Hybrid design derivation - 9 Original rules</th><th>User guide grammar for a hybrid design</th><th>Evaluation of the rule</th><th>Evaluation of the design</th></tr>
<tr><td>OR 14c</td><td></td><td>n7 = {n6\d11}
n6 = {d1', d2', d10'}
n7 = Ø
n7 = n7*
n7* = {m\d11}
n7* = {d3', d4', d5', d7', d9', d12'}</td><td>Rule Prevalence = 0.083
Geometrical difference = 0.583
Sequential difference = 0.833</td><td>Design diversity = 0.75
Design abundance = 1.571
Matching degree = 0.285
Geometrical difference = 0.678
Sequential difference = 0.583</td></tr>
<tr><td>OR 1e</td><td></td><td>n8 = {n7\d3}
n8 = {d1, d2, d10}</td><td>Rule Prevalence = 0.083
Geometrical difference = 0.916
Sequential difference = 0.916</td><td>Design diversity= 0.75
Design abundance= 1.5
Matching degree= 0.25
Geometrical difference= 0.707
Sequential difference= 0.624</td></tr>
<tr><td>OR 1f</td><td></td><td>n9 = 0</td><td>Rule Prevalence = 0.083
Geometrical difference = 0.916
Sequential difference = 0</td><td>Design diversity = 0.833
Design abundance = 1.444
Matching degree = 0.222
Geometrical difference = 0.731
Sequential difference = 0.555</td></tr>
<tr><td></td><td></td><td colspan="3">
Example 6
Hybrid design - OR
No. of Original Rules = 9

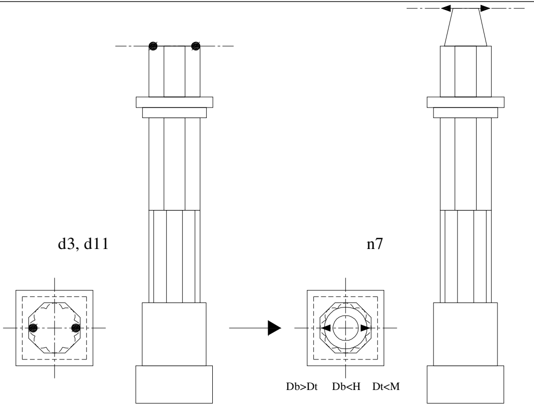
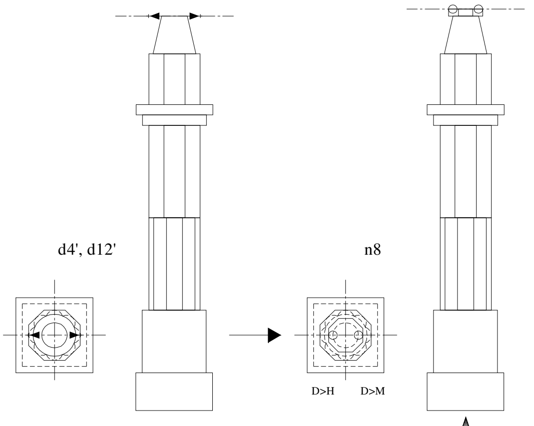
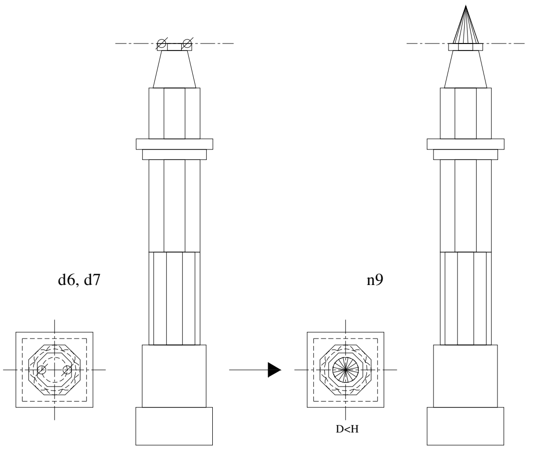
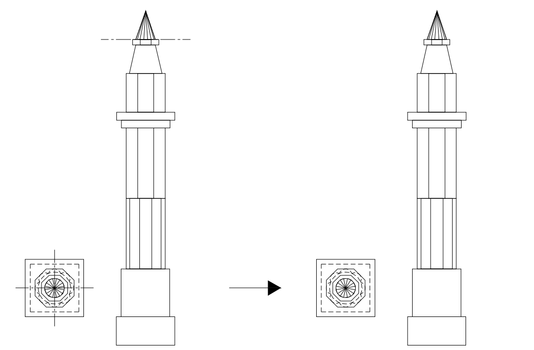
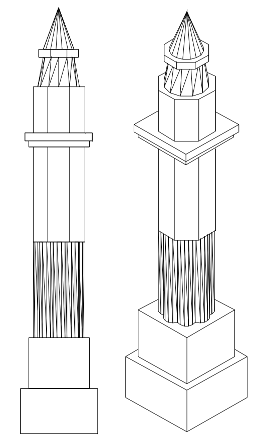
Evaluation Criteria:
Diversity= 0.833
Abundance= 1.444
Matching= 0.222
Geometrical diff.= 0.731
Sequential diff.= 0.555</td></tr>
</table>

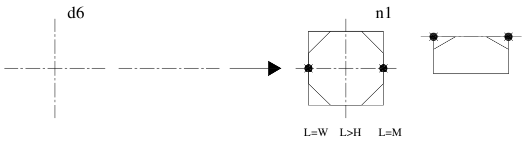
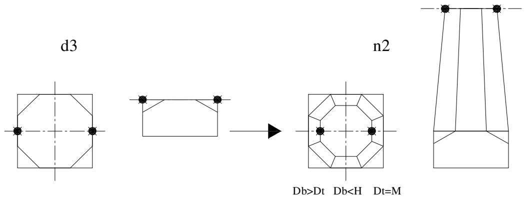
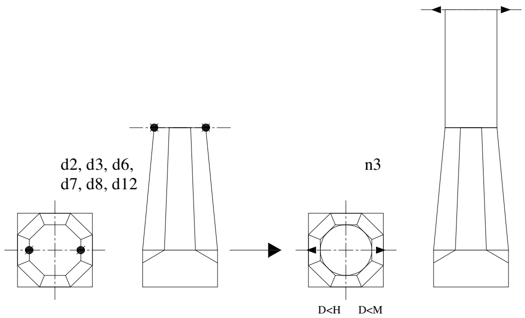
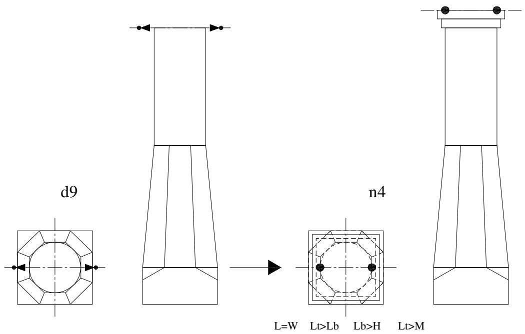
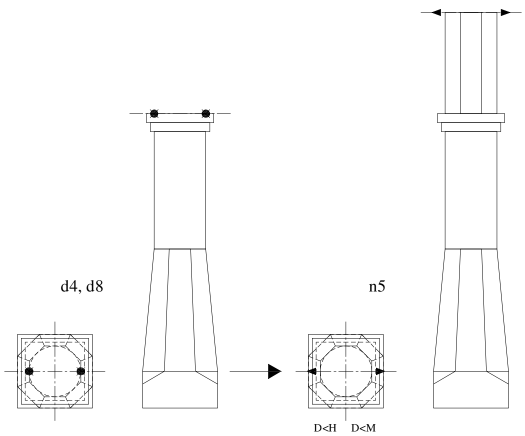
Rule no.	Example 7: Hybrid design derivation - 12 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 3a		n1 = {n\d3, d10} n1 = {d1, d2, d4, d5, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.583 Sequential difference = 0.5
OR 1b		n2 = {n1\d1, d4} n2 = {d2, d5, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 0.708 Sequential difference = 0.5
OR 14b		n3 = {n2\d12} n3 = {d2, d5, d6, d7, d8, d9, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.416 Design abundance = 1.666 Matching degree = 0.333 Geometrical difference = 0.777 Sequential difference = 0.5
OR 5b		n4={n3\d3, d11} n4={d2, d5, d6, d7, d8, d9}	Rule Prevalence=0.166 Geometrical difference=0.75 Sequential difference=0.333	Design diversity= 0.5 Design abundance= 1.75 Matching degree= 0.5 Geometrical difference= 0.77 Sequential difference= 0.458
OR 10c		n5 = {n4\d8} n5 = {d2, d5, d6, d7, d9}	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.833	Design diversity = 0.583 Design abundance = 1.6 Matching degree = 0.4 Geometrical difference = 0.783 Sequential difference = 0.533
OR 7b		n6 = {n5\d5} n6 = {d2, d6, d7, d9}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 1.5 Matching degree = 0.333 Geometrical difference = 0.749 Sequential difference = 0.527

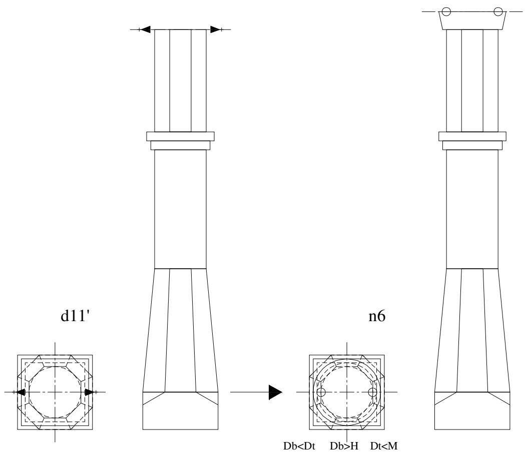
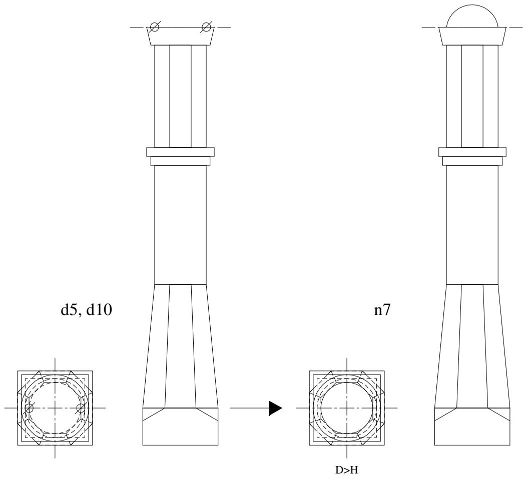
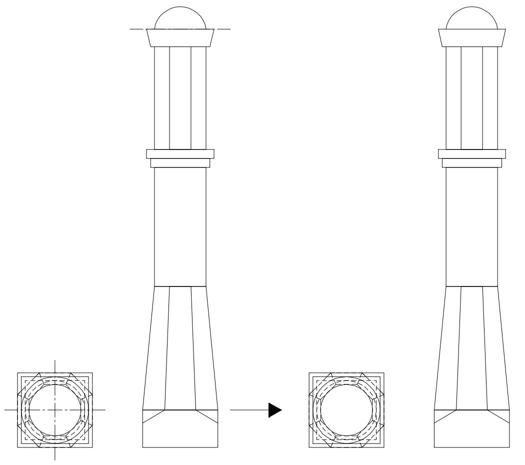
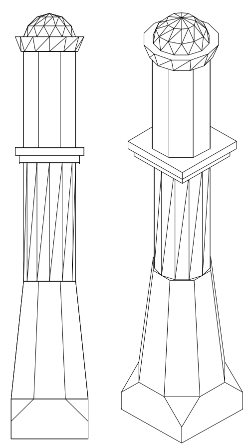
Rule no.	Example 7: Hybrid design derivation - 12 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 12b		n7 = {n6\d9} n7 = {d2, d6, d7}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.916	Design diversity = 0.75 Design abundance = 1.428 Matching degree = 0.285 Geometrical difference = 0.725 Sequential difference = 0.583
OR 2d		n8 = {n7\d2} n8 = {d6, d7} n8 = Ø n8 = n8* n8* = {m\d2} n9*={d1, d4, d5, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.833	Design diversity = 0.833 Design abundance = 1.375 Matching degree = 0.25 Geometrical difference = 0.707 Sequential difference = 0.614
OR 3e		n9 = {n8\d9} n9 = {d6, d7} n9 = Ø n9 = n9* n9* = {m\d9} n9* = {d1, d2, d4, d5, d8, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.75	Design diversity = 0.833 Design abundance = 1.333 Matching degree = 0.222 Geometrical difference = 0.73 Sequential difference = 0.629
OR 7c		n10 = {n9\d2} n10 = {d6', d7'} n10 = Ø n10 = n10* n10* = {m\d2} n10* = {d1', d4', d5', d8', d9', d10', d11', d12'} n10* = Ø n10* = n10** n10**= {n\(n10+n10*)} n10** = {d3', d2'}	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity= 0.833 Design abundance= 1.3 Matching degree= 0.2 Geometrical difference= 0.74 Sequential difference= 0.616

Rule no.	Example 7: Hybrid design derivation - 12 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 3d		n11 = {n10\d3} n11 = Ø n11 = n11* n11* = {m\d3} n11* = {d1, d2, d4, d5, d9, d10, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.75 Sequential difference = 0.916	Design diversity = 0.833 Design abundance = 1.272 Matching degree = 0.272 Geometrical difference = 0.741 Sequential difference = 0.643
OR 6f		n12 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 1.0 Design abundance = 1.333 Matching degree = 0.25 Geometrical difference = 0.749 Sequential difference = 0.589
			**Example 7** **Hybrid design - OR** **No. of Original Rules =12** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 1.333** **Matching= 0.25** **Geometrical diff.= 0.749** **Sequential diff.= 0.589**	

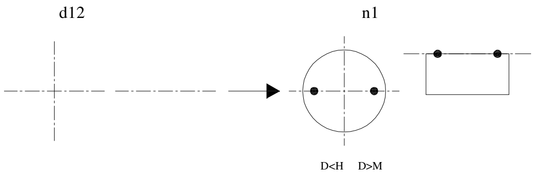
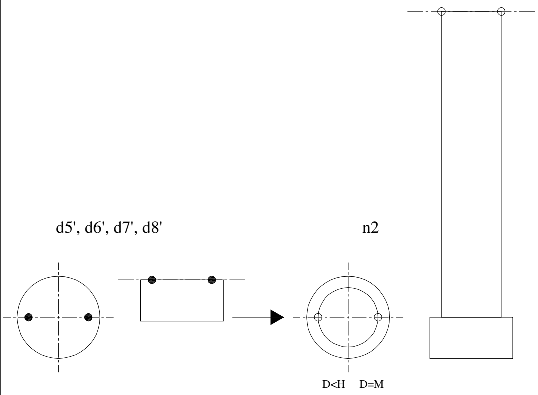
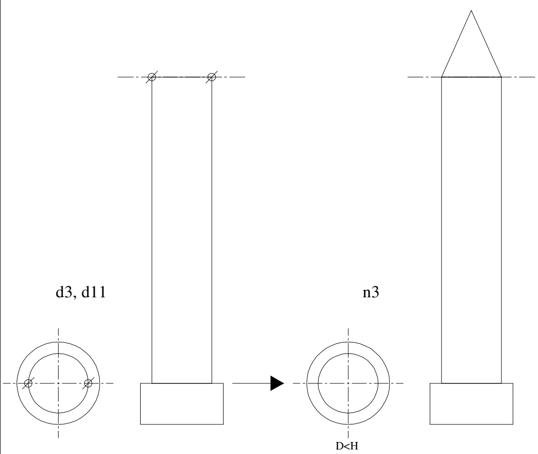
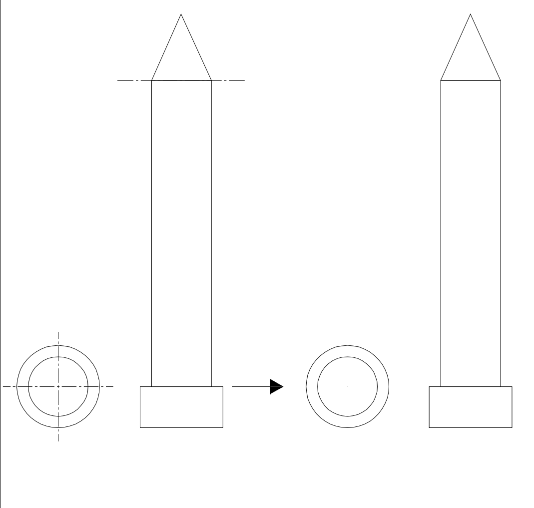
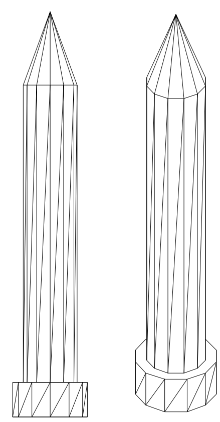
Rule no.	Example 8: Hybrid design derivation - 9 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 3a		n1 = {n\d3, d10} n1 = {d1, d2, d4, d5, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.583 Sequential difference = 0.5
OR 1b		n2 = {n1\d1, d4} n2 = {d2, d5, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 0.708 Sequential difference = 0.5
OR 11b		n3 = {n2\d6} n3 = {d2, d5, d7, d8, d9, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.416 Design abundance = 1.666 Matching degree = 0.333 Geometrical difference = 0.777 Sequential difference = 0.5
OR 6b		n4 = {n3\d4, d8} n4 = {d2, d5, d7, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.5 Design abundance = 1.75 Matching degree = 0.5 Geometrical difference = 0.728 Sequential difference = 0.458
OR 11c		n5 = {n4\d9} n5 = {d2, d5, d7, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.833	Design diversity = 0.583 Design abundance = 1.4 Matching degree = 0.4 Geometrical difference = 0.766 Sequential difference = 0.533
OR 7b		n6 = {n5\d5} n6 = {d2, d7, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 1.333 Matching degree = 0.333 Geometrical difference = 0.735 Sequential difference = 0.527

Rule no.	Example 8: Hybrid design derivation - 9 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 5b		n7={n6\d3, d11} n7={d2, d7, d12}	Rule Prevalence=0.166 Geometrical difference=0.75 Sequential difference=0.333	Design diversity = 0.75 Design abundance = 1.428 Matching degree = 0.285 Geometrical difference = 0.737 Sequential difference = 0.499
OR 9c		n8 = {n7\d4, d12} n8 = {d2, d7}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.666	Design diversity = 0.833 Design abundance = 1.5 Matching degree = 0.375 Geometrical difference = 0.749 Sequential difference = 0.52
OR 6f		n9 = 0	Rule Prevalence = 0.166 Geometrical difference =0.833 Sequential difference = 0	Design diversity = 0.916 Design abundance = 1.555 Matching degree = 0.333 Geometrical difference = 0.758 Sequential difference = 0.462
			**Example 8** **Hybrid design - OR** **No. of Original Rules = 9** **Evaluation Criteria:** **Diversity= 0.916** **Abundance= 1.555** **Matching= 0.333** **Geometrical diff.= 0.758** **Sequential diff.= 0.462**	

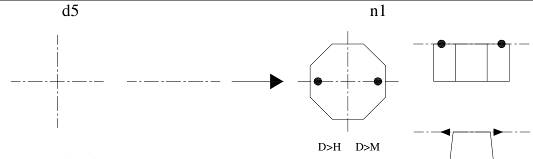
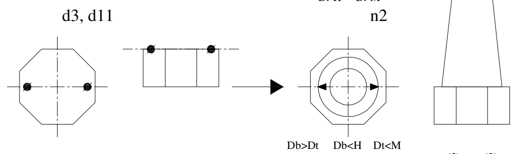
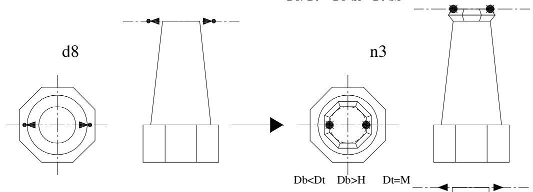
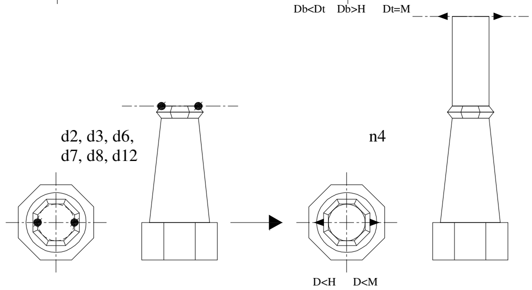
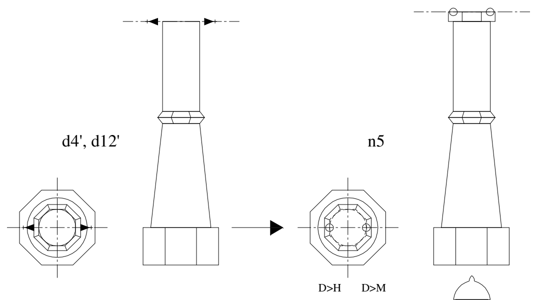
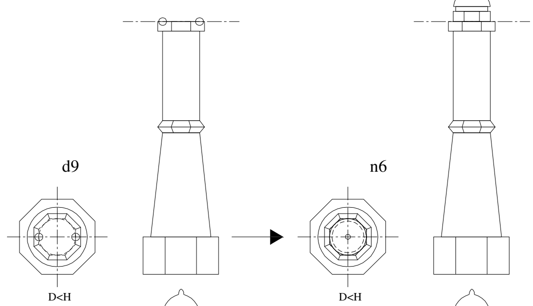
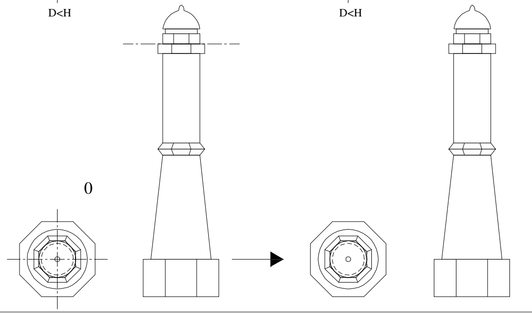
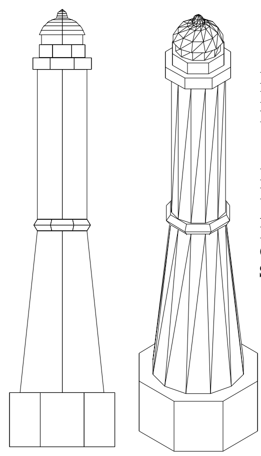
Rule no.	Example 9: Hybrid design derivation - 7 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 4a		n1 = {n\d6} n1 = {d1, d2, d3, d4, d5, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 4b		n2 = {n1\d3} n2 = {d1, d2, d4, d5, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.0 Matching degree = 0.5 Geometrical difference = 0.916 Sequential difference = 0.5
OR 3b		n3 = {n2\d2, d3, d6, d7, d8, d12} n3 = {d1, d4, d5, d9, d10, d11}	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.6 Design abundance = 2.666 Matching degree = 0.666 Geometrical difference = 0.749 Sequential difference = 0.444
OR 11c		n4 = {n3\d9} n4 = {d1, d4, d5, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.833	Design diversit y= 0.583 Design abundance = 2.25 Matching degree = 0.5 Geometrical difference = 0.791 Sequential difference = 0.541
OR 6b		n5 = {n4\d4, d8} n5 = {d1', d5', d10', d11'}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.666 Design abundance = 2.2 Matching degree = 0.4 Geometrical difference = 0.749 Sequential difference = 0.499

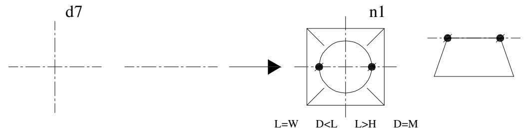
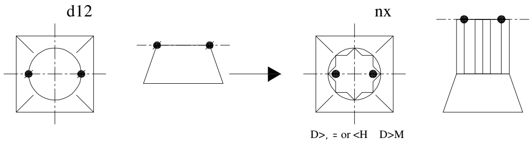
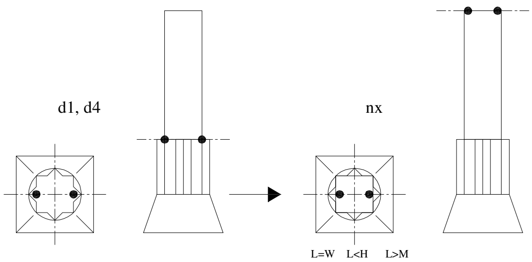
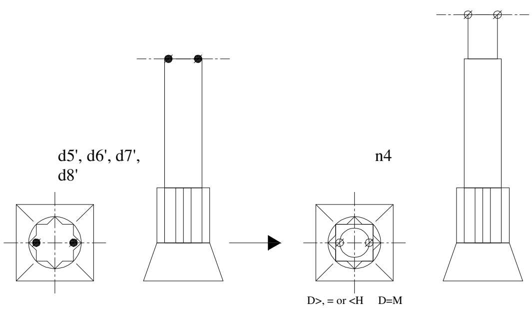
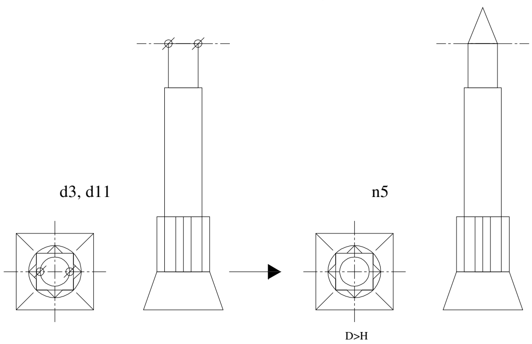
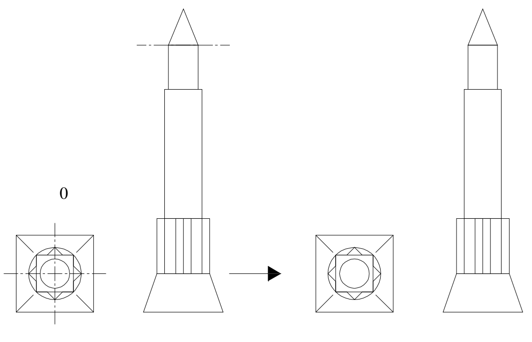
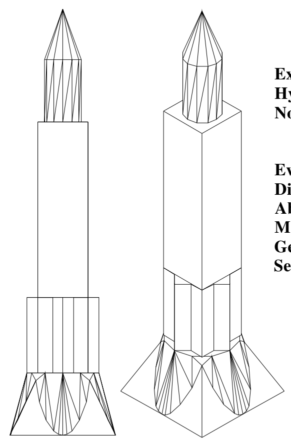
<table>
<tr><th>Rule no.</th><th>Example 9: Hybrid design derivation - 7 Original rules</th><th>User guide grammar for a hybrid design</th><th>Evaluation of the rule</th><th>Evaluation of the design</th></tr>
<tr><td>OR
15c</td><td>
</td><td>n6 = {n5\d11}
n6 = {d1, d5, d10}</td><td>Rule Prevalence = 0.083
Geometrical difference = 0.583
Sequential difference = 0.666</td><td>Design diversity = 0.75
Design abundance = 2.0
Matching degree = 0.333
Geometrical difference = 0.721
Sequential difference = 0.527</td></tr>
<tr><td>OR
5f</td><td>
</td><td>n7 = 0</td><td>Rule Prevalence = 0.166
Geometrical difference = 0.833
Sequential difference = 0</td><td>Design diversity = 0.916
Design abundance = 2.0
Matching degree = 0.285
Geometrical difference = 0.737
Sequential difference = 0.452</td></tr>
<tr><td></td><td></td><td colspan="3">
Example 9
Hybrid design - OR
No. of Original Rules = 7

Evaluation Criteria:
Diversity= 0.916
Abundance= 2.0
Matching= 0.285
Geometrical diff.= 0.737
Sequential diff.= 0.452</td></tr>
</table>

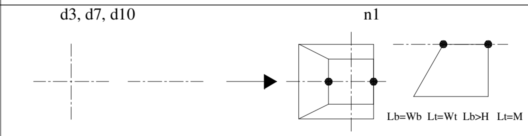
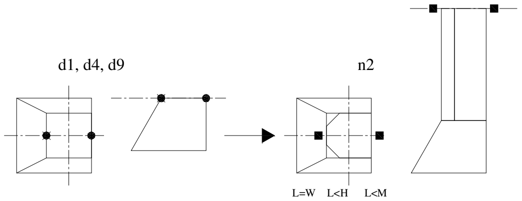
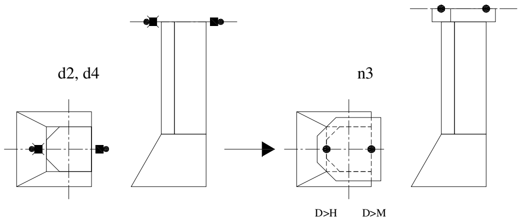
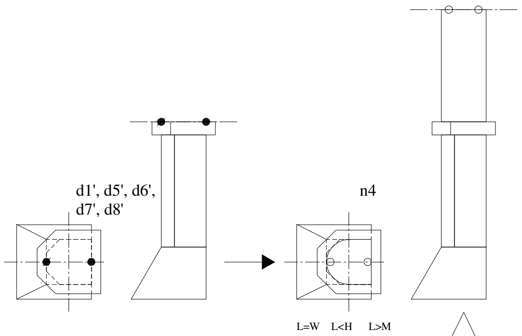
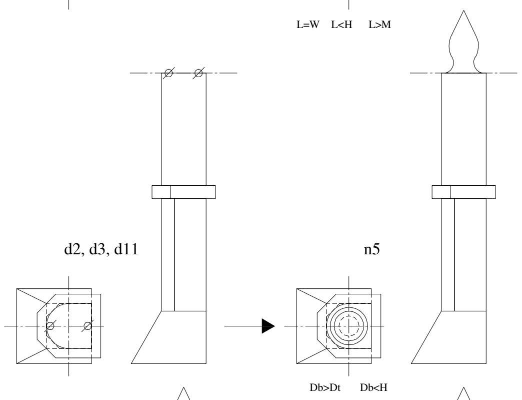
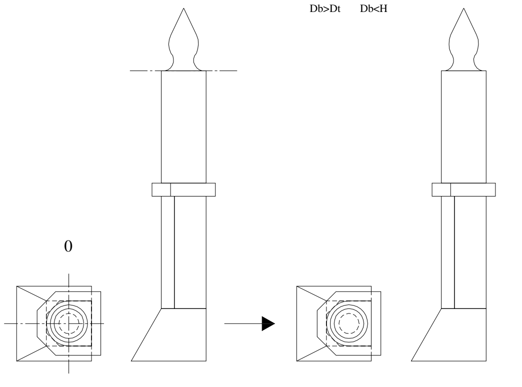
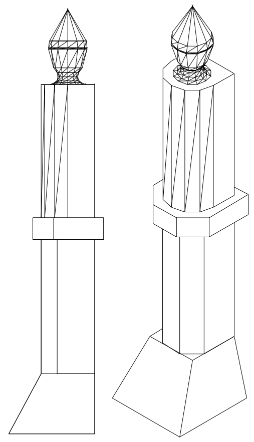
<table>
<tr><th>Rule no.</th><th>Example 10: Hybrid design derivation - 3 Original rules</th><th>User guide grammar for a hybrid design</th><th>Evaluation of the rule</th><th>Evaluation of the design</th></tr>
<tr><td>OR 7a</td><td></td><td>n1 = {n\d12}
n1 = {d1', d2', d3', d4', d5', d6', d7', d8', d9', d10', d11'}</td><td>Rule Prevalence = 0.083
Geometrical difference = 0.916
Sequential difference = 0.5</td><td>Design diversity = 0.083
Design abundance = 1.0
Matching degree = 1.0
Geometrical difference = 0.916
Sequential difference = 0.5</td></tr>
<tr><td>OR 8b</td><td></td><td>n2 = {n1\d5, d6,d7, d8}
n2 = {d1, d2, d3, d4, d9, d10, d11}</td><td>Rule Prevalence = 0.333
Geometrical difference = 0.416
Sequential difference = 0.5</td><td>Design diversity = 0.416
Design abundance = 2.5
Matching degree = 0.5
Geometrical difference = 0.666
Sequential difference = 0.5</td></tr>
<tr><td>OR 3f</td><td></td><td>n3 = 0</td><td>Rule Prevalence = 0.166
Geometrical difference = 0.833
Sequential difference = 0</td><td>Design diversity = 0.583
Design abundance = 2.333
Matching degree = 0.333
Geometrical difference = 0.721
Sequential difference = 0.333</td></tr>
<tr><td></td><td></td><td colspan="3">
Example 10
Hybrid design - OR
No. of Original Rules = 3

