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# CONTROL CO-DESIGN OF WAVE ENERGY CONVERTERS

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*It just works.*

– Todd Howard

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## *Abstract*

A novel optimisation model is presented that uses a control co-design (CCD) methodology to optimise the power take-off (PTO) and floater design of wave energy converters (WECs). The model is used to investigate the importance of applying a CCD methodology to WEC design.

CCD is a design philosophy that integrates the design stages of the physical device and controller to create a more effective overall design. Conventional design methodologies leave the controller design until after device design has been finalised; opening the possibility that the locked-in WEC design can cause difficulties for the subsequent controller design. If the two stages are integrated – as a part of CCD – the conflict could be corrected before the design is finalised.

The PTO uses a permanent magnet synchronous generator (PMSG) that is simulated using a lumped parameter model to estimate electromechanical losses within the drive train giving the output electrical power as a function of input mechanical force and speed. By using a genetic algorithm, the design of the PMSG (number of pole pairs, depth of magnets, rotor radius, *etc.*) is optimised to a user-selected control strategy, floater design, and wave climate. Candidate PMSG designs are tested in a suite of monochromatic-sea-state power simulations. Sea states are selected from historical wave data with the test sample found using a  $k$ -means clustering algorithm. The number of sea states required for a well-optimised design is investigated: finding that 20 sea states is the point of negligible improvement.

First the PMSG is optimised alone, to investigate differences when varying between: linear damping and impedance matching control; narrow- and broad-banded floaters; and a low-, medium-, and high-energy site. Results show that the optimal design is significantly different in each of the three investigations. Devices that can operate more effectively away from resonance tend to be larger in size and can extract more power. Devices that operate less effectively away from resonance perform better when smaller and cheaper, even at the cost of reduced electrical output: it is found that when using LD control material cost of the generator can be reduced by 60% with only a 5% reduction in electrical power output.

Then the model is then expanded to allow for Froude scaling of the floater: facilitating the novel co-optimisation of PTO design and floater scale. The comparison of performance of WECs designed with: PTO optimisation only; serial optimisation of PTO

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and floater scale; and co-optimisation of PTO and floater scale finds improved performance with a more integrated design process, further supporting the case of utilising CCD methodologies. It is found that co-optimisation of PTO and floater can lead to an up to 90% increase in electrical power output when compared to PTO optimisation alone.

Finally, the PTO designs are tested in a polychromatic-sea-state power simulation. Optimised designs outperform the non-optimised designs: showing that the monochromatic approximation is capturing an intrinsic property of the waves.

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## Introduction

In the transition to decarbonising electricity generation, power sources such as wind and solar have grown to be significant contributors to overall electricity production, whilst marine energy sources – such as wave and tidal – have not. The thesis therefore investigates whether a control co-design (CCD) methodology can be used to improve the effectiveness of wave energy converter (WEC) designs.

### OVERVIEW OF THE THESIS

The thesis is structured as follows:

- § I presents the background theory required to design a wave energy converter: starting with a description of the ocean energy resource before moving onto how WECs are modelled, controlled, and designed. The chapter concludes with the formation of the research question for the work.
- § II describes the methodology used to develop the model. The chapter begins by defining a lumped parameter model of a permanent magnet synchronous generator (PMSG) before incorporating it into a larger drive-train model. Finally, it details how the power take-off (PTO) model is integrated into a genetic algorithm to optimise the PMSG design.
- § III uses the model developed in § II to investigate differences in optimal PMSG design when varying the control strategy, WEC’s floater design, and the site location.
- § IV expands the model to allow for the co-optimisation of both the PMSG design and the scaling of the floater. The chapter first introduces the methodology for Froude scaling the floaters, then investigates the differences in optimal designs when varying: the control strategy used; the WEC’s floater design; and the site location. Both the serial and simultaneous optimisation of PTO design and floater scale are investigated here.
- § V details the creation of *synthetic* PTO designs for a wave-tank-scale WEC. These designs are then used in a polychromatic-sea-state power simulation to investigate the performance difference in monochromatic and polychromatic electrical power output, and between optimised and non-optimised PTO designs.
- § VI presents the final conclusions of the thesis, and options for future work.

## AIMS OF THE THESIS

As is explored in more detail in §I.3, there has been a growing interest in the control co-design (combining the physical device and controller design stages) of WECs but this has focussed primarily on the design of the WEC’s floater, with little consideration of the design of the PTO, or to the effect of the PTO’s dynamics and efficiencies on the optimal floater design.

This thesis therefore details the design of a novel PTO-optimisation model to optimise the internal geometry of a PMSG (*e.g.* number of pole pairs and rotor radius). It then expands the model for both the serial and simultaneous co-design of PTO and floater, investigating the performance benefits of integrating the design of distinct WEC components. Further details regarding the operation of the model and design of the test WEC are provided in §I.5, following the presentation of the background theory.

## CONTRIBUTIONS TO KNOWLEDGE

This thesis makes the following contributions to knowledge:

- The application of a higher-fidelity PTO model than has been used in CCD literature to date.
- The creation of a novel CCD optimisation model that co-optimises the PTO and floater of a WEC according to a defined control strategy.
- The rigorous testing of the optimiser to investigate changes in optimal WEC design when varying the control strategy, floater design, and site conditions.

## PUBLICATIONS

Matthew Onslow and Adam Stock. “Genetic algorithm based co-design of wave energy converter power take-off, floater geometry, and control”. In: *Proceedings of the 16<sup>th</sup> European Wave and Tidal Energy Conference*. 2025. DOI: [10.36688/ewtec-2025-1013](https://doi.org/10.36688/ewtec-2025-1013)

Matthew Onslow and Adam Stock. “Genetic Algorithms for the Control Co-Design of Permanent Magnet Synchronous Generators for Wave Energy Converters”. In: *Applied Ocean Research* (Under review)

David Campos-Gaona et al. “HAPiWEC – Progress update on the remote-lab for rapid testing of non-linear controllers for wave energy converters”. In: *Proceedings of OCEANS 2025*. 2025. DOI: [10.1109/OCEANS58557.2025.11104634](https://doi.org/10.1109/OCEANS58557.2025.11104634)

Matthew Onslow and Adam Stock. “Genetic Algorithms for Design of Optimal Velocity Tracking Controllers Including PTO Efficiencies”. In: *2024 UKACC 14<sup>th</sup> International Conference on Control*. 2024. DOI: [10.1109/CONTROL60310.2024.10532033](https://doi.org/10.1109/CONTROL60310.2024.10532033)

Alexandra Price et al. “HAPiGYM – Two Rapid Prototyping Environments for Wave Energy Control”. In: *Proceedings of the 15<sup>th</sup> European Wave and Tidal Energy Conference*. 2023. DOI: [10.36688/ewtec-2023-499](https://doi.org/10.36688/ewtec-2023-499).

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## *Mathematical Notation & Nomenclature*

The thesis covers topics in a wide variety of topics – from magnetic field analysis to control theory – therefore, the mathematical notation of the thesis has been chosen to help the reader follow which domain the current section of work is contained within. Any variable in an italic font (*e.g.*  $X$ ) is a mechanical, or numerical value; any in a roman font (*e.g.*  $X$ ), is an electrical variable; and any in a calligraphic font (*e.g.*  $\mathcal{X}$ ) is magnetic.

An alphabetised list of the acronyms and initialisms used at multiple points throughout the thesis is found in the table below; for convenience they are also defined the first time they are used in any section.

|      |                                        |
|------|----------------------------------------|
| BEM  | Boundary element method                |
| CCD  | Control co-design                      |
| DoF  | Degrees of freedom                     |
| EMF  | Electromotive force                    |
| EMEC | European Marine Energy Centre          |
| GA   | Genetic algorithm                      |
| IM   | Impedance matching (control)           |
| LCoE | Levelised cost of energy               |
| LD   | Linear damping (control)               |
| MMF  | Magnetomotive force                    |
| PMSG | Permanent magnet synchronous generator |
| PTO  | Power take-off                         |
| RMS  | Root mean squared                      |
| WEC  | Wave energy converter                  |

# CHAPTER I

---

## *Background & Theory of Wave Energy*

This chapter includes an in-depth review of the literature and theory, acting as a foundation for the rest of the thesis. The chapter builds up the theory: starting with a discussion on the wave resource; before moving onto how wave energy converters are modelled, designed, and controlled. The chapter ends with the formulation of the research question.

## *Ocean Waves*

Before discussing how energy is extracted from the oceans, a foundation of wave theory is required. This section covers the basics: starting with the origins of ocean waves; before moving onto the statistical and mathematical representations of the waves; and, ending with a comparison to other renewable energy sources.

### I.1.1 ORIGINS OF OCEAN WAVES

The most common waves in the ocean are called *wind waves* [1]: which, as the name suggests, result from interactions between the wind and sea. When the wind first blows over a still ocean surface, a phenomena known as *cat's paws* first appears [2]: a ruffling, and darkening of the surface, as if a cat was scratching at it. Should the wind keep blowing, ripples and then waves will begin to appear, the size of which is dependant on the amount of energy transferred into the water. The size of waves are therefore largely dependant on the strength, duration, and fetch (the distance the wind blows over) of the wind. The height and period of the waves grow in tandem at first, then the height hits a limit (dependant on limiting forces such as the viscosity of the water) at which point the period will increase alone.

As the waves grow larger, the wind can form new ripples upon them, which in turn can grow into waves; this creates a complex collection of waves which when combined with the fact that waves with different periods move at different speeds, results in a constantly changing interference pattern on the ocean's surface.

FIGURE I.1.1 is a recreation from work by Munk [1]. It shows a schematic spectrum of the frequency of different periods of ocean waves across the globe. The two highest energy density regions are from the wind waves (1 to 30 s) and from the tides (12 and 24 hr); these two regions are the focus of energy extraction by wave energy conversion and tidal energy conversion, respectively.

#### I.1.1.1 A WAVE OF ENERGY

Within an ocean wave individual particles of water do not move over any great distance. They instead move locally in a circular motion under the water column and it is energy that is transmitted over long distances. The radius of the circular motions is affected by the depth of the particles: at the still water level (the level the water would be at if

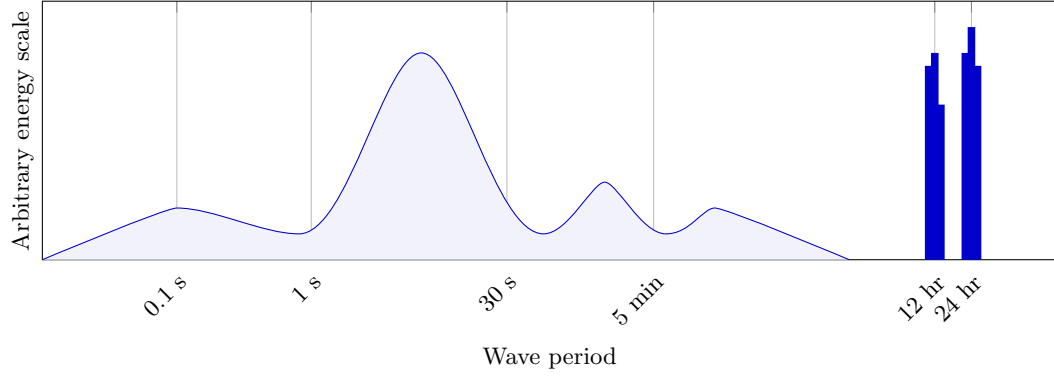


FIGURE I.1.1: Schematic spectrum of ocean waves as a function of wave period. (Recreated from [1])

there were no waves) the radius is the wave's amplitude; and this reduces going deeper into the water, reaching zero radius at a depth of around half the wavelength [3]. This point is called the *wave base*. Here, the majority of the wave power is contained within the top 10 to 20 m of the seas [4].

#### I.1.1.2 THE GEOGRAPHIC DISTRIBUTION

The schematic spectrum in FIGURE I.1.1 is the spatial average across the globe; however, the seas in different locations are dramatically different. Places exposed to longer fetches have larger waves and therefore more power. FIGURE I.1.2 shows the available wave power in UK waters, as can be seen the places exposed to the Atlantic have more power than those in the Irish sea.

The available energy is significant; in UK waters alone, the exploitable wave energy resource is estimated to be 40 TWhr/yr [6] (approximately 15% of current electricity demand [6]); if this could be effectively harnessed, wave energy could contribute significantly to the UK electricity supply.

### I.1.2 DESCRIBING SEA STATES

In order to effectively design devices to harness ocean wave power, a description of the power distribution, at any location, at any moment is required. Wave energy is generally described by two data types: wave spectra, and incidence matrices. These describe the waves in a moment, and over longer time periods, respectively.

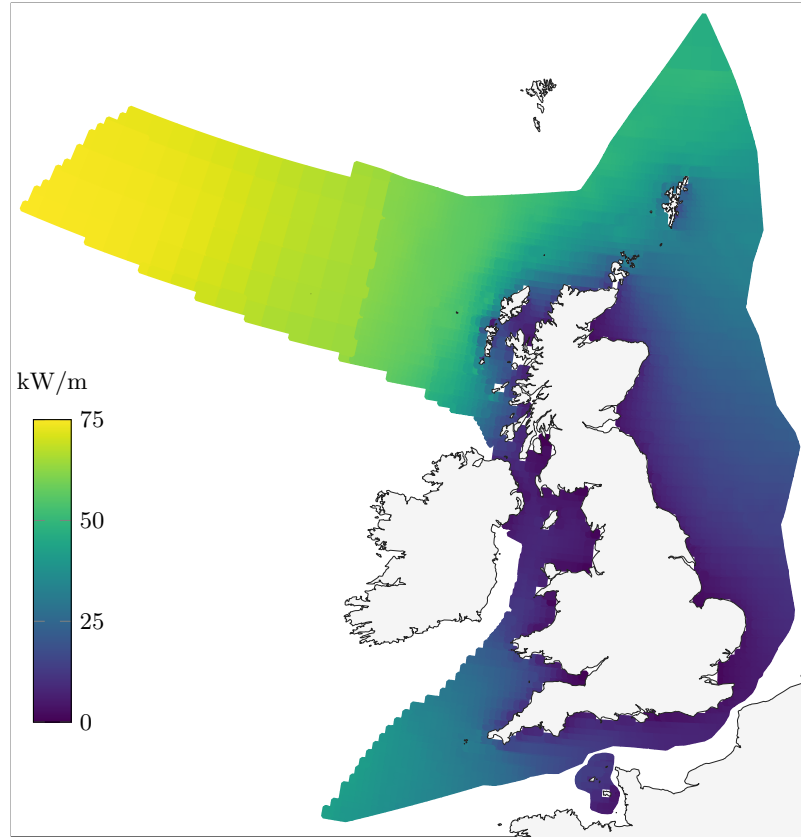


FIGURE I.1.2: Average annual wave power in UK waters. (Data from [5], averages were calculated from hourly recordings over seven years.)

#### I.1.2.1 WAVE SPECTRA

There are a variety of spectra used throughout the literature; two of the more commonly used spectra include the Pierson-Moskowitz spectrum [7], and the JONSWAP spectrum [8]. Here both spectra are described in turn; and then shortcomings of spectral definitions are examined.

**THE PIERSON-MOSKOWITZ SPECTRUM:** In 1964, Pierson and Moskowitz proposed their spectral representation of a fully-developed sea [7]:

$$S(\omega) = \frac{\alpha g^2}{\omega^5} \exp \left[ -\beta \left( \frac{\omega_0}{\omega} \right)^4 \right], \quad (\text{I.1.1})$$

where  $\omega$  is the wave frequency;  $g$  is the gravitational acceleration;  $\omega_0 = g/u_{19.5}$  is a wind-dependent frequency parameter where  $u_{19.5}$  is the wind speed measured at a height of 19.5 m above sea level<sup>i</sup>;  $\alpha$ , called the Phillips constant [10], and  $\beta$  are two

<sup>i</sup>The standard measurement height is 10 m to convert it is usually assumed that  $u_{19.5} = 1.026u_{\text{avg}}$  where  $u_{\text{avg}}$  is the average wind speed [9].

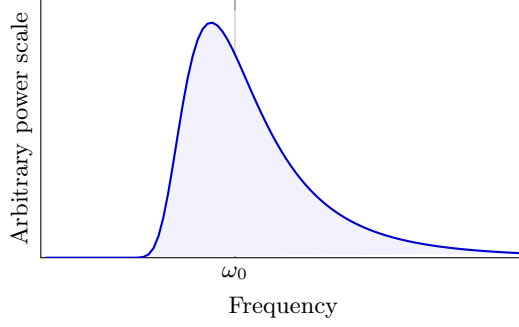


FIGURE I.1.3: An example Pierson-Moskowitz spectrum.

non-dimensional constants. The value of  $\alpha$  and  $\beta$  are variable in time and location; for their work, Pierson and Moskowitz used values of  $\alpha = 8.1 \times 10^{-3}$  and  $\beta = 0.74$ . An example of the spectra is shown in FIGURE I.1.3.

The Pierson-Moskowitz spectrum was developed only using only data from fully-developed seas: the waves have grown as large as the local wind is capable of growing them. Therefore, in a non-fully-developed sea the representation may be invalid.

**THE JONSWAP SPECTRUM:** The joint North Sea wave project (JONSWAP) took place in 1969, investigating the effect of fetch on wave spectra. As a part of the study, empirical studies by Hasselmann *et al.* found the following spectral definition [8],

$$S(\omega) = \frac{\alpha g^2}{\omega^5} \exp \left[ -\frac{5}{4} \left( \frac{\omega_p}{\omega} \right)^4 \right] \gamma^A, \quad (\text{I.1.2})$$

where  $\omega_p$  is the peak wave frequency;  $\gamma$  is the ratio of maximal spectral energy;

$$A = \exp \left[ \frac{-(\omega - \omega_p)^2}{2\sigma^2\omega_p^2} \right]; \quad (\text{I.1.3})$$

and,

$$\sigma = \begin{cases} \sigma_l & \text{if } \omega < \omega_p, \\ \sigma_r & \text{if } \omega > \omega_p; \end{cases} \quad (\text{I.1.4})$$

$\sigma_l$ ,  $\sigma_r$  are the left and right spectrum widths, respectively. The key difference between JONSWAP and Pierson-Moskowitz spectra is the inclusion of the *peak enhancement factor*,  $\gamma^A$ , which – as its name suggests – affects the size of the peak of the spectrum. Examples of the spectra, with a range of  $\gamma$  values are shown in FIGURE I.1.4. Note that when  $\gamma^A = 1$ , the JONSWAP spectrum reduces to a Pierson-Moskowitz spectrum. In their report, Hasselmann *et al.* found that the average sea state across the North Sea had shape parameters  $\gamma = 3.3$ ,  $\sigma_l = 0.07$ , and  $\sigma_r = 0.09$  [8]. By using the peak enhancement factor, the spectra can be tuned to represent non-fully-developed seas.

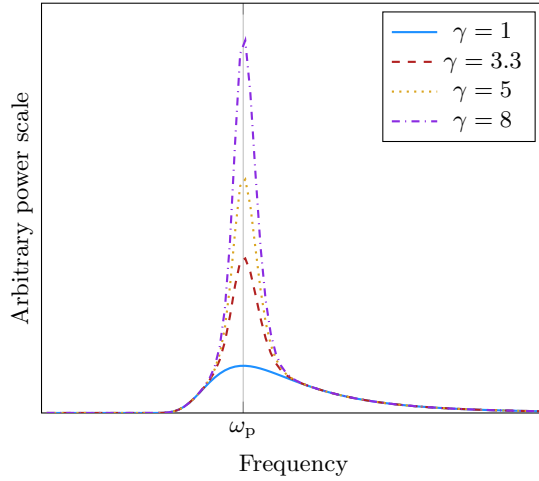


FIGURE I.1.4: A range of JONSWAP spectra, with different peak enhancement factors.

Another output of the project was the observation that the Phillips constant is in fact not a constant but is instead dependant on fetch; they empirically found that

$$\alpha = 0.076 \left( \frac{gx}{u_*^2} \right)^{-0.22}, \quad (\text{I.1.5})$$

where  $x$  is the fetch, and  $u_*$  is the friction velocity.

**SHORTCOMINGS OF SPECTRAL DEFINITIONS:** Both the Pierson-Moskowitz and JONSWAP spectra are unimodal spectra: meaning they only have one peak. However, the seas at a particular location often comprise both wind waves, formed locally, and swell, formed by wind elsewhere and propagated – often over long distances [11]; these separate wave systems often have different spectral peaks. Analysis has shown that in the North Atlantic, bimodal spectra occur on average 22% of the time, and up to 40% in low-wave-height sea states [12]. Work has therefore been done to define a spectrum that can describe multimodal sea states. One example is by Mackay which found sea states can be well described by simply summing two JONSWAP spectra [13]; similar work into multimodal spectra has also been done by others, for example in [11, 12, 14].

These different wave systems – as well as having differing spectral shapes – will also likely be moving in different directions. Hegermiller *et al.* worked on a methodology for separating a multimodal spectrum into a set of *wave families* which are independant spectra, each with their own peak period, significant wave height, and direction [11]. Whether or not this is important depends on the energy extraction technology: take a device that needs to be aligned to the direction of the waves; in the case of multidirectional waves, which direction should it face?

It should also be noted that multidirectional waves can also occur from a single wave system [15], albeit with less spreading; equally, in a multimodal system, the

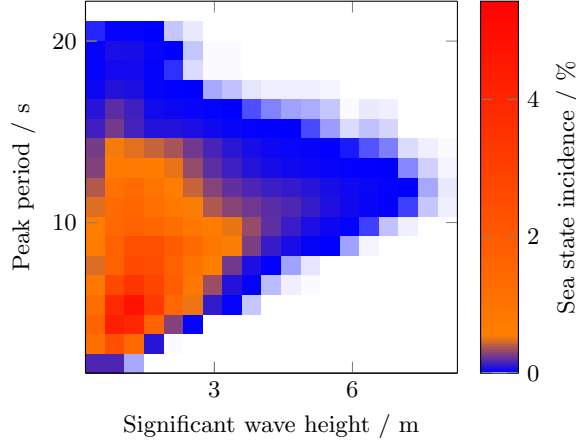


FIGURE I.1.5: Example of a sea state probability incidence plot. (Data from [17])

directionality is not a set of discrete wave directions. In both cases, there is an amount of direction spreading around the wind direction. Using collected directional data, Mitsuyasu *et al.* defined a directional spreading function,  $D(\omega, \theta)$  [15], such that:

$$S(\omega, \theta) = S(\omega)D(\omega, \theta), \quad (\text{I.1.6})$$

where  $S(\omega, \theta)$  is the directional wave spectrum; and

$$D(\omega, \theta) = Q(s) \cos^{2s} \left( \frac{|\theta - \theta_m(\omega)|}{2} \right), \quad (\text{I.1.7})$$

where  $\theta_m(\omega)$  is the mean wave direction at frequency  $\omega$ ;  $Q(s)$  is a normalisation factor; and  $s$  is the spreading parameter.  $s$  is defined differently in different works: see [16] for examples of different definitions, and for alternative forms of  $D(\omega, \theta)$ .

#### I.1.2.2 INCIDENCE MATRICES

As well as varying in space, sea states also vary in time; which is represented using an incidence matrix. Each sea state is reduced to its fundamental components – often just the significant wave height and peak period – then the matrix records the prevalence of every combination of values of the variables. FIGURE I.1.5 gives an example of a sea state probability incidence plot.

There is further loss of information in an incidence matrix. Taking the JONSWAP spectra as an example: which has the shape variables  $\gamma$ ,  $\sigma_l$ , and  $\sigma_r$ , information on these values are not recorded in most incidence matrices, so average values would have to be selected. Depending on how these values vary in reality, there may be a significant effect on device performance. The International Electrotechnical Commission's standards state that at minimum, the incidence matrix should include information about the significant wave height and peak period; but where possible, it should be expanded to include water depth; wave direction; spectral shape and directional spreading; water current; and tidal range [18].

### I.1.3 DESCRIBING OCEAN WAVES

A spectrum is useful for describing the sea state in the frequency domain, but a description is also needed in the time domain. This subsection discusses the conversion, to both a monochromatic (a single wave frequency), and a polychromatic (many wave frequencies) representation.

#### I.1.3.1 MONOCHROMATIC WAVES

Monochromatic waves simply represent the ocean's surface as a sine wave [19]:

$$\eta(t) = \text{imag} [Ae^{i\omega t}] , \quad (\text{I.1.8})$$

where  $\eta(t)$  is the ocean surface,  $A$  is the amplitude of the wave,  $\omega$  is the wave frequency,  $t$  is time; and  $\text{imag}[\cdot]$  takes the imaginary component of the argument.<sup>ii</sup>

There are multiple methods to finding the peak frequency of a spectrum. The simplest of which is to read the frequency at which the spectrum is at its maximum value; however, due to variability in measured wave spectra, other methods have been proposed. The International Association for Hydraulic Research (IAHR), have produced a list of wave parameters and how to calculate them, including a variety of methods for calculating the spectral peak frequency [20]. An example is the centroid method – also known as the Delft method – calculated via [21]

$$\omega_{\text{peak}} = \frac{\int_{\omega_b}^{\omega_a} \omega S(\omega) d\omega}{\int_{\omega_b}^{\omega_a} S(\omega) d\omega} , \quad (\text{I.1.9})$$

where  $\omega_a$  and  $\omega_b$  are the frequencies – above and below the spectral maximum frequency respectively – at which the spectra is at 80% its maximum value (60% has also been proposed as a threshold [21]). This method has been shown to have a 25% lower variability than reading the spectral peak [22].

Similarly to the peak frequency, there is a variety of methods to calculate different significant wave heights; again the IAHR produced a summary [20]. When starting from a spectral representation, one of the common definitions is [23]

$$H_{m_0} = 4\sqrt{m_0} , \quad (\text{I.1.10})$$

where

$$m_0 = \int_0^\infty S(\omega) d\omega . \quad (\text{I.1.11})$$

---

<sup>ii</sup>Alternatively, the surface could be represented by a cosine wave found by taking the real component of  $Ae^{i\omega t}$ .

Significant wave heights measure the peak-to-trough height of the waves, but the wave amplitude is from the centre-to-peak; converting between the two simply requires dividing by a factor of 2.

Modelling and testing with monochromatic seas is often the first step in the design of a wave energy device: to validate the underlying theory or to model of the device under simpler conditions. However, due to the inaccuracy of the representation, monochromatic waves should not be used as the only form of testing. The European Marine Energy Centre’s standards on wave tank testing states ‘*Whilst regular, monochromatic trials can be regarded as a method to learn how a device works, irregular, panchromatic trials begin to reveal how the device will perform.*’ [24].

#### I.1.3.2 POLYCHROMATIC WAVES

A point on the surface of an polychromatic sea state can be represented as [25]

$$\eta(t) = \sum_{j=1}^N A_j e^{i\omega_j t + \phi_j}; \quad (\text{I.1.12})$$

an extension of the monochromatic-wave sea to the sum of  $N$  monochromatic-waves, each with their own amplitude and frequency,  $A_j$  and  $\omega_j$ , and now including a phase shift,  $\phi_j$ .

To create a time-domain representation from a spectral definition: the spectra can be broken into  $N$  equally sized bins, each of a width  $\Delta\omega$ ; where the centre of the bin is the wave frequency,  $\omega_j$ . The amplitude of the waves can then be found via

$$A_j = \sqrt{2S(\omega_j)\Delta\omega}; \quad (\text{I.1.13})$$

and the phase shift should be randomly chosen from a uniform distribution in the interval  $[0, 2\pi)$  [25].

### I.1.4 IN COMPARISON TO OTHER RENEWABLE ENERGY SOURCES

Now that the method for modelling the wave energy resource and spectrum is described, this section briefly covers other renewable energy sources, and compares their energy distributions to that of ocean waves. Hence, the benefits and disadvantages of wave energy are discussed.

#### I.1.4.1 TIDAL ENERGY

As an energy resource, tidal energy is the most similar to wave energy: they both exist in the same medium, but at different periods. The tidal energy distribution is shown

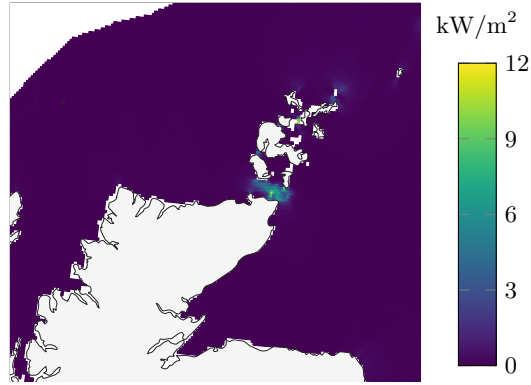


FIGURE I.1.6: Average annual tidal power around the north coast of Scotland and Orkney. (Data from [5])

in FIGURE I.1.1 of the ocean energy spectrum; tidal energy is focussed in two distinct peaks, at 12 and 24 hr periods, these arise from the relative positions, and motions of the Earth, Sun, and Moon. The mechanics of astronomical forcing is well known and this makes the tides the most predictable oceanographic phenomena; with root mean squared errors in elevation of 10 to 20 cm, and in the time of high water of the order of 10 min [26]. The high level of predictability gives tidal energy a benefit over wave (and wind) energy, and has led to work on using tidal energy as a base load for electricity grids [27]. However, the long periods mean that there are spans of time in which there is very little harvestable energy, so an electrical grid would struggle to operate off of tidal energy alone.

The biggest limiting factor for tidal energy is its geographic restrictions: tidal energy is best when constrained by landmasses into narrow waterways or basins that resonate with the tidal periods [28]. This means that tidal energy is very geographically restricted, as shown in FIGURE I.1.6 where the annual average tidal power is practically zero except for a few places around the Orkney islands.

#### I.1.4.2 WIND ENERGY

FIGURE I.1.7 shows the distribution of wind speeds as a function of period – also known as a van der Hoven spectrum [29]. There are two distinct peaks in the spectrum, at around 100 and 0.5 hours. The 100 hour peak corresponds to the transit period of a typical weather system defining the mean wind speed. The 0.5 hour peak corresponds to turbulence in the wind flow, causing short period variations in the wind speed. As a result, wind power also has to deal with the moment to moment variations in the available resource; however research has shown that wave energy is 23% more predictable than wind [30].

As mentioned in §I.1.1.1, ocean waves are a flow of energy and not matter<sup>iii</sup>: in-

<sup>iii</sup>In reality, there is some mass transport – known as Stoke’s drift – but it is primarily a flow of energy. See [31] for more information on Stoke’s drift.

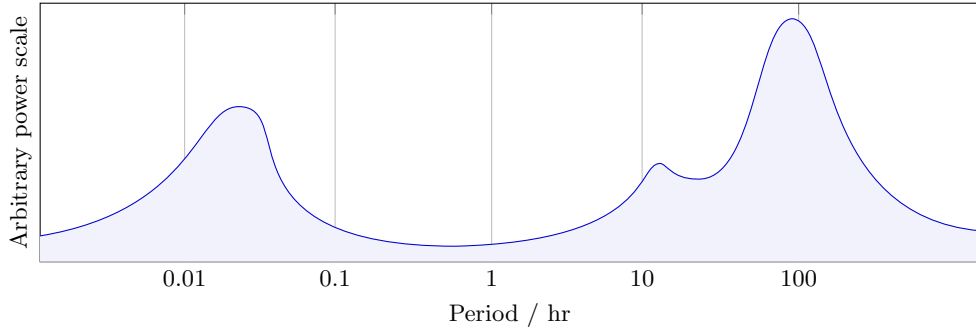


FIGURE I.1.7: Schematic power spectrum of wind speeds. (Recreated from [29])

dividual water particles oscillate locally in small circles, which when combined creates the overall flow of the wave. In wind energy however, individual air particles do move over long distances and it is their momentum that carries the energy. Wind energy devices therefore operate by the constant spinning of a rotor in the same direction<sup>iv</sup>; whereas wave energy devices typically oscillate with the waves and therefore have many loading cycles which are one of the main causes of fatigue in floating structures [33].

Another difference is that the two mediums, sea water and air, have vastly different mass densities: 1025 and 1.2 kg/m<sup>3</sup> respectively. The difference in density causes differences in the energy intensities of the two resources, where wave energy is roughly five times more energy intense than wind: 2 to 3 and 0.4 to 0.6 kW/m<sup>2</sup> respectively [34]; so a wind energy device must be much larger to harness the same amount of energy as a wave energy device. However, the increased density comes with the downside that the forces on the wave-energy devices are higher; increasing the forces on the wave-energy device and hence the cost to produce.

#### I.1.4.3 SOLAR ENERGY

The Sun is sometimes modelled as perfect blackbody radiator: it emits all wavelengths of light, in a spectrum defined by its temperature [35]. The spectral irradiance, arriving at Earth, assuming the Sun is a perfect blackbody is shown in FIGURE I.1.8.

In reality though, due to the Sun not being a perfect blackbody, and due to absorption and scattering from the atmosphere, the irradiance reaching the Earth's surface is better described by the BS ISO 9845-1:2022 reference spectrum, for designing solar energy devices [36] is also shown in FIGURE I.1.8.

Photovoltaic cells produce electrical power as long as the incident light has a higher energy than their band gap energy (a material property) [37]; so the actual spectral shape is of less importance than in other energy forms, only that the majority of the

<sup>iv</sup>There are some concept devices that do not work this way – *e.g.* [32] – but they are not the norm.

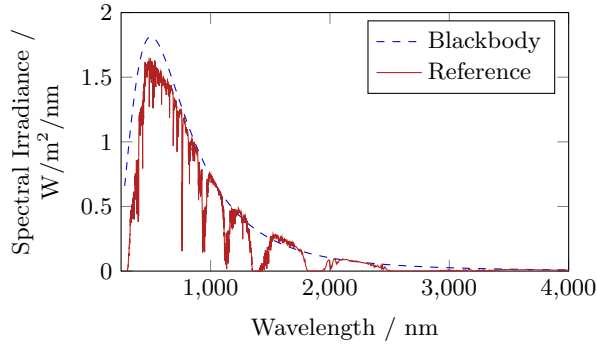


FIGURE I.1.8: Solar irradiance spectra. The blue dashed line assumes a black-body spectrum at 5800K; the red solid line is the BS ISO 9845-1:2022 reference spectrum [36].

energy is higher (at a smaller wavelength) than the band gap energy. A smaller band gap energy is not necessarily better however: larger band gaps increase the maximum achievable voltage; this comes at the cost of absorption, causing a reduction in the electrical current. The ideal band gap energy is in the range of 1-1.8 eV [37] (1240 to 690 nm wavelength).

#### I.1.5 IN SUMMARY

The wave energy resource exists in a midpoint between wind and tidal, exhibiting some of the benefits, and drawbacks of both. On the positive side, wave energy has the energy density of tidal energy, and the geographic freedom of wind. But, on the negative, it has the high forces of tidal and the unpredictability of wind energy.

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## *Wave Energy Converters*

Now that the waves are defined, the following section discusses how the energy in the waves is harnessed. Individuals have been working towards harnessing wave energy for a very long time: the first patent dates back to 1799 by a father and son named Girard in Paris. Their idea was to use a gigantic lever, forced by the motion of the waves to mechanically power machinery such as pumps and mills. However, nothing is known of any outcome of the idea [38]. In the UK, there has been an estimated 340 patents granted by 1973 [39] – showing a continued interest in harnessing wave energy.

The more modern movement of wave energy converters (WECs) for electrical power capture dates back to the mid-20<sup>th</sup> century in Japan, where work by Masuda – sometimes titled *the father of modern wave energy* – investigated powering navigation buoys using wave power [40]. In Europe, there was a surge in research and development following the 1973 oil crisis, when different European nations sought to increase their energy independence. Various governmental and private research programmes were set up throughout the continent, but research was mainly focussed in the UK, Portugal, Ireland, Norway, Sweden, and Denmark [41].

Of the flurry of European research, one of the best known outputs was by Salter [42] in the development of what became to be known as Salter’s duck. Salter’s duck had the theoretical capability to harness 90% of the oncoming wave energy – although in more realistic conditions this fell to around 50% [43]. By the early 1980s, oil prices had returned to their pre-crisis levels and interest in renewable energy research was falling, and therefore research budgets were also falling: the UK national renewable energy research budget was cut from £14 million to £11 million. And due to wind-energy research having more promising early results, wave-energy funding was hit the hardest by the cuts [44].

Since then, wind energy capture has continued to pull further and further ahead of wave energy. Across Europe there is now 285 GW of installed wind power capacity (both on- and off-shore), with 31.6 GW in the UK alone [45]. In contrast, there are no commercially operating wave energy plants anywhere on the planet. Forecasts from different policy advising groups suggest that this is not likely to change any time soon. The Supergen ORE hub<sup>i</sup> forecasts that the UK’s energy mix will include 6 GW of wave

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<sup>i</sup>The hub connects academia, industry, and policymakers to maximise the technological and societal outputs of offshore renewable energy research in the UK. see [46] for more details.



FIGURE I.2.1: Picture of the CorPower C4 WEC. (Picture taken on a site visit to Porto de Viana do Castelo)

power by 2050 [47]; by this date, the Climate Change Committee is recommending a wind energy capacity of 162 GW [48]. This is not to say there has been no progress in the installation of WECs, at the time of writing the CorPower C4 WEC has just been awarded the world's first DNV prototype certificate for a WEC [49]; FIGURE I.2.1 shows a picture of the C4 in the dock, ready to be installed at sea.

A problem with wave energy that is seen throughout this section is that WECs are made up of the coupling of many components, each with highly complex dynamics, and inefficiencies within a single component could cause the WEC's performance to massively drop off. To describe this problem, the remainder of this section first presents the various types of WEC designs before moving onto how they are modelled, and controlled.

### I.2.1 TYPES OF WAVE ENERGY CONVERTER

Due to the wave energy sector being in an early stage, there is yet to be any consensus on standard WEC design. Concept WEC devices can look extremely different to each other and interact with waves in completely different ways. FIGURE I.2.2 shows the relative popularity of different WEC design choices with developers. The wide variety in WEC designs means that there is no standard classification of WECs, different works use different distinctions, such as: how they interact with the waves (*eg.* [50]); or the type of power take-off (PTO) they use (*eg.* [51]). Each of the subplots in FIGURE I.2.2 could easily be its own classification.

The classification used in this work is of: point absorber, attenuator, or terminator; as defined by Falnes and Kurniawan [53]. The WECs are separated according to their geometry in relation to the waves – shown in FIGURE I.2.3. The point absorber is small in comparison to the wavelength, the terminator is large in the direction perpendicular to the wave direction, and the attenuator is large in the direction parallel to the wave direction. These groups are still very broad: they can be broken down into many subcategories, and there are WECs that fit into none of the categories; however, this classification provides a simple way define some of the more popular WEC designs.