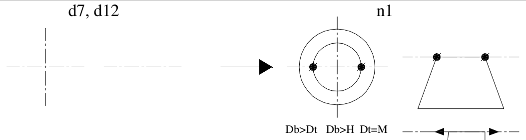
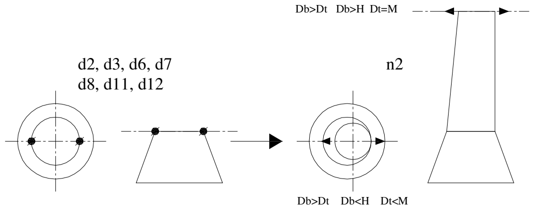
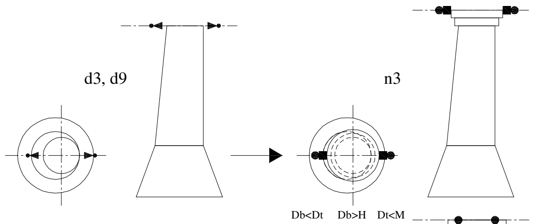
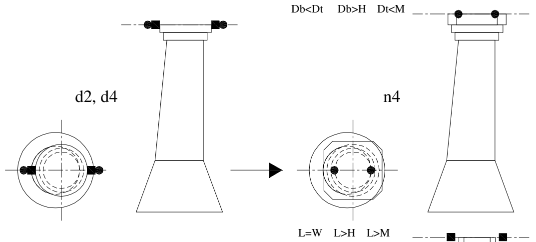
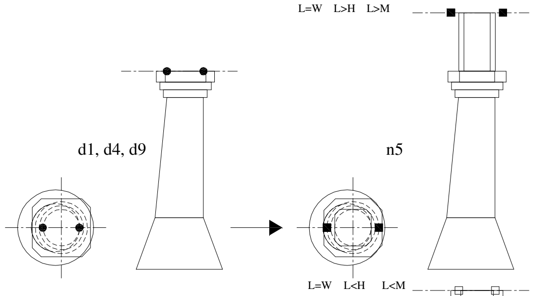
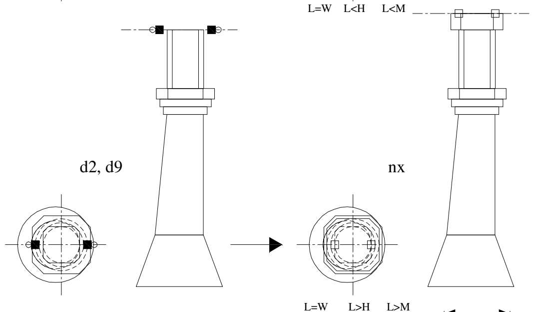
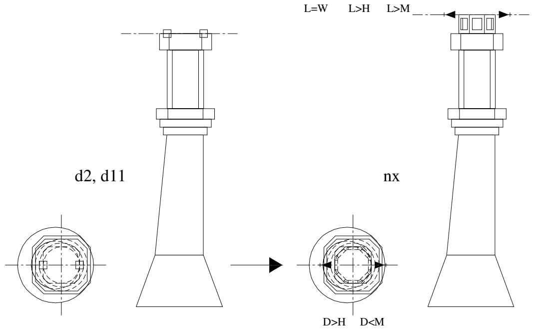
Evaluation Criteria:
Diversity= 0.583
Abundance= 2.333
Matching= 0.333
Geometrical diff.= 0.721
Sequential diff.= 0.333</td></tr>
</table>

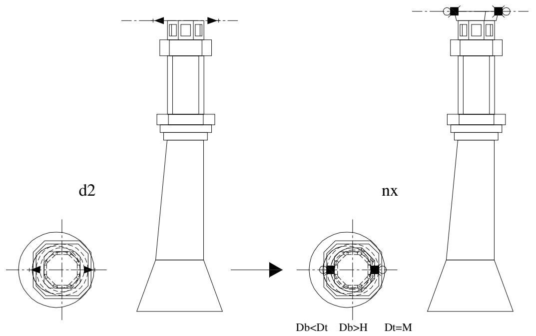
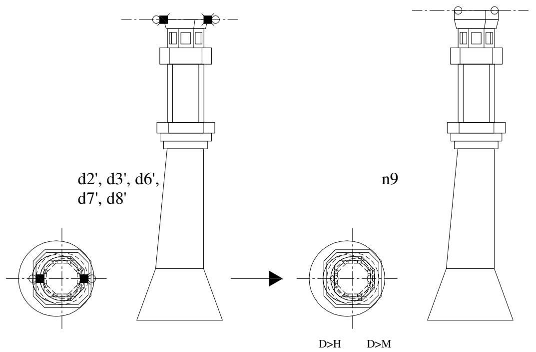
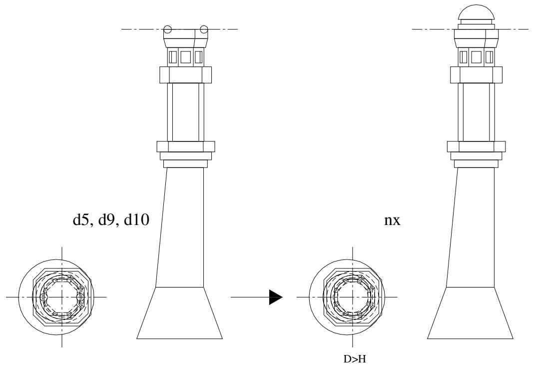
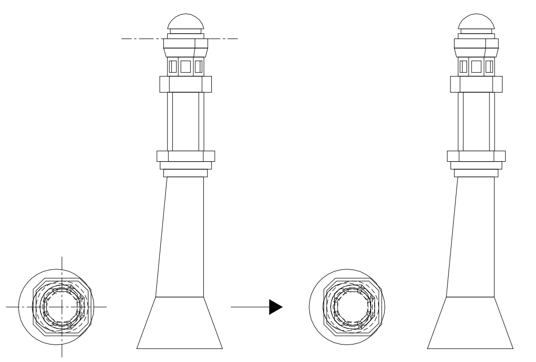
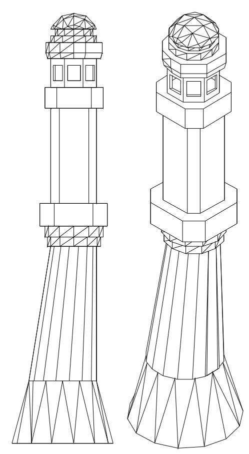
Rule no.	Example 11: Hybrid design derivation - 6 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 6a		n1 = {n\d5} n1 = {d1, d2, d3, d4, d6, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 5b		n2 = {n1\d3, d11} n2 = {d1, d2, d4, d6, d7, d8, d9, d10, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.75 Sequential difference = 0.333	Design diversity = 0.25 Design abundance = 1.5 Matching degree = 0.5 Geometrical difference = 0.833 Sequential difference = 0.416
OR 12c		n3 = {n2\d8} n3 = {d1, d2, d4, d6, d7, d9, d10, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.833	Design diversity = 0.333 Design abundance = 1.333 Matching degree = 0.333 Geometrical difference = 0.86 Sequential difference = 0.555
OR 3b		n4 = {n3\d2, d3, d6, d7, d8, d12} n4 = {d1, d4, d9, d10}	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.666 Design abundance = 2.5 Matching degree = 0.5 Geometrical difference = 0.749 Sequential difference = 0.499
OR 9c		n5 = {n4\d4, d12} n5 = {d1, d9, d10}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.666	Design diversity = 0.75 Design abundance = 2.4 Matching degree = 0.4 Geometrical difference = 0.766 Sequential difference = 0.533
OR 8f		n6 = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0	Design diversity = 0.833 Design abundance = 2.166 Matching degree = 0.333 Geometrical difference = 0.791 Sequential difference = 0.444
		 **Example 11** **Hybrid design - OR** **No. of Original Rules = 6** **Evaluation Criteria:** **Diversity= 0.833** **Abundance= 2.166** **Matching= 0.333** **Geometrical diff.= 0.791** **Sequential diff.= 0.444**		

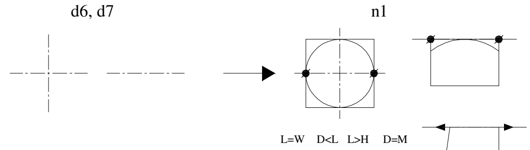
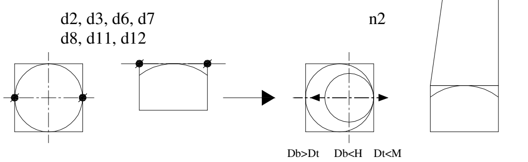
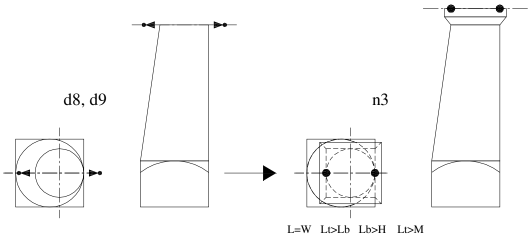
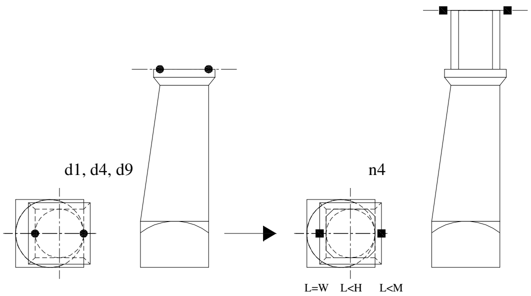
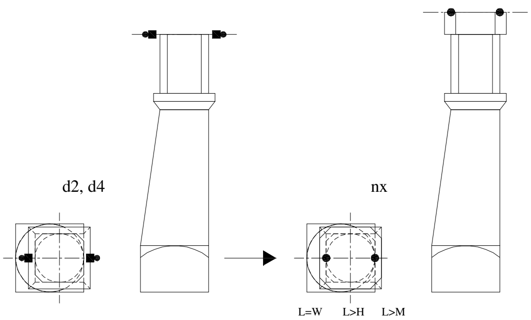
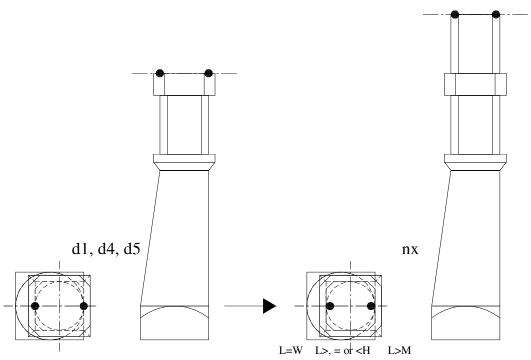
Rule no.	Example 12: Hybrid design derivation - 5 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 5a		n1 = {n\d7} n1 = {d1, d2, d3, d4, d5, d6, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 14b		n2 = {n1\d12} n2 = {d1, d2, d3, d4, d5, d6, d8, d9, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.0 Matching degree = 0.5 Geometrical difference = 0.916 Sequential difference = 0.5
OR 1b		n3 = {n2\d1, d4} n3 = {d2, d3, d5, d6, d8, d9, d10, d11}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 1.333 Matching degree = 0.333 Geometrical difference = 0.888 Sequential difference = 0.5
OR 8b		n4 = {n3\d5, d6, d7, d8} n4 = {d2, d3, d9, d10, d11}	Rule Prevalence = 0.333 Geometrical difference = 0.416 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 0.77 Sequential difference = 0.5
OR 5f		n5 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.75 Design abundance = 2.0 Matching degree = 0.4 Geometrical difference = 0.782 Sequential difference = 0.4
		 **Example 12** **Hybrid design - OR** **No. of Original Rules = 5** **Evaluation Criteria:** **Diversity= 0.75** **Abundance= 2.0** **Matching= 0.4** **Geometrical diff.= 0.782** **Sequential diff.= 0.4**		

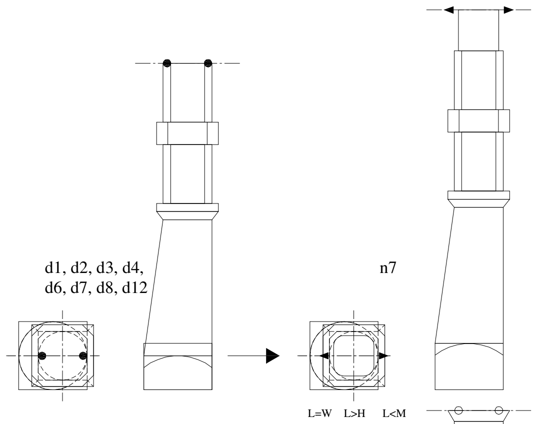
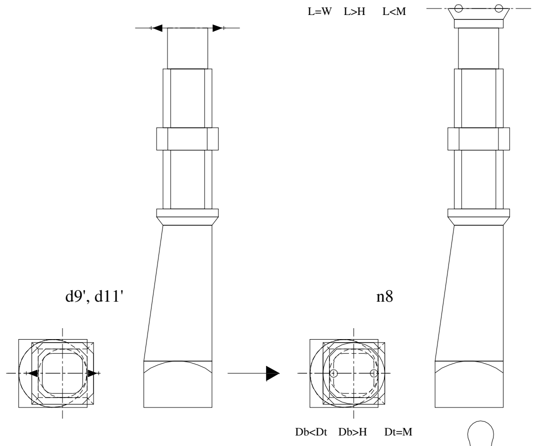
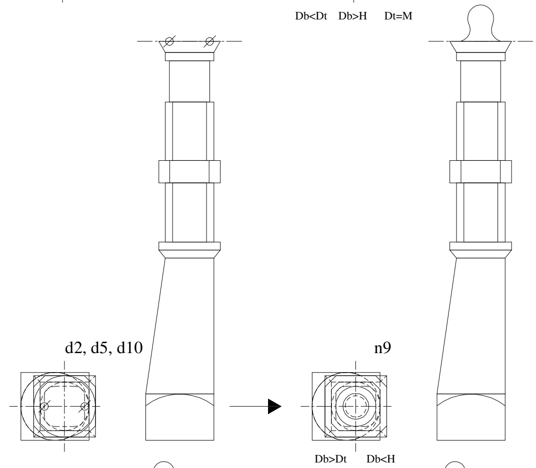
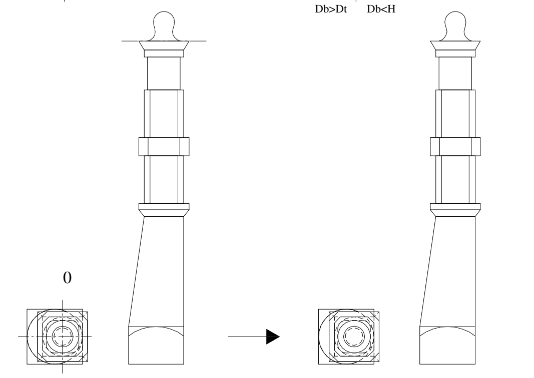
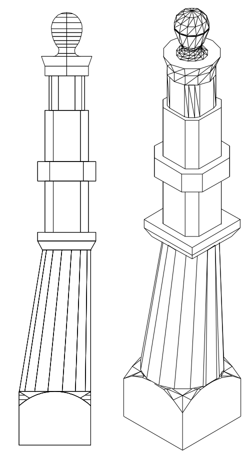
B-3 Hybrid designs composed of hybrid rules

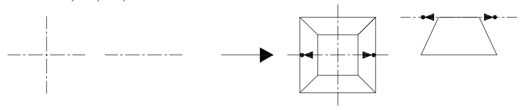
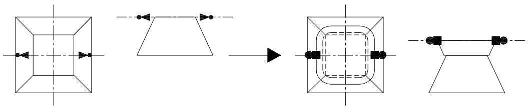
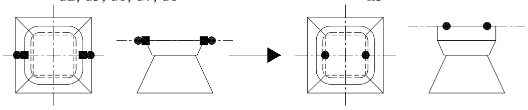
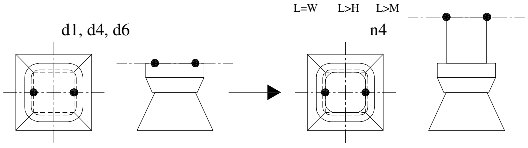
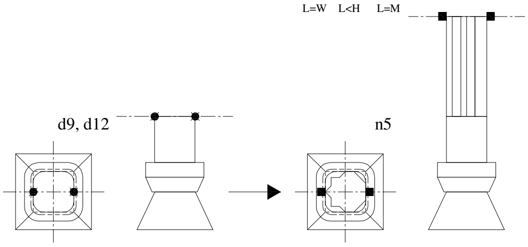
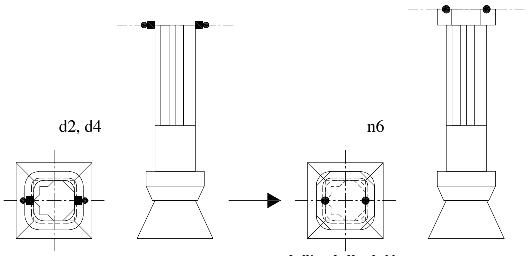
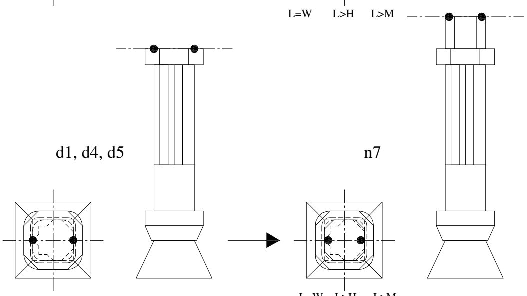
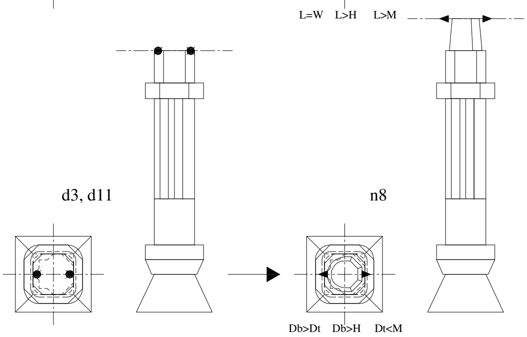
Rule no.	Example 1: Hybrid design derivation - 5 hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 12a		n1 = {n\d3, d7, d10} n1 = {d1, d2, d4, d5, d6, d8, d9, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 9b		n2 = {n1\d1, d4, d9} n2 = {d2, d5, d6, d8, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.5 Design abundance = 3.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.708
HR 3d		n3 = {n2\d2, d4} n3 = {d5, d6, d8, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 2.666 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.638
HR 12b		n4 = {n3\d1, d5, d6, d7, d8} n4 = {d11, d12}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.583	Design diversity = 0.833 Design abundance = 3.25 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.624
HR 1f		n5 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 0.916 Design abundance = 3.2 Matching degree = 0.2 Geometrical difference = 1.0 Sequential difference = 0.499
		 **Example 1** **Hybrid design - HR** **No. of Hybrid Rules = 5** **Evaluation Criteria:** **Diversity= 0.916** **Abundance= 3.2** **Matching= 0.2** **Geometrical diff.= 1.0** **Sequential diff.= 0.499**		

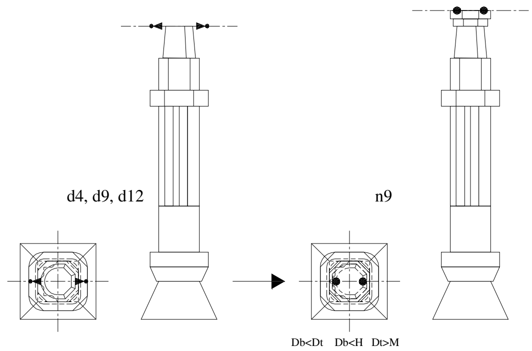
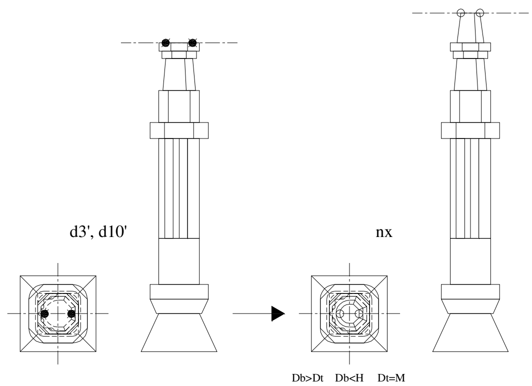
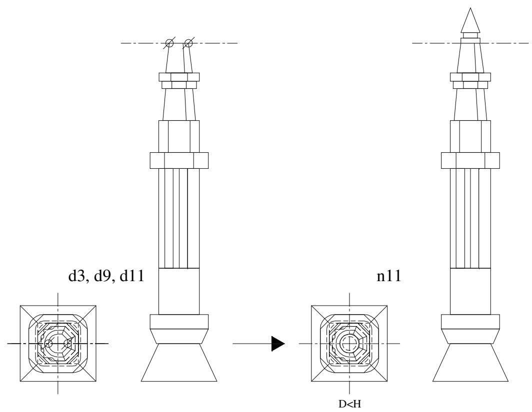
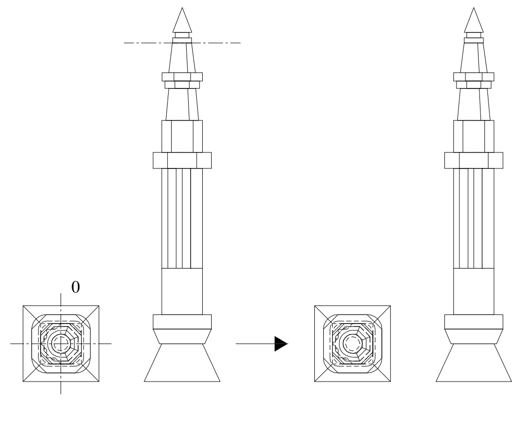
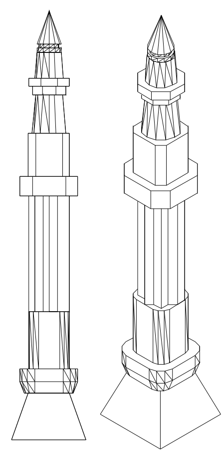
Rule no.	Example 2: Hybrid design derivation - 10 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 5a		n1 = {n\d7, d12} n1 = {d1, d2, d3, d4, d5, d6, d8, d9, d10, d11}	Rule Prevalence=0.166 Geometrical difference=1.0 Sequential difference=0.5	Design diversity = 0.25 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 5b		n2 = {n1\d2, d3, d6, d7, d8, d11, d12} n2 = {d1, d4, d5, d9, d10}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.583 Design abundance = 4.5 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.416
HR 20c		n3 = {n2\d3, d9} n3 = {d1, d4, d5, d10}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 3.666 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.444
HR 4d		n4 = {n3\d2, d4} n4 = {d1, d5, d10}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.75 Design abundance = 3.25 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.458
HR 15b		n5 = {n4\d1, d4, d9} n5 = {d5, d10} n5 = Ø n5 = n5* n5* = {m\d1, d4, d9} n5* = {d2, d6, d8, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.833 Design abundance = 3.2 Matching degree = 0.2 Geometrical difference = 1.0 Sequential difference = 0.549
HR 8d		n6 = {n5\d2, d9} n6 = {d5, d10} n6 = Ø n6 = n6* n6* = {m\d2, d9} n6* = {d1, d6, d8, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.833 Design abundance = 3.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.597
HR 10e		n7 = {n6\d2, d11} n7 = {d5, d10} n7 = Ø n7 = n7* n7* = {m\d2, d11} n7* = {d1, d6, d8} n7* = Ø n7* = n7** n7** = {n\(n7+n7*)} n7** = {d2, d3, d4, d7, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.75	Design diversity = 0.833 Design abundance = 2.857 Matching degree = 0.285 Geometrical difference = 1.0 Sequential difference = 0.618

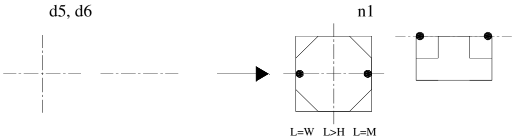
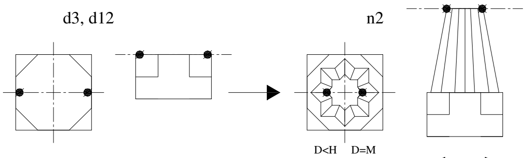
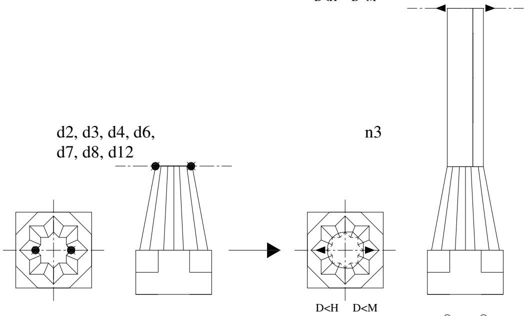
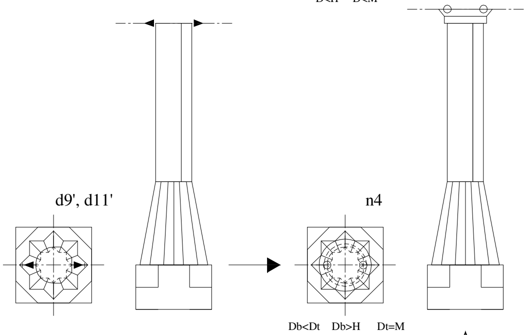
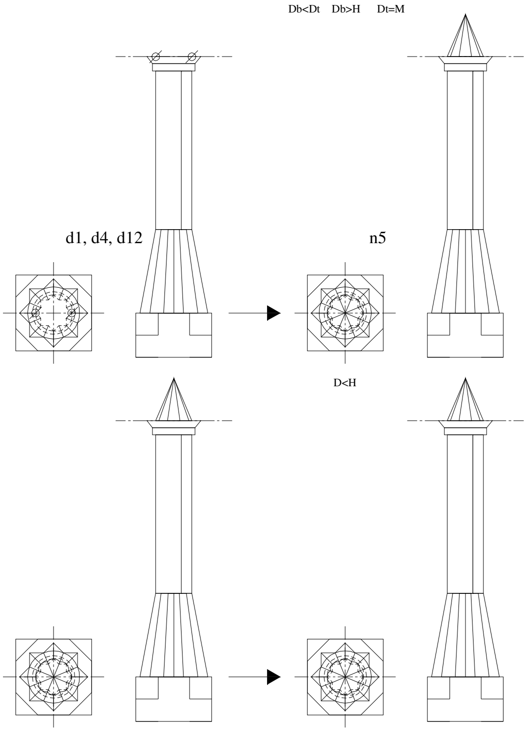
Rule no.	Example 2: Hybrid design derivation - 10 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 6c		n8 = {n7\d2} n8 = {d5', d10'} n8 = Ø n8 = n8* n8* = {m\d2} n8* = {d1', d6', d8'}	Rule Prevalence = 0.083 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.833 Design abundance = 2.625 Matching degree = 0.312 Geometrical difference = 1.0 Sequential difference = 0.604
HR 11d		n9 = {n8\d2, d3, d6, d7, d8} n9 = {d5, d10}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.833 Design abundance = 2.888 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.638
HR 6f		nx = 0	Rule Prevalence=0.25 Geometrical difference=1.0 Sequential difference=0.0	Design diversity = 1.0 Design abundance = 2.9 Matching degree = 0.3 Geometrical difference = 1.0 Sequential difference = 0.574
			**Example 2** **Hybrid design - HR** **No. of Hybrid Rules = 10** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 2.9** **Matching= 0.3** **Geometrical diff.= 1.0** **Sequential diff.= 0.574**	

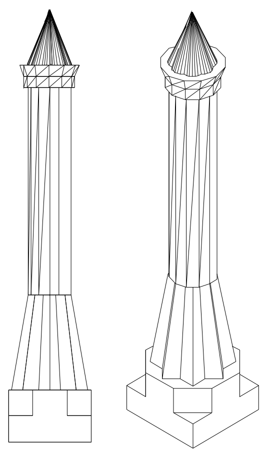
Rule no.	Example 3: Hybrid design derivation - 9 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 1a		n1 = {n\d6, d7} n1 = {d1, d2, d3, d4, d5, d8, d9, d10, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 5b		n2 = {n1\d2, d3, d6, d7, d8, d11, d12} n2 = {d1, d4, d5, d9, d10}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.583 Design abundance = 4.5 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.416
HR 9c		n3 = {n2\d8, d9} n3 = {d1, d4, d5, d10}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.666 Design abundance = 3.666 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.555
HR 15b		n4 = {n3\d1, d4, d9} n4 = {d5, d10} n4 = Ø n4 = n4* n4* = {m\d1, d4, d9} n4* = {d2, d3, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.833 Design abundance = 3.5 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.645
HR 4d		n5 = {n4\d2, d4} n5 = {d5, d10} n5 = Ø n5 = n5* n5* = {m\d2, d4} n5* = {d1, d3, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.833 Design abundance = 3.2 Matching degree = 0.2 Geometrical difference = 1.0 Sequential difference = 0.616
HR 16b		n6 = {n5\d1, d4, d5} n6 = {d10} n6 = Ø n6 = n6* n6* = {m\d2, d4} n6* = {d3, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.916 Design abundance = 3.166 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.597

Rule no.	Example 3: Hybrid design derivation - 9 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 17b		n7 = {n6\d1, d2, d3, d4, d6, d7, d8, d12} n7 = {d10'} n7 = Ø n7 = n6* n7* = {m\d1, d2, d3, d4, d6, d7, d8, d12} n7* = {d11'}	Rule Prevalence = 0.666 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.916 Design abundance = 3.857 Matching degree = 0.285 Geometrical difference = 1.0 Sequential difference = 0.559
HR 17c		n8 = {n7\ d9, d11} n8 = {d10}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.666	Design diversity = 0.916 Design abundance = 3.625 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.572
HR 2f		n9 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 3.555 Matching degree = 0.222 Geometrical difference = 1.0 Sequential difference = 0.509
		 **Example 3** **Hybrid design - HR** **No. of Hybrid Rules = 9** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 3.555** **Matching= 0.222** **Geometrical diff.= 1.0** **Sequential diff.= 0.509**		