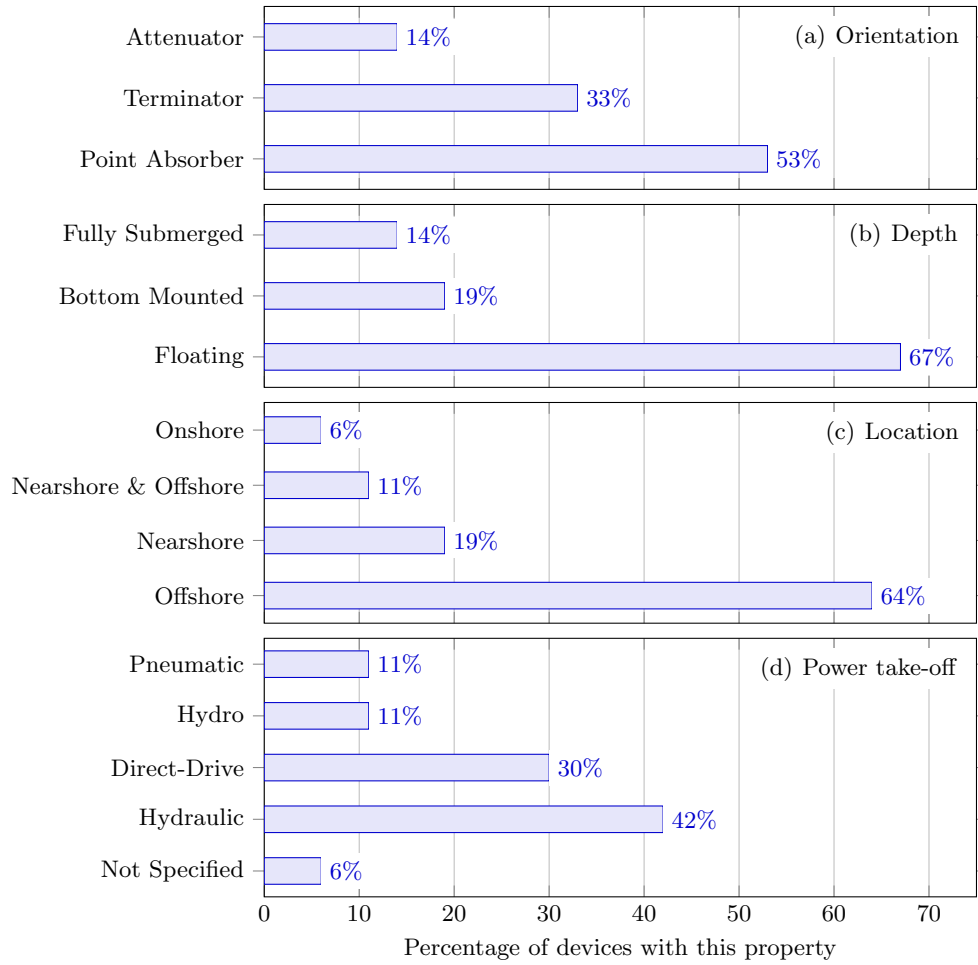


FIGURE I.2.2: The popularity of different wave energy converter technologies used by developers. (Data from [52])

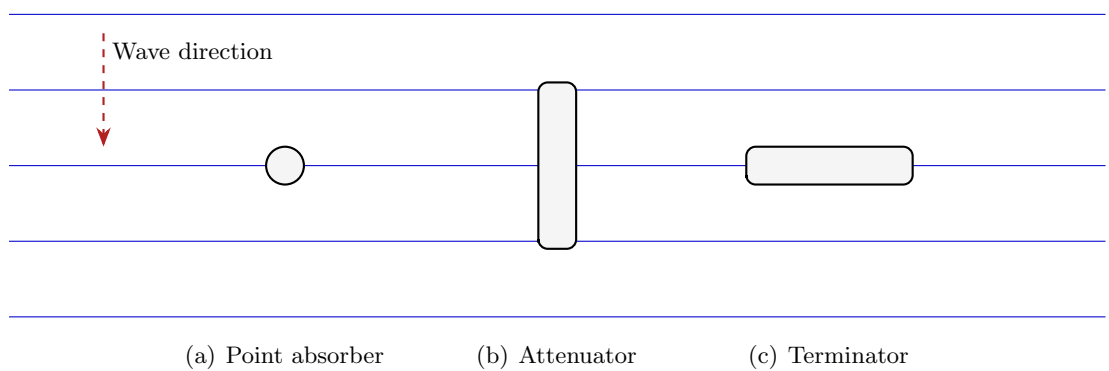


FIGURE I.2.3: Classification of wave energy converter.

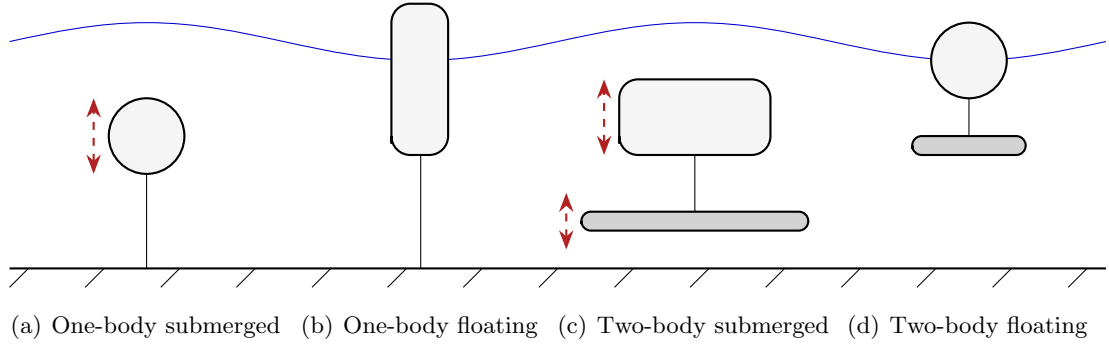


FIGURE I.2.4: Examples of point absorber wave energy converters.

### I.2.1.1 POINT ABSORBER

Point absorbers are one of the simpler WECs to design and manufacture [54], which has therefore made them very popular within the literature. In the modelling of point absorbers, the assumption is often made the device is sufficiently small enough that the diffraction of the waves around the device is negligible; this puts a limit on the size of point absorbers of around  $1/20^{\text{th}}$  the wavelength of the incident waves [55]. For devices larger than this, but still of a similar design, Falnes and Kurniawan suggest these devices be called *quasi-point absorbers* [56].

FIGURE I.2.4 gives a breakdown of various types of point absorber: for the single body devices, they extract energy from their motion relative to the sea bed; for the two body it is the relative motion of the two bodies. Although two-body devices add dynamical complexity, they are easier to install in deep-sea regions. Some examples of various types of point absorber devices include: CorPower C4 [57]; CETO 6 [58]; and, Seabased [59]. The shape of the buoys can also vary between devices, ranging from simple shapes like spheres and cylinders to more complicated, composite shapes.

A comparison by Sergiienko *et al.* [60] investigated the performance differences between floating and submerged point absorbers, finding the floating devices have a larger power output. This is due to the density of wave power falling off with increasing depth into the water (as discussed in § I.1.1.1). But this does mean that in times of extreme storms, a surface-piercing WEC is exposed to the harshest of conditions.

The relationship between wave force and depth is important to account for when considering the survivability of WECs. Different WEC designers have used it to come up with novel solutions to improve survivability. The CETO 6 device is a submerged WEC, and in extreme weather conditions it pulls itself even deeper to avoid the harsher conditions, and is still able to extract power. Alternatively, the WaveStar device hoists its floats out of the water, and into the air during a storm to protect them [61].

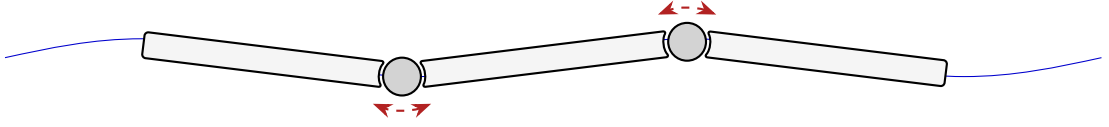


FIGURE I.2.5: Example of an attenuator wave energy converter.

#### I.2.1.2 ATTENUATOR

The first wave energy converter to reach a pre-commercial level of performance was an attenuator [51]: the Pelamis device [62]. Attenuators are long snake-like devices, generally consisting of multiple bodies connected by hinges, as shown in FIGURE I.2.5; energy is extracted from the relative motion in each of the hinges.

Designing an attenuator has some additional choices not present in the other WEC designs; namely in scaling up the device, do you increase the size of the bodies, or the number of bodies? Increasing the number of bodies adds more power take-off points, but that also adds in the cost of extra power take-offs. Work by Costello *et al.* [63], found that the devices with the best power performance were not necessarily the most economically viable.

An advantage of attenuators is their small cross-sectional area facing the waves, which increases the ability of the WEC to pierce through the waves in stormy conditions, reducing harmful loads [62].

#### I.2.1.3 TERMINATOR

One of the best-known terminator WECs is Salter's duck [42] – also known as the Edinburgh duck [64] – which can theoretically extract 90% of the energy from a wave (for a device of infinite width). In more realistic scenarios, the extraction efficiency fell to around 50%. Terminators were very popular in the early development of WECs, since they can be well approximated in two dimensions, and tested in narrow wave tanks [65].

FIGURE I.2.6 shows some examples of terminator WECs, and it is clear that designs vary much more than that of the point absorbers. Oscillating wave surge converters work by flapping in surge as waves force the device – for example: Aquamarine Power's Oyster [66]; or Sandia National Laboratories' floating FOSWEC 2 [67].

#### I.2.1.4 OTHER DEVICES

Owing to there being such a large variation in designs, no categorisation of WECs works perfectly for all designs. The WECs in FIGURE I.2.7 interact with the seas in completely different ways to that of the other designs seen so far.

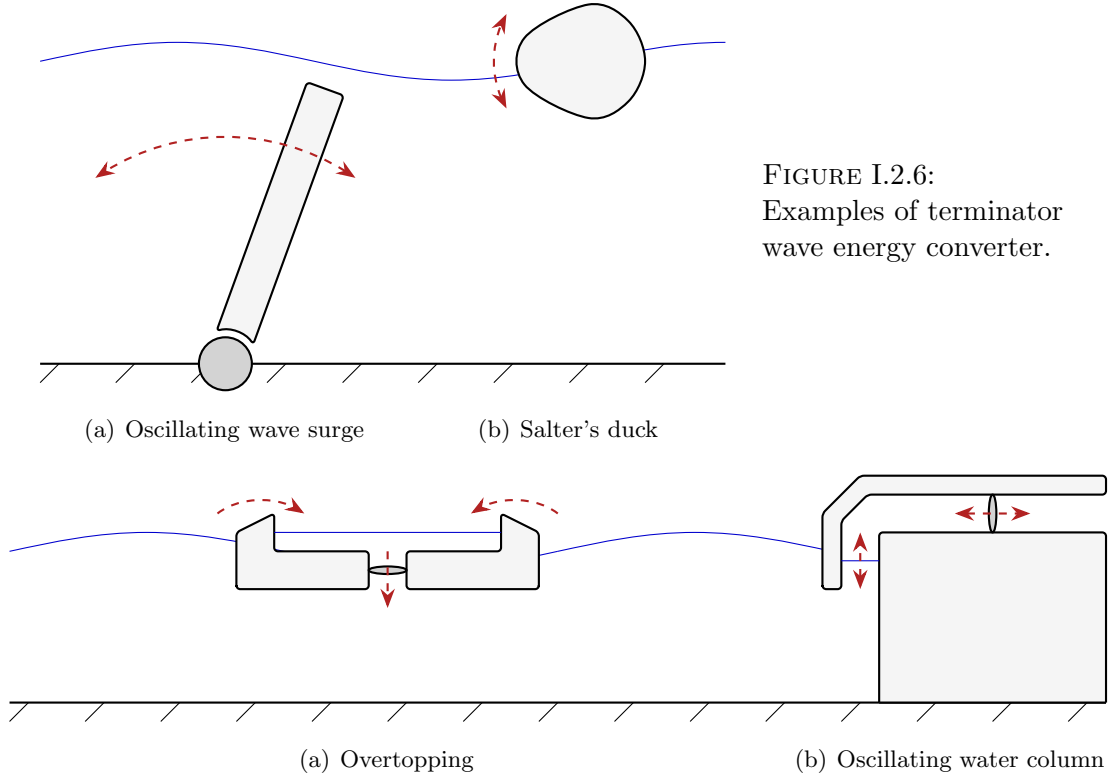


FIGURE I.2.7: Example of an overtopping, and an oscillating water column wave energy converter.

An overtopping device, FIGURE I.2.7(a), operates by having the water from the waves crash into a reservoir, then running that water through a turbine back into the sea – an example device is the WaveCat [68]. As well as the floating version shown in the diagram, onshore variations also exist, such as the SSG [69].

Oscillating water columns, FIGURE I.2.7(b), work by having water moving up and down in an enclosed column to force air forwards and backwards through a turbine [70]. Examples of oscillating water columns are: the Pico [71] for an onshore example; and, Wave Swell [72] for a floating example.

## I.2.2 ARRAYS OF WAVE ENERGY CONVERTERS

In the wind energy sector, the colocation of multiple wind turbines can have a negative effect on the individual performance of turbines, because the wakes created by one turbine reduce the power of another downwind [73]. In wave energy, the colocation can actually improve the individual performance of WECs; as the radiating and diffracted waves from one WEC can constructively interfere with the incident waves at another WEC increasing the available power. However, poor planning in the placement of WECs

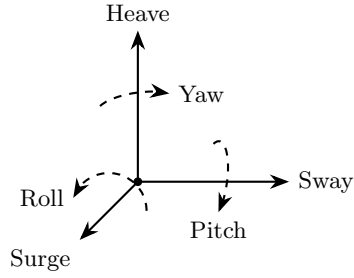


FIGURE I.2.8: Degrees of freedom for a single body. Surge is parallel with the wave direction.

can lead to destructive interference and less available power [74]. The performance of an array, compared to individual WECs, is often measured by a  $q$  factor (*eg.* in [75]) defined by

$$q = \frac{E_{\text{array}}}{N_{\text{wecs}} E_{\text{wec}}}, \quad (\text{I.2.1})$$

where  $E_{\text{array}}$  is the total energy absorbed by the array;  $N_{\text{wecs}}$  is the number of WECs in the array; and  $E_{\text{wec}}$  the energy absorbed by a single WEC, not in an array. Values of  $q$  greater than one signify constructive interference, and values less than one signify destructive interference.

### I.2.3 NUMERICAL MODELLING OF WAVE ENERGY CONVERTERS

Now that the designs of WECs has been laid out, this subsection discusses the mathematical theory behind how they are modelled, in both the frequency-, and time-domain. There are benefits and drawbacks to both: the former is more computationally efficient, but can not model nonlinear and transient phenomena; the latter can but at the cost of extra computational work [76]. Hence, frequency-domain models tend to be used in the early stages of WEC design before moving onto time-domain models at the later stages.

WECs can be modelled with varying degrees of freedom (DoF). At the simplest, a single DoF can be used to model the dominant axis of motion of the WEC. In a single rigid-body device, there is a maximum of six DoF: surge, sway, heave, roll, pitch, and yaw – these are shown relative to the wave direction in FIGURE I.2.8. However, for a multi-rigid-body device, there can be up to  $6(N - 1)$  DoF, where  $N$  is the number of bodies [77]; constraints between the bodies mean there will be less, and flexibility in the bodies will add more. Modelling more DoF obviously improves the accuracy of the model, but increases the computational workload. The equations in this work are formulated for a rigid, single-body WEC in the full six DoF, however the same equations apply for the modelling of any WEC, for any number of DoF, by expanding or reducing the size of the matrices.

## I.2.3.1 FREQUENCY-DOMAIN MODELS

The motion of a WEC is a highly nonlinear process; however under certain conditions, it can be simplified. In the case of small-amplitude WEC motion, the system is well described by a linear model [78]. The equations set forth in this section will be for a monochromatic, sinusoidal wave; but due to the assumed linearity, the principle of superposition applies and a more complex system could be described via

$$\hat{x}_{\text{tot}} = \sum_j \hat{x}(\omega_j), \quad (\text{I.2.2})$$

where  $\hat{x}(\omega)$  is the vector of complex amplitude of position, as a function of wave frequency,  $\omega$ . The generic equation to find  $\hat{x}(\omega)$  is given by [79]

$$\hat{x}(\omega) = A_\omega \hat{f}_{\text{exc}} \left[ -\omega^2(\mathbf{M} + \mathbf{A}) + i\omega\mathbf{R} + \mathbf{K}_{\text{PTO}} + i\omega\mathbf{B}_{\text{PTO}} + \mathbf{K}_{\text{moor}} + \mathbf{C} \right]^{-1}, \quad (\text{I.2.3})$$

where  $A_\omega$  is the wave amplitude;  $\hat{f}_{\text{exc}}(\omega)$  is the vector of complex amplitudes of the wave excitation forces in each DoF;

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 & mz_g & -my_g \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & my_g & -mx_g & 0 \\ 0 & -mz_g & my_g & J_{11} & J_{12} & J_{13} \\ mz_g & 0 & -mx_g & J_{21} & J_{22} & J_{23} \\ -my_g & mx_g & 0 & J_{31} & J_{32} & J_{33} \end{bmatrix} \quad (\text{I.2.4})$$

is the mass matrix, where  $m$  is the mass of the WEC;  $(x_g, y_g, z_g)$  is the coordinate of the centre of gravity of the WEC; and

$$J_{jk} = \rho V r_{jk} |r_{jk}| \quad (\text{I.2.5})$$

is the moment of inertia, where  $\rho$  is the density of the WEC,  $V$  is the volume, and  $r_{jk}$  is the radius of gyration. The added mass accounts for the fact that when the WEC is accelerated it also accelerates the water around it, creating an additional inertial effect – effectively adding to its mass: it is represented by the matrix  $\mathbf{A}$ . The hydrodynamic damping accounts for the energy transferred out of the WEC and into the water and is represented by the matrix  $\mathbf{R}$ . These two are found via [80]

$$\mathbf{R} + i\omega\mathbf{A} = -i\omega\rho \iint n_j \phi_k \, dS_{\text{wet}}, \quad (\text{I.2.6})$$

where  $n_j$  is the  $j^{\text{th}}$  component of  $\vec{n}$ , the unit normal vector;  $\phi_k$  is the  $k^{\text{th}}$  component of  $\phi$ , the vector of velocity potentials;  $S_{\text{wet}}$  is the wetted surface area (the surface

area submerged in the water).  $\mathbf{K}_{\text{PTO}}$  and  $\mathbf{B}_{\text{PTO}}$  are the spring and damping matrices of the power take-off (PTO) respectively; they define the characteristics of the PTO, and typically only have components on the main diagonal.  $\mathbf{K}_{\text{moor}}$  is the spring stiffness matrix for the mooring of the WEC. Finally,  $\mathbf{C}$  is the matrix of hydrostatic restoring coefficients, and its components are given by [79]

$$C_{33} = \rho g \int n_3 \, dS_{\text{wet}}, \quad (\text{I.2.7})$$

$$C_{34}, C_{43} = \rho g \int y n_3 \, dS_{\text{wet}}, \quad (\text{I.2.8})$$

$$C_{35}, C_{53} = -\rho g \int x n_3 \, dS_{\text{wet}}, \quad (\text{I.2.9})$$

$$C_{44} = \rho g \int y^2 n_3 \, dS_{\text{wet}} + \rho g V z_b - m g z_g, \quad (\text{I.2.10})$$

$$C_{45}, C_{54} = -\rho g \int x y n_3 \, dS_{\text{wet}}, \quad (\text{I.2.11})$$

$$C_{46} = -\rho g V x_b + m g x_g, \quad (\text{I.2.12})$$

$$C_{55} = \rho g \int x^2 n_3 \, dS_{\text{wet}} + \rho g V z_b - m g z_g, \quad (\text{I.2.13})$$

$$C_{56} = -\rho g V y_b + m g y_g, \quad (\text{I.2.14})$$

where the subscripts denote the component's position in the matrix;  $(x_b, y_b, z_b)$  is the coordinate of the centre of buoyancy of the WEC. For a WEC with no external forces  $(x_b, y_b, z_b) = (x_g, y_g, z_g)$ , where  $(x_g, y_g, z_g)$  is the coordinate of the centre of gravity of the WEC. Components without an equation above are equal to zero. The numbering of the components corresponds to the DoF. In order these are: surge, sway, heave, roll, pitch, yaw – as seen in FIGURE I.2.8. The component shows the effect from the first number's DoF to the second: *i.e.*  $C_{34}$  is the effect of motion in heave on roll.

Finding the coefficients for the equation of motion – EQUATION (I.2.3) – requires the application of boundary element methods (BEM): this involves dividing the WEC's surface into a mesh of small panels to numerically solve the surface integrals. There are a number of tools available that implement a BEM methodology, both commercial and open-source: examples of each include WAMIT [81] and NEMOH [82] respectively – these are the most popular BEM codes in literature [83].

The two codes have different implementations for solving Green's theorem<sup>ii</sup>: WAMIT makes various approximations (see [84] for more details); and NEMOH interpolates look-up tables. They broadly give the same results, but WAMIT does have some advantages over NEMOH: it consistently runs faster – between four and ten times faster, depending

<sup>ii</sup>Green's theorem is a mathematical theorem, which in this context transforms the three-dimensional equations governing the water's motion to the two-dimensional surface of the floater.

on the problem [83]; and it has extra features, such as irregular frequency removal (IFR). IFR is a purely numerical phenomena resulting in spikes of the hydrodynamic parameters at certain frequencies, caused by interactions between the edge of the WEC's mesh and the water's surface [85]; this can be accounted for when using NEMOH (or other BEM codes without IFR) with post-processing of the outputs.

One of the major factors for the slower computation in NEMOH is its lack of parallelisation (meaning the program cannot utilise more than one core in a computer's CPU cores for computation); this has resulted in the development of new generation of open-source BEM codes, such as Capytaine [86, 82] and HAMS [85]. The former being a rewrite of NEMOH in a modern computer language (Python versus FORTRAN 90); and the latter's implementation is closer to that of WAMIT. Both of these – as well as WAMIT – are capable of parallel processing.

In summary, there are a variety of BEM solvers available, each with differences in their implementation. Raghavan *et al.* undertook two comparisons of BEM solvers to compare their methodologies and outputs; the reader is directed to them for a more detailed comparison [83, 87].

### I.2.3.2 TIME-DOMAIN MODELS

Originally developed by Cummins [88] for use in modelling of ships, the Cummins equation has since become foundational in the time-domain modelling of WECs [89]:

$$(\mathbf{M} + \mathbf{A}_\infty)\ddot{\vec{x}}(t) + \int_{-\infty}^t \mathbf{K}(t - \tau)\dot{\vec{x}}(\tau) d\tau + \mathbf{C}\vec{x}(t) = \vec{f}_{\text{exc}}(t) - \vec{f}_{\text{ext}}(\vec{x}, \dot{\vec{x}}, t), \quad (\text{I.2.15})$$

where  $\vec{x}(t)$  is the vector of the time domain position of the WEC; and

$$\mathbf{A}_\infty = \lim_{\omega \rightarrow \infty} \mathbf{A}(\omega) \quad (\text{I.2.16})$$

is the added mass at infinite frequency.  $\mathbf{K}(t)$  is the radiation impulse response function, or memory function – as the name suggests this term accounts for the fact that previous motions of the WEC created waves and those waves have an effect on the WEC in the present. The fact that the integral only spans from time  $-\infty$  to  $t$  suggests the system is entirely causal (it only depends on previous events). This however is not true for control design, which is explained in §I.2.4.  $\vec{f}_{\text{exc}}(t)$  is the vector of the wave excitation forces and finally,  $\vec{f}_{\text{ext}}(\vec{x}, \dot{\vec{x}}, t)$  is the vector of external forces (PTO, mooring, *etc.*); examples of how these are modelled are presented in turn. The Cummins equation makes the assumption that the hydrodynamics is linear: the WEC's motions are small, as was the case for the frequency-domain model (see §I.2.3.1); whilst this may seem limiting, nonlinearities in the excitation and external forces can still be modelled [89].

**WAVE EXCITATION FORCE:** The wave excitation force is the force upon the WEC by the waves; it is a function of both the waves properties and the mechanical properties of the WEC's floater. The first-order formulation of the wave excitation force, which is usually sufficient – under the assumption of linear hydrodynamics – for a reliable estimate, is given by [90]

$$\vec{f}_{\text{exc}}(t) = \sum_j \mathbf{\Gamma}(\omega_j) \vec{A}_j \cos(\omega_j t + \vec{\phi}(\omega_j) + \vec{\varphi}) \quad (\text{I.2.17})$$

where  $\mathbf{\Gamma}(\omega_j)$  and  $\vec{\phi}(\omega_j)$  are the matrices of the excitation force coefficient and phase respectively; these are found by BEM solvers. The remaining terms are matrix equivalents of those defined in the description of a polychromatic ocean surface – EQUATION (I.1.12).<sup>iii</sup>

There are higher order representations of the wave excitation force – often referred to as the non-linear Froude-Krylov force – but the first order is generally suitable for most uses. The reader is directed to [91] for a detailed description of the higher-order forces.

**POWER TAKE-OFF SYSTEMS:** At its most fundamental, the function of a PTO can be described as

$$P_{\text{elec}} = \eta_{\text{PTO}} P_{\text{mech}} \quad (\text{I.2.18})$$

where  $\eta_{\text{PTO}} \in [0, 1]$  is the electrical efficiency of the PTO; this results in a lower electrical power output,  $P_{\text{elec}}$ , from the WEC than the mechanical power input,  $P_{\text{mech}}$ . How  $\eta_{\text{PTO}}$  is defined varies in the literature: the simplest is just a constant efficiency factor (*eg.* in [92]); a step up from that is using a look-up table to get efficiency as a function of operational variables (*eg.* in [93]); and at the highest fidelity, the inner workings of the PTO are modelled as part of the overall WEC simulation (*eg.* in [94]).

There are a variety of different PTO technologies used for the extraction of power. Returning to FIGURE I.2.2 on the popularity of various WEC designs, the two most popular PTO designs are hydraulic, and direct-drive PTOs. This subsection describes the two in turn. For a review of PTOs more generally, the reader is directed to [95, 96].

In hydraulic PTOs, the WEC's mechanical motion is used to drive a piston which forces a flow of hydraulic fluid to run through a hydraulic motor; a schematic example of which is shown in FIGURE I.2.9. The motion of the floater causes a bidirectional flow of fluid in the tubes between the cylinder and the control manifold, which then rectifies the flow such that beyond the manifold the flow is unidirectional. This flows through a hydraulic motor that in turn forces an electrical generator. High and low pressure

<sup>iii</sup>Note that removing  $\mathbf{\Gamma}(\omega_j)$  and  $\vec{\phi}(\omega_j)$ , reduces the EQUATION (I.2.17) to EQUATION (I.1.12) – but in matrix form to account for the different DoF.

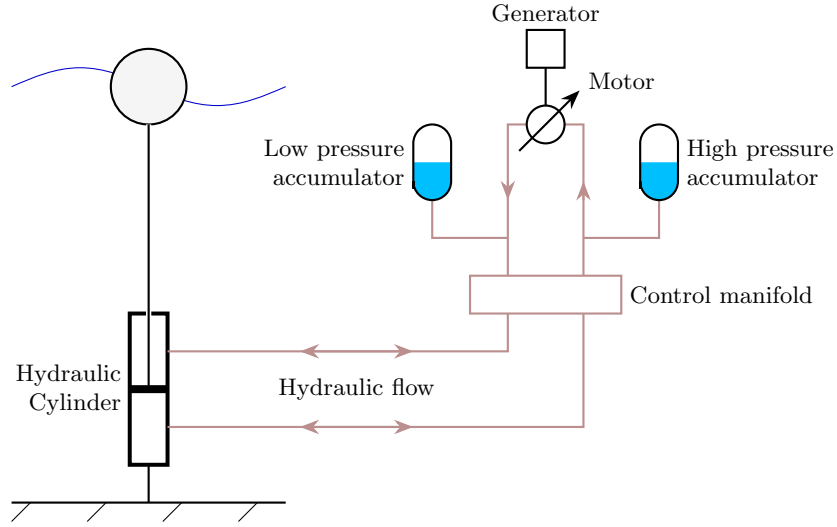


FIGURE I.2.9: Generic hydraulic circuit for a wave energy converter's power take-off. (Recreated from [34])

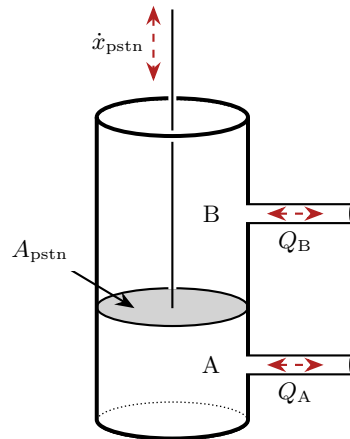


FIGURE I.2.10: Example of a hydraulic cylinder, showing the motions of the piston and fluid.

accumulators are often added as energy storage to regulate the electrical power output over a few wave periods [34]. The dynamics of hydraulic systems are well suited to WECS: the waves apply large forces at low speeds, which hydraulic systems are well suited to absorb, and store [97]. A hydraulic PTO system designed for the WaveStar WEC, was found to have an average efficiency of 80%, and in the smallest wave conditions tested its efficiency remained above 70% [98].

The modelling of a hydraulic PTO involves considering the motion and pressure of the fluid through the tubes and components. Take the cylinder in FIGURE I.2.10 as an example. The pressure dynamics of the fluid in the two chambers (A and B), can be

described via the continuity equation to be [99]

$$\dot{p}_A = \frac{\beta_{\text{eff}}(p_A)}{V_A + A_{\text{pstn}}x_{\text{pstn}}}(Q_A - \dot{x}_{\text{pstn}}A_{\text{pstn}}), \quad (\text{I.2.19})$$

$$\dot{p}_B = \frac{\beta_{\text{eff}}(p_B)}{V_B - A_{\text{pstn}}x_{\text{pstn}}}(\dot{x}_{\text{pstn}}A_{\text{pstn}} - Q_B), \quad (\text{I.2.20})$$

where  $\beta_{\text{eff}}(p)$  is the bulk modulus of the fluid;  $Q_A$  and  $Q_B$  is the flows exiting each chamber (ignoring leakages);  $x_{\text{pstn}}$ ,  $\dot{x}_{\text{pstn}}$ , and  $A_{\text{pstn}}$  are the position, velocity, and area of the piston; and  $V_A$  and  $V_B$  are the volumes of each chamber when chamber A is at a minimum. The PTO force a floater experiences is [99]

$$F_{\text{PTO}} = A_{\text{pstn}}(p_B - p_A) + F_{\text{fric}} + F_{\text{in}}, \quad (\text{I.2.21})$$

where  $F_{\text{fric}}$  and  $F_{\text{in}}$  are the friction and inertial forces within the cylinder. Similar modelling takes place throughout all components of the hydraulic circuit to create a full model of the PTO: see [99] for a full description of the model.

In direct-conversion PTOs, the mechanical motion of the WEC is directly used to force an electrical generator; sometimes with a mechanical transmission system to converter between linear and rotational motion. Linear to rotational transmission systems can be incredibly efficient: reaching efficiencies of up to 97% for rack and pinion systems, although they can have issues with durability [100]. The direct coupling of mechanical and electrical subsystems also has benefits for efficiency and there is one less energy conversion step when compared to hydraulics – so one less stage for an energy loss [96]. However, electrical generators are at their least efficient when experiencing high forces at low speeds [101] – the region in which WECs operate most often.

Modelling of direct-conversion PTOs varies in the literature. Stock [93] uses a look-up table where the efficiency of the WEC has been parameterised as a function of operating variables: mechanical force and velocity. Grasberger *et al.* [102] models the PTO as using a two-port network, connecting the mechanical and electrical systems. An example is shown in FIGURE II.2.19: on the left is an equivalent mechanical circuit, where instead of current,  $I$ , and voltage,  $v$ ; there is instead angular speed,  $\omega$ , and torque,  $T$ ; which is then connected to the electrical circuit on the right. In the figure:  $Z_i$ ,  $Z_d$ ,  $Z_g$ ,  $Z_l$  are the impedances of the floater, drive train, generator, and load respectively. Mathematically, the system can be described as

$$\begin{bmatrix} T \\ v \end{bmatrix} = \begin{bmatrix} Z_{T\omega} & Z_{TI} \\ Z_{v\omega} & Z_{vI} \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix}; \quad (\text{I.2.22})$$

where the impedances  $Z_{xy}$  give the impedance from  $y$  to  $x$ . The instantaneous electrical power is then simply

$$P_{\text{elec}} = \text{real}[i]\text{real}[v], \quad (\text{I.2.23})$$

where  $\text{real}[\cdot]$  returns the real component of the argument.

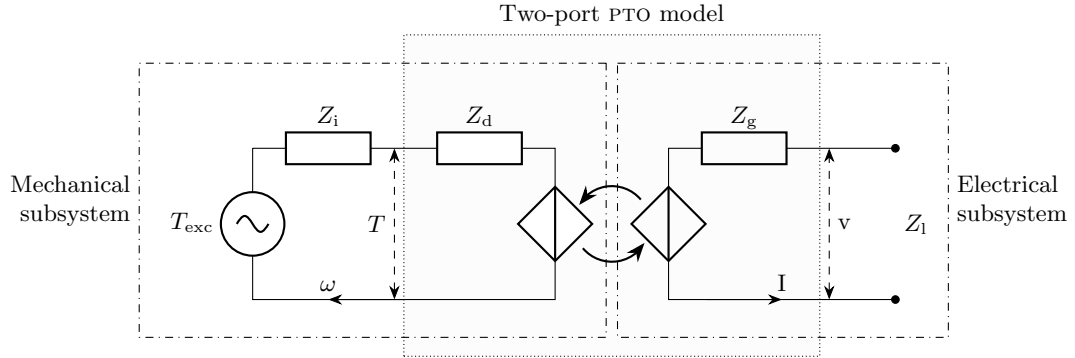


FIGURE I.2.11: Example circuit for a two-port direct drive power take-off model. (Adapted from [102])

**MOORINGS:** The purpose of the moorings is to keep the WEC in place; this is the antithesis of what is wanted from a power production viewpoint – where the more the WEC moves, the more power can be produced. Therefore the idealised mooring system would be one that fully constrains the motion of the WEC in DoF that are not being used for power generation, and is fully unconstrained in DoF that are. Of course, in reality such a design is not possible and compromises have to be made.

A variety of mooring systems have been developed over time for applications for a wide variety of floating structures. Harris *et al.* [103] reviewed a selection of mooring types to assess their suitability for use with WECs; they analysed the different configurations according to installation cost, and reliability. One of the configurations they ranked as highly suitable is shown in FIGURE I.2.12: it is a catenary mooring system, in which the lines are free hanging, and arrive at the anchor points on the sea bed horizontally – this means they do not exert any vertical forces on the anchors. The restoring forces on the WEC generally come from the weight and elasticity of the mooring lines trying to return to equilibrium.

When it is not the focus of the work, the modelling of moorings is often neglected in the literature. In these works the moorings are either not mentioned, or the WEC is said to be moored perfectly such that it is unconstrained in the DoF used for power extraction, and fully constrained in the others. Functionally, these two approaches are the same; it is just that one is implicit in its assumptions, and the other is explicit.

### I.2.3.3 COMPUTATIONAL FLUID DYNAMICS

Another method of modelling WECs is by using computational fluid dynamics (CFD) simulations: these are the highest fidelity numerical simulations that can be undertaken. CFD implements the Navier-Stokes and continuity equations to fully describe

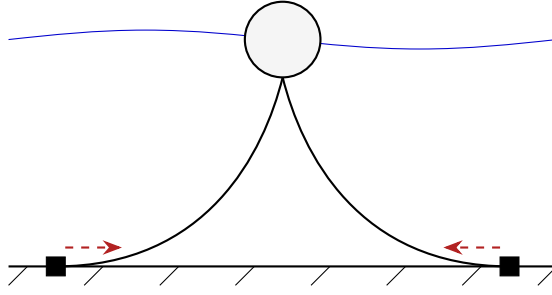


FIGURE I.2.12: Example of a catenary mooring configuration.

the interactions between wave and WEC. Generally, these equations are systems of partial differential equations and can not be solved analytically. Therefore numerical solutions are used, which get very computationally expensive [104] and can make them unattractive to researchers, especially in optimisation work which requires running a model many times under different scenarios. In a review of geometric optimisation of WEC floaters, it was found that only 9% of the reviewed work utilised CFD techniques at any point in their optimisation [105].

The use of CFD is better aligned to tank testing than other numerical methods. They are on the more expensive side of techniques – monetarily for tank tests and computationally for CFD (therefore typically requiring a lot of money to acquire the computational resources) – but they have the best physical accuracy, beaten only by sea trials. Advantages that CFD has over tank testing includes being able to test on a full-sized device, so scaling effects do not have to be accounted for; and being able to obtain data from any point in the simulation. In a tank test, to measure data at a point a probe would need to be added, further increasing the monetary cost and complexity. Conversely, CFD does not properly model every physical phenomena due to scaling effects (see §IV.2.1 for more details), so if the WECs motion relies upon that phenomena and the CFD model does not account for it results will be inaccurate [104].

An in-depth review of CFD modelling is beyond the scope of this thesis. For that, the reader is directed to [106].