Rule no.	Example 4: Hybrid design derivation - 11 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 4a	d2, d7, d8, d9 n1  Lb=Wb Lt=Wt Lb>Lt Lb>H Lt<M	n1 = {n\d2, d7, d8, d9} n1 = {d1, d3, d4, d5, d6, d10, d11, d12}	Rule Prevalence = 0.333 Geometrical difference = 1.0 Sequential difference = 0.75	Design diversity = 0.333 Design abundance = 4.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.75
HR 4c	d2, d6, d7, d8 n2  L=W Lt>Lb Lb>H Lt=M	n2 = {n1\d2, d6, d7, d8} n2 = {d1, d3, d4, d5, d10, d11, d12}	Rule Prevalence = 0.333 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.416 Design abundance = 4.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.625
HR 2d	d2, d3, d6, d7, d8 n3  L=W L>H L>M	n3 = {n2\d2, d3, d6, d7, d8} n3 = {d1, d4, d5, d10, d11, d12}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 4.333 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.583
HR 18b	d1, d4, d6 n4  L=W L<H L=M	n4 = {n3\d1, d4, d6} n4 = {d1, d5, d10, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 4.0 Matching degree = 0.375 Geometrical difference = 1.0 Sequential difference = 0.562
HR 20b	d9, d12 n5 	n5 = {n4\d9, d12} n5 = {d1, d5, d10, d11} n5 = Ø n5 = n5* n5* = {m\d9, d12} n5* = {d3, d4}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.666 Design abundance = 3.6 Matching degree = 0.3 Geometrical difference = 1.0 Sequential difference = 0.633
HR 4d	d2, d4 n6  L=W L>H L>M	n6 = {n5\d2, d4} n6 = {d1, d5, d10, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 3.333 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.611
HR 16b	d1, d4, d5 n7  L=W L>H L>M	n7 = {n6\d1, d4, d5} n7 = {d10, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.833 Design abundance = 3.285 Matching degree = 0.285 Geometrical difference = 1.0 Sequential difference = 0.595
HR 22b	d3, d11 n8  Db>Dt Db>H Dt<M	n8 = {n7\d3, d11} n8 = {d10} n8 = Ø n8 = n8* n8* = {m\d3, d11} n8* = {d5, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.916 Design abundance = 3.125 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.562

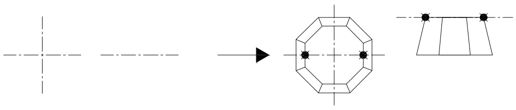
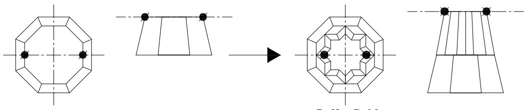
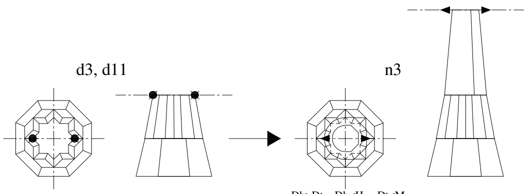
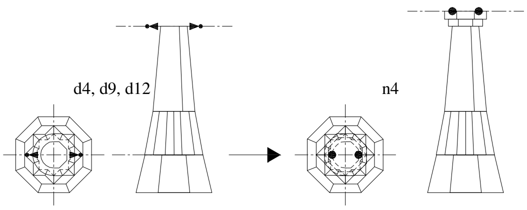
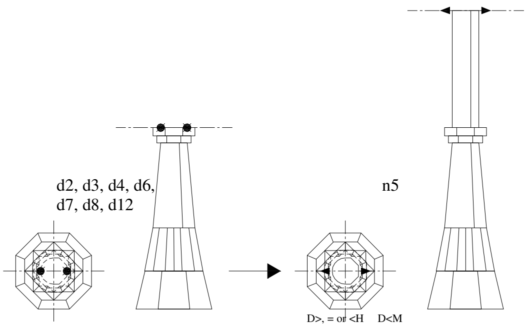
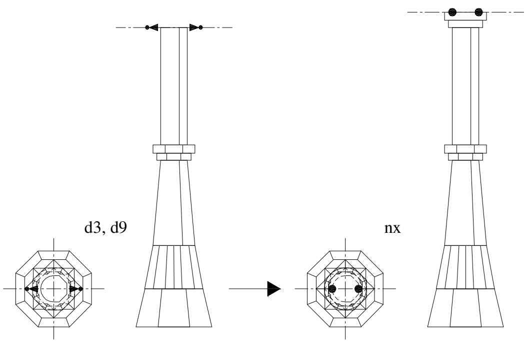
Rule no.	Example 4: Hybrid design derivation - 11 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 13c		n9 = {n8\d4, d9, d12} n9 = {d10}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.916 Design abundance = 3.111 Matching degree = 0.222 Geometrical difference = 1.0 Sequential difference = 0.592
HR 23b		n10 = {n9\d3, d10} n10 = Ø n10 = n10* n10* = {m\d3, d10} n5* = {d5, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.583	Design diversity = 1.0 Design abundance = 3.0 Matching degree = 0.2 Geometrical difference = 1.0 Sequential difference = 0.591
HR 4f		n11 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 3.0 Matching degree = 0.181 Geometrical difference = 1.0 Sequential difference = 0.537
		 **Example 4** **Hybrid design - HR** **No. of Hybrid Rules = 11** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 3.0** **Matching= 0.181** **Geometrical diff.= 1.0** **Sequential diff.= 0.537**		

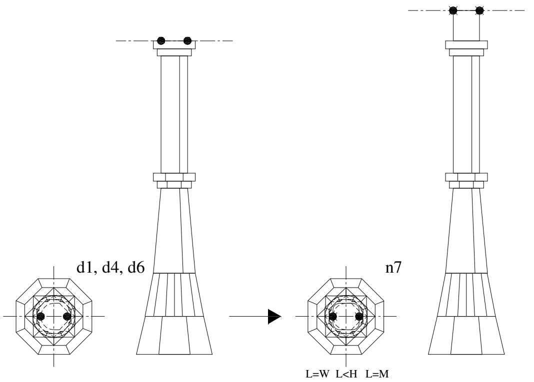
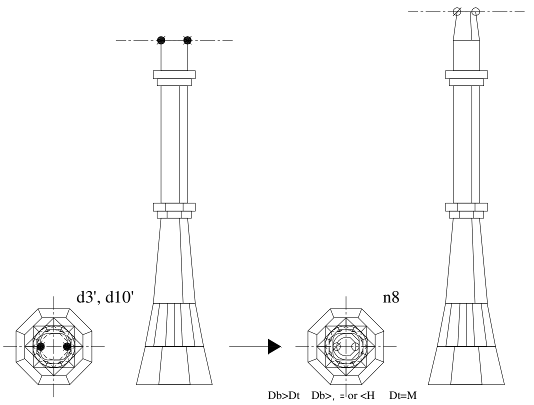
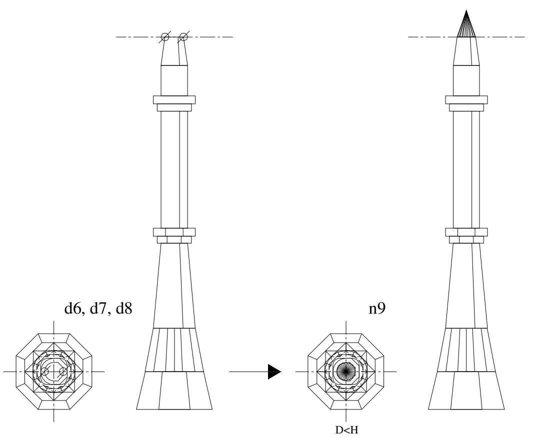
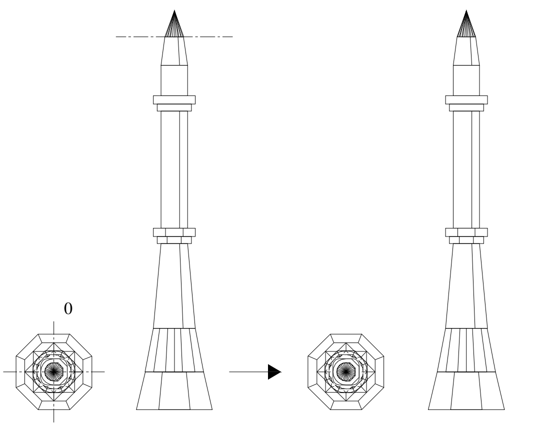
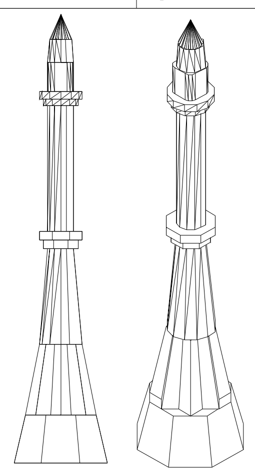
Rule no.	Example 5: Hybrid design derivation - 5 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 6a		n1 = {n\d5, d6} n1 = {d1, d2, d3, d4, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 4b		n2 = {n1\d3, 12} n2 = {d1, d2, d4, d7, d8, d9, d10, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 2.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5
HR 13b		n3 = {n2\d2, d3, d4, d6, d7, d8, d12} n3 = {d1, d9, d10, d11}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity= 0.666 Design abundance= 3.666 Matching degree= 0.333 Geometrical difference= 1.0 Sequential difference= 0.444
HR 17c		n4 = {n3\ d9, d11} n4 = {d1, d10}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.666	Design diversity = 0.833 Design abundance = 3.25 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.499
HR 5f		n5 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 0.916 Design abundance = 3.2 Matching degree = 0.3 Geometrical difference = 1.0 Sequential difference = 0.399

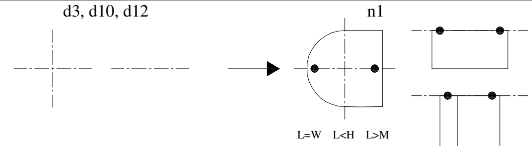
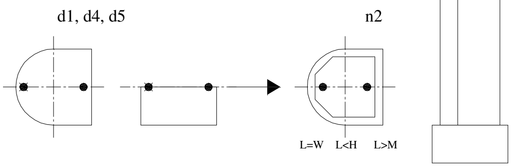
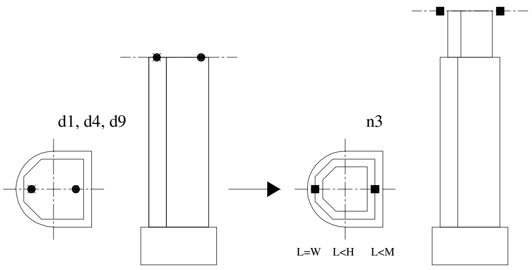
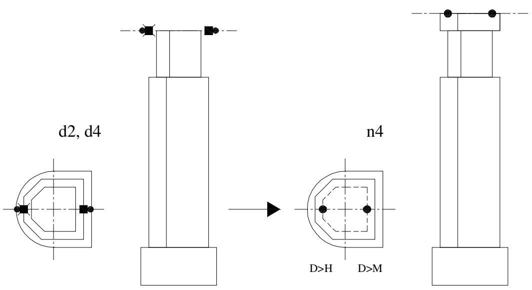
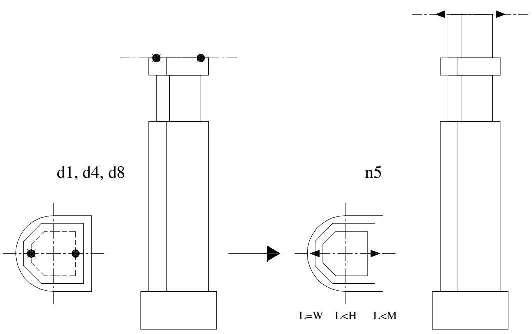
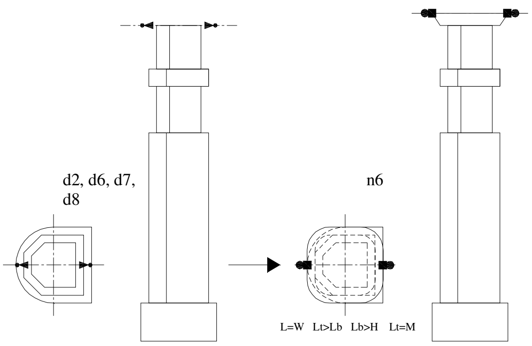


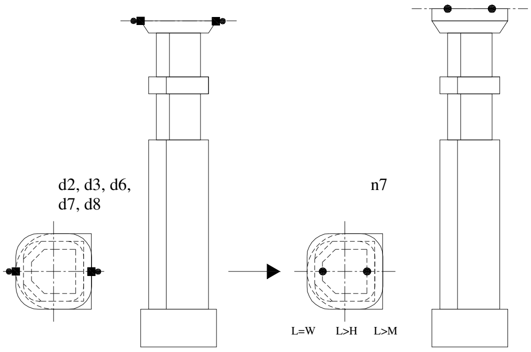
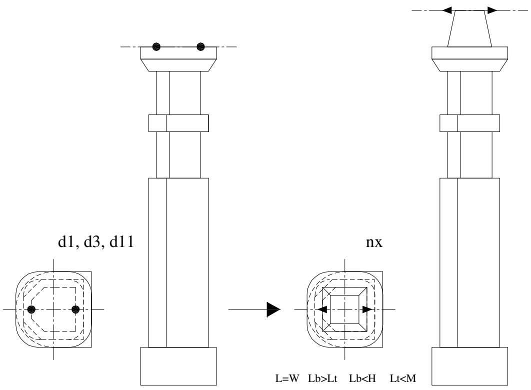
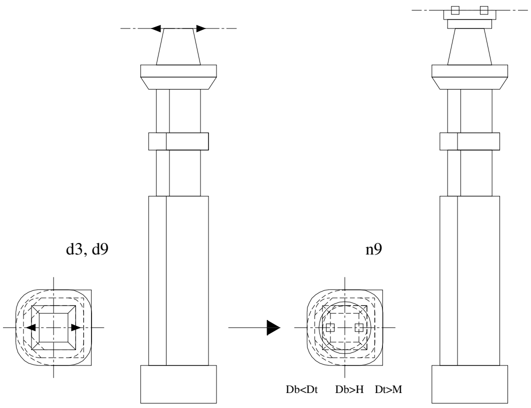
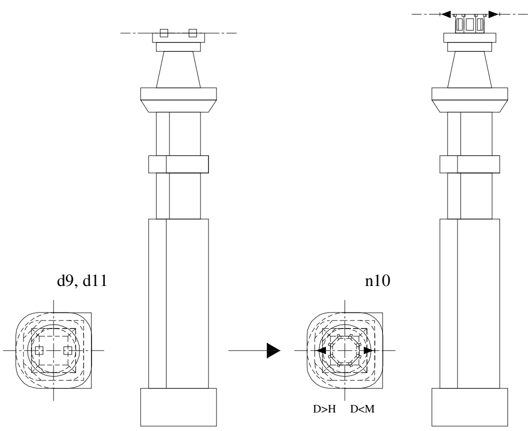
Example 5
Hybrid design - HR
No. of Hybrid Rules = 5

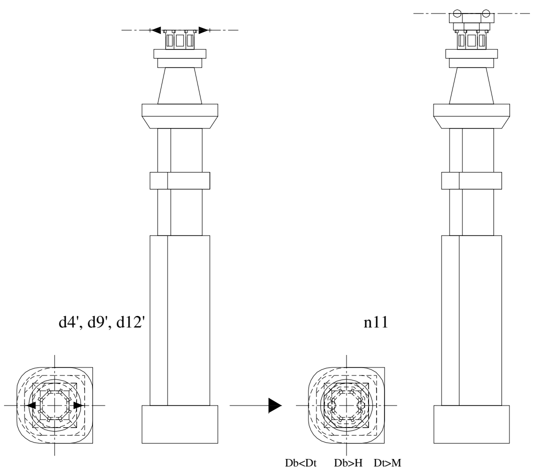
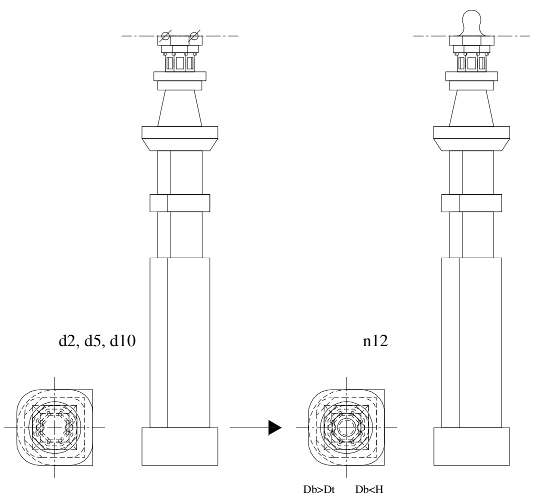
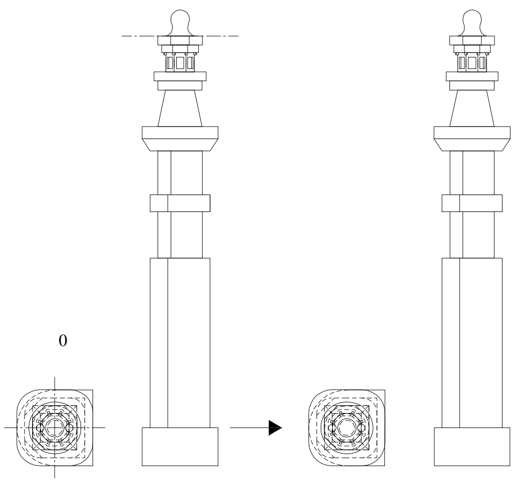
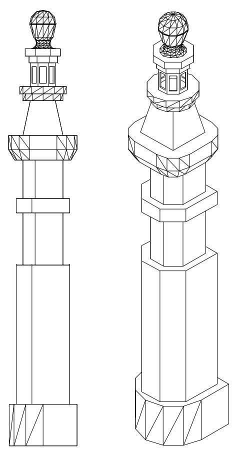
Evaluation Criteria:
Diversity= 0.916
Abundance= 3.2
Matching= 0.3
Geometrical diff.= 1.0
Sequential diff.= 0.399

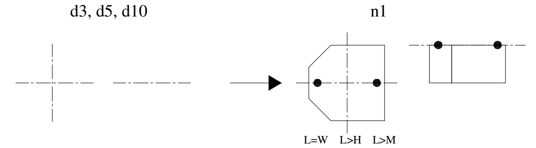
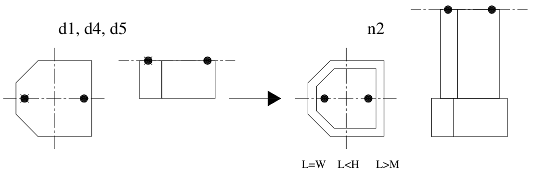
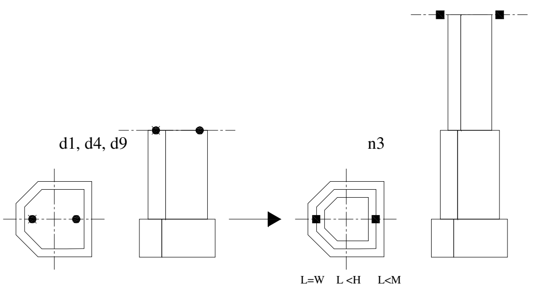
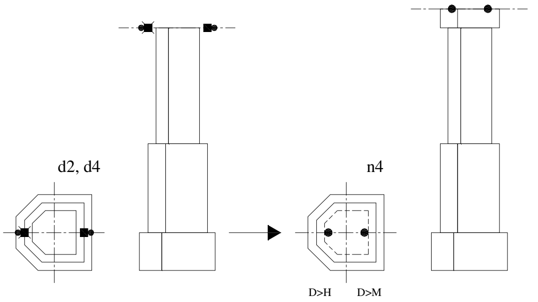
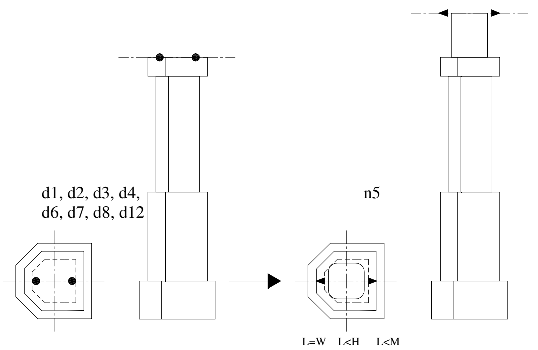
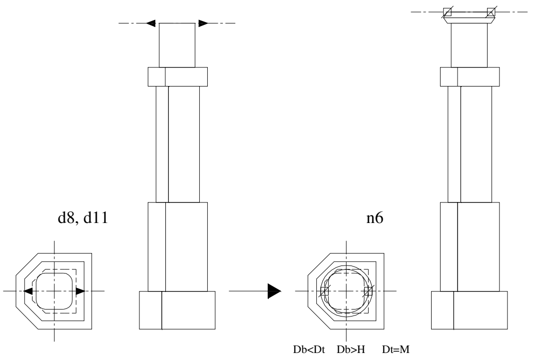
Rule no.	Example 6: Hybrid design derivation - 9 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 3a	d5, d7 n1  Dt<Db Db>H Dt=M	n1 = {n\d5, d7} n1 = {d1, d2, d3, d4, d6, d8, d9, d10, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 4b	d3, d12 n2  D>H D=M	n2 = {n1\d3, 12} n2 = {d1, d2, d4, d6, d8, d9, d10, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 2.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5
HR 22b	d3, d11 n3  Db>Dt Db<H Dt<M	n3 = {n2\d3, d11} n3 = {d1, d2, d4, d6, d8, d9, d10}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.416 Design abundance = 2.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.444
HR 13c	d4, d9, d12 n4  Db<Dt Db>H Dt>M	n4 = {n3\d4, d9, d12} n4 = {d1, d2, d6, d8, d10}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.583 Design abundance = 2.25 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.541
HR 13b	d2, d3, d4, d6, d7, d8, d12 n5  D>, = or <H D<M	n5 = {n4\d2, d3, d4, d6, d7, d8, d12} n5 = {d1, d10} n5 = Ø n5 = n5* n5* = {m\d2, d3, d4, d6, d7, d8, d12} n5* = {d5, d9, d11}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.833 Design abundance = 3.2 Matching degree = 0.3 Geometrical difference = 1.0 Sequential difference = 0.499
HR 19c	d3, d9 nx  Db<Dt Db>H Dt>M	n6 = {n5\d3, d9} n6 = {d1, d10}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.833 Design abundance = 3.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.499

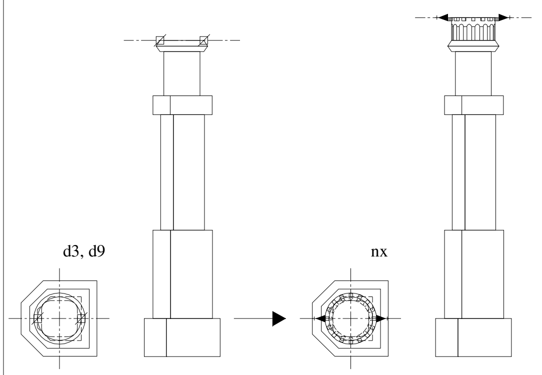
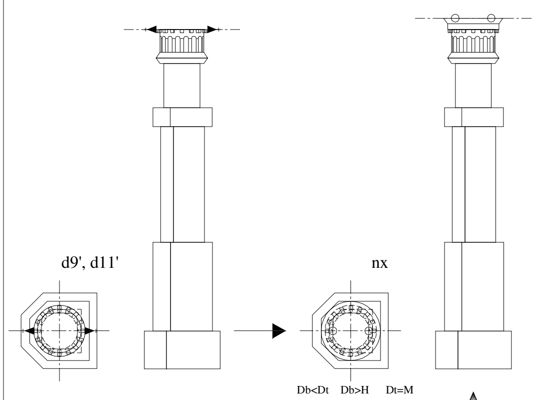
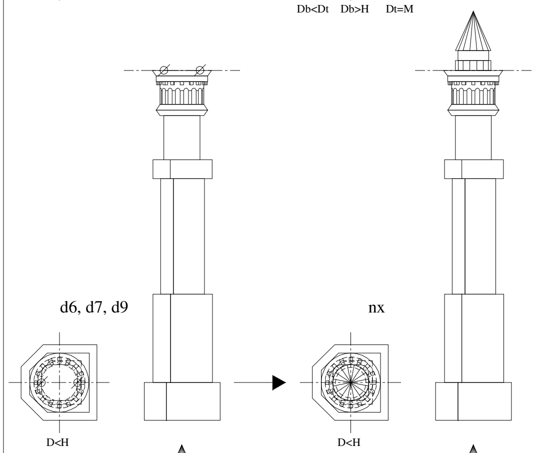
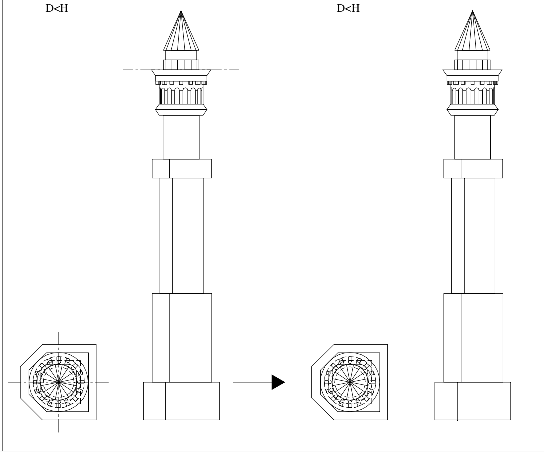
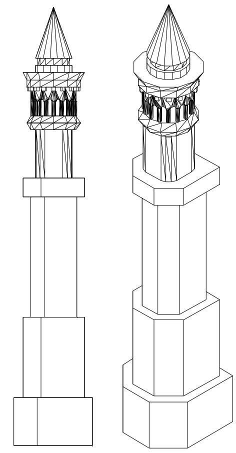
Rule no.	Example 6: Hybrid design derivation - 9 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 18b		n7 = {n6\d1, d4, d6} n7 = {d10'}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity 0.916 Design abundance = 3.0 Matching degree = 0.285 Geometrical difference = 1.0 Sequential difference = 0.499
HR 23b		n8 = {n7\d3, d10} n8 = {Ø} n8 = n8* n8* = {m\d3, d10} n8* = {d1, d2, d5, d8, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.583	Design diversity = 1.0 Design abundance = 2,875 Matching degree = 0.312 Geometrical difference = 1.0 Sequential difference = 0.51
HR 8f		n9 = 0	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 2.888 Matching degree = 0.277 Geometrical difference = 1.0 Sequential difference = 0.453
				**Example 6** **Hybrid design - HR** **No. of Hybrid Rules = 9** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 2.888** **Matching= 0.277** **Geometrical diff.= 1.0** **Sequential diff.= 0.453**

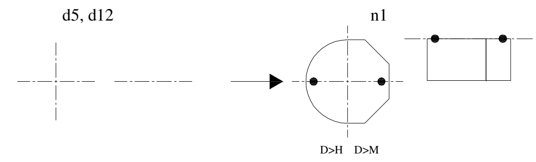
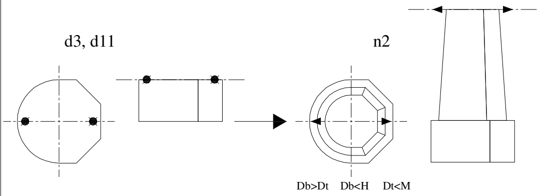
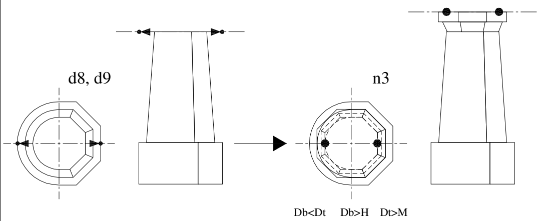
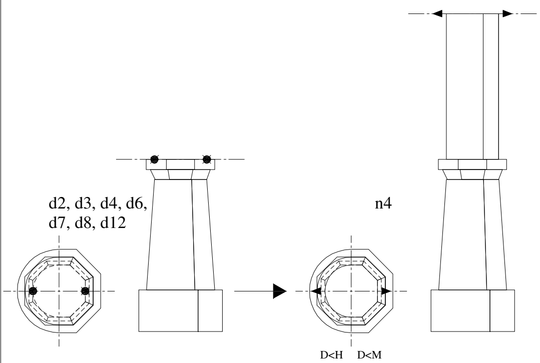
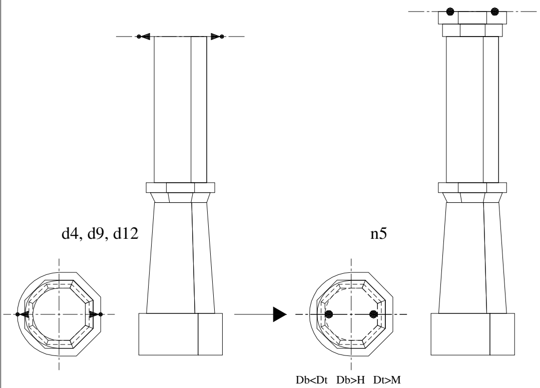
Rule no.	Example 7: Hybrid design derivation - 12 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 10a		n1 = {n\d3, d10, d12} n1 = {d1, d2, d4, d5, d6, d7, d8, d9, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity= 0.25 Design abundance= 3.0 Matching degree= 0.5 Geometrical difference= 1.0 Sequential difference= 0.5
HR 8b		n2 = {n1\d1, d4, d5} n2 = {d2, d6, d7, d8, d9, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 3.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5
HR 9b		n3 = {n2\d1,d4, d9} n3 = {d2, d6, d7, d8, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.583 Design abundance = 3.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.638
HR 3d		n4 = {n3\d2, d4} n4 = {d6, d7, d8, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 2.75 Matching degree = 0.375 Geometrical difference = 1.0 Sequential difference = 0.604
HR 7b		n5 = {n4\d1, d4, d8} n5 = {d6, d7, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.75 Design abundance = 2.8 Matching degree = 0.4 Geometrical difference = 1.0 Sequential difference = 0.549
HR 4c		n6 = {n5\d2, d6, d7, d8} n6 = {d11} n6 = {Ø} n6 = n6* n6* = {m\d2, d6, d7, d8} n6* = {d3, d5, d9, d10, d12}	Rule Prevalence = 0.333 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.916 Design abundance = 3.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.541

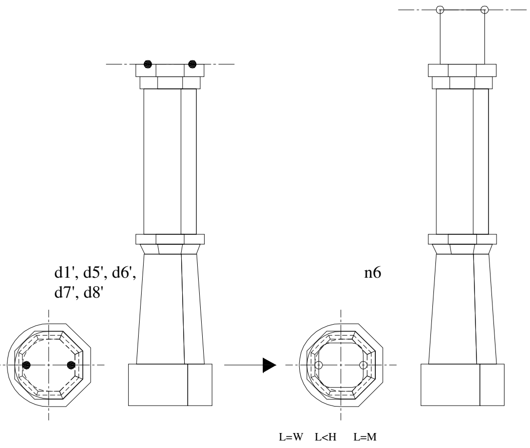
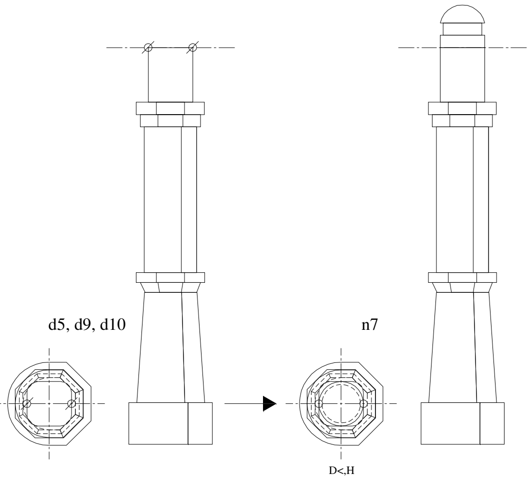
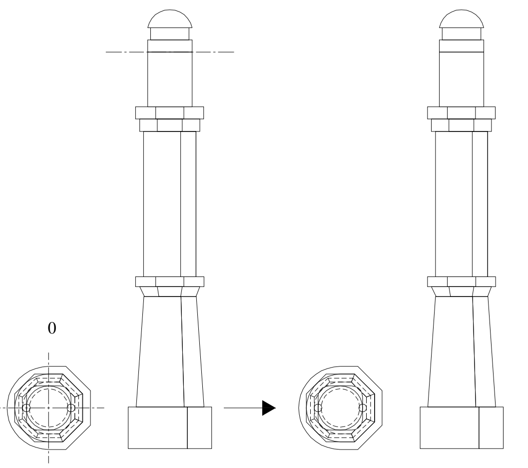
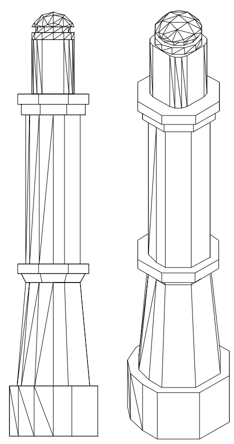
Rule no.	Example 7: Hybrid design derivation - 12 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 2d		n7 = {n6\d2, d3, d6, d7, d8} n7 = {d11}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.916 Design abundance = 3.285 Matching degree = 0.285 Geometrical difference = 1.0 Sequential difference = 0.535
HR 1b		n8 = {n7\d1, d3, d11} n8 = {Ø} n8 = n8* n8* = {m\d1, d3, d11} n8* = {d5, d9, d10, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 1.0 Design abundance = 3.25 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.51
HR 18c		n9 = {n8\d3, d9} n9 = {Ø} n9 = n9* n9* = {m\d3, d9} n9* = {d5, d10, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 1.0 Design abundance = 3.111 Matching degree = 0.222 Geometrical difference = 1.0 Sequential difference = 0.546
HR 12e		n10 = {n9\d9, d11} n10 = {Ø} n10 = n10* n10* = {m\d9, d11} n10* = {d5', d10', d12'}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.75	Design diversity = 1.0 Design abundance = 3.0 Matching degree = 0.2 Geometrical difference = 1.0 Sequential difference = 0.566

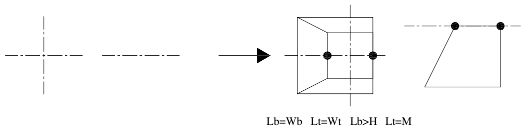
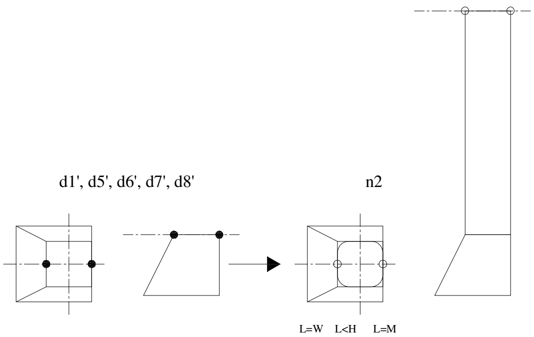
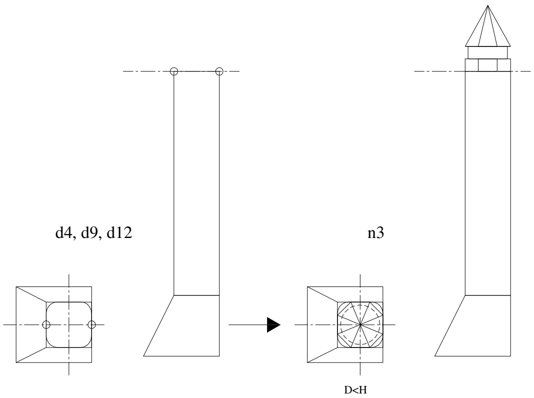
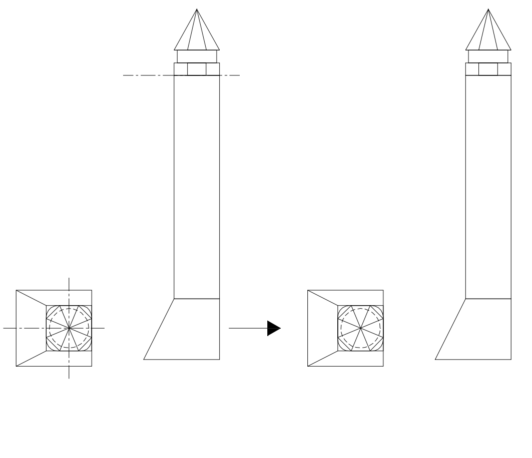
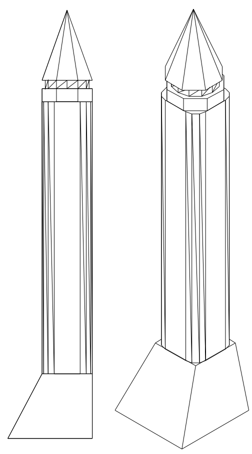
Rule no.	Example 7: Hybrid design derivation - 12 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 14c		n11 = {n10\d4, d9, d12} n11 = {Ø} n11 = n11* n11* = {m\d4, d9, d12} n11* = {d5, d10}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.666	Design diversity = 1.0 Design abundance = 3.0 Matching degree = 0.227 Geometrical difference = 1.0 Sequential difference = 0.575
HR 2f		n12 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 0.208 Geometrical difference = 1.0 Sequential difference = 0.527
			**Example 7** **Hybrid design - HR** **No. of Hybrid Rules =12**	**Evaluation Criteria:** **Diversity= 1.0** **Abundance= 3.0** **Matching= 0.208** **Geometrical diff.= 1.0** **Sequential diff.= 0.527**

Rule no.	Example 8: Hybrid design derivation - 9 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 7a		n1 = {n\d3, d5, d10} n1 = {d1, d2, d4, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 8b		n2 = {n1\d1, d4, d5} n2 = {d2, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.416 Design abundance = 3.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 9b		n3 = {n2\d1, d9} n3 = {d2, d6, d7, d8, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.5 Design abundance = 3.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.638
HR 3d		n4 = {n3\d2, d4} n4 = {d6, d7, d8, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 2.75 Matching degree = 0.375 Geometrical difference = 1.0 Sequential difference = 0.604
HR 17b		n5 = {n4\d1, d2, d3, d4, d6, d7, d8, d12} n5 = {d11}	Rule Prevalence = 0.666 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.916 Design abundance = 3.8 Matching degree = 0.4 Geometrical difference = 1.0 Sequential difference = 0.549
HR 21c		n6 = {n5\d8, d11} n6 = {Ø} n6 = n6* n6* = {m\d8, d11} n6* = {d5, d6, d7, d9, d10, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 1.0 Design abundance = 3.5 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.597

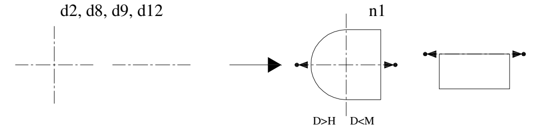
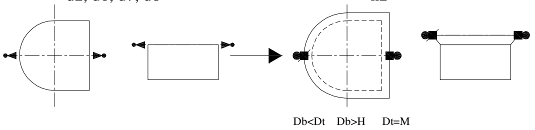
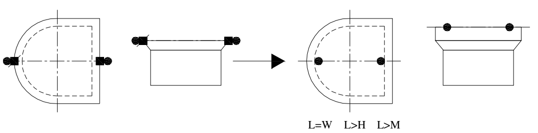
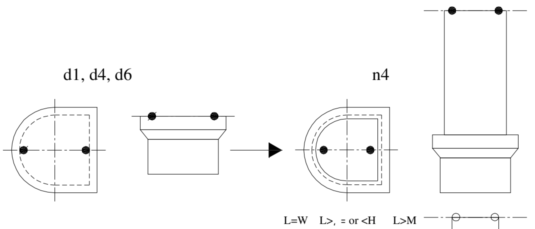
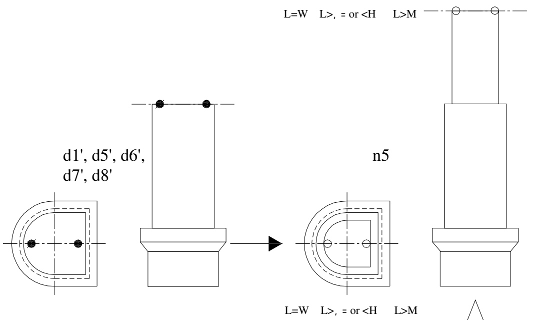
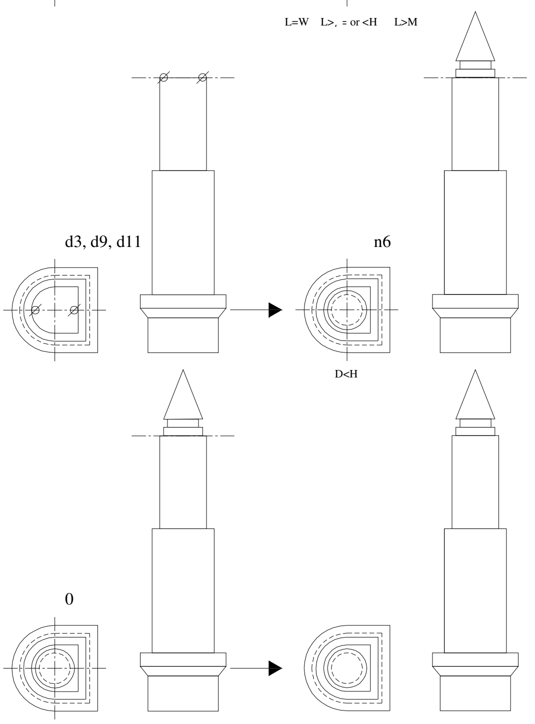
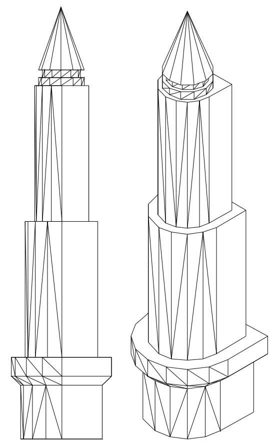
Rule no.	Example 8: Hybrid design derivation - 9 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 6e		n7 = {n6\d3, d9} n7 = {Ø} n7 = n7* n7* = {m\d8, d11} n7* = {d5', d6', d7', d10', d11', d12'}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.75	Design diversity = 1.0 Design abundance = 3.285 Matching degree = 0.285 Geometrical difference = 1.0 Sequential difference = 0.618
HR 17c		n8 = {n7\ d9, d11} n8 = {Ø} n8 = n8* n8* = {m\d8, d11} n8* = {d5, d6, d7, d10, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.666	Design diversity = 1.0 Design abundance = 3.125 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.624
HR 3f		n9 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 3.111 Matching degree = 0.222 Geometrical difference = 1.0 Sequential difference = 0.555
			**Example 8** **Hybrid design - HR** **No. of Hybrid Rules = 9** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 3.111** **Matching= 0.222** **Geometrical diff.= 1.0** **Sequential diff.= 0.555**	

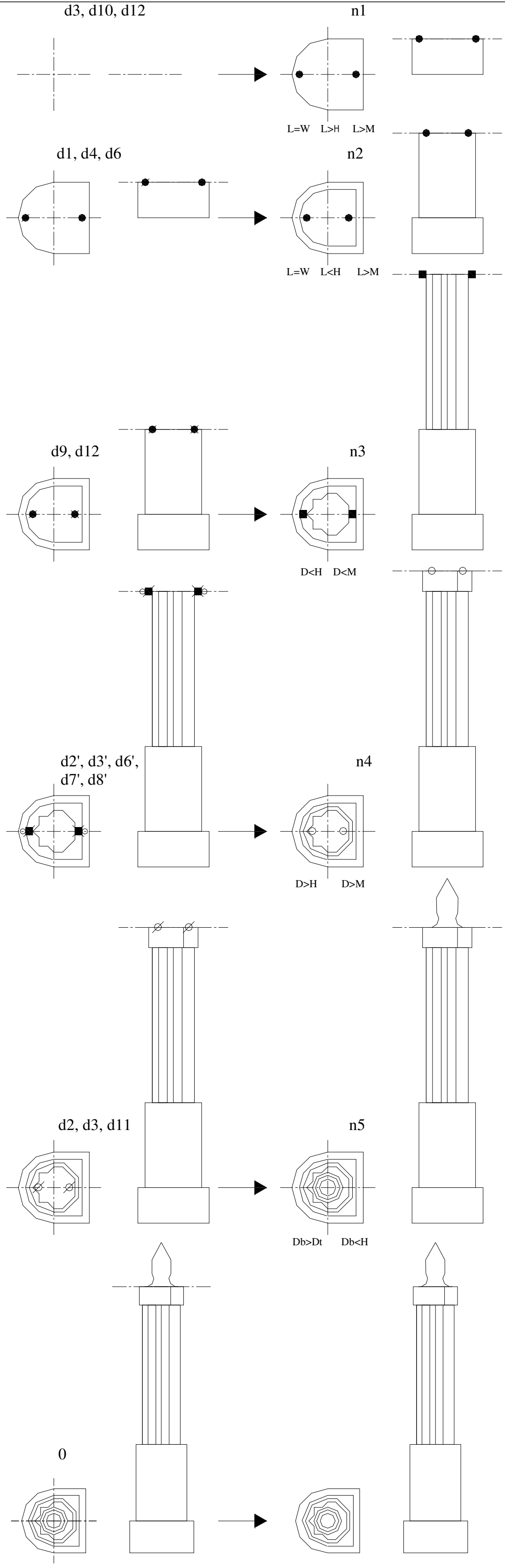
Rule no.	Example 9: Hybrid design derivation - 7 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 11a		n1 = {n\d5, d12} n1 = {d1, d2, d3, d4, d6, d7, d8, d9, d10, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 22b		n2 = {n1\d3, d11} n2 = {d1, d2, d4, d6, d7, d8, d9, d10}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.333 Design abundance = 2.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.416
HR 12c		n3 = {n2\d8, d9} n3 = {d1, d2, d4, d6, d7, d10}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.5 Design abundance = 2.0 Matching degree = 0.166 Geometrical difference = 1.0 Sequential difference = 0.555
HR 13b		n4 = {n3\d2, d3, d4, d6, d7, d8, d12} n4 = {d1, d10} n4 = {Ø} n4 = n4* n4* ={m\d2, d3, d4, d6, d7, d8, d12} n4* = {d5, d9, d11}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.833 Design abundance = 3.25 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.499
HR 13c		n5 = {n4\d4, d9, d12} n5 = {d1', d10'}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.833 Design abundance = 3.2 Matching degree = 0.3 Geometrical difference = 1.0 Sequential difference = 0.566

Rule no.	Example 9: Hybrid design derivation - 7 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 19b		n6 = {n5\d1, d5, d6, d7, d8} n6 = {d10}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.583	Design diversity = 0.916 Design abundance = 3.5 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.569
HR 6f		nx = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 3.428 Matching degree = 0.214 Geometrical difference = 1.0 Sequential difference = 0.487
		 **Example 9** **Hybrid design - HR** **No. of Hybrid Rules = 7** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 3.428** **Matching= 0.214** **Geometrical diff.= 1.0** **Sequential diff.= 0.487**		

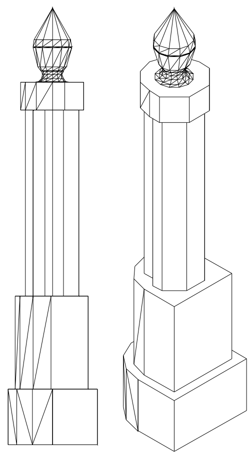
<table>
<tr><th>Rule no.</th><th>Example 10: Hybrid design derivation - 3 Hybrid rules</th><th>User guide grammar for a hybrid design</th><th>Evaluation of the rule</th><th>Evaluation of the design</th></tr>
<tr><td>HR 12a</td><td>d3, d7, d10 n1
</td><td>n1 = {n\d3, d7, d10}
n1 = {d1, d2, d4, d5, d6, d8, d9, d11, d12}</td><td>Rule Prevalence = 0.25
Geometrical difference = 1.0
Sequential difference = 0.5</td><td>Design diversity = 0.25
Design abundance = 3.0
Matching degree = 0.5
Geometrical difference = 1.0
Sequential difference = 0.5</td></tr>
<tr><td>HR 19b</td><td>d1', d5', d6', d7', d8' n2
</td><td>n2 = {n1\d1, d5, d6, d7, d8}
n2 = {d2, d4, d9, d11, d12}</td><td>Rule Prevalence = 0.416
Geometrical difference = 1.0
Sequential difference = 0.583</td><td>Design diversity = 0.583
Design abundance = 4.0
Matching degree = 0.5
Geometrical difference = 1.0
Sequential difference = 0.541</td></tr>
<tr><td>HR 7f</td><td>d4, d9, d12 n3
</td><td>n3 = 0</td><td>Rule Prevalence = 0.25
Geometrical difference = 1.0
Sequential difference = 0.0</td><td>Design diversity = 0.833
Design abundance = 3.666
Matching degree = 0.333
Geometrical difference = 1.0
Sequential difference = 0.361</td></tr>
<tr><td></td><td></td><td colspan="3">
Example 10
Hybrid design - HR
No. of Hybrid Rules = 3

Evaluation Criteria:
Diversity= 0.833
Abundance= 2.666
Matching= 0.333
Geometrical diff.= 1.0
Sequential diff.= 0.361</td></tr>
</table>

Rule no.	Example 11: Hybrid design derivation - 6 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 9a		n1 = {n\d2, d8, d9, d12} n1 = {d1, d3, d4, d5, d6, d7, d10, d11}	Rule Prevalence = 0.333 Geometrical difference = 1.0 Sequential difference = 0.75	Design diversity = 0.333 Design abundance = 4.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.75
HR 3c		n2 = {n1\d2, d6, d7, d8} n2 = {d1, d3, d4, d5, d10, d11}	Rule Prevalence = 0.333 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 4.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.625
HR 1d		n3 = {n2\d2, d3, d6, d7, d8} n3 = {d1, d4, d5, d10, d11}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 4.333 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.583
HR 11b		n4 = {n3\d1, d4, d6} n4 = {d3', d5', d10', d11'}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 4.0 Matching degree = 0.375 Geometrical difference = 1.0 Sequential difference = 0.562
HR 12b		n5 = {n4\d1, d5, d6, d7, d8} n5 = {d3, d10, d11}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.583	Design diversity = 0.75 Design abundance = 4.2 Matching degree = 0.4 Geometrical difference = 1.0 Sequential difference = 0.566
HR 4f		n6 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 0.916 Design abundance = 4.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.472
			**Example 11** **Hybrid design - HR** **No. of Hybrid Rules = 6** **Evaluation Criteria:** **Diversity= 0.916** **Abundance= 4.0** **Matching= 0.333** **Geometrical diff.= 1.0** **Sequential diff.= 0.472**	



Rule no.	Example 12: Hybrid design derivation - 5 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 10a		n1 = {n\d3, d10, d12} n1 = {d1, d2, d4, d5, d6, d7, d8, d9, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 11b		n2 = {n1\d1, d4, d6} n2 = {d2, d5, d7, d8, d9, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 3.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5
HR 20b		n3 = {n2\d9, d12} n3 = {d2, d5, d7, d8, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.583 Design abundance = 2.666 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.638
HR 11d		n4 = {n3\d2, d3, d6, d7, d8} n4 = {d5, d11}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.833 Design abundance = 3.25 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.708
HR 1f		n5 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 0.916 Design abundance = 3.2 Matching degree = 0.3 Geometrical difference = 1.0 Sequential difference = 0.566



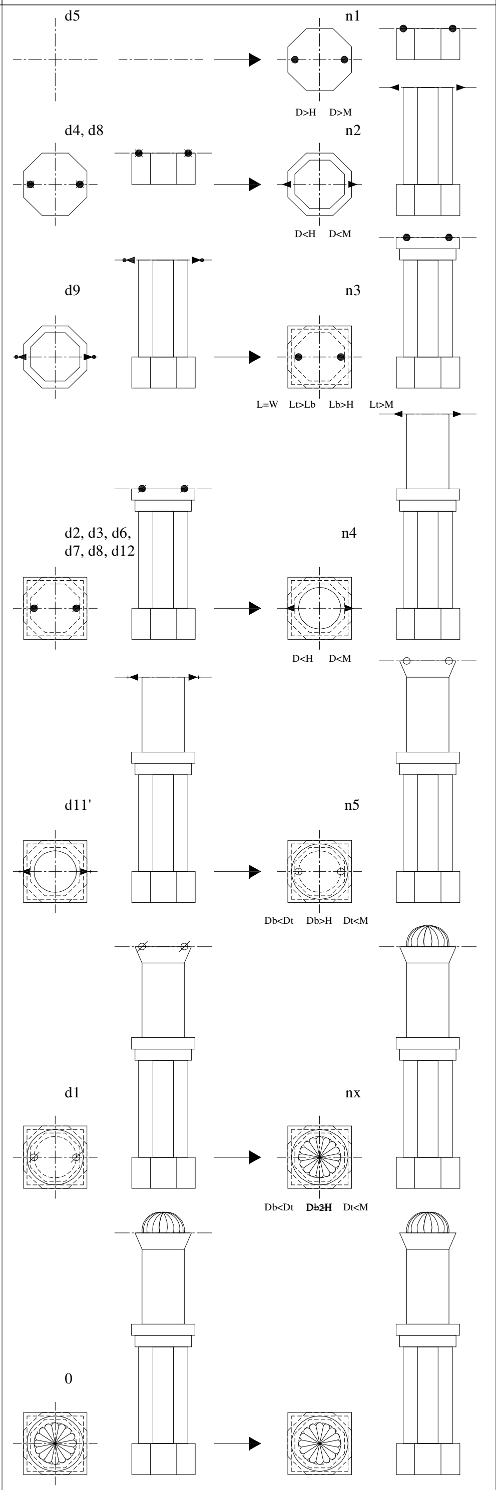
Example 12
Hybrid design - HR
No. of Hybrid Rules = 5

Evaluation Criteria:
Diversity= 0.916
Abundance= 3.2
Matching= 0.3
Geometrical diff.= 1.0
Sequential diff.= 0.566

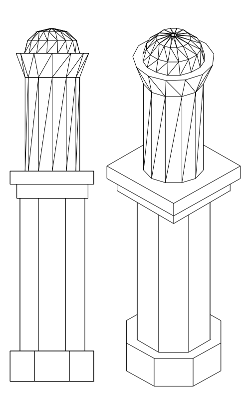
Appendix–C: Pairs of hybrid designs

C-1 Five pairs of hybrid designs composed of original rules (OR)

Pair 1: Hybrid design derivation - 6 Original rules

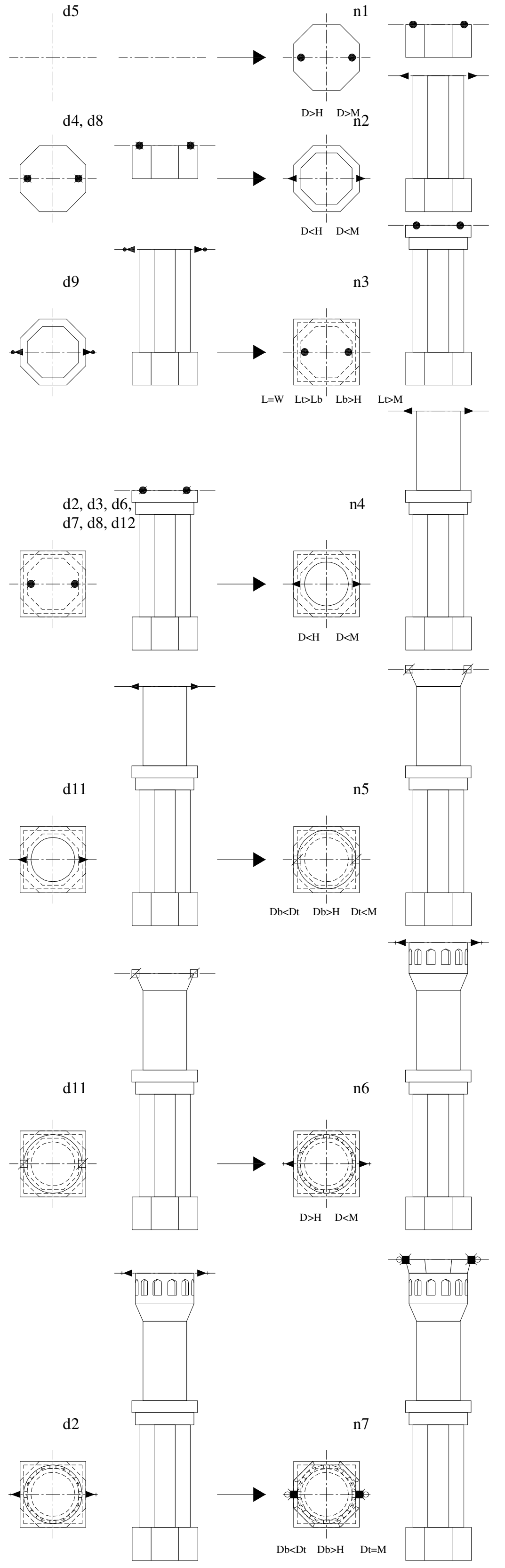


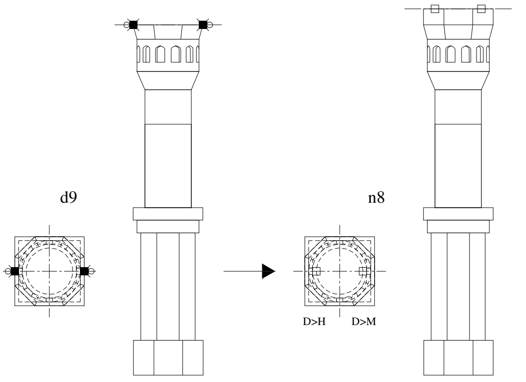
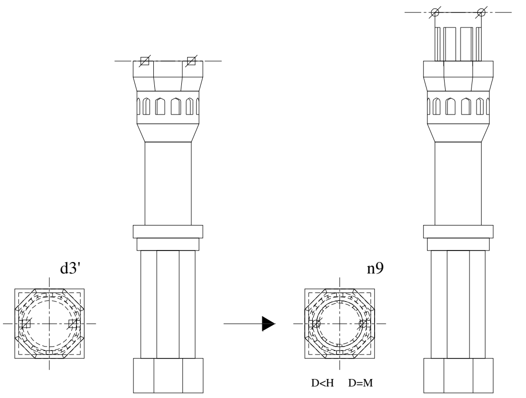
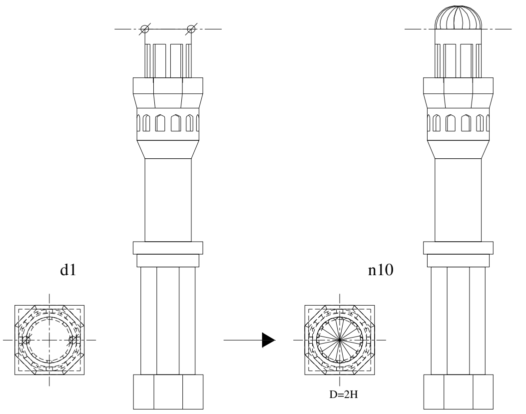
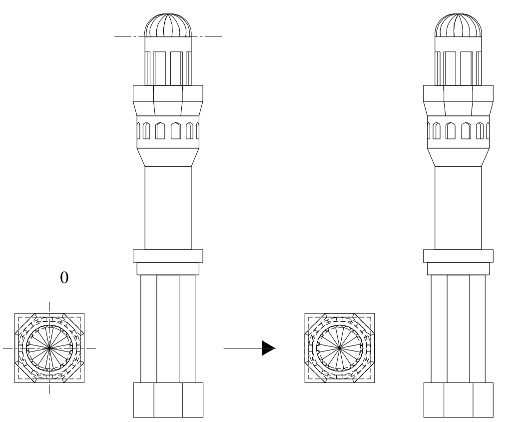
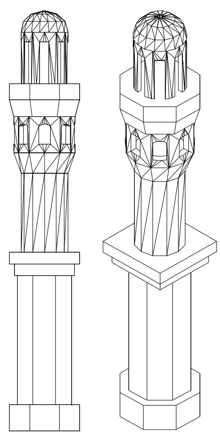
Rule no.	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 6a	n1 = {n\d5} n1 = {d1, d2, d3, d4, d6, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 6b	n2 = {n1\d4, d8} n2 = {d1, d2, d3, d6, d7, d9, d10, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.25 Design abundance = 1.5 Matching degree = 0.5 Geometrical difference = 0.749 Sequential difference = 0.416
OR 11c	n3 = {n2\d9} n3 = {d1, d2, d3, d6, d7, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.833	Design diversity = 0.333 Design abundance = 1.333 Matching degree = 0.333 Geometrical difference = 0.805 Sequential difference = 0.555
OR 3b	n4 = {n3\d2, d3, d6, d7, d8, d12} n4 = {d1', d10', d11'}	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.75 Design abundance = 2.5 Matching degree = 0.5 Geometrical difference = 0.707 Sequential difference = 0.499
OR 15c	n5 = {n4\d11} n5 = {d1, d10}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.666	Design diversity = 0.833 Design abundance = 2.2 Matching degree = 0.4 Geometrical difference = 0.682 Sequential difference = 0.533
OR 1f	n6 = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0	Design diversity = 0.916 Design abundance = 2.0 Matching degree = 0.333 Geometrical difference = 0.721 Sequential difference = 0.444