#### I.2.4 CONTROL OF WAVE ENERGY CONVERTERS

The control of WECs is a complex problem: take a WEC that is perfectly moored such that it is free to move in heave, but fixed in every other DoF. The equation of motion of this WEC is

$$Z(\omega)\dot{x}(\omega) = F_{\text{exc}}(\omega) + F_{\text{PTO}}(\omega), \quad (\text{I.2.24})$$

$$z(t)\dot{x}(t) = f_{\text{exc}}(t) + f_{\text{PTO}}(t), \quad (\text{I.2.25})$$

where  $Z(\omega)$  and  $z(t)$  are the mechanical impedances of the WEC, in the frequency-, and time-domain respectively. It is assumed that  $F_{\text{PTO}}(\omega) < 0$  and  $f_{\text{PTO}}(t) < 0$  to oppose

the natural motion of the WEC. The extracted mechanical power is found via

$$P_{\text{mech}} = f_{\text{PTO}}(t)\dot{x}(t), \quad (\text{I.2.26})$$

where a large negative value is desired. To increase power capture, a simple solution is to increase the magnitude of the resistive PTO force; however the increased resistive force causes the WEC to slow down which reduces (or even removes) any gains from the increased force. This is a well known problem in wave energy control for which a theoretical optimal has been found. Start by considering the harnessed energy

$$W = \int_{-\infty}^{\infty} f_{\text{PTO}}(t)\dot{x}(t) dt, \quad (\text{I.2.27})$$

by utilising the frequency convolution and algebraic manipulation, one can find that

$$W = -\frac{2}{\pi} \int_0^{\infty} \frac{|F_{\text{exc}}(\omega)|^2 - \alpha(\omega)}{8R(\omega)} d\omega, \quad (\text{I.2.28})$$

where

$$\alpha(\omega) = F_{\text{exc}}(\omega)F_{\text{exc}}^*(\omega) + 2R(\omega)[F_{\text{PTO}}(\omega)\dot{x}(\omega) + F_{\text{PTO}}^*(\omega)\dot{x}(\omega)], \quad (\text{I.2.29})$$

$$R(\omega) = \frac{Z(\omega) + Z^*(\omega)}{2}, \quad (\text{I.2.30})$$

and  $*$  represents the complex conjugate. From observation of EQUATION (I.2.28), it is seen that to maximise the energy extracted,  $\alpha(\omega)$  must be minimised. Re-arranging EQUATION (I.2.24), and multiplying itself by its complex conjugate gives<sup>iv</sup>

$$F_{\text{exc}}F_{\text{exc}}^* = F_{\text{PTO}}F_{\text{PTO}}^* + ZZ^*\dot{x}\dot{x}^* - F_{\text{PTO}}Z^*\dot{x}^* - F_{\text{PTO}}^*Z\dot{x}. \quad (\text{I.2.31})$$

Inputting EQUATIONS (I.2.30) and (I.2.31) into EQUATION (I.2.29) and simplifying finds

$$\alpha(\omega) = |F_{\text{PTO}}(\omega) + Z^*(\omega)\dot{x}(\omega)|^2, \quad (\text{I.2.32})$$

so  $\alpha(\omega)$  is minimised, to zero, when

$$F_{\text{PTO}}(\omega) = -Z^*(\omega)\dot{x}(\omega). \quad (\text{I.2.33})$$

Owing from the  $Z^*(\omega)$  term, this is often called *complex-conjugate control*. For a more thorough derivation, the reader is directed to [107].

Now, consider the inverse Fourier transform of  $Z(\omega)$  and  $Z^*(\omega)$ :

$$\mathcal{F}^{-1}[Z(\omega)] = z(t), \quad (\text{I.2.34})$$

$$\mathcal{F}^{-1}[Z^*(\omega)] = z(-t). \quad (\text{I.2.35})$$

---

<sup>iv</sup>For readability, the argument  $\omega$  is omitted; each variable in the equation is frequency dependent.

$z(t)$  corresponds to a causal impulse response, *i.e.* an impulse on the WEC causes an effect on its future motions. Conversely,  $z(-t)$  corresponds to a anti-causal impulse response: *i.e.* future waves have an effect on the optimal WEC motion in the present. For the case of a monochromatic wave, calculating this is trivial; but for a polychromatic wave, one would require foreknowledge of the future conditions to track the optimal control force. It is therefore evident that approximations and estimations are required.

The various controllers can be split into two groups: resistive, and reactive. The former only extracts power from the WEC; whereas the latter extracts and inputs power at opportune times – if implemented correctly, this results in better energy capture [108].

#### I.2.4.1 RESISTIVE CONTROL

Resistive controllers are the simpler class of controller. They involve purely extracting power at opportune rates and times to extract the highest average power. Linear damping – also called resistive loading – is often used as a starting or reference point in the design of WECS. Under this regime, the PTO force is defined as

$$F_{\text{PTO}} = -R\dot{x}, \quad (\text{I.2.36})$$

where  $R > 0$  is a constant over short time scales; but often varies on longer time scales, to be the *optimal* value for a given sea state [108].

Latching and clutching controllers can be seen as an add-on to the linear damping controller. When in wave conditions far away from the WEC’s resonant frequency, the WEC’s motion can move away from resonance with the excitation frequency, hindering its performance; so a phase correction can be useful to get the WEC back into a good operating point. Latching and clutching implement phase correction differently: latching holds the position of the WEC at an extreme point in its motion, then releases at the right moment; clutching sets  $F_{\text{PTO}} = 0$  to allow the WEC to get back into phase before re-engaging the PTO force. Once again in the case of monochromatic waves, calculating the optimal times to do this is trivial; in the case of polychromatic it gets more complicated. See [108] for a more in depth discussion of the timings, and how each of these controllers operate more generally.

#### I.2.4.2 REACTIVE CONTROL

Reactive control allows both the extraction and injection of power from and to the WEC; having this freedom allows the controller to better track  $Z^*(\omega)$  and hence improve the power capture. There are many controllers that fit into the classification of reactive control, two examples are optimal velocity tracking, and model predictive control. They are discussed in turn.

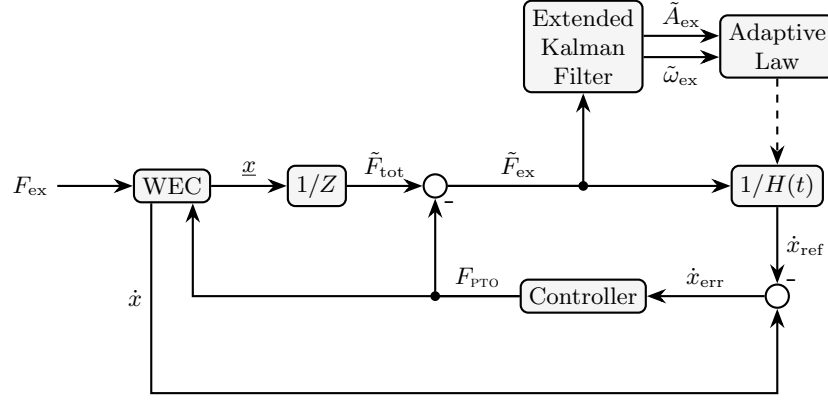


FIGURE I.2.13: Structure of a (mechanically focussed) optimal velocity tracking controller. Tildes represent estimated values.

**OPTIMAL VELOCITY TRACKING:** Optimal velocity tracking (OVT) estimates the optimal velocity the WEC should be travelling at and attempts to track that. First implemented by Fusco and Ringwood [109], the controller tracked the optimal velocity for mechanical power capture. Stock [93] later developed an alternative that tracks for optimal electrical power capture. Only Fusco and Ringwood’s methodology is covered here; see [93] for Stock’s methodology.

FIGURE I.2.13 shows the structure of a mechanically-focussed OVT controller, the position, velocity, and acceleration – collectively  $\vec{x}$  – of the WEC are fed into an inverse model of the WEC to estimate the total force on the WEC,  $\tilde{F}_{\text{tot}}$ . An extended Kalman filter is then used to estimate both the amplitude and force of the excitation force –  $\tilde{A}_{\text{ex}}$ , and  $\tilde{\omega}_{\text{ex}}$  respectively; these are then used to set the gain of  $1/H(t)$  such that

$$\dot{x}_{\text{ref}} = \frac{\tilde{F}_{\text{ex}}}{H(t)} \approx \frac{F_{\text{ex}}}{2B(\omega)} = \dot{x}_{\text{opt}}, \quad (\text{I.2.37})$$

where  $\dot{x}_{\text{opt}}$  is the theoretical optimal velocity. Finally, a standard error correcting controller can be used to minimise the error between  $\dot{x}_{\text{ref}}$  and  $\dot{x}$ .

Instead of an extended Kalman filter, there is a variety of methods for estimating the excitation force: see [110] for a review. The estimators tend to have an accuracy in the region of 80-90%, but another – often competing – property of an estimator is the time to get to the estimation, as the WEC can not respond to the waves until the force estimation is evaluated; therefore the highest accuracy estimator may not be the best performing. There has also been some work on measuring the force up-wave of the WEC (*e.g.* [111, 112]<sup>v</sup>), but estimates are a lot more popular as they do not require installing another device.

<sup>v</sup>Note that these are for oscillating water columns, and therefore would not work with OVT, and are presented as a solution to other control problems.

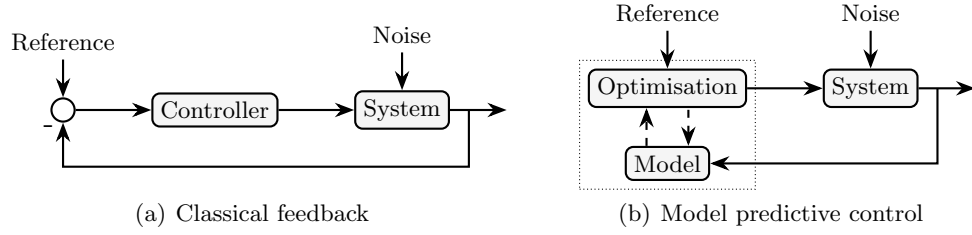


FIGURE I.2.14: Comparison between a classical feedback, and a model predictive control loop.

**MODEL PREDICTIVE CONTROL:** Model predictive control (MPC) is a class of controller with uses throughout many disciplines – from the optimal heating of residential buildings [113] to controlling free-floating space robots [114]. MPC controllers use models of the system to predict what the optimal control would be to maximise performance, both in the present and the future. FIGURE I.2.14 gives a comparison between classical feedback loops and MPC. In the classical feedback, the controller has to determine a reasonable value to minimise the error between the reference and measured value; finding this value relies solely on current and past information, with no consideration of the future conditions of the system. This is where MPC differs; by using a mathematical model of the system the controller optimises the control signal to follow – what it believes is – the optimal trajectory to reach the reference [115]. The model is also constantly updated with the true response of the system as a form of continual validation and error correction to ensure it is following the optimal trajectory. As could be expected, including a model of the system comes at a computational cost; therefore, depending on the complexity of the system, it may not be suitable for all applications. In fact, the original developers of MPC controllers have stated that classical controllers suit more than 90% of problems perfectly [116].

Nevertheless, MPC has been applied to WEC control. In the control of a the WaveSub WEC – a mutli-DoF, submerged point absorber with variable geometry – it was found that MPC increased power capture by 200% in idealised conditions when compared to a linear damping controller [117]. In more realistic simulations, this fell to 30% – showing the controller’s reliance on the quality of its model.

## I.2.5 MEASURING PERFORMANCE

Now that the dynamics and modelling of WECs has been defined, the final piece needed in the discussion of WECs is a way to compare the performance of different WECs. This subsection describes a few popular metrics.

### I.2.5.1 CAPTURE WIDTH RATIO

Capture width ratio (CWR) is a measure of the hydrodynamic performance of a WEC. It is defined by [118]

$$\text{CWR} = \frac{P_{\text{abs}}}{E_{\text{wave}}}, \quad (\text{I.2.38})$$

where  $P_{\text{abs}}$  (in kW) is the absorbed wave power, and  $E_{\text{wave}}$  is the wave resource (in kW/m). This gives the CWR units of metres, and can be thought of as the width of a wave crest that can be completely absorbed by the WEC. A database of CWRs for various WECs can be found in [118].

Dividing the CWR by a characteristic dimension of the WEC (*eg.* the diameter of a cylindrical or spherical floater) gives a hydrodynamic efficiency of the device: a measure of how much of the wave energy is absorbed by the WEC, and how much passes through.

### I.2.5.2 RESPONSE AMPLITUDE OPERATOR

The response amplitude operator (RAO) is used widely in the design of ships [119] and floating structures more generally [120]. It is a measure of the object's motion in relation to the water's motion; a unique RAO exists for every DoF. In terms of the heaving motion of a WEC, the RAO is

$$\text{RAO}(\omega) = \frac{A_{\text{WEC}}}{A_{\text{wave}}} = \frac{\Gamma(\omega)}{C - (M + A(\omega))\omega^2 + iB(\omega)\omega}. \quad (\text{I.2.39})$$

where  $A_{\text{WEC}}$  is the amplitude of the WEC's motion, and  $A_{\text{wave}}$  is the amplitude of the wave;  $\Gamma(\omega)$  is the amplitude of the wave excitation force – see EQUATION (I.2.17); and the denominator comes from the equation of motion of a WEC – EQUATION (I.2.3).

The RAO does not directly measure power capture like the CWR does but a WEC that moves more likely produces more power so it can be used as a proxy. It is also a function of the wave frequency; therefore it gives an insight into how the performance of the WEC varies in varying wave conditions.

### I.2.5.3 LEVELISED COST OF ENERGY

Popular across the power production industry, the levelised cost of energy (LCoE) is defined as [121]

$$\text{LCoE} = \frac{C_{\text{cap}} + C_{\text{op}} + C_{\text{dec}}}{E}, \quad (\text{I.2.40})$$

where  $C_{\text{cap}}$ ,  $C_{\text{op}}$ , and  $C_{\text{dec}}$  are the capital, operational, and decommissioning expenditures of the WEC; and  $E$  is the total energy harvested over the lifetime of the device. It is a measure of the average cost over a device's lifetime to produce one unit of energy. The main benefit of LCoE is it makes for easy comparison between different energy

sources as it does not directly depend on the resource as the other two performance metrics do.

The downside of LCoE is the requirement of knowledge about the operational and decommissioning costs of running any power station. This is especially hard for WECs because there has not been any used in commercial operation to date. This therefore brings a lot of uncertainty into the value.

#### I.2.6 IN SUMMARY

The concept of a WEC is introduced, as well as methods for modelling its various components. Independently, each component's interaction with the waves is fairly complex, and these complexities compound when accounting for multiple components at once. Therefore it is very challenging to create an effective WEC design; made evident by the fact there is currently no commercially operating WEC.

## Control Co-design

In a conventional design methodology, the physical system tends to be designed first, then the controller is designed only once the design of the physical system is finalised; a schematic example methodology is shown in FIGURE I.3.1(a). The serial design process is the simplest design methodology, but it does not necessarily produce the most optimal design (for any definition of optimal: efficient, fastest, long lasting, powerful, *etc.*). This is because a decision in the physical system design could cause difficulties in the controller design, resulting in a decrease in overall performance. Therefore, making tweaks to the physical design could result in much higher performance – even if it makes the device mechanically less optimal – due to improvements in controller performance. In the type of device that regularly updates (*e.g.* yearly releases of smartphones) knowledge of flaws can be used to improve the next model, in a form of iterated serial design, FIGURE I.3.1(b); however, this both requires a potentially flawed product be released for at least one generation, and does not help in the design of more bespoke products (*e.g.* a power station). Therefore, a more holistic design philosophy could be useful, especially for more complex design problems; of which the design of wave energy converters (WECs) is one.

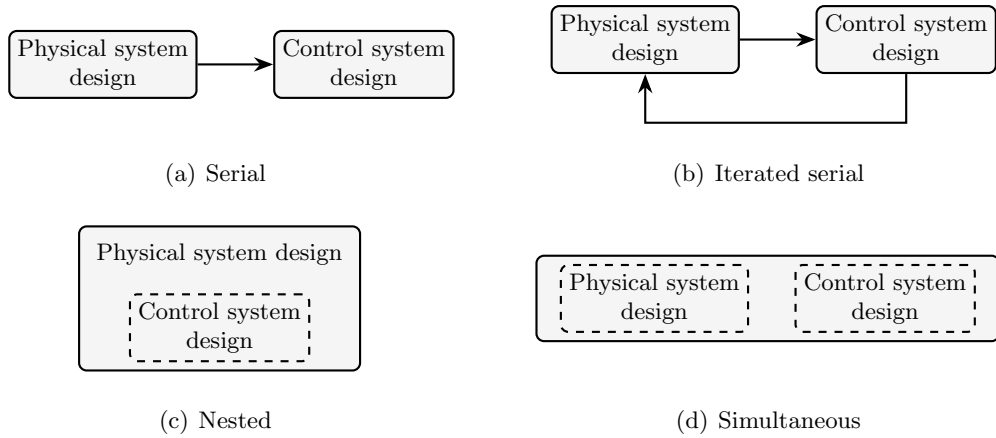


FIGURE I.3.1: Schematic examples of different design processes.

## I.3.1 WHAT IS CONTROL CO-DESIGN?

The idea of control co-design (CCD) is to combine the physical and control system design steps to help solve any potential conflicts between the two before any designs get finalised. CCD philosophies have previously been applied to a wide variety of design challenges: from bicycles [122] to chemical reactors [123, 124]. It should be noted however, this is not a new design philosophy and used to be standard: well known designers of the late 19<sup>th</sup> century – such as Nikola Tesla, and the Wright brothers – tended to have a multidisciplinary approach to their work, resulting in them applying CCD-esque methodologies [125]. The popularity of CCD later fell off as engineering problems have become much more complex, and individual engineers specialised into separate disciplines.

FIGURES I.3.1(c), and I.3.1(d) show two types of CCD methodologies: nested and simultaneous design. The nested design process involves designing a bespoke controller for a candidate physical design; then tweaking and iterating the physical design, creating a new controller for each new iteration of the candidate physical design. Whilst on the surface this seems very similar to the iterated serial design process, the nested design does not produce a final product before analysing and evaluating design choices – data would generally come from numerical models (*e.g.* [126, 127]), as the production of physical prototypes for each small change would quickly become cost-prohibitive.

The final design process is the simultaneous design: where both the physical and control systems are altered in tandem. As could be expected, integrating the design of the two components can make the problem much more complex, and greatly increase the size of the problem [128]. But in some problems, where the design of plant and controller are highly coupled, the simultaneous approach may converge faster to an optimal solution [129].

## I.3.2 CONTROL CO-DESIGN OF WAVE ENERGY CONVERTERS

As is shown in § I.2, WECs are dynamically complex; there has therefore been increasing interest in applying CCD methodologies to the design of WECs, in areas such as: the geometry of the WEC’s floater; positioning of WECs in an array; power take-off (PTO) design; and mooring design. The popularity of literature in each of these areas decreases in the order listed [130]. This section will give an overview of some examples in each area, investigating what questions have been asked, and what answers they found. More in depth reviews into PTO and floater co-design methodologies and models take place in § II.1 and § IV.1 respectively.

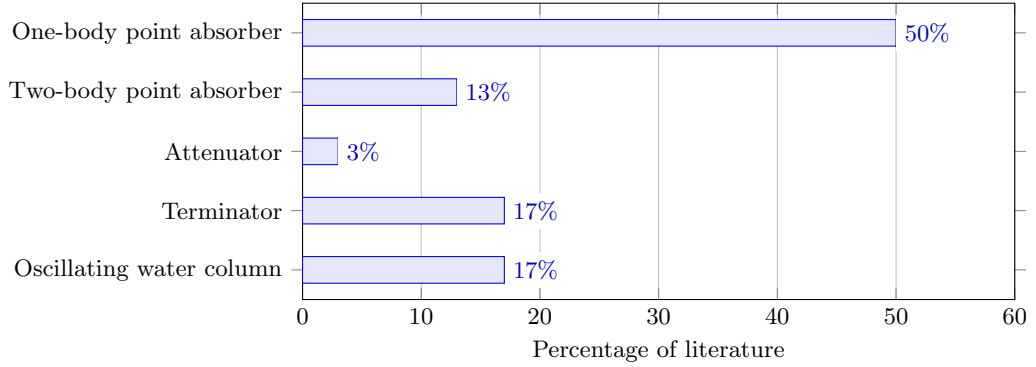


FIGURE I.3.2: Popularity of different wave energy converter types in floater geometry control co-design literature. (Data from [105])

#### I.3.2.1 FLOATER GEOMETRY CO-DESIGN

Floater geometry co-design involves the optimisation of the physical shape of the bodies that interact with the waves. FIGURE I.3.2 shows the relative popularity of different WEC types in floater CCD literature; one-body point absorbers dominate the field, but within this section, there is a wide variety of work with different levels of detail for the floater design.