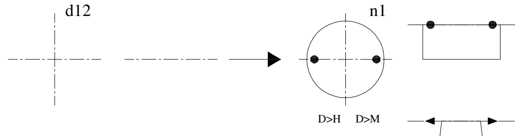
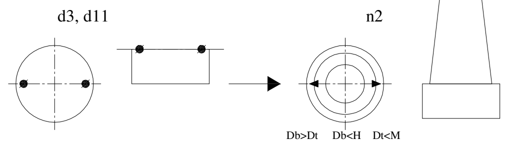
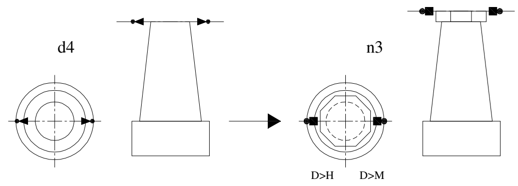
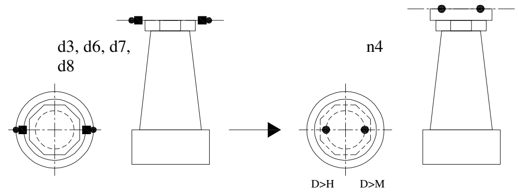
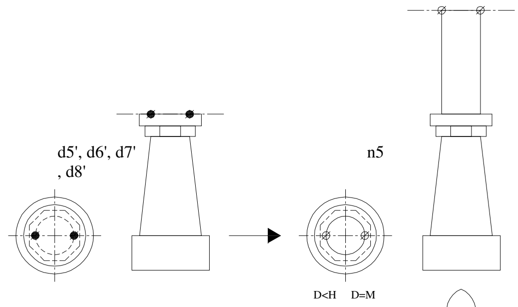
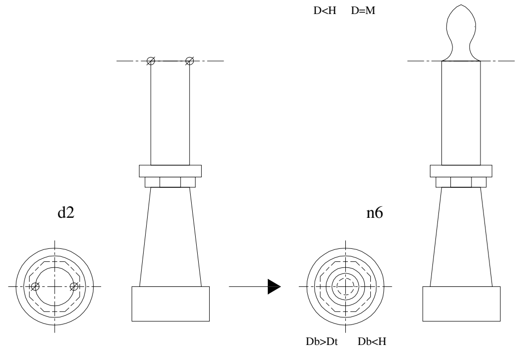
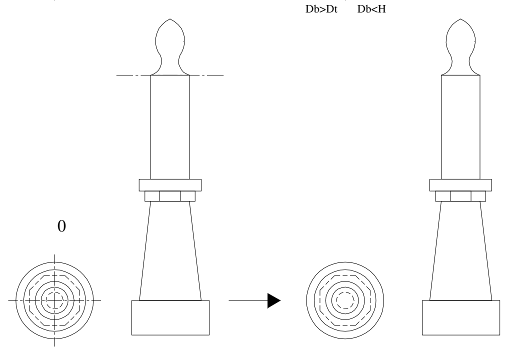
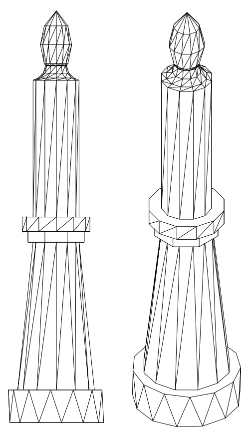


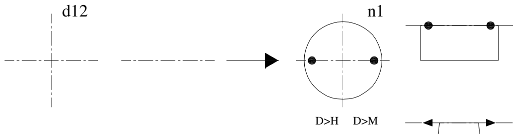
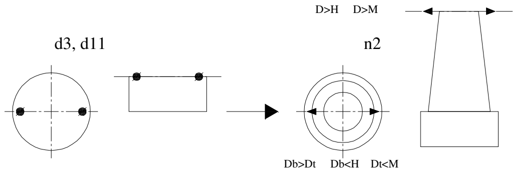
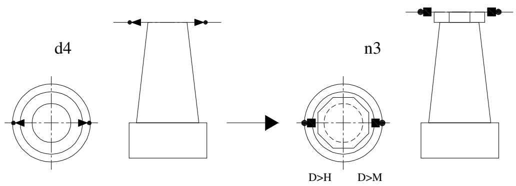
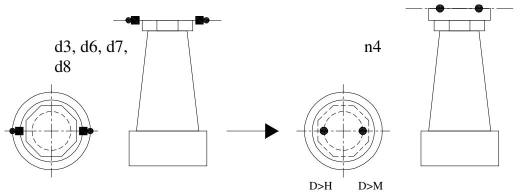
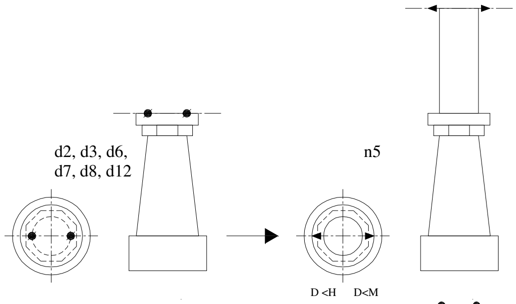
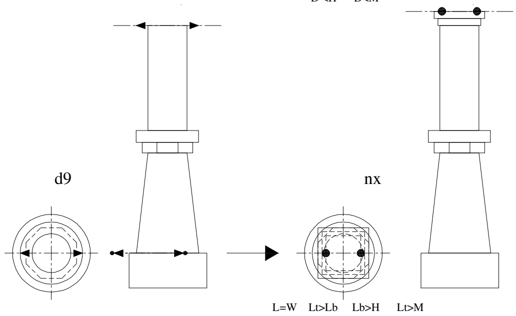
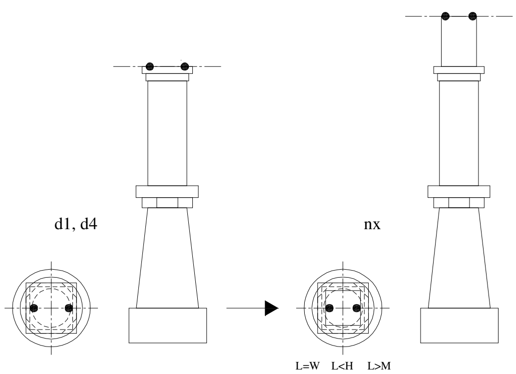
Pair 1
Hybrid design - OR
No. of Original Rules =6

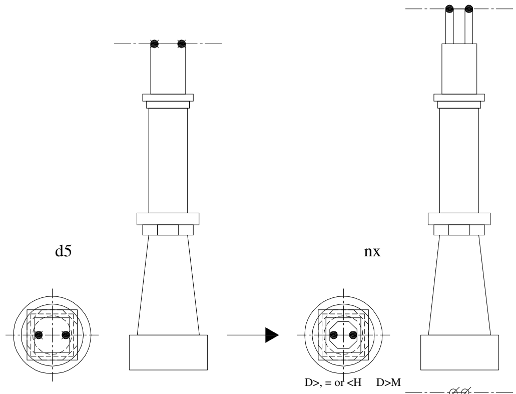
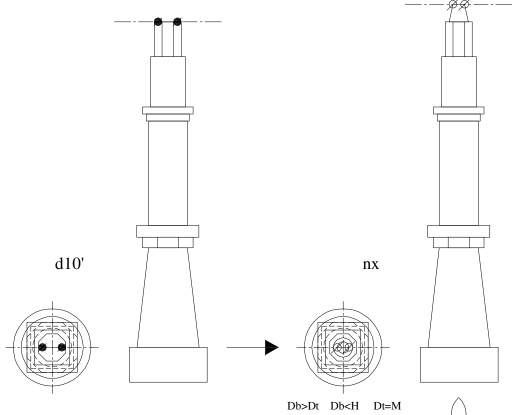
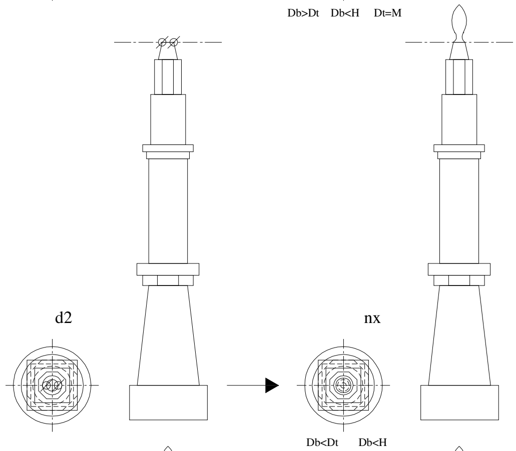
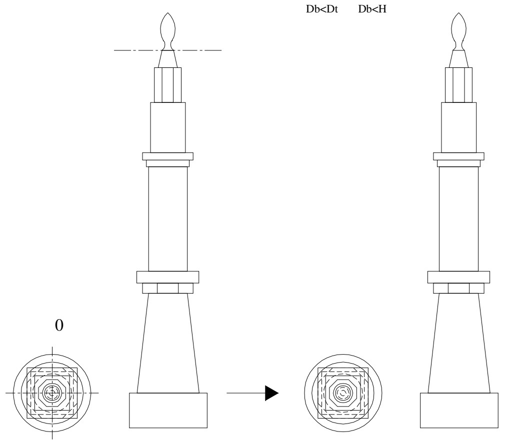
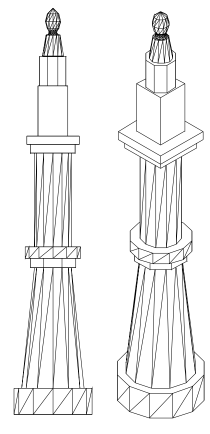
Evaluation Criteria:
Diversity= 0.916
Abundance= 2.0
Matching= 0.333
Geometrical diff.= 0.721
Sequential diff.= 0.444

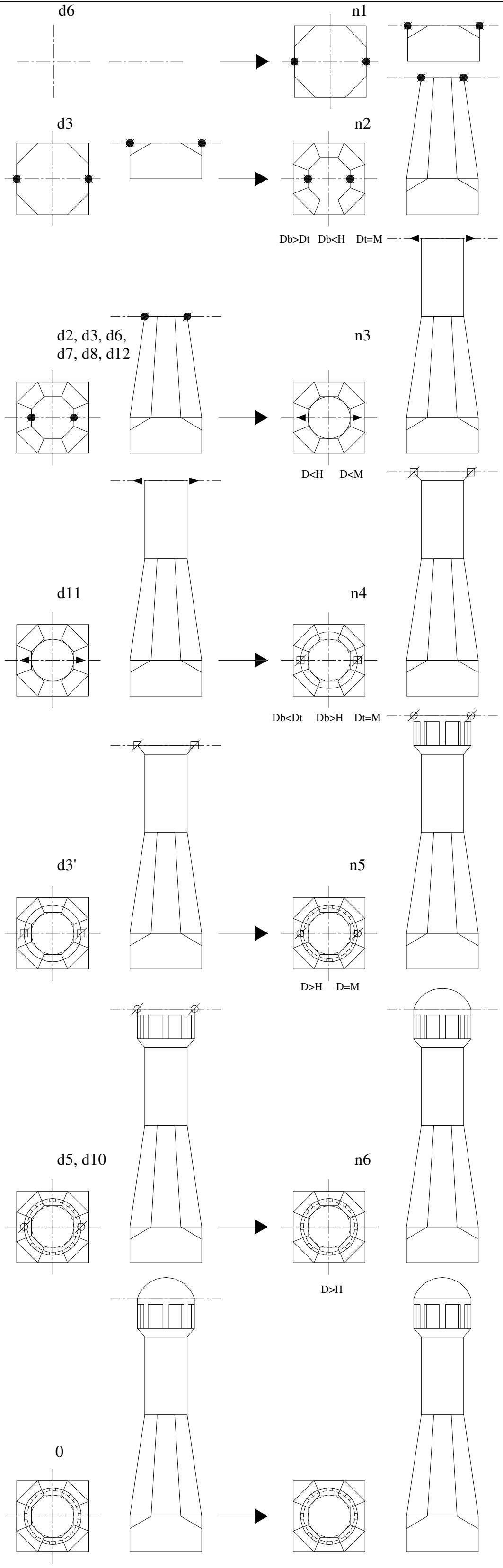
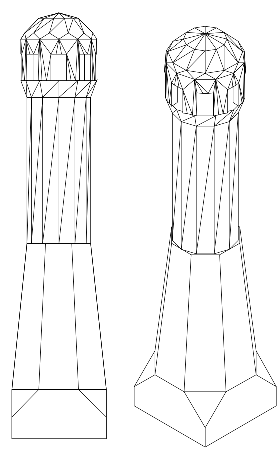
Rule no.	Pair 1: Hybrid design derivation - 10 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 6a	 d5 n1 D>H D>M	n1 = {n\d5} n1 = {d1, d2, d3, d4, d6, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 6b	d4, d8 n2 D<H D<M	n2 = {n1\d4, d8} n2 = {d1, d2, d3, d6, d7, d9, d10, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.25 Design abundance = 1.5 Matching degree = 0.5 Geometrical difference = 0.749 Sequential difference = 0.416
OR 11c	d9 n3 L=W Lt>Lb Lb>H Lt>M	n3 = {n2\d9} n3 = {d1, d2, d3, d6, d7, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.833	Design diversity = 0.333 Design abundance = 1.333 Matching degree = 0.333 Geometrical difference = 0.805 Sequential difference = 0.555
OR 3b	d2, d3, d6, d7, d8, d12 n4 D<H D<M	n4 = {n3\d2, d3, d6, d7, d8, d12} n4 = {d1, d10, d11}	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.75 Design abundance = 2.5 Matching degree = 0.5 Geometrical difference = 0.707 Sequential difference = 0.499
OR 14c	d11 n5 Db<Dt Db>H Dt<M	n5 = {n4\d11} n5 = {d1, d10} n5 = {Ø} n5 = n5* n5* = {m\d11} n5* = {d2, d3, d4, d5, d6, d7, d9, d12} n5* = {Ø} n5* = n5** n5** = {n\(n5+n5*)} n5** = {d8, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.833	Design diversity = 0.833 Design abundance = 2.2 Matching degree = 0.4 Geometrical difference = 0.682 Sequential difference = 0.566
OR 4e	d11 n6 D>H D<M	n6 = {n5\d11} n6 = {d1, d10} n6 = {Ø} n6 = n6* n6* = {m\d11} n6* = {d2, d3, d4, d5, d6, d7, d9, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.75	Design diversity = 0.833 Design abundance = 2.0 Matching degree = 0.333 Geometrical difference = 0.721 Sequential difference = 0.597
OR 7c	d2 n7 Db<Dt Db>H Dt=M	n7 = {n6\d2} n7 = {d1, d10} n7 = {Ø} n7 = n7* n7* = {m\d2} n7* = {d3, d4, d5, d6, d7, d9, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.833 Design abundance = 1.857 Matching degree = 0.285 Geometrical difference = 0.737 Sequential difference = 0.583

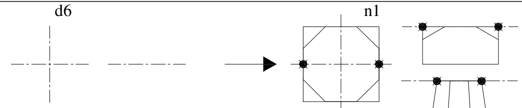
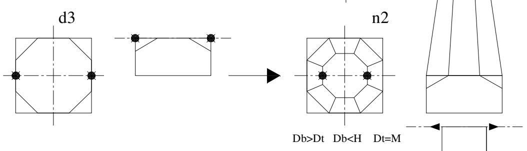
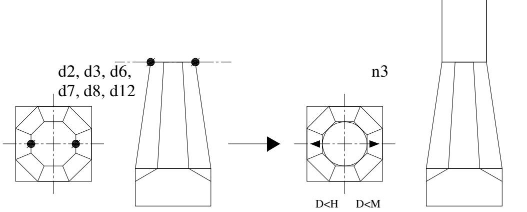
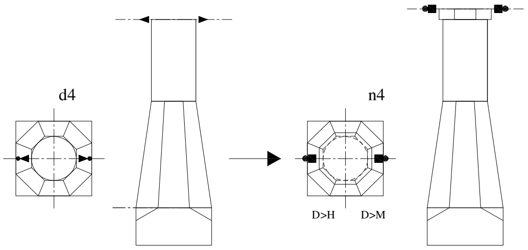
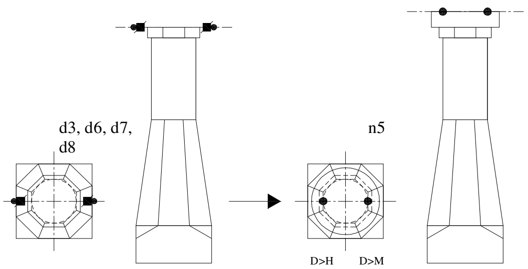
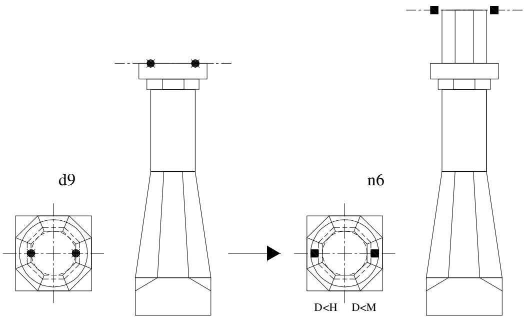
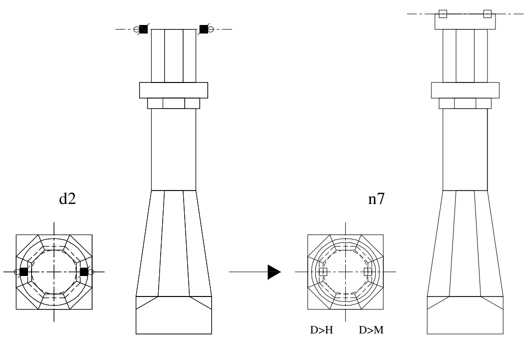
Rule no.	Pair 1: Hybrid design derivation - 10 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 6d		n8 = {n7\d9} n8 = {d1', d10'} n8 = {Ø} n8 = n8* n8* = {m\d9} n8* = {d3', d4', d5', d6', d7', d12'}	Rule Prevalence = 0.083 Geometrical difference = 0.75 Sequential difference = 0.833	Design diversity = 0.833 Design abundance = 1.75 Matching degree = 0.25 Geometrical difference = 0.739 Sequential difference = 0.635
OR 1e		n9 = {n8\d3} n9 = {d1, d10}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.916	Design diversity = 0.833 Design abundance = 1.666 Matching degree = 0.222 Geometrical difference = 0.758 Sequential difference = 0.666
OR 1f		n10 = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0	Design diversity = 0.916 Design abundance = 1.6 Matching degree = 0.2 Geometrical difference = 0.774 Sequential difference = 0.599
			**Pair 1** **Hybrid design - OR** **No. of Original Rules = 10**	**Evaluation Criteria:** **Diversity= 0.916** **Abundance= 1.6** **Matching= 0.2** **Geometrical diff.= 0.774** **Sequential diff.= 0.599**

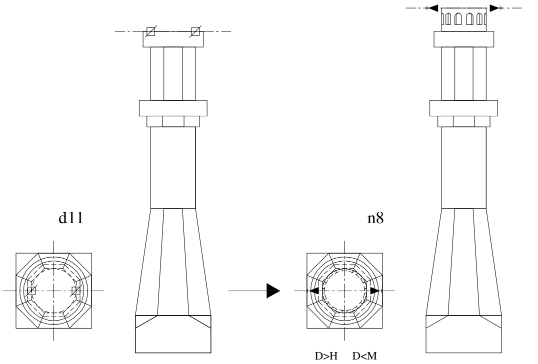
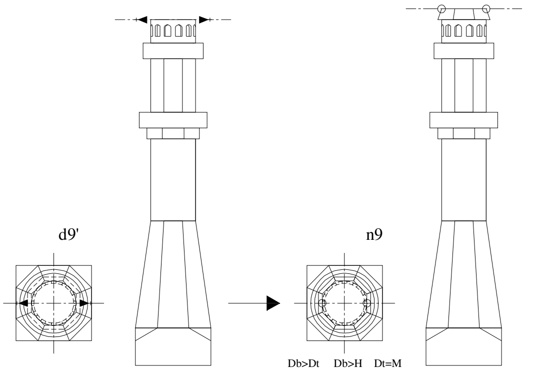
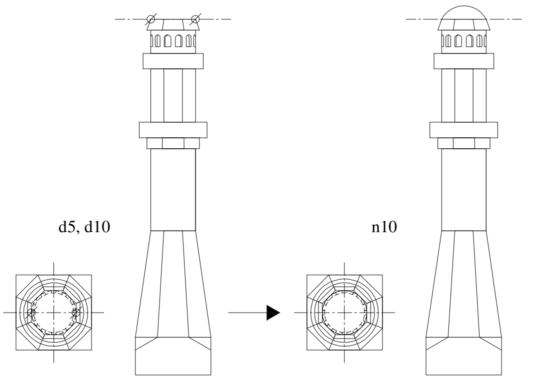
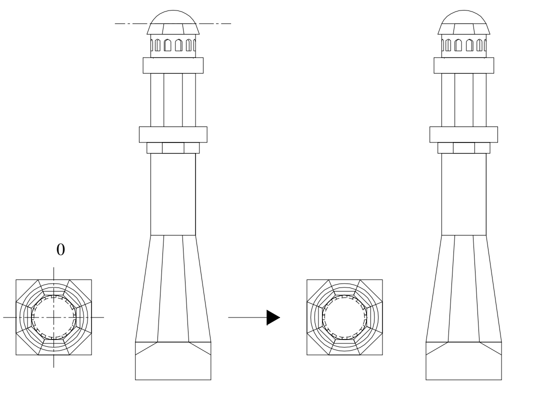
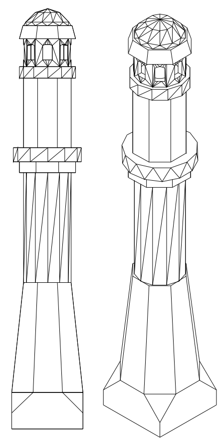
Rule no.	Pair 2: Hybrid design derivation - 6 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 7a		n1 = {n\d12} n1 = {d1, d2, d3, d4, d5, d6, d7, d8, d9, d10, d11}	Rule prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 5b		n2 = {n1\d3, d11} n2 = {d1, d2, d4, d5, d6, d7, d8, d9, d10}	Rule Prevalence = 0.166 Geometrical difference = 0.75 Sequential difference = 0.333	Design diversity = 0.25 Design abundance = 1.5 Matching degree = 0.5 Geometrical difference = 0.833 Sequential difference = 0.416
OR 8c		n3={n2\d4} n3={d1, d2, d5, d6, d7, d8, d9, d10}	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 1.333 Matching degree = 0.333 Geometrical difference = 0.833 Sequential difference = 0.444
OR 4d		n4 = {n3\d3, d6, d7, d8} n4 = {d1, d2, d5, d9, d10}	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 0.77 Sequential difference = 0.458
OR 8b		n5 = {n4\d5, d6, d7, d8} n5 = {d1, d2, d9, d10}	Rule Prevalence = 0.333 Geometrical difference = 0.416 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 2.4 Matching degree = 0.4 Geometrical difference = 0.699 Sequential difference = 0.466
OR 2f		n6 = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0	Design diversity = 0.75 Design abundance = 2.166 Matching degree = 0.333 Geometrical difference = 0.735 Sequential difference = 0.388
			**Pair 2** **Hybrid design - OR** **No. of Original Rules =6** **Evaluation Criteria:** **Diversity= 0.75** **Abundance= 2.166** **Matching= 0.333** **Geometrical diff.= 0.735** **Sequential diff.= 0.388**	

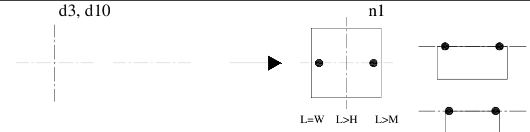
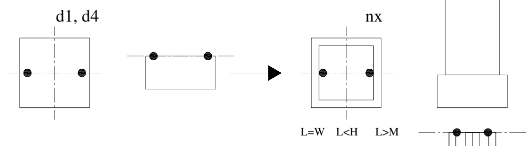
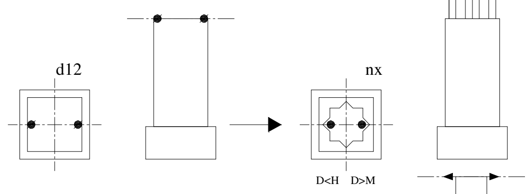
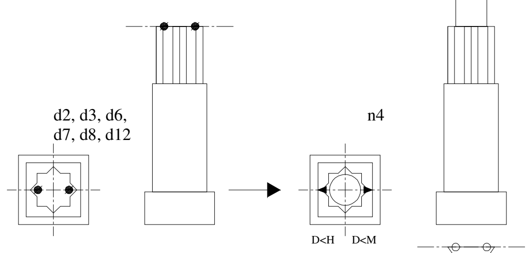
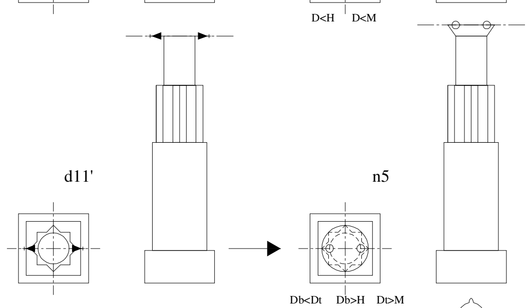
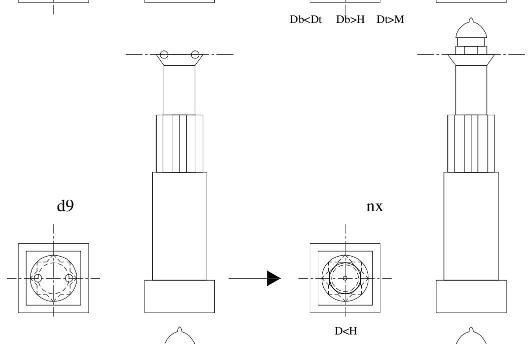
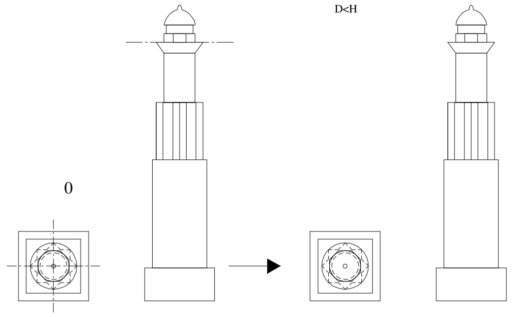
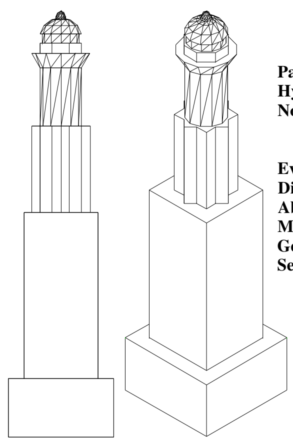
Rule no.	Pair 2: Hybrid design derivation - 10 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 7a		n1 = {n\d12} n1 = {d1, d2, d3, d4, d5, d6, d7, d8, d9, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 5b		n2 = {n1\d3, d11} n2 = {d1, d2, d4, d5, d6, d7, d8, d9, d10}	Rule Prevalence = 0.166 Geometrical difference = 0.75 Sequential difference = 0.333	Design diversity = 0.25 Design abundance = 1.5 Matching degree = 0.5 Geometrical difference = 0.833 Sequential difference = 0.416
OR 8c		n3 = {n2\d4} n3 = {d1, d2, d5, d6, d7, d8, d9, d10}	Rule Prevalence=0.083 Geometrical difference=0.833 Sequential difference=0.5	Design diversity = 0.333 Design abundance = 1.333 Matching degree = 0.333 Geometrical difference = 0.833 Sequential difference = 0.444
OR 4d		n4 = {n3\d3, d6, d7, d8} n4 = {d1, d2, d5, d9, d10}	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 0.77 Sequential difference = 0.458
OR 3b		n5 = {n4\d2, d3, d6, d7, d8, d12} n5 = {d1, d5, d9, d10}	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.666 Design abundance = 2.8 Matching degree = 0.6 Geometrical difference = 0.699 Sequential difference = 0.433
OR 11c		n6 = {n5\d9} n6 = {d1, d5, d10}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.833	Design diversity = 0.75 Design abundance = 2.5 Matching degree = 0.5 Geometrical difference = 0.735 Sequential difference = 0.499
OR 1b		n7 = {n6\d1, d4} n7 = {d5, d10}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.833 Design abundance = 2.285 Matching degree = 0.428 Geometrical difference = 0.749 Sequential difference = 0.499

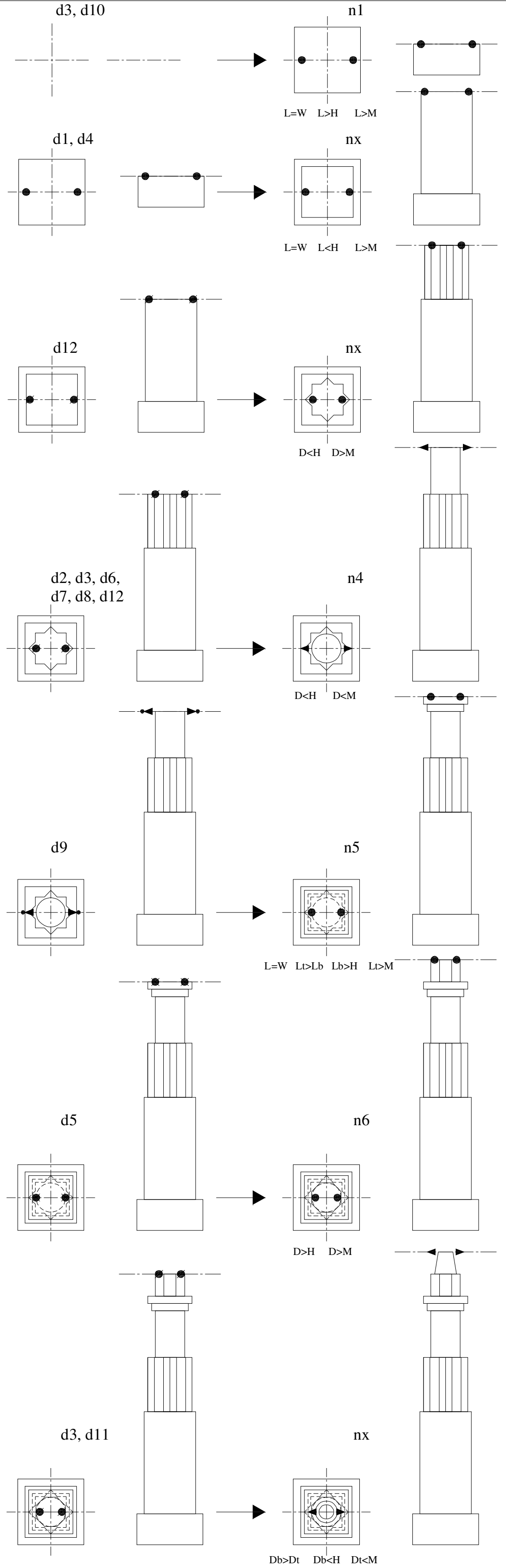
Rule no.	Pair 2: Hybrid design derivation - 10 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 7b		n8 = {n7\d5} n8 = { d10'}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.916 Design abundance = 2.25 Matching degree = 0.375 Geometrical difference = 0.728 Sequential difference = 0.499
OR 13b		n9 = {n8\d10} n9 = {Ø} n9 = n9* n9* = {m\d10} n9* = {d1, d2, d5, d9}	Rule Prevalence = 0.083 Geometrical difference = 0.75 Sequential difference = 0.5	Design diversity = 1.0 Design abundance = 2.111 Matching degree = 0.333 Geometrical difference = 0.73 Sequential difference = 0.499
OR 2f		nx = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0	Design diversity = 1.0 Design abundance = 2.0 Matching degree = 0.3 Geometrical difference = 0.749 Sequential difference = 0.449
			**Pair 2** **Hybrid design - OR** **No. of Original Rules = 10** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 2.0** **Matching= 0.3** **Geometrical diff.= 0.749** **Sequential diff.= 0.449**	