On the lower end of floater design fidelity, optimising the radius and draught of a heaving cylinder has been well covered in the literature, *eg.* [131, 132, 133]; of these, Garcia-Rosa and Ringwood [133] went to the most detail in their controller designs. They investigated how the optimal design – for mechanical power capture – varied between using linear-damping, latching, declutching, and MPC controllers. It was found that the optimal design of a linear-damping controlled WEC tried to align its resonant frequency with that of the peak period of the irregular waves, whereas an optimal latching controlled WEC has its resonant frequency slightly below the peak period, and the optimal declutching slightly higher: these differences arise due to how the two controllers work to get the WEC back into phase with the waves (see §I.2.4.1). Finally, MPC control was (not unexpectedly) able to extract a significant amount of power from the waves regardless of the floater design; but favoured wider, shorter WECs for maximising power capture. The resonant frequency was closer to the peak period than both latching and declutching controlled devices. Garcia-Rosa and Ringwood concluded that the nature of the control type must be included when designing the WEC to get the optimal performance.

An example of a step up in geometry detail is work by Edwards [134], who created axisymmetric point absorbers about the heave axis. Edwards defines the profile of the WEC, then revolves the profile around the heave axis to design the floater. Through the

selection of the parameters, this method allows for the creation of piecewise continuous profiles, and therefore any desired profile. Edwards used linear-damping controllers and found that the more complex shapes could produce the same maximum power as optimised cylinders with: 72% less surface area and 93% less volume in a heave-only case; 31% less surface area and 56% less volume in a heave-surge-pitch case. It was also found that a complex shape can produce nine times more power than a cylinder whose radius and draught are equal to that of the maximum points in the complex shape.

At the highest level of geometry fidelity there is work by Garcia-Teruel [135]; who builds upon work by McCabe [136]. In these works, the shape is defined by a number of control points that are placed freely within three-dimensional space. A smooth surface is then drawn between the points to create a floater for the WEC. McCabe investigated WECs that moved only in surge and had a plane of symmetry about the heave-sway axis; finding that when no penalties were placed on the size of the floater, very large, almost hemispherical floaters performed best; but once penalties on size were included smaller, asymmetric shapes with pronounced prows and sterns were favoured. Garcia-Teruel investigated varying DoF (different combinations of surge, heave, and pitch) finding that, when used with perfect complex-conjugate control the floaters had an increased surface area perpendicular to their modes of energy extraction; including volume penalties led to smaller designs that had larger percentages of their power coming from heave; and including surface area penalties led to rounder designs that had higher pitch power contributions.

For all these works: once their floater geometries have been defined, they follow very similar methodologies to calculate the WEC's response to the waves. That is using boundary element method (BEM) software to calculate the hydrostatic and hydrodynamic properties of the floater, and using Cummins' equation to find the WEC's motion – see §I.2.3 for definitions of both of these.

### I.3.2.2 ARRAY CO-DESIGN

The effects of controllers on array layout is shown well by Garcia-Rosa *et al.* in [75]. FIGURE I.3.3 is recreated from that paper, showing the optimal array spacing, for different types of layouts, for different controllers: a linear-damping controller and a global pseudospectral controller (one that works to maximise array performance). The linear damping controller was found to have a better  $q$  factor<sup>i</sup> at very small spacings but, using a global controller, had a higher overall power than when using linear-damping. As the spacing increases, the effect of radiated waves diminishes, and for all controllers  $q \rightarrow 1$ . Garcia-Rosa *et al.* found that the optimal orientation of the array relative to

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<sup>i</sup>The ability of the array to extract energy when compared to the performance of an equivalent number of individual WECs – see §I.2.2.

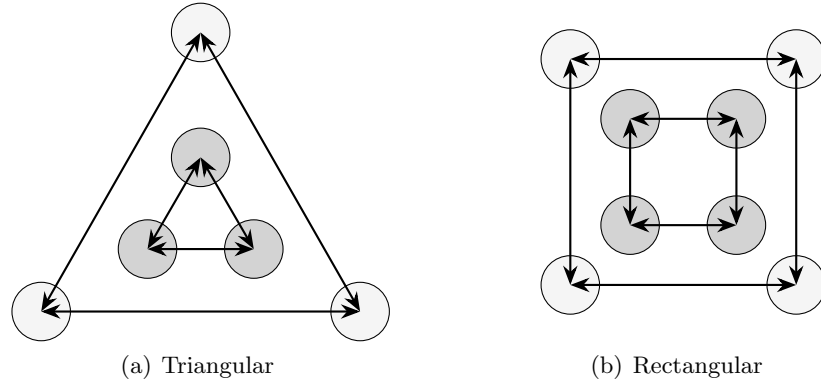


FIGURE I.3.3: Optimal spacings for wave energy converter arrays with varying controllers. Light WECs use a linear damping controller; dark WECs use a globally aware controller. (Recreated from [75])

the waves is independent of control type, but optimal array spacing was affected; and independent of array design, the performance of the array can improve by up to 40% by using a controller designed for array performance.

There has also been work on the simultaneous co-design of floater geometry and arrays, *eg.* [137, 138]. Ermakov *et al.* [138] optimised both WEC positioning in the array, and the scale of the floater geometry – via Froude scaling – of each individual WEC in the array. They found that the optimal design involved having smaller WECs surrounding larger ones.

### I.3.2.3 POWER TAKE-OFF CO-DESIGN

Within the PTO co-design literature, there are many differing definitions of what a PTO is. Peña-Sanchez *et al.* [139] modelled the PTO purely as a limit on the maximum force and displacement, adding a cost penalty to expand those limits; these constraints are the limits in the space the controller operates in. Whereas Michelén Ströfer *et al.* [140] model the internal mechanical and electrical impedances of the drive train to estimate losses; and select an optimal sprocket out of a predefined list. Their methodologies are not discussed here, that is the focus of § II.1

Again, literature exists that combines multiple component categories, namely PTO design and array layout by Neshat *et al.* [141] who modelled each of their WEC's PTOS as a spring and damper – finding the optimal coefficients of each of those. For the two locations tested, and with four WECs being positioned, average power with PTO optimisation was increased by 4.5% and 2.5% when compared to the manufacture's default values.

## I.3.2.4 MOORING CO-DESIGN

As previously mentioned, mooring co-design has been covered the least in the literature, but there has been some work done. Paduano *et al.* [142] investigated the optimal mooring pattern for a three-tethered point absorber WEC with a proportional-derivative controller approximating complex conjugate control. They found that neglecting the mooring system in the design of the controller can reduce the WEC's average power output by up to 15%; and optimising the angular positions of the tethers gave an improvement in power capture of 5.6% when compared to equally spaced tethers.

## I.3.3 RESEARCH GAPS

The majority of the literature has focussed on the optimisation of floater designs; and within the other components, there has been nothing of the fidelity of the work by Garcia-Teruel [135], and Edwards [134] in floater design.

Edwards found improvements in power production of 800% from optimising the floater geometry. Whilst Neshat *et al.* [141] found improvements of only 2.5% from PTO optimisation, and Paduano *et al.* [142] found improvements of 5.6% from mooring optimisation. The fidelity of the simulations by Neshat *et al.* and Paduano *et al.*, was much less than that of Edwards. The problem of WEC design has been shown in the previous section to be very complex, therefore a higher-fidelity CCD study into PTO or mooring designs could be found to show improvements above that of Neshat *et al.* or Paduano *et al.* Work by Stock [93] supports this theory: when designing controllers which account for PTO efficiency electrical output increased when compared to those optimised for mechanical power capture in all sea states tested – and in some sea states, the mechanically optimised controller absorbed electrical power rather than outputted it. So whilst Stock did not directly optimise the PTO design, he found that accounting for it at a higher fidelity improves the overall performance of the WEC.

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## *Optimisation Algorithms*

The design space of a control co-design problem is typically so large and complex that an optimal design can not be found analytically. This therefore necessitates the use of an optimisation algorithm. They work by defining a *cost function*, then using an algorithm to find the set of inputs – subject to constraints – that give the optimal cost. It is convention in optimisation for an algorithm to work to minimise a cost; so if the aim is to maximise an output: say the power output from a system, convention would be to define that power as negative, and seek the largest negative value. This section explores examples of different optimisation algorithms, with increasing levels of complexity.

### I.4.1 EXHAUSTIVE SEARCH

Exhaustive search (also known as brute-force search) is the simplest of the algorithms: it involves systematically testing every possible combination of inputs to find the best cost. For a problem that only takes a discretised input, the exhaustive search will return the optimal value. However, for a problem that takes a continuous input, the inputs must be discretised, meaning that the search will return the closest tested value to the optimal; but a sufficiently high enough resolution of test points will ensure that this is not a problem.

Their ease of implementation makes exhaustive searches a popular choice as a first attempt at optimisation. However, for a problem that has: a high dimensionality (number of inputs), a large bound on its inputs, or requires a particularly high resolution of input; the number of test points can quickly become prohibitively high, resulting in very long computation times, and is the point at which one should look into other types of algorithm.

### I.4.2 HILL-CLIMB SEARCH

The hill-climb search breaks the convention of minimising the cost, and instead maximises – working to reach the top of the hill. Take a random starting point in the design space, the algorithm searches for a change in a single input that improves the cost; it then searches from this new position and does the same procedure; this repeats until no changes in the inputs give an improved cost [143]. There are variations in

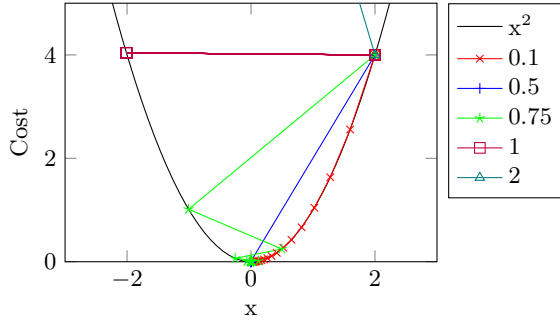


FIGURE I.4.1: Example of how the scale factor affects the convergence of a gradient descent algorithm. All runs start at  $x = 2$ , with a scale factor shown in the legend.

the algorithm that affect how the algorithm chooses an improvement. For example the algorithm could take the first improvement it finds, or calculate all possible moves then select the best.

For a cost function that monotonically increases to a single maxima, the selection of the starting point only affects how long the algorithm will take to complete. However, in a problem with multiple local maxima, an algorithm will simply find the closest maxima. This is because the hill-climb search is a *greedy algorithm*: it always takes a move for the best improvement in the present, with no consideration for larger improvements in the future. One simple solution to this is the random-restart hill climb: simply running the algorithm multiple times, each with a randomly generated starting position.

### I.4.3 GRADIENT DESCENT

Gradient descent is another popular optimisation algorithm; it is similar to the hill climb search but where the hill climb searches in the cost function, gradient descent searches in the derivative of the cost function.

Starting from a location,  $\underline{x}_1$ , the algorithm iterates according to:

$$\underline{x}_{j+1} = \underline{x}_j - \alpha \nabla \underline{x}_j, \quad (\text{I.4.1})$$

where  $j$  is the iteration number,  $\alpha$  is a step size factor, and  $\nabla$  is the differential operator.<sup>i</sup> Another difference to the hill climb is that it alters all elements of the input at once rather than just a single one.

Choosing the value of  $\alpha$  can have large effects on the computational speed of the algorithm, but can prevent the algorithm converging. FIGURE I.4.1 shows the effect of changing the scale factor for a simple problem: having a scale factor less than one affects how quickly the algorithm converges on the minima; having a scale factor equal to one causes the algorithm to oscillate between two and negative two, never making any progress; and having a value greater than one causes the algorithm to diverge.

<sup>i</sup>Note the minus in the equation, this is for finding the local minima; to find a maxima, this will become a plus.

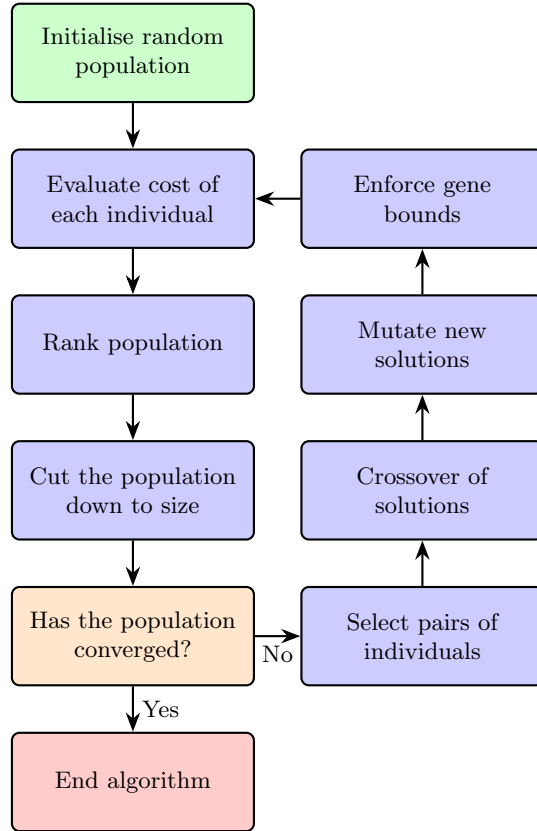


FIGURE I.4.2: Flowchart of the operation of a genetic algorithm.

Note that these values are unique to this problem, and will likely not be true for other problems.

#### I.4.4 GENETIC ALGORITHMS

Genetic algorithms (GAs) are more complex than the algorithms seen so far: they utilise Darwinian evolutionary theory to iterate towards an optimal solution. This is done by *breeding* and *mutating* candidate solutions over many generations; a schematic flowchart of the operation of a GA is shown in FIGURE I.4.2.

In GAs, a solution to an optimisation problem is represented as a genome (array of inputs), where each individual gene (element) represents a property of the solution. GAs have been utilised across a wide variety of applications, from the design of wind turbine blades [144] to credit risk analysis of supply chain finance [145] and have even being used to guide the editing of real genes in CPRISPR studies [146].

First, a population – of size  $N_{\text{pop}}$  – of candidate solutions are randomly initialised, within defined bounds; these are then evaluated using a cost function to find the relative ranking of the designs.

Then on each iteration, new designs are created via breeding of existing solutions,

then mutation of the genome of the new solutions. The first step of this process is the selection of parent solutions. There are a variety of methods to do this; one of the more common methods is called roulette wheel selection; in which the solutions are randomly selected, but weighted towards selecting the better performing solutions. Using the roulette wheel method, the probability of selecting individual  $i$  is

$$P(i) = e^{-\beta \bar{C}(i)}; \quad (\text{I.4.2})$$

with

$$\bar{C}(i) = \frac{C(i)}{\sum_{j=1}^{N_{\text{pop}}} C(j)} \quad (\text{I.4.3})$$

being the cost of individual  $i$ ; and  $\beta$  is a selection pressure factor (where a higher value leads to a more uniform distribution<sup>ii</sup>). This is just one of many selection methods, for a review of different types, the reader is directed to [147].

Whilst GAS are more resilient to local minima, there is still a chance they become stuck; this has resulted in the development of various methodologies to try and reduce the chance of premature convergence, or stagnation. One such method is of an *incest prevention algorithm*, in which an extra factor is added to the probability of selecting the second parent, to encourage the selection of genetically diverse parents. This extra term can be defined as

$$S(j) = \frac{\left( \sum_{a=1}^{N_{\text{gene}}} |g(j, a) - p(a)| \right)^\gamma}{\sum_{k=1}^{N_{\text{pop}}} \sum_{b=1}^{N_{\text{gene}}} |g(k, b) - p(b)|}, \quad (\text{I.4.4})$$

where  $S(j)$  is the selection factor to individual  $j$ 's probability;  $N_{\text{gene}}$  is the number of genes;  $g(j, a)$  is the  $a^{\text{th}}$  gene of individual  $j$ ;  $p(a)$  is the  $a^{\text{th}}$  gene of the first parent; finally  $\gamma$  is a selection factor, where a higher value favours the selection of genetically distinct second parent from the first. The probability of selecting the second parent would then be

$$P(i) = e^{-\beta S(i) \bar{C}(i)}. \quad (\text{I.4.5})$$

A review of other approaches to preventing early convergence is found in [148].

Now that two parents have been selected, the next step is creating the offspring. One method for how this can be achieved – for a pair of offspring – is via an arithmetic crossover

$$c_1(i) = \alpha p_1(i) + (1 - \alpha) p_2(i), \quad (\text{I.4.6})$$

$$c_2(i) = (1 - \alpha) p_1(i) + \alpha p_2(i); \quad (\text{I.4.7})$$

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<sup>ii</sup>  $\beta = \infty$  would result in a uniform distribution: *i.e.* parents are selected randomly.

where  $c_1(i)$ , and  $c_2(i)$  are the  $i^{\text{th}}$  genes of child 1 and 2 respectively;  $p_1(i)$ , and  $p_2(i)$  are the  $i^{\text{th}}$  genes of parent 1 and 2 respectively; and  $\alpha \in [-\beta, 1+\beta]$  is randomly chosen. For integer genes, the values would need to be rounded. A value of  $\beta = 0$  causes the search space to slowly reduce through the iterations of the algorithm: the children's genomes must be contained between the parent's genomes, so the difference between the two children must be equal to, or smaller than the gap between the parents, resulting in a reduction of the design space without necessarily honing in on the optimal solution. Therefore, making  $\beta > 0$  gives the algorithm more ability to explore the design space, by increasing the average difference in children's genomes. Typically a value of  $\beta = 0.25$  is recommended [149]. Again, there are a variety of methods to undergo crossover; see [150] for a summary.

Finally, the children are mutated. The aim of mutating is to further help the GA explore the design space. Typically, the operation looks like this

$$\underline{g}' = \underline{g} + \underline{\psi}(\mu, \sigma), \quad (\text{I.4.8})$$

where  $\underline{g}$  and  $\underline{g}'$  are the genomes pre- and post-mutation.  $\underline{\psi}(\mu, \sigma)$  is a statistical function for selecting a random number, usually a normal distribution [151]. It is recommended that the mutation rate (number of mutated genes) be equal to the inverse of the number of genes [149]; this is so that, on average, a single gene gets mutated. If multiple mutations happen, then a very poor mutation in one gene could overshadow a slightly good mutation in another.

The new generation are combined with the existing population and the worse performing cut such that the combined population is still of size  $N_{\text{pop}}$  – whether or not they are from the new or previous generation. It is generally recommended to keep the size of the new generation of children less than  $N_{\text{pop}}$  such that there are always some designs continuing between generations.

The GA iterates until either the population has converged on a solution, or a pre-defined maximum number of iterations has been reached; the former being a more rigorous way of ensuring the optimal solution has been found.

## *The Research Problem*

Throughout the chapter, it has been shown that the ocean waves are complex and unpredictable, with wave energy having to deal with similar strong forces as to tidal energy, but with the unpredictability of wind energy. This means that the design of wave energy converters (WECs) involves the interaction of many complex components. As a result, increasing numbers of researchers have been implementing control co-design (CCD) methodologies into their work, finding that this improves the performance of WECs; but so far investigations in to power take-off (PTO) CCD has been lacking.

This work introduces a fundamental, low-level co-optimisation framework that increases the fidelity of PTO CCD methodology within WEC CCD literature. The internal permanent magnet synchronous generator geometry parameters (rotor radius, number of pole pairs, *etc.*) are optimised alongside the floater scale to account for nonlinear electromechanical losses early in the design stage.

GAs have previously been shown to work in WEC CCD of floater geometry [135], arrays [137], and moorings [142]. Using GAs for this work introduces the possibility of integrating previously developed methods for the CCD of other components by expanding the size of the genome.

### I.5.1 THE CHOSEN WAVE ENERGY CONVERTER

The main focus of the work is the development of a model of the PTO, but because it is part of a CCD methodology, it is important to account for the designs of the other components of the WEC. Therefore, the overall WEC design is chosen to be detailed

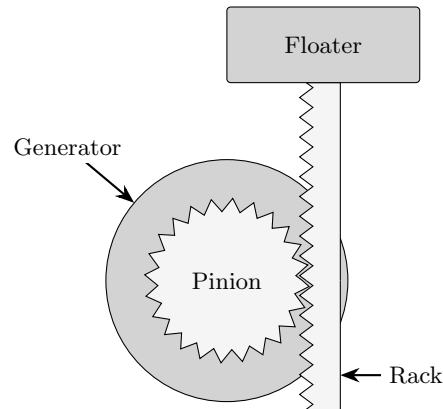


FIGURE I.5.1: Schematic of the chosen wave energy converter's drive train.

enough to be sufficiently accurate, but not too complex that its dynamics obscure the dynamics of the PTO. The chosen WEC is shown in FIGURE I.5.1. It is a point absorber; assumed to be perfectly moored such that its motion is unconstrained in heave, but it is fixed in all other DoF; and the floater is connected to a permanent magnet synchronous generator via a rack-and-pinion drive train.

The rack-and-pinion drive train was selected for its practical applicability, as the design is used at the core of both industrial (*e.g.* CorPower C4 [57]) and academic (*e.g.* Liang *et al.* [152]) designs. The CorPower C4 [57] uses a ‘cascade gearbox’ which consists of many smaller pinions connected to a single rack. Liang *et al.* designed a WEC with a ‘mechanical motion rectifier’ gearbox which at its core has two separate rack-and-pinions, one engages in the rising motion and one in the falling motion of the floater such that the bidirectional motion of the floater is converted into a unidirectional input to the generator.

## I.5.2 OVERVIEW OF THE REMAINDER OF THE THESIS

The remainder of the thesis is structured as follows:

- § II develops the model, detailing the theory behind the model and the implementation into a genetic algorithm. A flow chart of the model’s operation is shown in FIGURE I.5.2.
- § III uses the model to investigate differences in the optimal PTOs design when varying: control strategy; floater design; and location.
- § IV discusses previous methods of optimising the floater design expands the model to allow for the novel co-optimisation of PTO and floater, the altered flow chart is shown in FIGURE I.5.3. The chapter ends with an investigation into differences in optimal WEC designs when varying the level of CCD integration.
- § V adapts the model to create *synthetic* PTO designs for a wave-tank scale WEC. Then the monochromatic-sea-state power simulation is replaced with a polychromatic-sea-state power simulation to investigate how the power output changes in more realistic conditions.
- § VI gives the final conclusions, recommendations for WEC design in the future, and avenues for future work.

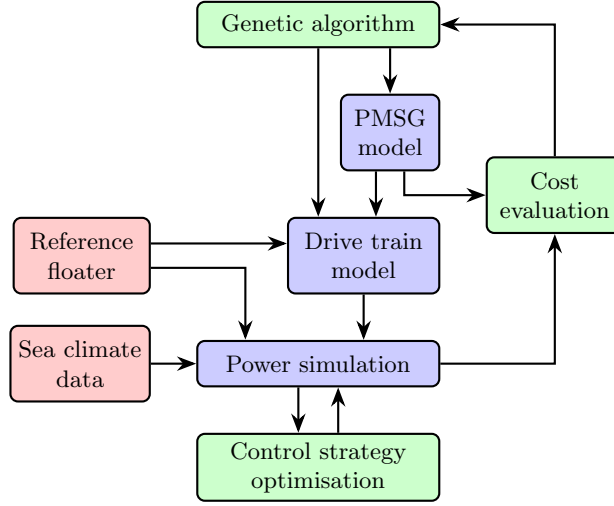


FIGURE I.5.2: Flow chart of the §II model. Red blocks show data inputs; blue blocks show physical models; and green blocks show optimisation tools.

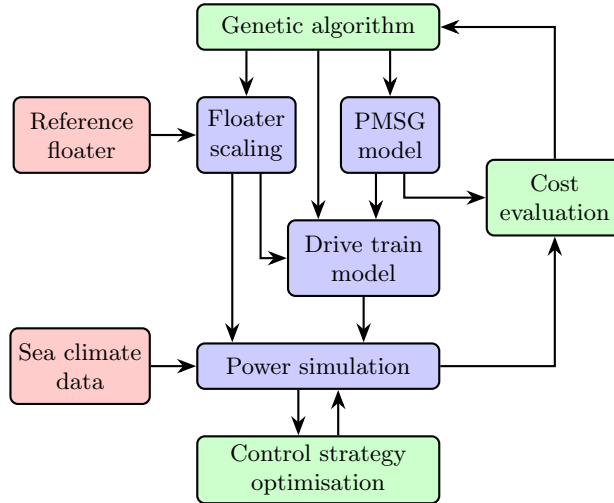


FIGURE I.5.3: Flow chart of the §IV model. Red blocks show data inputs; blue blocks show physical models; and green blocks show optimisation tools.

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## Bibliography

- [1] Walter H. Munk. “Origin and Generation of Waves”. In: *Proceedings of First Conference on Coastal Engineering*. Long Beach, Californina, 1950 (cited on pages 2, 3).
- [2] Blair Kinsman. “Wind Waves: Their Generation and Propagation on the Ocean Surface”. In: 1st ed. New York: Dover Publications, 1965. Chap. 1. ISBN: 0-486-64652-1 (cited on page 2).
- [3] Steven Earle. “Physical Geology”. In: 2nd ed. BCcampus, 2019. Chap. 17.1. ISBN: 978-1-77420-028-5 (cited on page 3).
- [4] Denis Mollison. “Wave climate and the wave power resource”. In: *IUTAM Symposium Lisbon*. 1985 (cited on page 3).
- [5] *Atlas of UK Marine Renewable Energy Resources*. Last Accessed: 19/7/26. 2008. URL: <http://www.renewables-atlas.info/> (cited on pages 4, 10).
- [6] *Wave Energy Road Map*. Tech. rep. Supergen Offshore Renewable Energy Hub, 2020 (cited on page 3).
- [7] Willard J. Pierson Jr. and Lionel Moskowitz. “A Proposed Spectral Form for Fully Developed Wind Seas Based on the Similarity Theory of S. A. Kitaigorodskii”. In: *Journal of Geophysical Research* (1964). DOI: <https://doi.org/10.1029/JZ069i024p05181> (cited on page 4).
- [8] K. Hasselmann et al. *Measurements of Wind-Wave Growth and Swell Decay during the Joint North Sea Wave Project (JONSWAP)*. Tech. rep. Deutsches Hydrographisches Institut, 1973 (cited on pages 4, 5).
- [9] Nikolaos D. Katopodes. “Free-Surface Flow: Shallow-water dynamics”. In: Oxford: Butterworth-Heinemann, 2018. Chap. 2.5.4.1. ISBN: 978-0-12-815487-8 (cited on page 4).
- [10] O. M. Phillips. “The equilibrium range in the spectrum of wind-generated waves”. In: *Journal of Fluid Mechanics* (1958). DOI: <https://doi.org/10.1017/S0022112058000550> (cited on page 4).

- [11] C. A. Hegermiller et al. “A Multimodal Wave Spectrum–Based Approach for Statistical Downscaling of Local Wave Climate”. In: *Journal of Physical Oceanography* (2017). DOI: [10.1175/JPO-D-16-0191.1](https://doi.org/10.1175/JPO-D-16-0191.1) (cited on page 6).
- [12] C. Guedes Soares. “Representation of double-peaked sea wave spectra”. In: *Ocean Engineering* 11 (1984) (cited on page 6).
- [13] Edward B.L. Mackay. “Modelling and Description of Omnidirectional Wave Spectra”. In: *Proceedings of 9th European Wave and Tidal Conference*. Southampton, 2011 (cited on page 6).
- [14] David W. Wang and Paul A. Hwang. “An Operational Method for Separating Wind Sea and Swell from Ocean Wave Spectra”. In: *Journal of Atmospheric and Oceanic Technology* (2001). DOI: [10.1175/1520-0426\(2001\)018<2052:AOMFSW>2.0.CO;2](https://doi.org/10.1175/1520-0426(2001)018<2052:AOMFSW>2.0.CO;2) (cited on page 6).
- [15] Hisashi Mitsuyasu et al. “Observations of the Directional Spectrum of Ocean Waves Using a Cloverleaf Buoy”. In: *Journal of Physical Oceanography* (1975). DOI: [10.1175/1520-0485\(1975\)005<0750:00TDS0>2.0.CO;2](https://doi.org/10.1175/1520-0485(1975)005<0750:00TDS0>2.0.CO;2) (cited on pages 6, 7).
- [16] Ian R. Young. “Wind Generated Ocean Waves”. In: 1st ed. Oxford: Elsevier, 1999. Chap. 5.5. ISBN: 0-08-043317-0 (cited on page 7).
- [17] Centre for Environment Fisheries and Aquaculture Science. *Tyne/Tees WaveNet Site*. Last Accessed: 19/7/26. URL: <https://wavenet.cefas.co.uk/details/TYNETEESWN/INT> (cited on page 7).
- [18] International Electrotechnical Commission. *PD IEC TS 626000-100:2024: Marine energy - Wave, tidal and other water current converters - Part 100: Electricity producing wave energy converters - Power performance assessment*. 2024 (cited on page 7).
- [19] Blair Kinsman. “Wind Waves: Their Generation and Propagation on the Ocean Surface”. In: 1st ed. New York: Dover Publications, 1965. Chap. 3.1. ISBN: 0-486-64652-1 (cited on page 8).
- [20] International Association for Hydraulic Research Working Group on Wave Generation and Analysis. “List of Sea-State Parameters”. In: *Journal of Waterway, Port, Coastal, and Ocean Engineering* (1989). DOI: [10.1061/\(ASCE\)0733-950X\(1989\)115:6\(793\)](https://doi.org/10.1061/(ASCE)0733-950X(1989)115:6(793)) (cited on page 8).
- [21] Subrata Kumar Chakrabarti. “Offshore Structure Modeling”. In: 1st. Singapore: World Scientific, 1994. Chap. 10.3.4. ISBN: 981-02-1512-6 (cited on page 8).

- [22] E.P.D. Mansard and E.R. Funke. “On the fitting of parametric models to measured wave spectra”. In: *Proceedings of 2nd International Symposium on Wave Research and Coastal Engineering*. 1988 (cited on page 8).
- [23] Johannes Falnes and Adi Kurniawan. “Ocean Waves and Oscillating Systems: Linear interactions Including Wave-Energy Extraction”. In: 2nd. Cambridge: Cambridge University Press, 2020. Chap. 4.5. DOI: [978-1-108-48166-3](https://doi.org/10.1017/9781108481663) (cited on page 8).
- [24] Brian Holmes. *Tank Testing of Wave Energy Conversion Systems*. Tech. rep. European Marine Energy Centre, 2009 (cited on page 9).
- [25] Mohammad Reza Tabeshpour and Navid Belvasi. “Ocean waves time-series generation: minimum required artificial wave time-series for wave energy converter analysis”. In: *Journal of Marine Engineering & Technology* (2023). DOI: [10.1080/20464177.2023.2197280](https://doi.org/10.1080/20464177.2023.2197280) (cited on page 9).
- [26] Gary D. Egbert and Richard D. Ray. “Tidal Prediction”. In: *Journal of Marine Research* 75 (2017), pp. 189–237. DOI: [10.1357/002224017821836761](https://doi.org/10.1357/002224017821836761) (cited on page 10).
- [27] G. Todeschini et al. “Medium-term variability of the UK’s combined tidal energy resource for a net-zero carbon grid”. In: *Energy* 238.121990 (2022). DOI: [10.1016/j.energy.2021.121990](https://doi.org/10.1016/j.energy.2021.121990) (cited on page 10).
- [28] Roger H. Charlier and Charles W. Finkl. “Ocean Energy: Tide and Tidal Power”. In: 1st ed. Springer Berlin, 2010. Chap. 3.5. DOI: [10.1007/978-3-540-77932-2](https://doi.org/10.1007/978-3-540-77932-2) (cited on page 10).
- [29] Isaac Van der Hoven. “Power spectrum of horizontal wind speed in the frequency range from 0.0007 to 900 cycles per hour”. In: *Journal of Meteorology* 14.2 (Apr. 1956), pp. 160–164. DOI: [10.1175/1520-0469\(1957\)014<0160:PSOHWS>2.0.CO;2](https://doi.org/10.1175/1520-0469(1957)014<0160:PSOHWS>2.0.CO;2) (cited on pages 10, 11).
- [30] J. F. Chozas, J. P. Kofoed, and H. C. Sørensen. *Predictability and Variability of Wave and Wind*. Tech. rep. Aalborg University, 2013 (cited on page 10).
- [31] M. S. Longuet-Higgins. “Mass Transport in Water Waves”. In: *Philosophical Transactions of The Royal Society A* 245.903 (1953), pp. 535–581. DOI: [10.1098/rsta.1953.0006](https://doi.org/10.1098/rsta.1953.0006) (cited on page 10).
- [32] David Jesús Yáñez Villarreal. *VIV resonant wind generators*. Tech. rep. Vortex Bladeless, 2018 (cited on page 11).

- 
- [33] P. Corigliano and F. Frisone. “On the Fatigue of Ship-Structures under Wave Loads”. In: *Journal of Civil Engineering Research & Technology* 6.5 (2024), pp. 1–7. DOI: [10.47363/JCERT/2024\(6\)163](https://doi.org/10.47363/JCERT/2024(6)163) (cited on page 11).
- [34] Iraide López et al. “Review of wave energy technologies and the necessary power-equipment”. In: *Renewable and Sustainable Energy Reviews* 27 (2013). DOI: [10.1016/j.rser.2013.07.009](https://doi.org/10.1016/j.rser.2013.07.009) (cited on pages 11, 24).
- [35] Hugh D. Young and Roger A. Freedman. “University Physics with Modern Physics”. In: 14th ed. Pearson Education Limited, 2016. Chap. 39.5. ISBN: 978-1-292-10031-9 (cited on page 11).
- [36] The British Standards Institution. *BS ISO 9845-1:2022 - Solar energy. Reference solar spectral irradiance at the ground at different receiving conditions. Direct normal and hemispherical solar irradiance for air mass 1,5*. 2022 (cited on pages 11, 12).
- [37] Brandon R. Sutherland. “Solar Materials Find Their Band Gap”. In: *Joule* 4.5 (2020), pp. 984–985. DOI: [10.1016/j.joule.2020.05.001](https://doi.org/10.1016/j.joule.2020.05.001) (cited on pages 11, 12).
- [38] David Ross. “Power from the waves”. In: Oxford University Press, 1995. Chap. 2. ISBN: 0198565119 (cited on page 13).
- [39] J. M. Leishman and G. Scobie. *The Development of Wave Power: A Techno-Economic Study*. Tech. rep. National Engineering Laboratory, 1976 (cited on page 13).
- [40] António F.O. Falcão. “Wave energy utilization: A review of the technologies”. In: *Renewable and Sustainable Energy Reviews* 14 (2010), pp. 899–918. DOI: [10.1016/j.rser.2009.11.003](https://doi.org/10.1016/j.rser.2009.11.003) (cited on page 13).
- [41] Alain Clément et al. “Wave energy in Europe: current status and perspectives”. In: *Renewable and Sustainable Energy Reviews* 6 (2002), pp. 405–431. DOI: [10.1016/S1364-0321\(02\)00009-6](https://doi.org/10.1016/S1364-0321(02)00009-6) (cited on page 13).
- [42] S. H. Salter. “Wave Power”. In: *Nature* 249 (1974), pp. 720–724. DOI: [10.1038/249720a0](https://doi.org/10.1038/249720a0) (cited on pages 13, 17).
- [43] Roger Henri Charlier and John R. Justus. “Ocean energies: environmental, economic, and technological aspects of alternative power sources”. In: vol. 56. Elsevier, 1993. Chap. 5. ISBN: 9780080870946 (cited on page 13).

- 
- [44] Ron Iphofen. “The UK Wave Power Project: Salter’s Duck”. In: *Ethics, Integrity and Policymaking: The Value of the Case Study*. Ed. by Dónal O’Mathúna and Ron Iphofen. Springer International Publishing, 2022. Chap. 8, pp. 101–110. ISBN: 978-3-031-15746-2. DOI: [10.1007/978-3-031-15746-2\\_8](https://doi.org/10.1007/978-3-031-15746-2_8) (cited on page 13).
  - [45] Giuseppe Costanzo, Guy Brindley, and Pierre Tardieu. *Wind energy in Europe: 2024 Statistics and the outlook for 2025-2030*. Tech. rep. Wind Europe, 2025 (cited on page 13).
  - [46] Supergen Offshore Renewable Energy Hub. *Welcome to the Supergen ORE Hub*. Last Accessed: 19/7/26. URL: <https://supergen-ore.net> (cited on page 13).
  - [47] Adam Thirkill, Uwe Remme, and Grant Allan. *Delivering Net Zero: Forecasting Wave and Tidal Stream Deployment in UK Waters by 2050*. Tech. rep. Supergen Offshore Renewable Energy Hub, 2023 (cited on page 14).
  - [48] Emma Pinchbeck et al. *The Seventh Carbon Budget*. Tech. rep. Climate Change Committee, 2025 (cited on page 14).
  - [49] CorPower Ocean. *CorPower Ocean secures world-first DNV certification for wave energy technology*. Last Accessed: 23/7/26. URL: <https://corpowerocean.com/corpower-ocean-secures-world-first-dnv-certification-for-wave-energy-technology/> (cited on page 14).
  - [50] Jan Steinkohl et al. *Innovation outlook: Ocean energy technologies*. Tech. rep. International Renewable Energy Agency, 2020 (cited on page 14).
  - [51] “Encyclopedia of Ocean Engineering”. In: ed. by Weicheng Cui, Shixiao Fu, and Zhiqiang Hu. 1st ed. Springer Singapore, 2022. Chap. Wave Energy Converters. DOI: <https://doi.org/10.1007/978-981-10-6946-8> (cited on pages 14, 17).
  - [52] Linus Mofor, Jarett Goldsmith, and Fliss Jones. *Ocean Energy: Technology Readiness, Patents, Deployment Status and Outlook*. Tech. rep. International Renewable Energy Agency, 2014 (cited on page 15).
  - [53] Johannes Falnes and Adi Kurniawan. “Ocean Waves and Oscillating Systems: Linear interactions Including Wave-Energy Extraction”. In: 2nd. Cambridge: Cambridge University Press, 2020. Chap. 6.1.1. DOI: [978-1-108-48166-3](https://doi.org/10.1017/9781108481663.006) (cited on page 14).
  - [54] Bingyong Guo et al. “A Review of Point Absorber Wave Energy Converters”. In: *Journal of Marine Science and Engineering* 10 (2022). DOI: <https://doi.org/10.3390/jmse10101534> (cited on page 16).