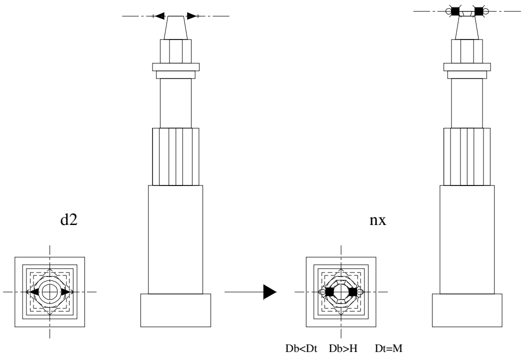
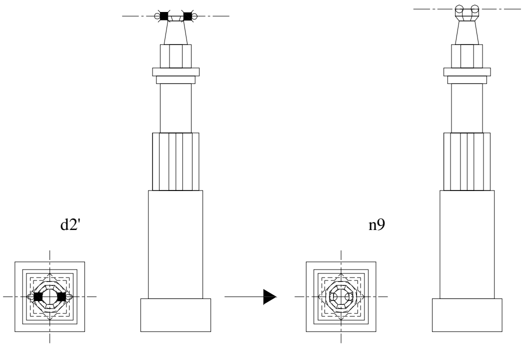
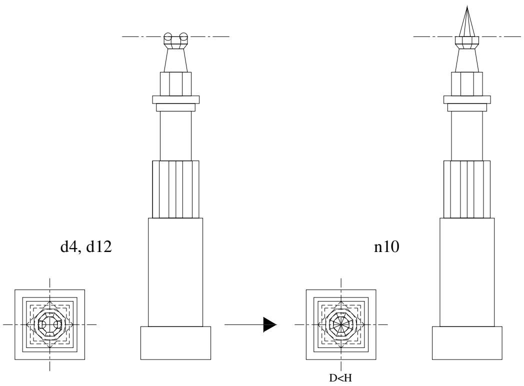
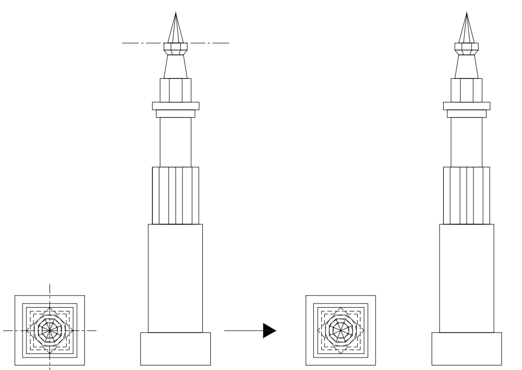
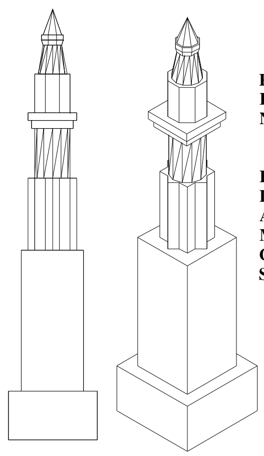
Rule no.	Pair 3: Hybrid design derivation - 6 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 4a	 d6 → n1	n1 = {n\d6} n1 = {d1, d2, d3, d4, d5, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversit y= 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 4b	d3 → n2 Db>Dt Db<H Dt=M	n2 = {n1\d3} n1 = {d1, d2, d4, d5, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.0 Matching degree = 0.5 Geometrical difference = 0.916 Sequential difference = 0.5
OR 3b	d2, d3, d6, d7, d8, d12 → n3 D<H D<M	n3 = {n2\d2, d3, d6, d7, d8, d12} n3 = {d1, d4, d5, d9, d10, d11}	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.5 Design abundance = 2.666 Matching degree = 0.666 Geometrical difference = 0.749 Sequential difference = 0.444
OR 14c	d11 → n4 Db<Dt Db>H Dt=M	n4 = {n3\d11} n4 = {d1', d4', d5', d9', d10'} n4 = {Ø} n4 = n4* n4* = {m\d11} n4* = {d2', d6', d7', d8', d12'} n4* = n4** n4** = {n\(n4+n4*)} n4** = {d3'}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.833	Design diversity = 0.583 Design abundance = 2.25 Matching degree = 0.5 Geometrical difference = 0.707 Sequential difference = 0.541
OR 1e	d3' → n5 D>H D=M	n5 = {n4\d3} n5 = {d1, d4, d5, d9, d10}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.916	Design diversity = 0.583 Design abundance = 2.0 Matching degree = 0.4 Geometrical difference = 0.749 Sequential difference = 0.616
OR 5f	d5, d10 → n6 D>H 0	n6 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.75 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 0.763 Sequential difference = 0.513
			**Pair 3** **Hybrid design - OR** **No. of Original Rules =6** **Evaluation Criteria:** **Diversity= 0.75** **Abundance= 2.0** **Matching= 0.5** **Geometrical diff.= 0.763** **Sequential diff'.= 0.513**	

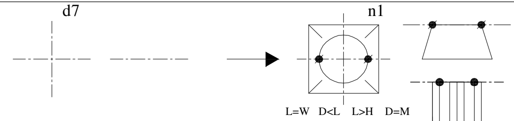
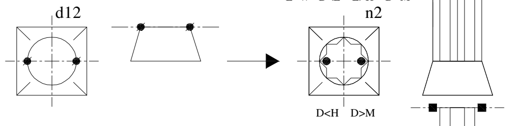
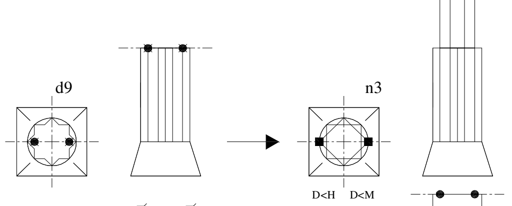
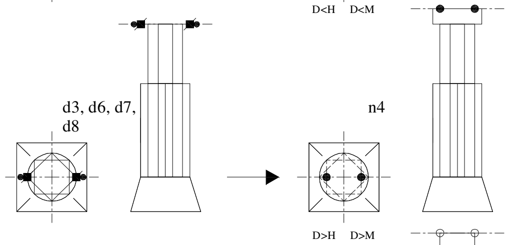
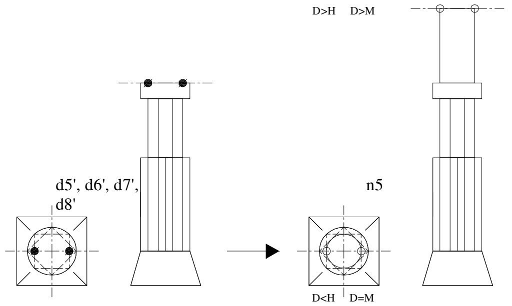
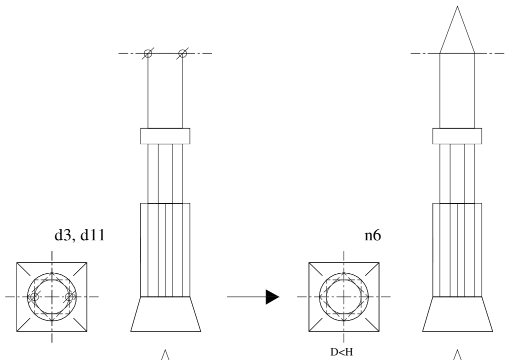
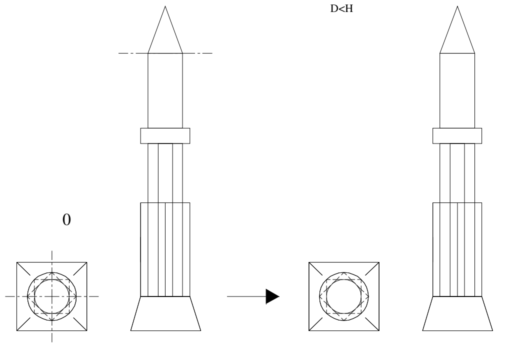
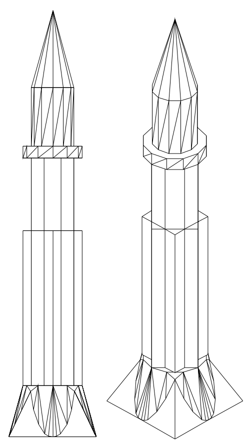
Rule no.	Pair 3: Hybrid design derivation - 10 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 4a		n1 = {n\d6} n1 = {d1, d2, d3, d4, d5, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 4b		n2 = {n1\d3} n1 = {d1, d2, d4, d5, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.0 Matching degree = 0.5 Geometrical difference = 0.916 Sequential difference = 0.5
OR 3b		n3 = {n2\d2, d3, d6, d7, d8, d12} n3 = {d1, d4, d5, d9, d10, d11}	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.5 Design abundance = 2.666 Matching degree = 0.666 Geometrical difference = 0.749 Sequential difference = 0.444
OR 8c		n4 = {n3\d4} n4 = {d1, d5, d9, d10, d11} n4 = {Ø} n4 = n4* n4* = {m\d4} n4* = {d2, d6, d7, d8, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 2.25 Matching degree = 0.5 Geometrical difference = 0.77 Sequential difference = 0.458
OR 4d		n5 = {n4\d3, d6, d7, d8} n5 = {d1, d5, d9, d10, d11}	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 2.6 Matching degree = 0.6 Geometrical difference = 0.732 Sequential difference = 0.466
OR 12b		n6 = {n5\d9} n6 = {d1, d5, d10, d11} n6 = {Ø} n6 = n6* n6* = {m\d9} n6* = {d2, d4, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.916	Design diversity = 0.666 Design abundance = 2.333 Matching degree = 0.5 Geometrical difference = 0.707 Sequential difference = 0.541
OR 2d		n7 = {n6\d2} n7 = {d1, d5, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.833	Design diversity = 0.666 Design abundance = 2.142 Matching degree = 0.428 Geometrical difference = 0.69 Sequential difference = 0.583

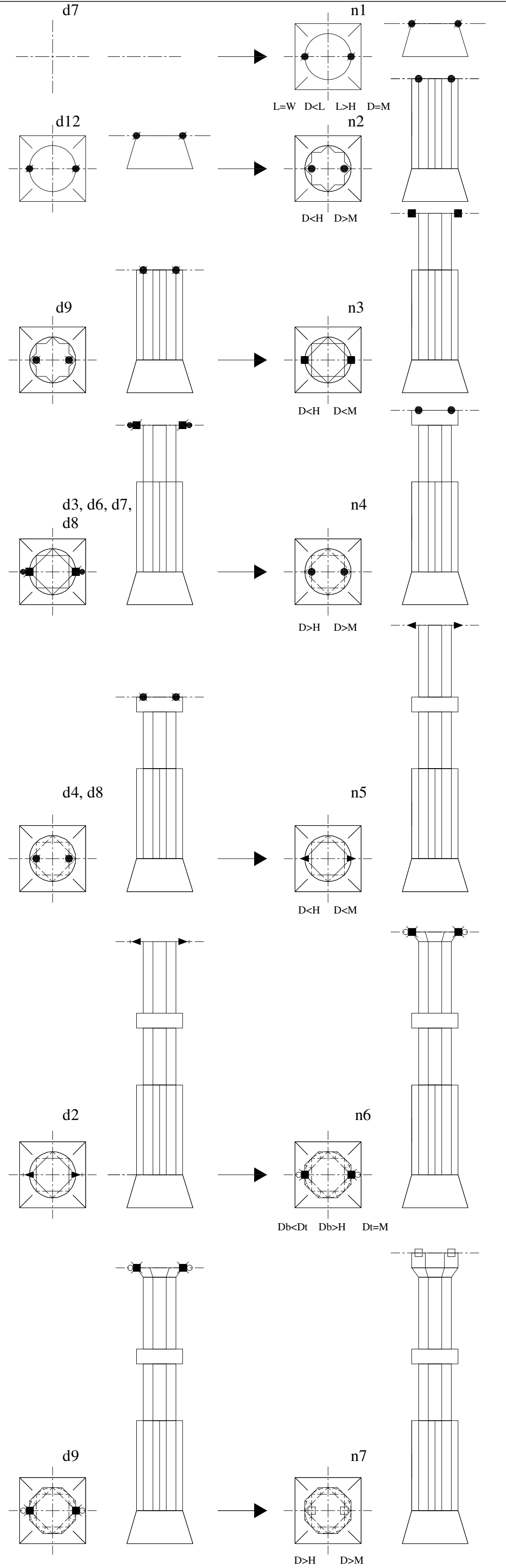
Rule no.	Pair 3: Hybrid design derivation - 10 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 4e		n8 = {n7\d11} n8 = {d1', d5', d10'} n8 = {Ø} n8 = n8* n8* = {m\d11} n8* = {d4', d12', d9'}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.75	Design diversity = 0.75 Design abundance = 2.0 Matching degree = 0.375 Geometrical difference = 0.718 Sequential difference = 0.604
OR 13c		n9 = {n8\d9} n9 = {d1, d5, d10}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.666	Design diversity = 0.75 Design abundance = 1.888 Matching degree = 0.333 Geometrical difference = 0.74 Sequential difference = 0.61
OR 5f		n10 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.916 Design abundance = 1.9 Matching degree = 0.3 Geometrical difference = 0.749 Sequential difference = 0.549
			**Pair 3** **Hybrid design - OR** **No. of Original Rules = 10** **Evaluation Criteria:** **Diversity= 0.916** **Abundance= 1.9** **Matching= 0.3** **Geometrical diff.= 0.749** **Sequential diff.= 0.549**	

Rule no.	Pair 4: Hybrid design derivation - 6 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 3a		n1 = {n\d3, d10} n1 = {d1, d2, d4, d5, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.583 Sequential difference = 0.5
OR 1b		n2 = {n1\d1, d4} n2 = {d2, d5, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 0.708 Sequential difference = 0.5
OR 14b		n3 = {n2\d12} n3 = {d2, d5, d6, d7, d8, d9, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.416 Design abundance = 1.666 Matching degree = 0.333 Geometrical difference = 0.777 Sequential difference = 0.5
OR 3b		n4 = {n3\d2, d3, d6, d7, d8, d12} n4 = {d5', d9', d11'}	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.75 Design abundance = 2.75 Matching degree = 0.5 Geometrical difference = 0.687 Sequential difference = 0.458
OR 15c		n5 = {n4\d11} n5 = {d5, d9}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.666	Design diversity = 0.833 Design abundance = 2.4 Matching degree = 0.4 Geometrical difference = 0.666 Sequential difference = 0.499
OR 8f		n6 = 0	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0	Design diversity = 0.916 Design abundance = 2.166 Matching degree = 0.5 Geometrical difference = 0.707 Sequential difference = 0.416
			**Pair 4** **Hybrid design - OR** **No. of Original Rules =6** **Evaluation Criteria:** **Diversity= 0.916** **Abundance= 2.166** **Matching= 0.5** **Geometrical diff.= 0.707** **Sequential diff.= 0.416**	

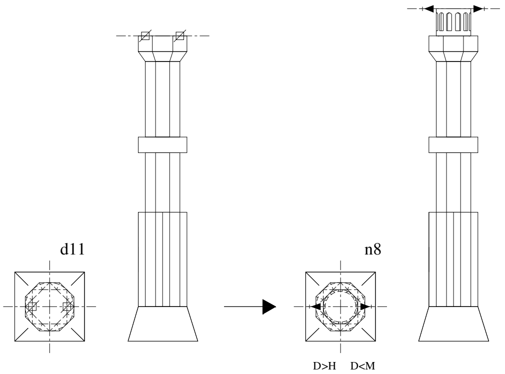
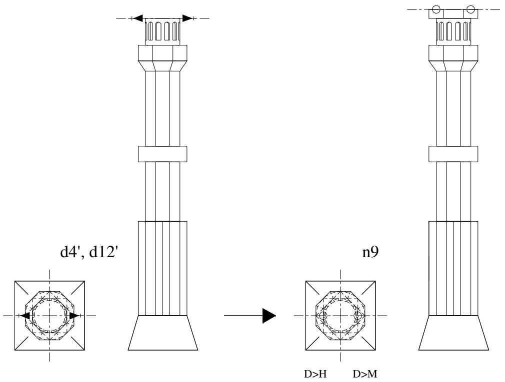
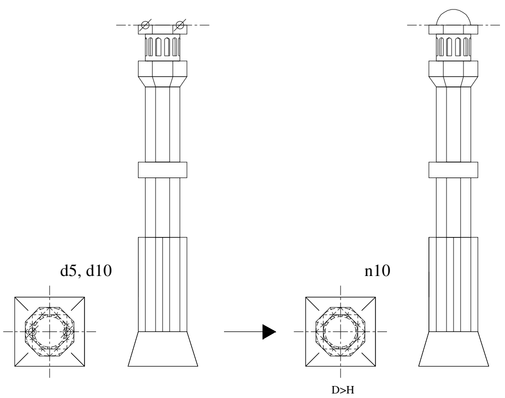
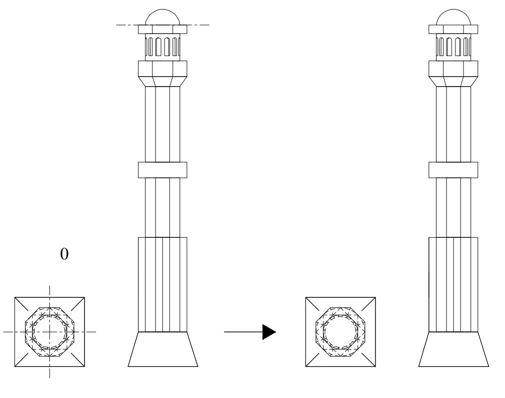
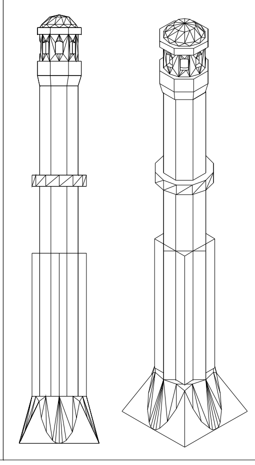
Rule no.	Pair 4: Hybrid design derivation - 10 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 3a		n1 = {n\d3, d10} n1 = {d1, d2, d4, d5, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 1.0 Geometrical difference = 0.583 Sequential difference = 0.5
OR 1b		n2 = {n1\d1, d4} n2 = {d2, d5, d6, d7, d8, d9, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 0.708 Sequential difference = 0.5
OR 14b		n3 = {n2\d12} n3 = {d2, d5, d6, d7, d8, d9, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.416 Design abundance = 1.666 Matching degree = 0.333 Geometrical difference = 0.777 Sequential difference = 0.5
OR 3b		n4 = {n3\d2, d3, d6, d7, d8, d12} n4 = {d5, d9, d11}	Rule Prevalence = 0.5 Geometrical difference = 0.416 Sequential difference = 0.333	Design diversity = 0.75 Design abundance = 2.75 Matching degree = 0.5 Geometrical difference = 0.687 Sequential difference = 0.458
OR 11c		n5 = {n4\d9} n5 = {d5, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.833	Design diversity = 0.833 Design abundance = 2.4 Matching degree = 0.4 Geometrical difference = 0.732 Sequential difference = 0.533
OR 7b		n6 = {n5\d5} n6 = {d11}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity= 0.916 Design abundance= 2.166 Matching degree= 0.333 Geometrical difference= 0.707 Sequential difference= 0.527
OR 5b		n7 = {n6\d3, d11} n7 = {Ø} n7 = n7* n7* = {m\d3, d11} n7* = {d1, d2, d4, d5, d6, d7, d8, d9, d10}	Rule Prevalence = 0.166 Geometrical difference = 0.75 Sequential difference = 0.333	Design diversity = 1.0 Design abundance = 2.142 Matching degree = 0.428 Geometrical difference = 0.713 Sequential difference = 0.499

Rule no.	Pair 4: Hybrid design derivation - 10 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 7c		n8 = {n7\d2} n8 = {Ø} n8 = n8* n8* = {m\d2} n8* = {d1', d4', d5', d6', d7', d8', d9', d10', d11'} n8* = {Ø} n8* = n8** n8** = {n\(n8+n8*)} n8** = {d2', d3', d12'}	Rule Prevalence = 0.083 Geometrical difference = 0.833 Sequential difference = 0.5	Design diversity = 1.0 Design abundance = 2.0 Matching degree = 0.375 Geometrical difference = 0.728 Sequential difference = 0.499
OR 3d		n9 = {n8\d2} n9 = {Ø} n9 = n9* n9* = {m\d2} n9* = {d1, d4, d5, d6, d7, d8, d9, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.75 Sequential difference = 0.916	Design diversity = 1.0 Design abundance = 1.888 Matching degree = 0.333 Geometrical difference = 0.731 Sequential difference = 0.546
OR 4f		n10 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 1.0 Design abundance = 1.9 Matching degree = 0.3 Geometrical difference = 0.741 Sequential difference = 0.491
		 **Pair 4** **Hybrid design - OR** **No. of Original Rules = 10** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 1.9** **Matching= 0.3** **Geometrical diff.= 0.741** **Sequential diff.= 0.491**		

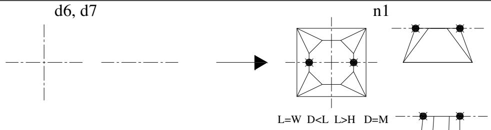
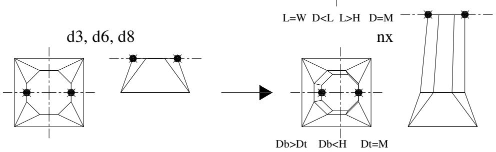
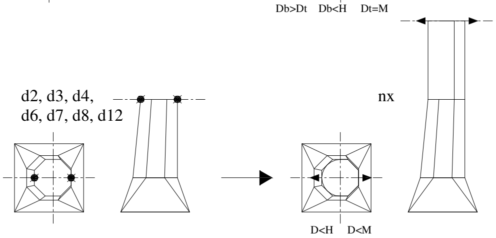
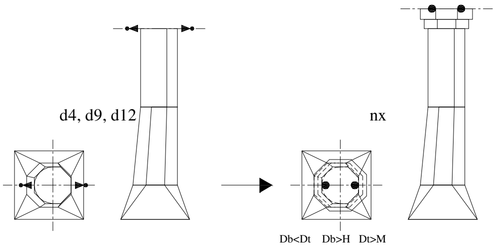
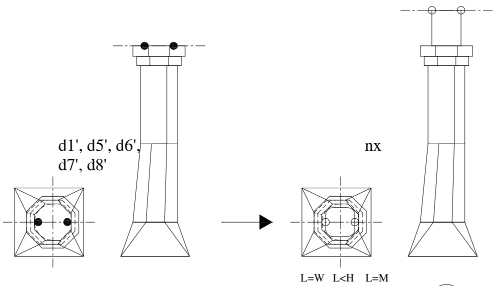
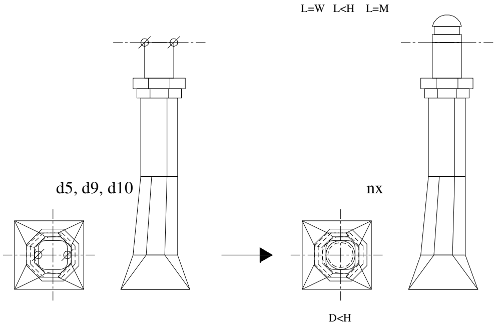
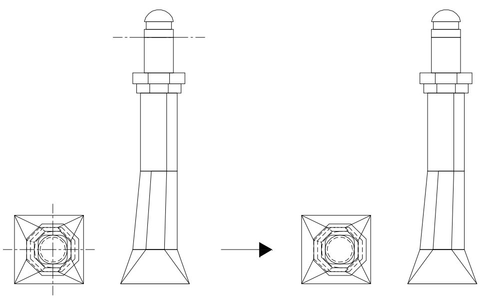
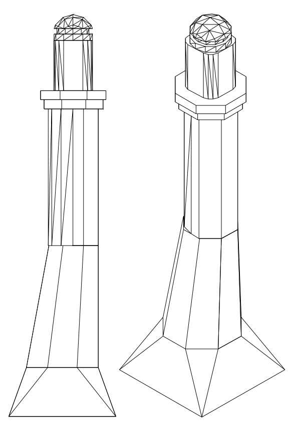
Rule no.	Pair 5: Hybrid design derivation - 6 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 5a		n1 = {n\d7} n1 = {d1, d2, d3, d4, d5, d6, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 14b		n2 = {n1\d12} n2 = {d1, d2, d3, d4, d5, d6, d8, d9, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.0 Matching degree = 0.5 Geometrical difference = 0.916 Sequential difference = 0.5
OR 12b		n3 = {n2\d9} n3 = {d1, d2, d3, d4, d5, d6, d8, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.916	Design diversity = 0.25 Design abundance = 1.0 Matching degree = 0.333 Geometrical difference = 0.805 Sequential difference = 0.638
OR 4d		n4 = {n3\d3, d6, d7, d8} n4 = {d1, d2, d4, d5, d10, d11}	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 1.75 Matching degree = 0.5 Geometrical difference = 0.749 Sequential difference = 0.604
OR 8b		n5 = {n4\d5, d6, d7, d8} n5 = {d1, d2, d4, d10, d11}	Rule Prevalence = 0.333 Geometrical difference = 0.416 Sequential difference = 0.5	Design diversity = 0.583 Design abundance = 2.2 Matching degree = 0.666 Geometrical difference = 0.682 Sequential difference = 0.583
OR 3f		n6 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.666 Design abundance = 2.166 Matching degree = 0.5 Geometrical difference = 0.707 Sequential difference = 0.486
		 **Pair 5** **Hybrid design - OR** **No. of Original Rules =6** **Evaluation Criteria:** **Diversity= 0.666** **Abundance= 2.166** **Matching= 0.5** **Geometrical diff.= 0.707** **Sequential diff.= 0.486**		

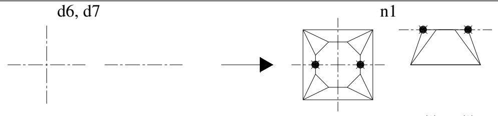
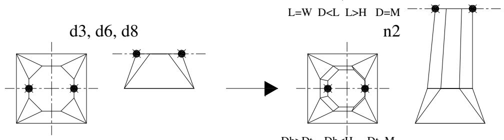
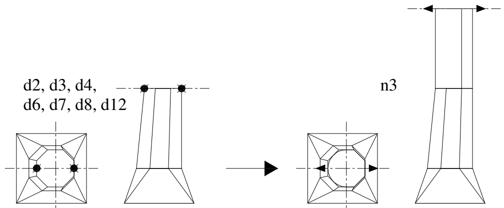
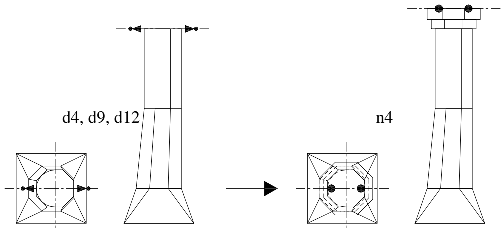
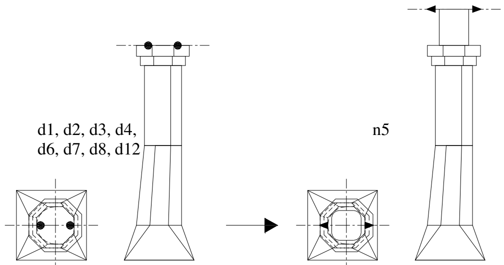
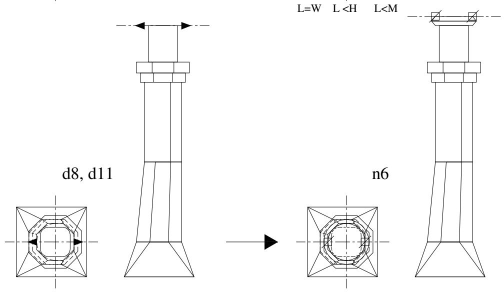
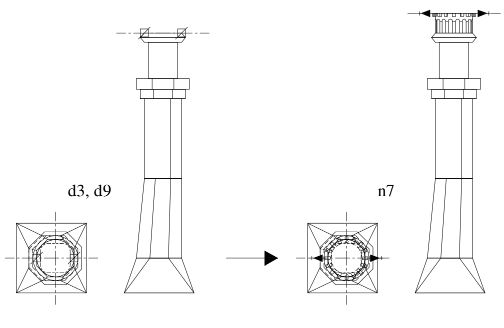


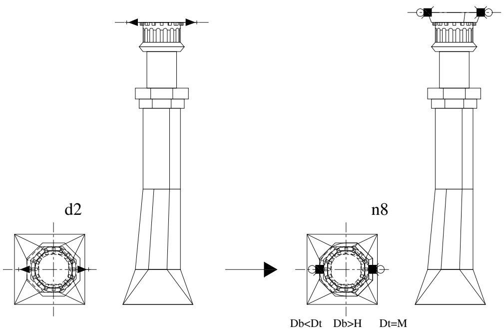
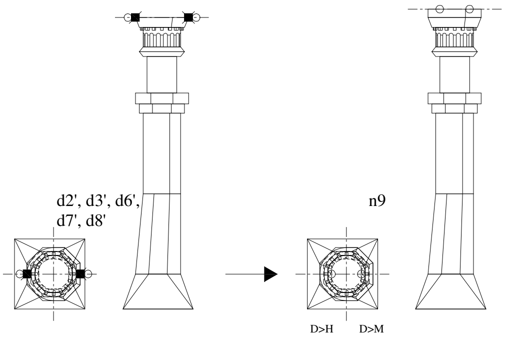
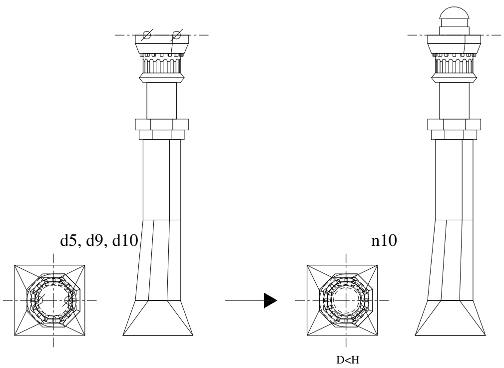
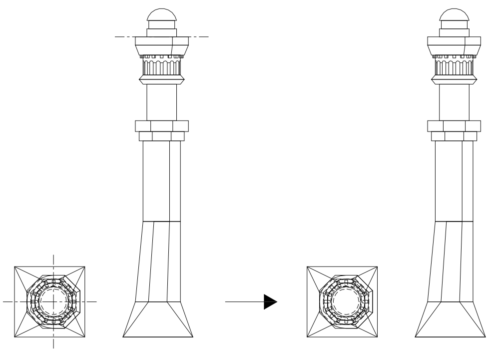
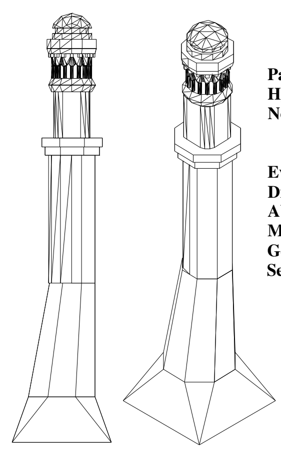
Rule no.	Pair 5: Hybrid design derivation - 10 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 5a		n1 = {n\d7} n1 = {d1, d2, d3, d4, d5, d6, d8, d9, d10, d11, d12}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.083 Design abundance = 1.0 Matching degree = 1.0 Geometrical difference = 0.916 Sequential difference = 0.5
OR 14b		n2 = {n1\d12} n2 = {d1, d2, d3, d4, d5, d6, d8, d9, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 1.0 Matching degree = 0.5 Geometrical difference = 0.916 Sequential difference = 0.5
OR 12b		n3 = {n2\d9} n3 = {d1, d2, d3, d4, d5, d6, d8, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.583 Sequential difference = 0.916	Design diversity= 0.25 Design abundance= 1.0 Matching degree= 0.333 Geometrical difference= 0.805 Sequential difference= 0.638
OR 4d		n4 = {n3\d3, d6, d7, d8} n4 = {d1, d2, d4, d5, d10, d11}	Rule Prevalence = 0.333 Geometrical difference = 0.583 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 1.75 Matching degree = 0.5 Geometrical difference = 0.749 Sequential difference = 0.604
OR 6b		n5 = {n4\d4, d8} n5 = {d1, d2, d5, d10, d11}	Rule Prevalence = 0.166 Geometrical difference = 0.583 Sequential difference = 0.333	Design diversity = 0.583 Design abundance = 1.8 Matching degree = 0.4 Geometrical difference = 0.716 Sequential difference = 0.549
OR 7c		n6 = {n5\d2} n6 = {d1, d5, d10, d11} n6 = {Ø} n6 = n6* n6* = {m\d2} n6* = {d3, d4, d6, d9, d12}	Rule Prevalence = 0.083 Geometrical difference =0.833 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 1.666 Matching degree = 0.333 Geometrical difference = 0.735 Sequential difference = 0.541
OR 6d		n7 = {n6\d9} n7 = {d1, d5, d10, d11}	Rule Prevalence = 0.083 Geometrical difference = 0.75 Sequential difference = 0.833	Design diversity = 0.666 Design abundance = 1. Matching degree = 0. Geometrical difference = 0.737 Sequential difference = 0.583

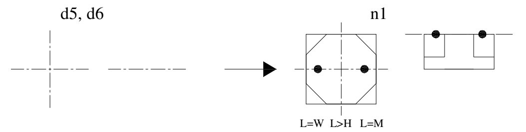
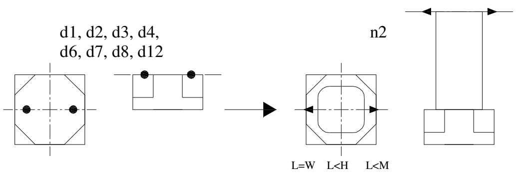
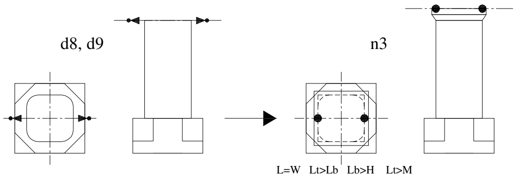
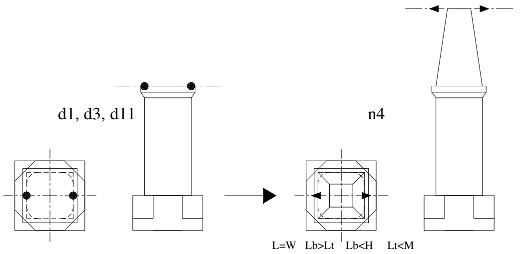
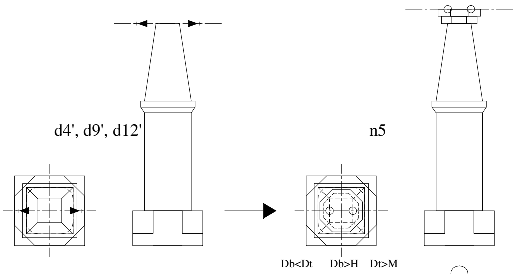
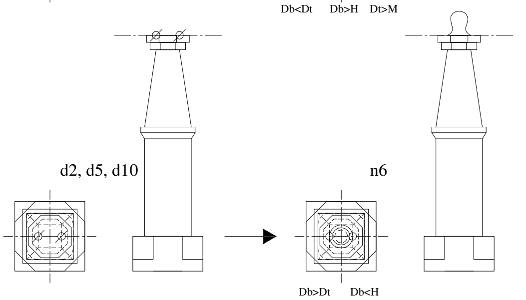
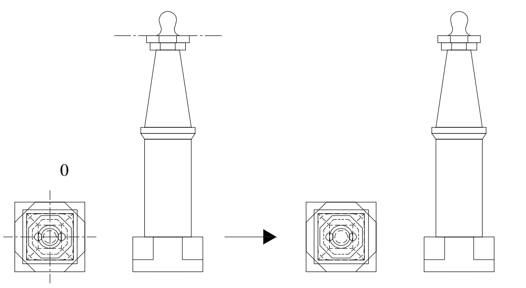
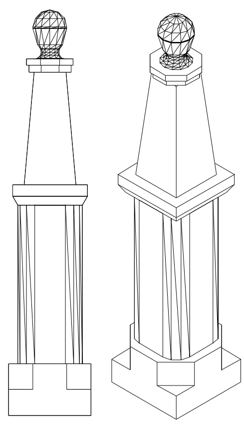
Rule no.	Pair 5: Hybrid design derivation - 10 Original rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
OR 4e		n8 = {n7\d11} n8 = {d1', d5', d10'} n8 = {Ø} n8 = n8* n8* = {m\d11} n8* = {d2', d3', d4', d6', d12'}	Rule Prevalence = 0.083 Geometrical difference = 0.916 Sequential difference = 0.75	Design diversity = 0.75 Design abundance = 1.5 Matching degree = 0.25 Geometrical difference = 0.76 Sequential difference = 0.604
OR 9c		n9 = {n8\d4, d12} n9 = {d1, d5, d10}	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference=0.666	Design diversity= 0.75 Design abundance= 1.555 Matching degree= 0.222 Geometrical difference= 0.768 Sequential difference= 0.61
OR 5f		n10 = 0	Rule Prevalence = 0.166 Geometrical difference = 0.833 Sequential difference = 0	Design diversity = 0.916 Design abundance = 1.6 Matching degree = 0.2 Geometrical difference = 0.774 Sequential difference = 0.549
		 **Pair 5** **Hybrid design - OR** **No. of Original Rules = 10** **Evaluation Criteria:** **Diversity= 0.916** **Abundance= 1.6** **Matching= 0.2** **Geometrical diff.= 0.774** **Sequential diff.= 0.549**		

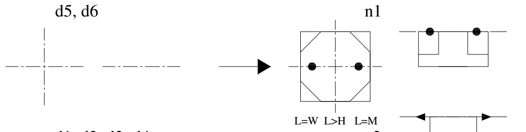
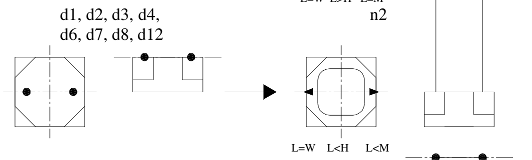
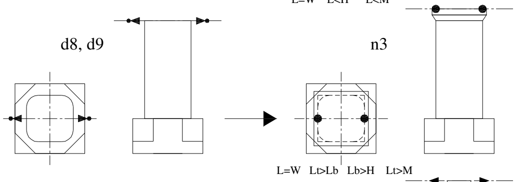
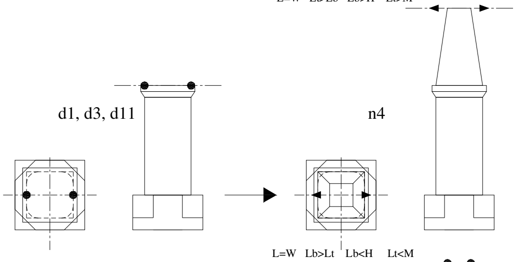
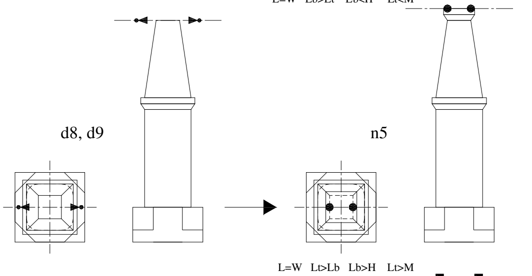
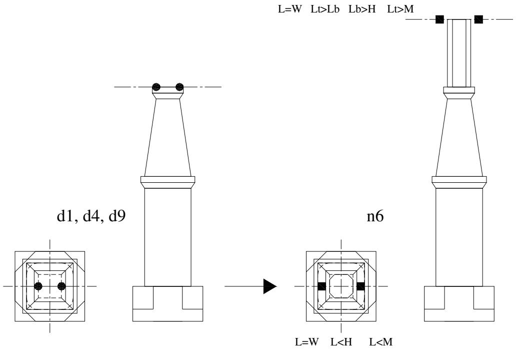
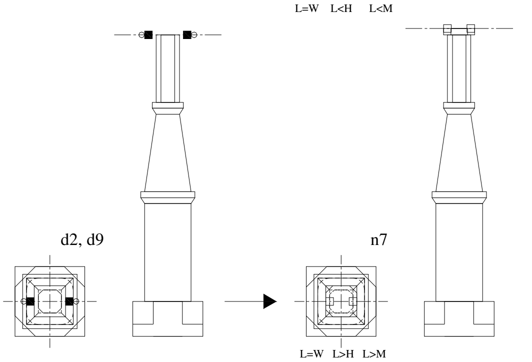
C-2 Five pairs of hybrid designs composed of hybrid rules (HR)

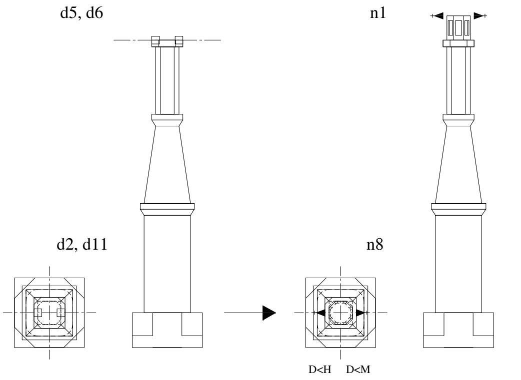
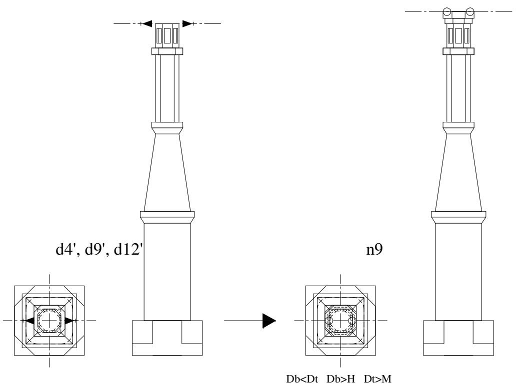
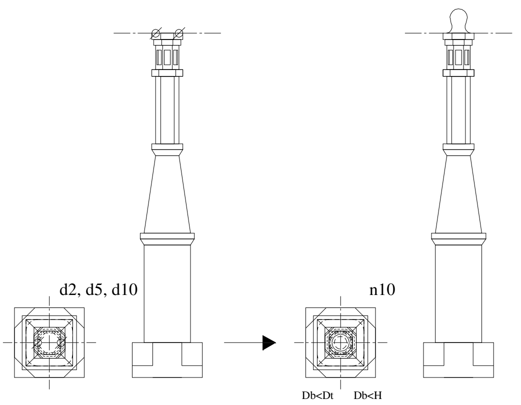
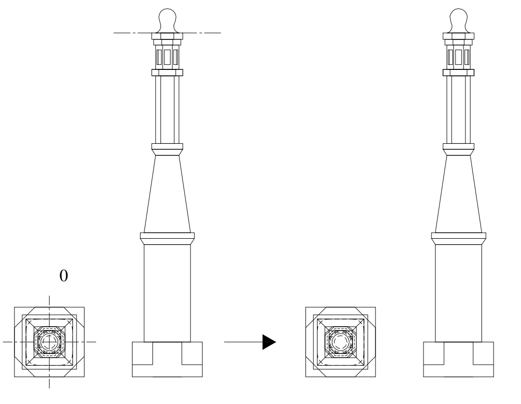
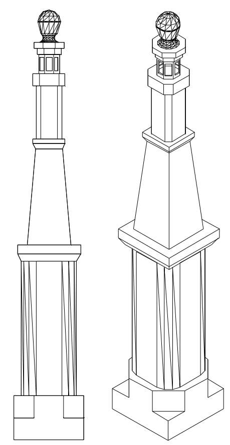
Rule no.	Pair 1: Hybrid design derivation - 6 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 2a		n1 = {n\d6, d7} n1 = {d1, d2, d3, d4, d5, d8, d9, d10, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 6b		n2 = {n1\d3, d6, d8} n2 = {d1, d2, d4, d5, d9, d10, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 2.5 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 13b		n3 = {n2\d2, d3, d4, d6, d7, d8, d12} n3 = {d1, d5, d9, d10, d11}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.583 Design abundance = 4.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.444
HR 13c		n4 = {n3\d4, d9, d12} n4 = {d1', d5', d10', d11'}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.666 Design abundance = 3.75 Matching degree = 0.375 Geometrical difference = 1.0 Sequential difference = 0.541
HR 19b		n5 = {n4\d1, d5, d6, d7, d8} n5 = {d10, d11}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.583	Design diversity = 0.833 Design abundance = 4.0 Matching degree = 0.4 Geometrical difference = 1.0 Sequential difference = 0.549
HR 6f		n6 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 0.916 Design abundance = 3.833 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.458
				**Pair 1** **Hybrid design - HR** **No. of Hybrid Rules = 6** **Evaluation Criteria:** **Diversity= 0.916** **Abundance= 3.833** **Matching= 0.333** **Geometrical diff.= 1.0** **Sequential diff.= 0.458**

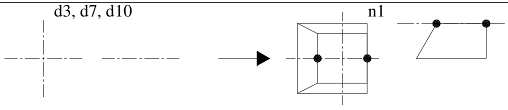
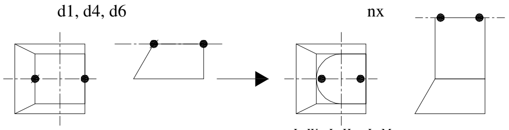
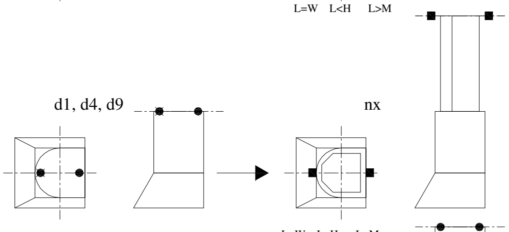
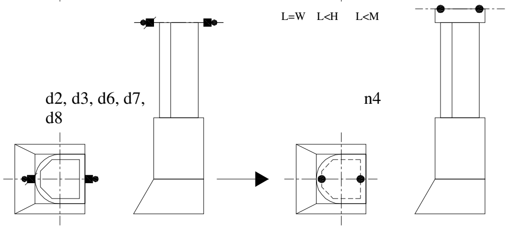
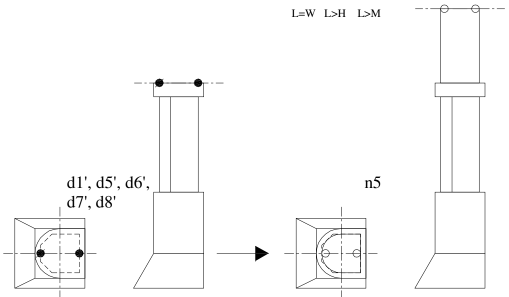
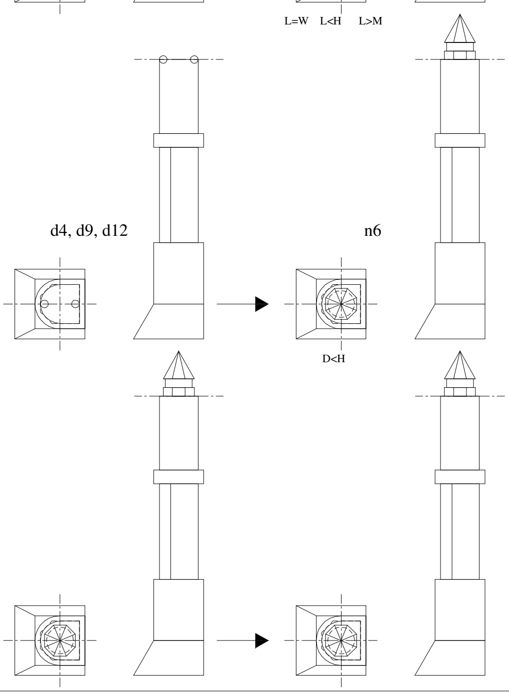
Rule no.	Pair 1: Hybrid design derivation - 10 Hybrid rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
HR 2a	d6, d7 n1  L=W D<L L>H D=M	n1 = {n\d6, d7} n1 = {d1, d2, d3, d4, d5, d8, d9, d10, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 6b	d3, d6, d8 n2  Db>Dt Db<H Dt=M	n2={n1\d3, d6, d8} n2={d1, d2, d4, d5, d9, d10, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.333 Design abundance = 2.5 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 13b	d2, d3, d4, d6, d7, d8, d12 n3  D<H D<M	n3={n2\d2, d3, d4, d6, d7, d8, d12} n3={d1, d5, d9, d10, d11}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.583 Design abundance = 4.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.444
HR 13c	d4, d9, d12 n4  Db<Dt Db>H Dt>M	n4={n3\d4, d9, d12} n4={d1, d5, d10, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.666 Design abundance = 3.75 Matching degree = 0.375 Geometrical difference = 1.0 Sequential difference = 0.541
HR 17b	d1, d2, d3, d4, d6, d7, d8, d12 n5  L=W L <H L<M	n5 = {n(x-1)\d1, d2, d3, d4, d6, d7, d8, d12} n5 = {d5, d10, d11}	Rule Prevalence = 0.666 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.75 Design abundance = 4.6 Matching degree = 0.4 Geometrical difference = 1.0 Sequential difference = 0.499
HR 21c	d8, d11 n6  Db<Dt Db>H Dt=M	n6 = {n5\d8, d11} n6 = {d5, d10} n6 = Ø n6 = n6* n6* = {m\d8, d11} n6* = {d1, d9}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.833 Design abundance = 3.833 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.555
HR 6e	d3, d9 n7 	n7 = {n6\d3, d9} n7 = {d5, d10} n7 = Ø n7 = n7* n7* = {m\d3, d9} n7* = {d1, d11} n7* = Ø n7* = n7** n7** = {n\(n7+n7*)] n7** = {d2, d3, d4, d6, d7, d8, d9, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.75	Design diversity = 0.833 Design abundance = 3.857 Matching degree = 0.285 Geometrical difference = 1.0 Sequential difference = 0.583

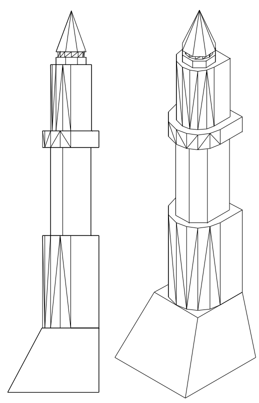
Rule no.	Pair 1: Hybrid design derivation - 10 Hybrid rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
HR 6c		n8 = {n7\d2} n8 = {d5, d10} n8 = Ø n8 = n8* n8* = {m\d2} n8* = {d1, d11} n8* = Ø n8* = n8** n7** = {n\(n7+n7*)] n7** = {d2', d3', d4', d6', d7', d8', d9', d12'}	Rule Prevalence = 0.083 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.833 Design abundance = 3.5 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.572
HR 11d		n9 = {n8\d2, d3, d6, d7, d8} n9 = {d5, d10}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.833 Design abundance = 3.666 Matching degree = 0.277 Geometrical difference = 1.0 Sequential difference = 0.610
HR 6f		n10 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 3.6 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.549
			**Pair 1** **Hybrid design - HR** **No. of Hybrid Rules = 10** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 3.6** **Matching= 0.25** **Geometrical diff.= 1.0** **Sequential diff.= 0.549**	

Rule no.	Pair 2: Hybrid design derivation - 6 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 6a		n1 = {n\d5, d6} n1 = {d1, d2, d3, d4, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 17b		n2 = {n1\d1, d2, d3, d4, d6, d7, d8, d12} n2 = {d9, d10, d11}	Rule Prevalence = 0.666 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.75 Design abundance = 5.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.416
HR 9c		n3 = {n2\d8, d9} n3 = {d10, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.833 Design abundance = 4.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.555
HR 1b		n4 = {n3\d1, d3, d11} n4 = {d10'} n4 = Ø n4 = n4* n4* = {m\d8, d11} n4* = {d2', d4', d5', d7', d9', d12'}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.916 Design abundance = 3.75 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.499
HR 14c		n5 = {n4\d4, d9, d12} n5 = {d10}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.666	Design diversity = 0.916 Design abundance = 3.6 Matching degree = 0.2 Geometrical difference = 1.0 Sequential difference = 0.533
HR 2f		n6 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 3.5 Matching degree = 0.166 Geometrical difference = 1.0 Sequential difference = 0.444
			**Pair 2** **Hybrid design - HR** **No. of Hybrid Rules = 6** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 3.5** **Matching= 0.166** **Geometrical diff.= 1.0** **Sequential diff.= 0.444**	

Rule no.	Pair 2: Hybrid design derivation - 10 Hybrid rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
HR 6a		n1 = {n\d5, d6} n1 = {d1, d2, d3, d4, d7, d8, d9, d10, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 17b		n2 = {n1\d1, d2, d3, d4, d6, d7, d8, d12} n2 = {d9, d10, d11}	Rule Prevalence = 0.666 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.75 Design abundance = 5.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.416
HR 9c		n3 = {n2\d8, d9} n3 = {d10, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.833 Design abundance = 4.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.555
HR 1b		n4 = {n3\d1, d3, d11} n4 = {d10} n4 = Ø n4 = n4* n4* = {m\d8, d11} n4* = {d2, d4, d5, d7, d9, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.916 Design abundance = 3.75 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.499
HR 9c		n5 = {n4\d8, d9} n5 = {d10} n5 = Ø n5=n5* n5* = {m\d8, d9} n5* = {d2, d4, d5, d7, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.916 Design abundance = 3.4 Matching degree = 0.3 Geometrical difference = 1.0 Sequential difference = 0.566
HR 15b		n6 = {n5\d1, d4, d9} n6 = {d10} n6 = Ø n6 = n6* n6* = {m\d1, d4, d9} n6* = {d2, d5, d7, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.916 Design abundance = 3.333 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.624
HR 8d		n7 = {n6\d2, d9} n7 = {d10} n7 = Ø n7 = n7* n7* = {m\d2, d9} n7* = {d5, d7, d11, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.916 Design abundance = 3.142 Matching degree = 0.285 Geometrical difference = 1.0 Sequential difference = 0.654

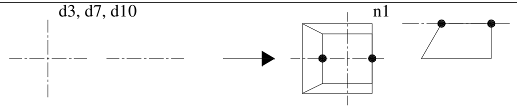
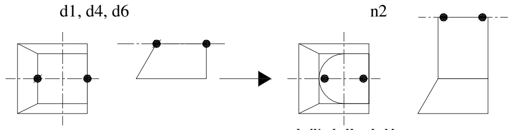
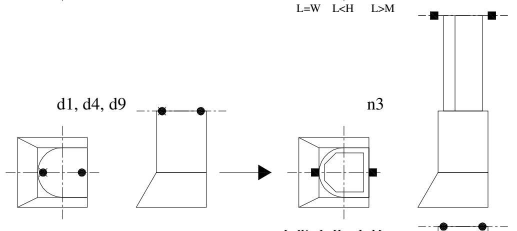
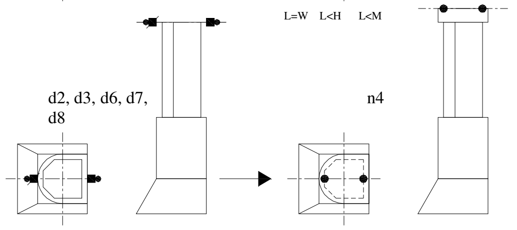
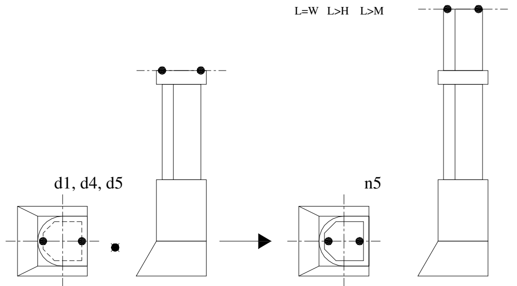
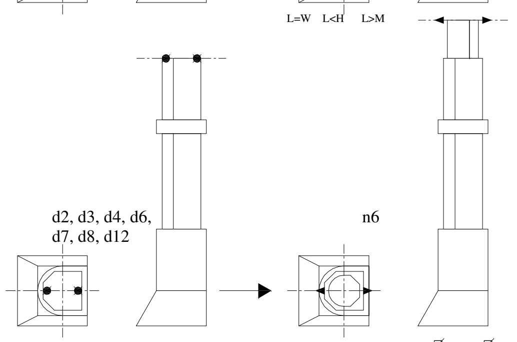
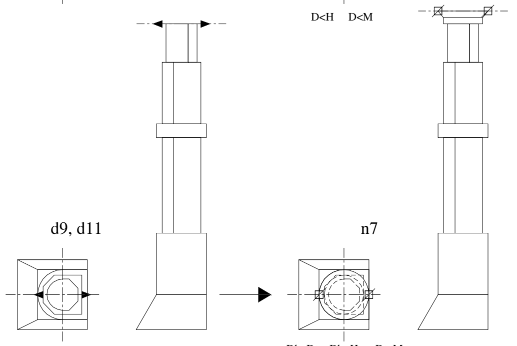
Rule no.	Pair 2: Hybrid design derivation - 10 Hybrid rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
HR 10e		n8 = {n8\d2, d11} n8 = {d10} n8 = Ø n8 = n8* n8* = {m\d2, d11} n8* = {d5', d7', d12'}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.75	Design diversity = 0.916 Design abundance = 3.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.666
HR 14c		n9 = {n8\d4, d9, d12} n9 = {d10}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.666	Design diversity = 0.916 Design abundance = 3.0 Matching degree = 0.277 Geometrical difference = 1.0 Sequential difference = 0.666
HR 2f		n10 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 3.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.599
		 **Pair 2** **Hybrid design - HR** **No. of Hybrid Rules = 10** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 3.0** **Matching= 0.25** **Geometrical diff.= 1.0** **Sequential diff.= 0.599**		

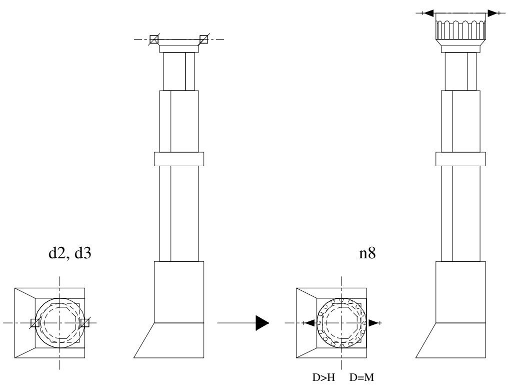
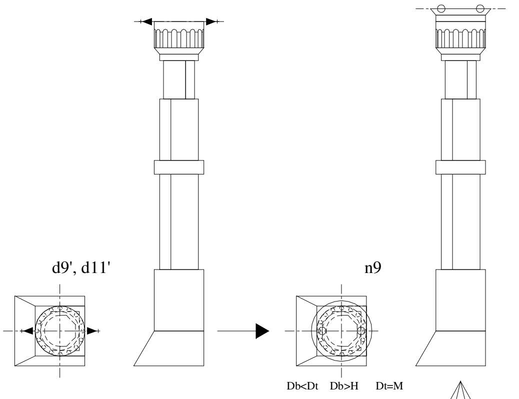
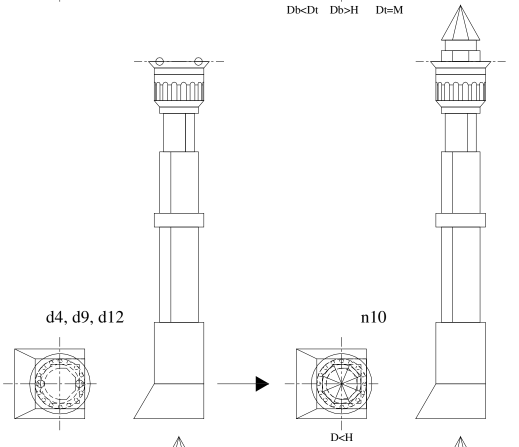
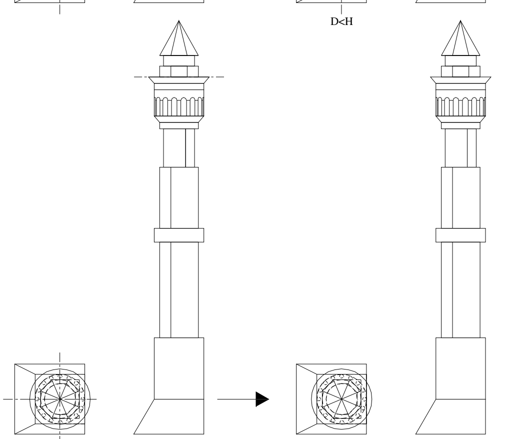
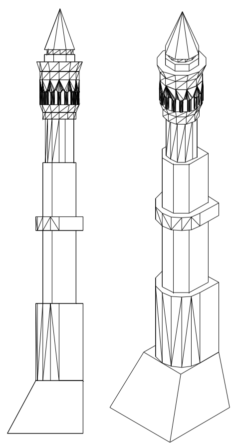
Rule no.	Pair 3: Hybrid design derivation - 6 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 12a	d3, d7, d10 → n1  Lb=Wb Lt=Wt Lb>H Lt=M	n1 = {n\d3, d7, d10} n1 = {d1, d2, d4, d5, d6, d8, d9, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 11b	d1, d4, d6 → nx  L=W L<H L>M	n2 = {n1\d1, d4, d6} n2 = {d2, d5, d8, d9, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 3.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5
HR 9b	d1, d4, d9 → nx  L=W L<H L<M	n3 = {n2\d1, d9} n3 = {d2, d5, d8, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.666 Design abundance = 3.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.638
HR 1d	d2, d3, d6, d7, d8 → n4  L=W L>H L>M	n4 = {n3\d2, d3, d6, d7, d8} n4 = {d5, d11, d12}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.75 Design abundance = 3.5 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.604
HR 12b	d1', d5', d6', d7', d8' → n5  L=W L<H L>M	n5 = {n4\d1, d5, d6, d7, d8} n5 = {d11, d12}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.583	Design diversity = 0.833 Design abundance = 3.8 Matching degree = 0.3 Geometrical difference = 1.0 Sequential difference = 0.599
HR 7f	d4, d9, d12 → n6  D<H	n6 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 0.916 Design abundance = 3.666 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.499

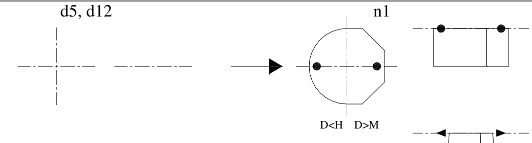
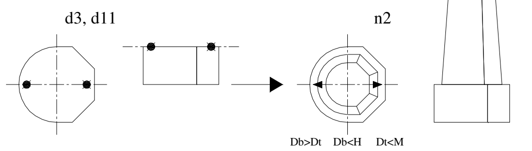
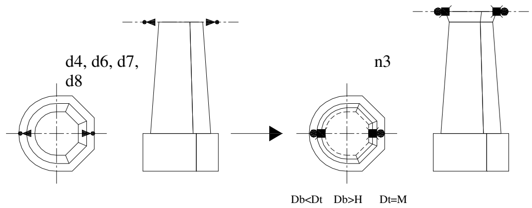
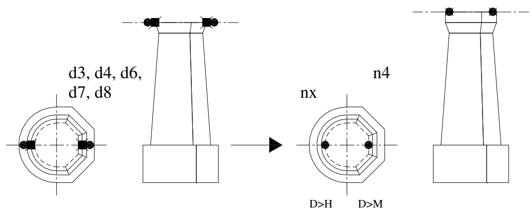
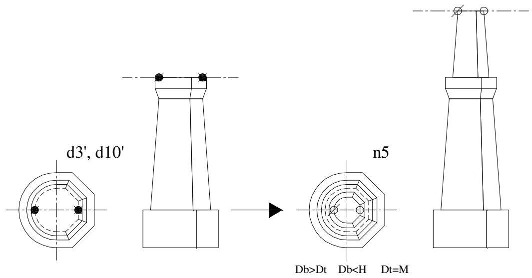
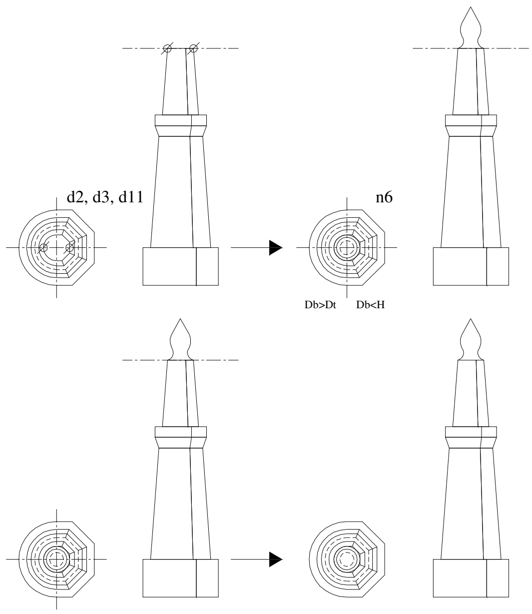
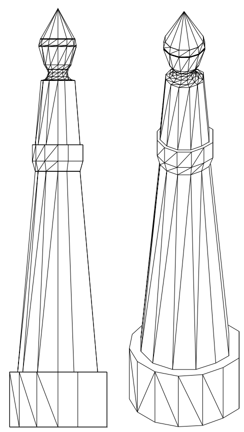


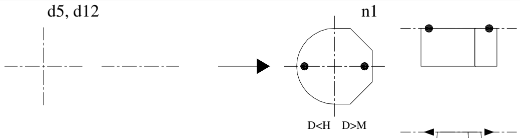
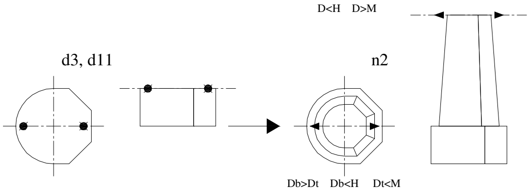
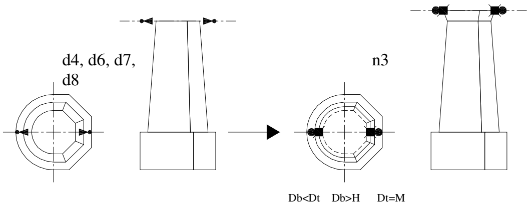
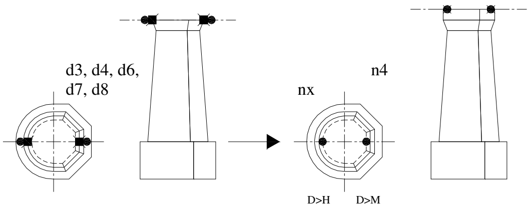
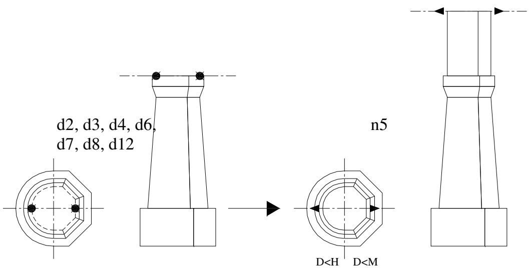
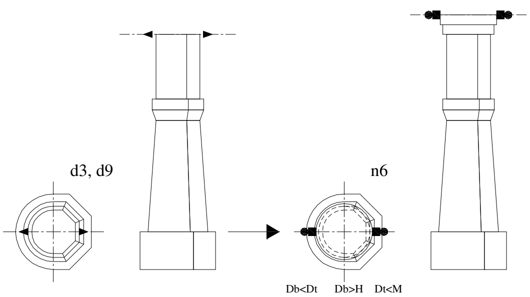
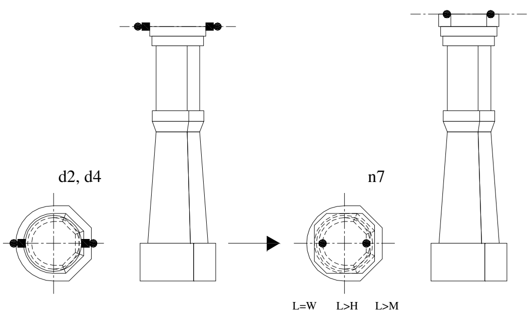
Pair 3
Hybrid design - HR
No. of Hybrid Rules = 6

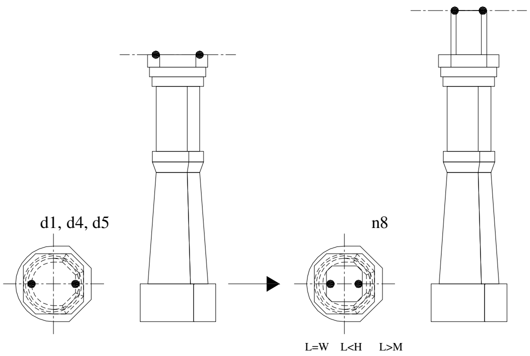
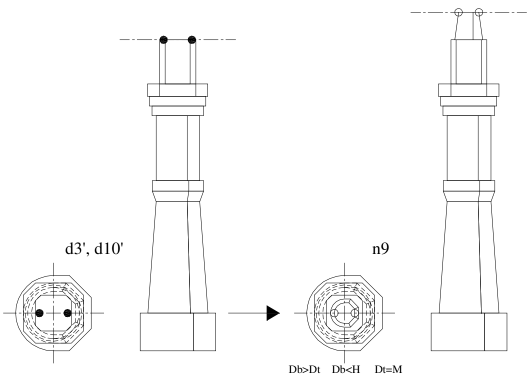
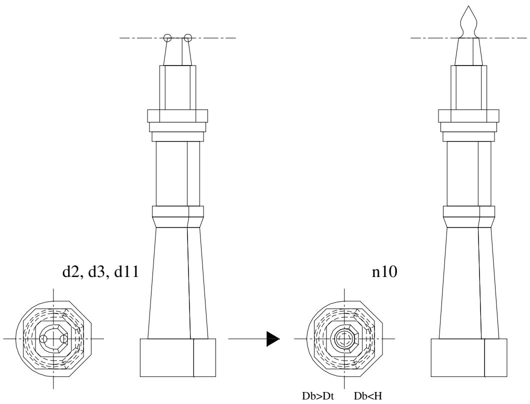
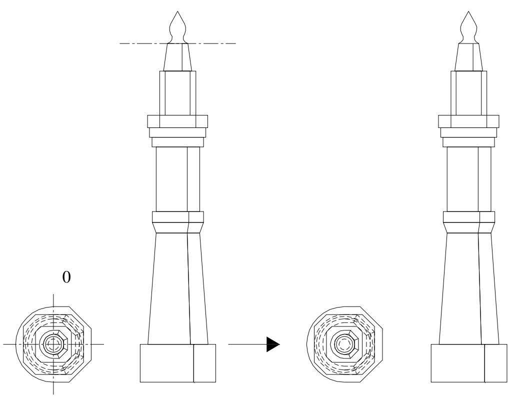
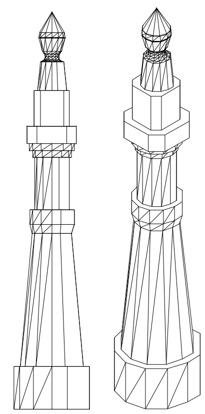
Evaluation Criteria:
Diversity= 0.916
Abundance= 3.666
Matching= 0.25
Geometrical diff.= 1.0
Sequential diff.= 0.499

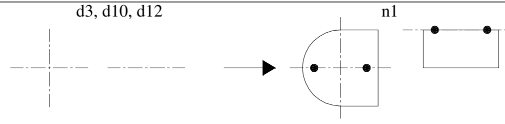
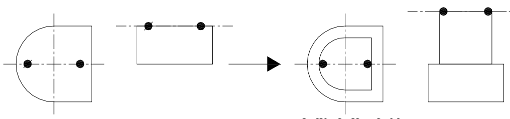
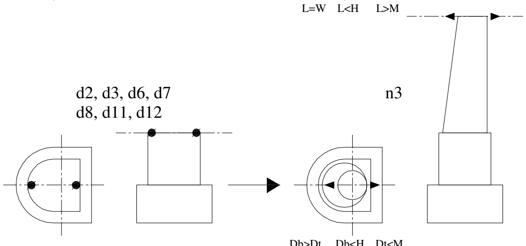
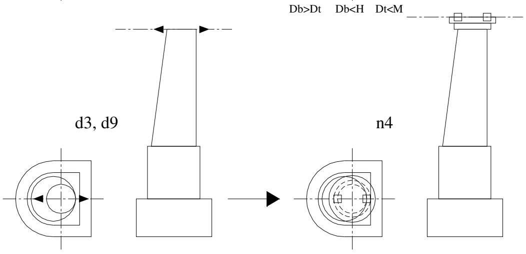
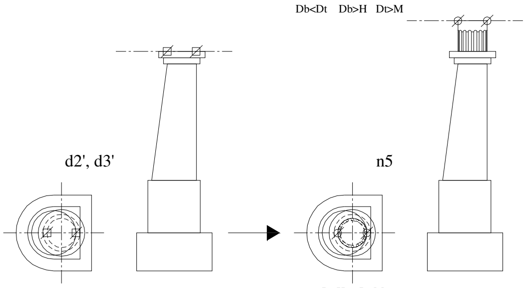
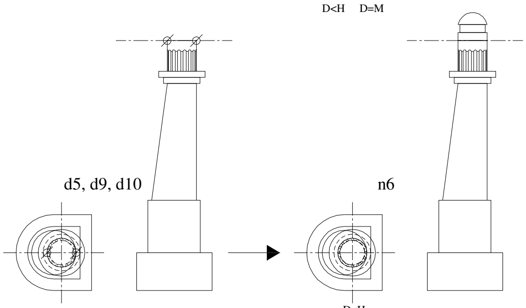
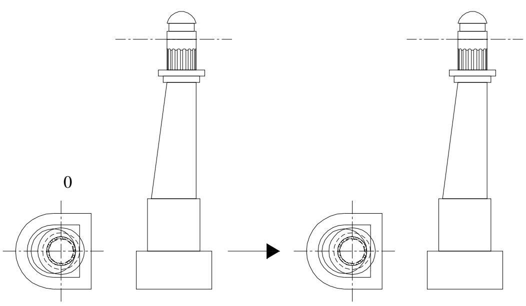
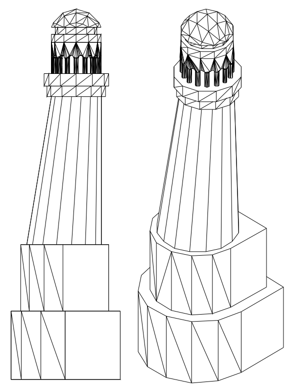
Rule no.	Pair 3: Hybrid design derivation - 10 Hybrid rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
HR 12a	d3, d7, d10 n1  Lb=Wb Lt=Wt Lb>H Lt=M	n1 = {n\d3, d7, d10} n1 = {d1, d2, d4, d5, d6, d8, d9, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 11b	d1, d4, d6 n2  L=W L<H L>M	n2 = {n1\d1, d4, d6} n2 = {d2, d5, d8, d9, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 3.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5
HR 9b	d1, d4, d9 n3  L=W L<H L<M	n3 = {n2\d1, d9} n3 = {d2, d5, d8, d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.666 Design abundance = 3.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.638
HR 1d	d2, d3, d6, d7, d8 n4  L=W L>H L>M	n4 = {n3\d2, d3, d6, d7, d8} n4 = {d5, d11, d12}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.75 Design abundance = 3.5 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.604
HR 8b	d1, d4, d5 n5  L=W L<H L>M	n5 = {n4\d1, d4, d5} n5 = {d11, d12}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.833 Design abundance = 3.4 Matching degree = 0.3 Geometrical difference = 1.0 Sequential difference = 0.583
HR 13b	d2, d3, d4, d6, d7, d8, d12 n6  D<H D<M	n6 = {n5\d2, d3, d4, d6, d7, d8, d12} n6 = {d11}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity= 0.916 Design abundance= 4.0 Matching degree= 0.333 Geometrical difference= 1.0 Sequential difference= 0.541
HR 16c	d9, d11 n7  Db<Dt Db>H Dt=M	nx = {n(x-1)\ d9, d11} n7 = {Ø} n7 = n7* n7* = {m\d9, d11} n7* = {d5, d10, d12} n7* = {Ø} n7* = n7** n7** = {n\(n7+n7*)] n7** = {d1, d2, d3, d4, d6, d7, d8, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 1.0 Design abundance = 3.714 Matching degree = 0.285 Geometrical difference = 1.0 Sequential difference = 0.583

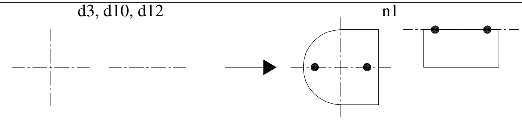
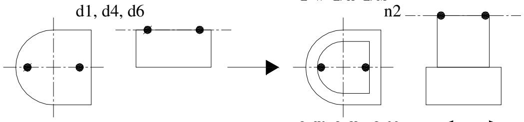
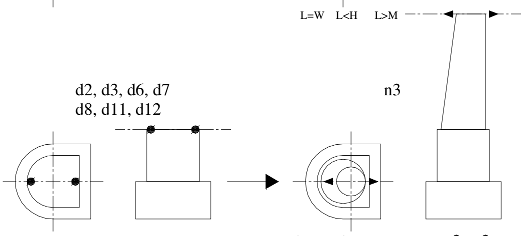
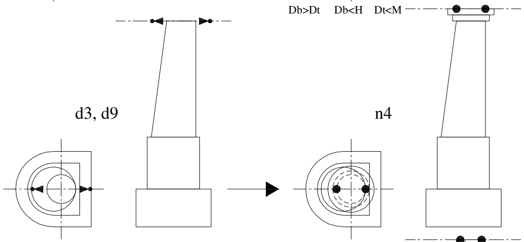
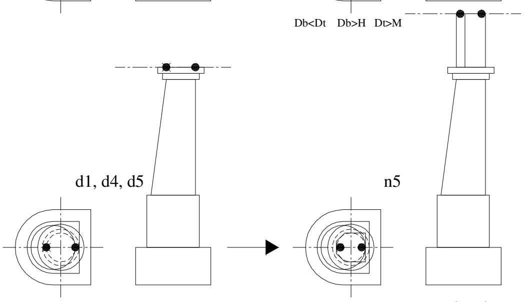
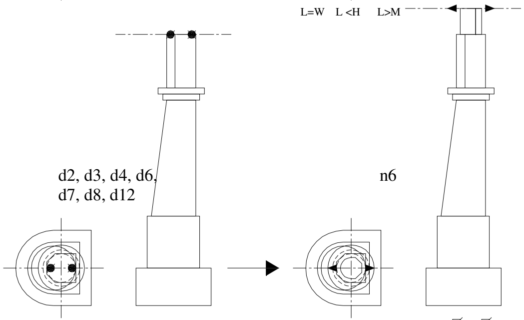
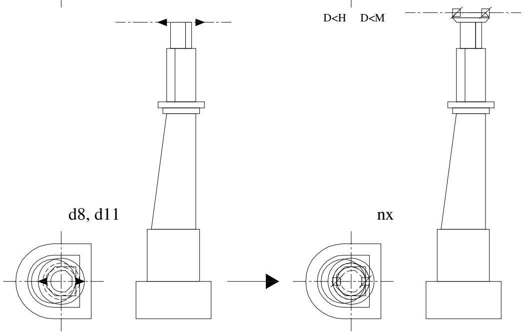
Rule no.	Pair 3: Hybrid design derivation - 10 Hybrid rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
HR 2e		n8 = {n7\d2, d3} n8 = {Ø} n8 = n8* n8* = {m\d2, d3} n8* = {d5', d10', d11', d12'}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.75	Design diversity = 1.0 Design abundance = 3.5 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.604
HR 17c		n9 = {n8\ d9, d11} n9 = {Ø} n9 = n9* n9* = {m\d9, d11} n9* = {d5, d10, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.666	Design diversity = 1.0 Design abundance = 3.333 Matching degree = 0.222 Geometrical difference = 1.0 Sequential difference = 0.61
HR 7f		n10 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 3.3 Matching degree = 0.2 Geometrical difference = 1.0 Sequential difference = 0.549
			**Pair 3** **Hybrid design - HR** **No. of Hybrid Rules = 10** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 3.3** **Matching= 0.2** **Geometrical diff.= 1.0** **Sequential diff.= 0.549**	

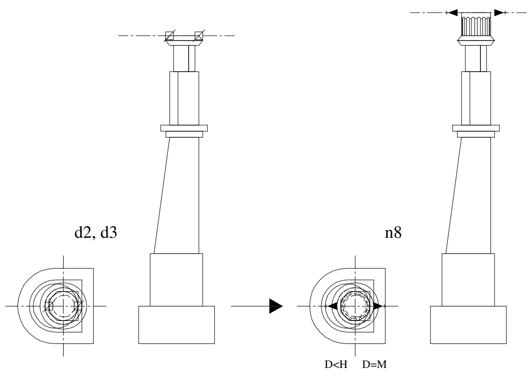
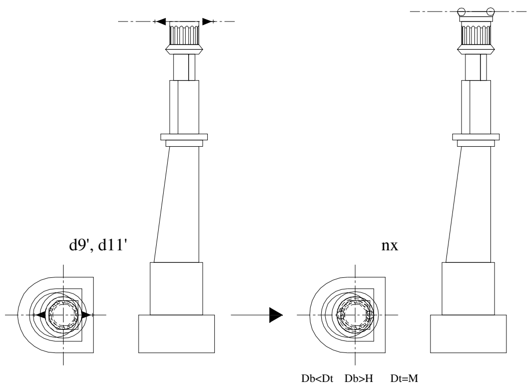
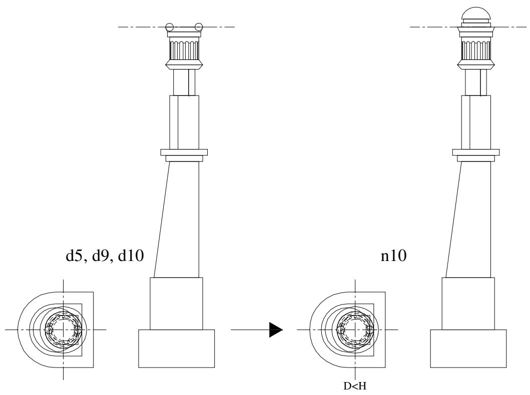
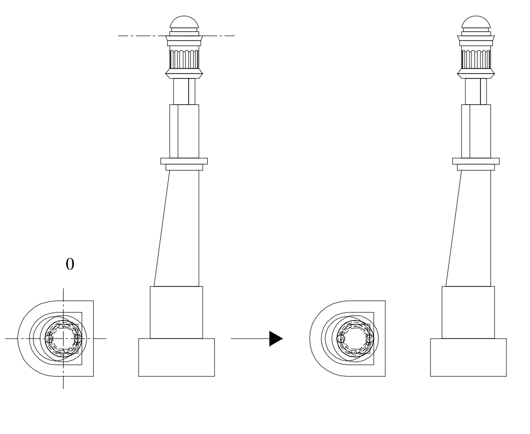
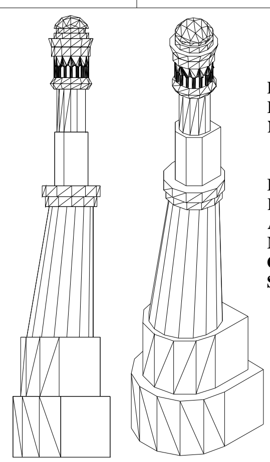
Rule no.	Pair 4: Hybrid design derivation - 6 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 11a		n1 = {n\d5, d12} n1 = {d1, d2, d3, d4, d6, d7, d8, d9, d10, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 22b		n2 = {n1\d3, d11} n2 = {d1, d2, d4, d6, d7, d8, d9, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity= 0.333 Design abundance= 2.0 Matching degree= 0.25 Geometrical difference= 1.0 Sequential difference= 0.416
HR 5c		n3 = {n2\d4, d6, d7, d8} n3 = {d1, d2, d9, d10} n3 = {Ø} n3 = n3* n7* = {m\d4, d6, d7, d8} n7* = {d3, d5, d11, d12}	Rule Prevalence = 0.333 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 2.666 Matching degree = 0.166 Geometrical difference = 1.0 Sequential difference = 0.444
HR 12d		n4 = {n3\d3, d4, d6, d7, d8} n4 = {d1, d2, d9, d10}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 3.25 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.458
HR 23b		n5 = {n4\d3, d10} n5 = {d1, d2, d9}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.583	Design diversity = 0.75 Design abundance = 3.0 Matching degree = 0.3 Geometrical difference = 1.0 Sequential difference = 0.483
HR 1f		n6 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 0.833 Design abundance = 3.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.402
			**Pair 4** **Hybrid design - HR** **No. of Hybrid Rules = 6** **Evaluation Criteria:** **Diversity= 0.833** **Abundance= 3.0** **Matching= 0.333** **Geometrical diff.= 1.0** **Sequential diff.= 0.402**	

Rule no.	Pair 4: Hybrid design derivation - 10 Hybrid rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
HR 11a		n1 = {n\d5, d12} n1 = {d1, d2, d3, d4, d6, d7, d8, d9, d10, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.166 Design abundance = 2.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 22b		n2 = {n1\d3, d11} n2 = {d1, d2, d4, d6, d7, d8, d9, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.333 Design abundance = 2.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.416
HR 5c		n3 = {n2\d4, d6, d7, d8} n3 = {d1, d2, d9, d10} n3 = {Ø} n3 = n3* n3* = {m\d4, d6, d7, d8} n3* = {d3, d5, d11, d12}	Rule Prevalence = 0.333 Geometrical difference = 1.0 Sequential difference= 0.5	Design diversity = 0.666 Design abundance = 2.666 Matching degree = 0.166 Geometrical difference = 1.0 Sequential difference = 0.444
HR 12d		n4 = {n3\d3, d4, d6, d7, d8} n4 = {d1, d2, d9, d10}	Rule Prevalence = 0.416 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.666 Design abundance = 3.25 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.458
HR 13b		n5 = {n4\d2, d3, d4, d6, d7, d8, d12} n5 = {d1, d9, d10}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.75 Design abundance = 4.0 Matching degree = 0.3 Geometrical difference = 1.0 Sequential difference = 0.433
HR 20c		n6 = {n5\d3, d9} n6 = {d1, d10} n6 = {Ø} n6 = n6* n6* = {m\d3, d9} n6* = {d2, d5, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.833 Design abundance = 3.666 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.444
HR 4d		n7 = {n6\d2, d4} n7 = {d1, d10}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.833 Design abundance = 3.428 Matching degree = 0.285 Geometrical difference = 1.0 Sequential difference = 0.452

Rule no.	Pair 4: Hybrid design derivation - 10 Hybrid rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
HR 16b		n8 = {n7\d1, d4, d5} n8 = {d10'}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.916 Design abundance = 3.375 Matching degree = 0.312 Geometrical difference = 1.0 Sequential difference = 0.458
HR 23b		n9 = {n8\d3, d10} n9 = {Ø} n9 = n9* n9* = {m\d3, d10} n9* = {d1, d9, d11}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.583	Design diversity= 1.0 Design abundance= 3.222 Matching degree= 0.277 Geometrical difference= 1.0 Sequential difference= 0.472
HR 1f		n10 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 3.2 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.424
				**Pair 4** **Hybrid design - HR** **No. of Hybrid Rules = 10** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 3.2** **Matching= 0.25** **Geometrical diff.= 1.0** **Sequential diff.= 0.424**

Rule no.	Pair 4: Hybrid design derivation - 6 Hybrid rules	User guide grammar for a hybrid design	Evaluation of the rule	Evaluation of the design
HR 10a	d3, d10, d12 n1 	n1 = {n\d3, d10, d12} n1 = {d1, d2, d4, d5, d6, d7, d8, d9, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 11b	d1, d4, d6 n2 	n2 = {n1\d1, d4, d6} n2 = {d2, d5, d7, d8, d9, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 3.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5
HR 5b	d2, d3, d6, d7 d8, d11, d12 n3 	n3 = {n2\d2, d3, d6, d7, d8, d11, d12} n3 = {d5, d9}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.833 Design abundance = 4.333 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.444
HR 18c	d3, d9 n4 	n4 = {n3\d3, d9} n4 = {d5} n4 = {Ø} n4 = n4* n4* = {m\d3, d9} n4* = {d1', d2', d4', d7', d8', d10', d11'}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 0.916 Design abundance = 3.75 Matching degree = 0.375 Geometrical difference = 1.0 Sequential difference = 0.541
HR 1e	d2', d3' n5 	n5 = {n4\d3, d2} n5 = {d5}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.916	Design diversity = 0.916 Design abundance = 3.4 Matching degree = 0.4 Geometrical difference = 1.0 Sequential difference = 0.616
HR 6f	d5, d9, d10 n6 	n6 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 3.333 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.513
	0 		**Pair 5** **Hybrid design - HR** **No. of Hybrid Rules = 6**	**Evaluation Criteria:** **Diversity= 1.0** **Abundance= 3.333** **Matching= 0.333** **Geometrical diff.= 1.0** **Sequential diff.= 0.513**

Rule no.	Pair 5: Hybrid design derivation - 10 Hybrid rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
HR 10a	d3, d10, d12 n1  L=W L>H L>M	n1 = {n\d3, d10, d12} n1 = {d1, d2, d4, d5, d6, d7, d8, d9, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.25 Design abundance = 3.0 Matching degree = 0.5 Geometrical difference = 1.0 Sequential difference = 0.5
HR 11b	d1, d4, d6 n2  L=W L<H L>M	n2 = {n1\d1, d4, d6} n2 = {d2, d5, d7, d8, d9, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.5 Design abundance = 3.0 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5
HR 5b	d2, d3, d6, d7 d8, d11, d12 n3  Db>Dt Db<H Dt<M	n3 = {n2\d2, d3, d6, d7, d8, d11, d12} n3 = {d5, d9}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 0.833 Design abundance = 4.333 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.444
HR 19c	d3, d9 n4  Db<Dt Db>H Dt>M	n4 = {n3\d3, d9} n4 = {d5}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 0.916 Design abundance = 3.75 Matching degree = 0.375 Geometrical difference = 1.0 Sequential difference = 0.458
HR 8b	d1, d4, d5 n5  L=W L <H L>M	n5 = {n4\d1, d4, d5} n5 = {Ø} n5 = n5* n5* = {m\d1, d4, d5} n5* = {d2, d7, d8, d9, d10, d11}	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.5	Design diversity = 1.0 Design abundance = 3.4 Matching degree = 0.3 Geometrical difference = 1.0 Sequential difference = 0.466
HR 13b	d2, d3, d4, d6, d7, d8, d12 n6  D<H D<M	n6 = {n5\d2, d3, d4, d6, d7, d8, d12} n6 = {Ø} n6= n6* n6* = {m\d2, d3, d4, d6, d7, d8, d12} n6* = {d5, d9, d10, d11}	Rule Prevalence = 0.583 Geometrical difference = 1.0 Sequential difference = 0.333	Design diversity = 1.0 Design abundance = 4.0 Matching degree = 0.333 Geometrical difference = 1.0 Sequential difference = 0.444
HR 21c	d8, d11 nx  Db<Dt Db>H Dt=M	n7 = {n6\d8, d11} n7 = {Ø} n7 = n7* n7* = {m\d8, d11} n7* = {d5, d9, d10} n7* = {Ø} n7* = n7** n7** = {n\(n7+n7*)} n7** = {d1, d2, d3, d4, d6, d7, d12}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.833	Design diversity = 1.0 Design abundance = 3.857 Matching degree = 0.285 Geometrical difference = 1.0 Sequential difference = 0.499

Rule no.	Pair 5: Hybrid design derivation - 10 Hybrid rules	User guide grammar for original design	Evaluation of the rule	Evaluation of the design
HR 2e		n8 = {n7\d3, d2} n8 = {Ø} n8 = n8* n8* = {m\d 2, d3} n8* = {d5, d9, d10}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.75	Design diversity = 1.0 Design abundance = 3.5 Matching degree = 0.25 Geometrical difference = 1.0 Sequential difference = 0.531
HR 17c		n9 = {n8\ d9, d11} n9 = {Ø} n9 = n9* n9* = {m\d9, d11} n9* = {d5, d10}	Rule Prevalence = 0.166 Geometrical difference = 1.0 Sequential difference = 0.666	Design diversity = 1.0 Design abundance = 3.333 Matching degree = 0.222 Geometrical difference = 1.0 Sequential difference = 0.546
HR 6f		n10 = 0	Rule Prevalence = 0.25 Geometrical difference = 1.0 Sequential difference = 0.0	Design diversity = 1.0 Design abundance = 3.3 Matching degree = 0.2 Geometrical difference = 1.0 Sequential difference = 0.491
			**Pair 5** **Hybrid design - HR** **No. of Hybrid Rules = 10** **Evaluation Criteria:** **Diversity= 1.0** **Abundance= 3.3** **Matching= 0.2** **Geometrical diff.= 1.0** **Sequential diff.= 0.491**	