- [55] B. M. Count. “Power from Sea Waves”. In: ed. by Brian Count. Academic Press, 1980. Chap. 2.3. ISBN: 0-12-193550-7 (cited on page 16).
- [56] Johannes Falnes and Jørgen Hals. “Heaving buoys, point absorbers and arrays”. In: *Philosophical Transactions of The Royal Society A* 370 (2012). DOI: <https://doi.org/10.1098/rsta.2011.0249> (cited on page 16).
- [57] CorPower Ocean. Last Accessed: 19/7/26. URL: <https://corpowerocean.com/> (cited on pages 16, 46).
- [58] Carnegie Clean Energy. *CETO*. Last Accessed: 19/7/26. URL: <https://www.carnegiece.com/ceto-technology/> (cited on page 16).
- [59] Seabased. Last Accessed: 19/7/26. URL: <https://seabased.com/the-technology> (cited on page 16).
- [60] N.Y. Sergiienko et al. “Performance comparison of the floating and fully submerged quai-point absorber wave energy converters”. In: *Renewable Energy* 108 (2017). DOI: <http://dx.doi.org/10.1016/j.renene.2017.03.002> (cited on page 16).
- [61] Morten M. Kramer, Laurent Marquis, and Peter Frigaard. “Performance Evaluation of the Wavestar Prototype”. In: *Proceedings of 9th European Wave and Tidal Conference*. 2011 (cited on page 16).
- [62] Richard Yemm et al. “Pelamis: experience from concept to connection”. In: *Philosophical Transactions of The Royal Society A* 370 (2012) (cited on page 17).
- [63] Ronan Costello et al. “Techno-Economic Optimisation for Wave Energy Converters”. In: *4th International Conference on Ocean Energy*. 2012 (cited on page 17).
- [64] Peter B. Williams et al. *Updated report on the Edinburgh - Liang wave energy device (The Duck)*. Tech. rep. University of Edinburgh, 1981. DOI: <http://hdl.handle.net/1842/23534> (cited on page 17).
- [65] B. M. Count. “Power from Sea Waves”. In: ed. by Brian Count. Academic Press, 1980. Chap. 2.1. ISBN: 0-12-193550-7 (cited on page 17).
- [66] European Marine Energy Centre Ltd. *Aquamarine Power*. Last Accessed: 19/7/26. URL: <https://www.emec.org.uk/about-us/wave-clients/aquamarine-power/> (cited on page 17).
- [67] Dominic D. Forbush et al. “Design and testing of a free floating dual flap wave energy converter”. In: *Energy* 240 (2022). DOI: <https://doi.org/10.1016/j.energy.2021.122485> (cited on page 17).
- [68] H. Fernandez et al. “The new wave energy converter WaveCat: Concept and laboratory tests”. In: *Marine Structures* 29 (2012) (cited on page 18).

- [69] L. Margheritini, D. Vicinanza, and P. Frigaard. “SSG wave energy converter: Design, reliability and hydraulic performance of an innovative overtopping device”. In: *Renewable Energy* 34 (2009) (cited on page 18).
- [70] B. M. Count. “Power from Sea Waves”. In: ed. by Brian Count. Academic Press, 1980. Chap. 2.1.4. ISBN: 0-12-193550-7 (cited on page 18).
- [71] António F.O. Falcão et al. “The Pico OWC wave power plant: Its lifetime from conception to closure 1986-2018”. In: *Applied Ocean Research* 98 (2020). DOI: <https://doi.org/10.1016/j.apor.2020.102104> (cited on page 18).
- [72] Wave Swell. Last Accessed: 19/7/26. URL: <https://www.waveswell.com> (cited on page 18).
- [73] A. Crespo, J. Hernández, and S. Frandsen. “Survey of Modelling Methods for Wind Turbine Wakes and Wind Farms”. In: *Wind Energy* 2.1 (1999). DOI: [10.1002/\(SICI\)1099-1824\(199901/03\)2:1<1::AID-WE16>3.0.CO;2-7](https://doi.org/10.1002/(SICI)1099-1824(199901/03)2:1<1::AID-WE16>3.0.CO;2-7) (cited on page 18).
- [74] A. D. de Andrés et al. “Factors that influence array layout on wave energy farms”. In: *Ocean Engineering* 82 (2014), pp. 32–41. DOI: [10.1016/j.oceaneng.2014.02.027](https://doi.org/10.1016/j.oceaneng.2014.02.027) (cited on page 19).
- [75] Paula B. Garcia-Rosa, Giorgio Bacelli, and John V. Ringwood. “Control-Informed Optimal Array Layout for Wave Farms”. In: *IEEE Transactions on Sustainable Energy* 6.2 (2015), pp. 575–582. DOI: [10.1109/TSTE.2015.2394750](https://doi.org/10.1109/TSTE.2015.2394750) (cited on pages 19, 37, 38).
- [76] P. Ricci. “Numerical Modelling of Wave Energy Converters”. In: ed. by Matt Folley. Academic Press, 2016. Chap. 3.1. ISBN: 9780128032107 (cited on page 19).
- [77] Joseph Edward Shigley and John Joseph Uicker Jr. “Theory of Machines and Mechanisms”. In: McGraw-Hill, Inc., 1980. Chap. 1.6 (cited on page 19).
- [78] M. Alves. “Numerical Modelling of Wave Energy Converters”. In: ed. by Matt Folley. Academic Press, 2016. Chap. 2.1. ISBN: 9780128032107 (cited on page 20).
- [79] M. Alves. “Numerical Modelling of Wave Energy Converters”. In: ed. by Matt Folley. Academic Press, 2016. Chap. 2.5. ISBN: 9780128032107 (cited on pages 20, 21).
- [80] M. Alves. “Numerical Modelling of Wave Energy Converters”. In: ed. by Matt Folley. Academic Press, 2016. Chap. 2.4.1.1. ISBN: 9780128032107 (cited on page 20).
- [81] WAMIT. Last Accessed: 19/7/26. URL: <https://www.wamit.com> (cited on page 21).

- 
- [82] A. Babarit and G. Delhommeau. “Theoretical and numerical aspects of the open source BEM solver NEMOH”. In: *Proceedings of 11th European Wave and Tidal Energy Conference*. 2015 (cited on pages 21, 22).
  - [83] V. Raghavan et al. “A comparative study on BEM solvers for Wave Energy Converters”. In: *Trends in Renewable Energies Offshore* (2022). DOI: [10.1201/9781003360773-50](https://doi.org/10.1201/9781003360773-50) (cited on pages 21, 22).
  - [84] *WAMIT User Manual - Version 7.5*. Tech. rep. WAMIT, Inc., 2023 (cited on page 21).
  - [85] Yingyi Liu. “HAMS: A Frequency-Domain Preprocessor for Wave-Structure Interactions – Theory, Development, and Application”. In: *Journal of Marine Science and Engineering* 7 (2019). DOI: [10.3390/jmse7030081](https://doi.org/10.3390/jmse7030081) (cited on page 22).
  - [86] Matthieu Ancellin and Frédéric Dias. “Capytaine: a Python-based linear potential flow solver”. In: *Journal of Open Source Software* 4 (2019). DOI: [10.21105/joss.01341](https://doi.org/10.21105/joss.01341) (cited on page 22).
  - [87] Vaibhav Raghavan and George Lavidas. “Comparing BEM solvers for analyzing wave energy converters”. In: *Journal of OTEC (Departmental Bulletin Paper)* 27 (2023) (cited on page 22).
  - [88] W. E. Cummins. “The Impulse Response Function and Ship Motions”. In: *Schiffstechnik* 47 (1962) (cited on page 22).
  - [89] P. Ricci. “Numerical Modelling of Wave Energy Converters”. In: ed. by Matt Folley. Academic Press, 2016. Chap. 3.2. ISBN: 9780128032107 (cited on page 22).
  - [90] P. Ricci. “Numerical Modelling of Wave Energy Converters”. In: ed. by Matt Folley. Academic Press, 2016. Chap. 3.3.2. ISBN: 9780128032107 (cited on page 23).
  - [91] P. Ricci. “Numerical Modelling of Wave Energy Converters”. In: ed. by Matt Folley. Academic Press, 2016. Chap. 3.3.4. ISBN: 9780128032107 (cited on page 23).
  - [92] Stefan G. Siegel. “Numerical benchmarking study of a Cycloidal Wave Energy Converter”. In: *Renewable Energy* 134 (2019). DOI: [10.1016/j.renene.2018.11.041](https://doi.org/10.1016/j.renene.2018.11.041) (cited on page 23).
  - [93] Adam Stock. “Accounting for power take-off efficiency in optimal velocity tracking control of wave energy converters”. In: *Ocean Engineering* 312 (2024). DOI: [10.1016/j.oceaneng.2024.119191](https://doi.org/10.1016/j.oceaneng.2024.119191) (cited on pages 23, 25, 30, 39).
  - [94] Y. Huang et al. “A CFD-FEM analysis for Anaconda WEC with mooring lines”. In: *Proceedings of 15th European Wave and Tidal Energy Conference*. Bilbao, 2023. DOI: [10.36688/ewtec-2023-163](https://doi.org/10.36688/ewtec-2023-163) (cited on page 23).

- 
- [95] Nathan Tom. “Review of Wave Energy Converter Power Take-Off Systems, Testing Practices and Evaluation Metrics”. In: *Proceedings of ASME 2022 International Mechanical Engineering Congress and Exposition*. 2023. DOI: [10.1115/IMECE2022-94077](#) (cited on page 23).
- [96] Raju Ahamed, Kristoffer McKee, and Ian Howard. “Advancements of wave energy converters based on power take off (PTO) systems: A review”. In: *Ocean Engineering* 204 (2020). DOI: [10.1016/j.oceaneng.2020.107248](#) (cited on pages 23, 25).
- [97] Ross Henderson. “Design, simulation, and testing of a novel hydraulic power take-off system for the Pelamis wave energy converter”. In: *Renewable Energy* 31 (2006). DOI: [10.1016/j.renene.2005.08.021](#) (cited on page 24).
- [98] Rico H. Hansen, Morten M. Kramer, and Enrique Vidal. “Discrete Displacement Hydraulic Power Take-off System for the Wavestar Wave Energy Converter”. In: *Energies* 6 (2013). DOI: [10.3390/en6084001](#) (cited on page 24).
- [99] Markel Penalba et al. “Validating a Wave-to-Wire Model for a Wave Energy Converter—Part I: The Hydraulic Transmission System”. In: *Energies* 10.977 (2017). DOI: [10.3390/en10070977](#) (cited on page 25).
- [100] Markel Penalba and John V. Ringwood. “A Review of Wave-to-Wire Models for Wave Energy Converters”. In: *Energies* 9.506 (2016). DOI: [10.3390/en9070506](#) (cited on page 25).
- [101] *PowerPod Datasheet*. Tech. rep. Trident Energy Ltd. (cited on page 25).
- [102] Jeff Grasberger et al. “Control co-design and optimization of oscillating-surge wave energy converter”. In: *Renewable Energy* 225.120234 (Feb. 2024). DOI: [10.1016/j.renene.2024.120234](#) (cited on pages 25, 26).
- [103] Robert Harris, Lars Johanning, and Julian Wolfram. “Mooring systems for wave energy converters: A review of design issues and choices”. In: *Proceedings of 3rd International Conference on Marine Renewable Energy*. 2004 (cited on page 26).
- [104] C. Mingham, L. Qian, and D. Causon. “Numerical Modelling of Wave Energy Converters”. In: ed. by Matt Folley. Academic Press, 2016. Chap. 6.1. ISBN: 9780128032107 (cited on page 27).
- [105] Bingyong Guo and John V. Ringwood. “Geometric optimisation of wave energy conversion devices: A survey”. In: *Applied Energy* 297 (2021). DOI: [10.1016/j.apenergy.2021.117100](#) (cited on pages 27, 36).

- 
- [106] C. Mingham, L. Qian, and D. Causon. “Numerical Modelling of Wave Energy Converters”. In: ed. by Matt Folley. Academic Press, 2016. Chap. 6. ISBN: 9780128032107 (cited on page 27).
- [107] Johannes Falnes and Adi Kurniawan. “Ocean Waves and Oscillating Systems: Linear interactions Including Wave-Energy Extraction”. In: 2nd. Cambridge: Cambridge University Press, 2020. Chap. 6.3. DOI: [978-1-108-48166-3](https://doi.org/10.1017/9781108481663) (cited on page 28).
- [108] Jørgen Hals, Johannes Falnes, and Torgeir Moan. “A Comparison of Selected Strategies for Adaptive Control of Wave Energy Converters”. In: *Journal of Offshore Mechanics and Arctic Engineering* (2011). DOI: [10.1115/1.4002735](https://doi.org/10.1115/1.4002735) (cited on page 29).
- [109] Francesco Fusco and John V. Ringwood. “A Simple and Effective Real-Time Controller for Wave Energy Converters”. In: *IEEE Transactions on Sustainable Energy* 4.1 (2013). DOI: [10.1109/TSTE.2012.2196717](https://doi.org/10.1109/TSTE.2012.2196717) (cited on page 30).
- [110] Yeraí Peña-Sánchez et al. “A Critical Comparison of Excitation Force Estimators for Wave-Energy Devices”. In: *IEEE Transactions on Control Systems Technology* (2020). DOI: [10.1109/TCST.2019.2939092](https://doi.org/10.1109/TCST.2019.2939092) (cited on page 30).
- [111] Francesco Paparella et al. “Benefits of up-wave measurements in linear short-term wave forecasting for wave energy applications”. In: *2014 IEEE Conference on Control Applications*. Juan Les Antibes, France, 2014. DOI: [10.1109/CCA.2014.6981604](https://doi.org/10.1109/CCA.2014.6981604) (cited on page 30).
- [112] K. Monk et al. “Pneumatic Power Regulation by Wave Forecasting and Real-Time Relief Valve Control for an OWC”. In: *Proceedings of 10th European Wave and Tidal Energy Conference*. Aalborg, Denmark, 2013 (cited on page 30).
- [113] Hyeonaseok Lee and Yeonsook Heo. “Simplified data-driven models for model predictive control of residential buildings”. In: *Energy & Buildings* 265.112067 (2022). DOI: [10.1016/j.enbuild.2022.112067](https://doi.org/10.1016/j.enbuild.2022.112067) (cited on page 31).
- [114] Renhao Mao et al. “Deep Koopman-operator-based model predictive control for free-floating space robots with disturbance observer”. In: *Aerospace Science and Technology* 154.109515 (2024). DOI: [10.1016/j.ast.2024.109515](https://doi.org/10.1016/j.ast.2024.109515) (cited on page 31).
- [115] Max Schwenzer et al. “Review on model predictive control: an engineering perspective”. In: *The International Journal of Advanced Manufacturing Technology* 117 (2021). DOI: [10.1007/s00170-021-07682-3](https://doi.org/10.1007/s00170-021-07682-3) (cited on page 31).

- 
- [116] Carlos E. Garcia and Manfred Morari. “Internal model control. A unifying review and some new results”. In: *Industrial & Engineering Chemistry Process Design and Development* 21.2 (1982). DOI: [10.1021/i200017a016](https://doi.org/10.1021/i200017a016) (cited on page 31).
- [117] A. J. Hillis et al. “Model predictive control of a multi-degree-of-freedom wave energy converter with model mismatch and prediction errors”. In: *Ocean Engineering* 212.107724 (2020). DOI: [10.1016/j.oceaneng.2020.107724](https://doi.org/10.1016/j.oceaneng.2020.107724) (cited on page 31).
- [118] A. Babarit. “A database of capture width ratio of wave energy converters”. In: *Renewable Energy* 80 (2015), pp. 610–628. DOI: [10.1016/j.renene.2015.02.049](https://doi.org/10.1016/j.renene.2015.02.049) (cited on page 32).
- [119] J. J. Jensen. “Load and Global Response of Ships”. In: 1st ed. Vol. 4. Elsevier Science, 2001. Chap. 4. ISBN: 0 08 043953 5 (cited on page 32).
- [120] Peter Aird. “Deepwater Drilling: Well Planning, Design, Engineering, Operations, and Technology Application”. In: Gulf Professional Publishing, 2018. Chap. 4. ISBN: 978-0-08-102282-5 (cited on page 32).
- [121] Grace Chang et al. “A comprehensive evaluation of factors affecting the levelized cost of wave energy conversion projects”. In: *Renewable Energy* 127 (2018), pp. 334–354. DOI: [10.1016/j.renene.2018.04.071](https://doi.org/10.1016/j.renene.2018.04.071) (cited on page 32).
- [122] K. J. Astrom, R. E. Klein, and A. Lennartsson. “Bicycle dynamics and control: adapted bicycles for education and research”. In: *IEEE Control Systems Magazine* 25.4 (2005), pp. 26–47. DOI: [10.1109/MCS.2005.1499389](https://doi.org/10.1109/MCS.2005.1499389) (cited on page 35).
- [123] M. L. Luyben and C. A. Floudas. “Analyzing the interaction of design and control—1. A multiobjective framework and application to binary distillation synthesis”. In: *Computers & Chemical Engineering* 18.10 (1994), pp. 933–969. DOI: [10.1016/0098-1354\(94\)E0013-D](https://doi.org/10.1016/0098-1354(94)E0013-D) (cited on page 35).
- [124] M. L. Luyben and C. A. Floudas. “Analyzing the interaction of design and control—2. reactor-separator-recycle system”. In: *Computers & Chemical Engineering* 18.10 (1994), pp. 971–993. DOI: [10.1016/0098-1354\(94\)85006-2](https://doi.org/10.1016/0098-1354(94)85006-2) (cited on page 35).
- [125] Mario Garcia-Sanz. “Control Co-Design: An engineering game changer”. In: *Advanced Control for Applications* 1.1 (2019). DOI: [10.1002/adc2.18](https://doi.org/10.1002/adc2.18) (cited on page 35).

- 
- [126] Saeid Bayata, Yong Hoon Lee, and James T. Allison. “Nested control co-design of a spar buoy horizontal-axis floating offshore wind turbine”. In: *Ocean Engineering* 328.121037 (2025). DOI: [10.1016/j.oceaneng.2025.121037](https://doi.org/10.1016/j.oceaneng.2025.121037) (cited on page 35).
- [127] Jingjie Wu and Lei Zhou. “Control Co-design of Actively Controlled Lightweight Structures for High-acceleration Precision Motion Systems”. In: *2022 American Control Conference (ACC)*. 2022. DOI: [10.23919/ACC53348.2022.9867780](https://doi.org/10.23919/ACC53348.2022.9867780) (cited on page 35).
- [128] Julie A. Reyer et al. “Comparison of combined embodiment design and control comitisation strategies using optimality conditions”. In: *Proceedings of the Design Engineering Technical Conference*. 2001 (cited on page 35).
- [129] Daniel R. Herber and James T. Allison. “Nested and Simultaneous Solution Strategies for General Combined Plant and Control Design Problems”. In: *Journal of Mechanical Design* 141.1 (2019). DOI: [10.1115/1.4040705](https://doi.org/10.1115/1.4040705) (cited on page 35).
- [130] John V. Ringwood. “Control co-design for wave energy systems”. In: *Applied Ocean Research* 158.104514 (2025). DOI: [10.1016/j.apor.2025.104514](https://doi.org/10.1016/j.apor.2025.104514) (cited on page 35).
- [131] Jean-Christophe Gilloteaux and John Ringwood. “Control-informed geometric optimisation of wave energy converters”. In: *8th IFAC Conference on Control Applications in Marine Systems*. 2010. DOI: [10.3182/20100915-3-DE-3008.00072](https://doi.org/10.3182/20100915-3-DE-3008.00072) (cited on page 36).
- [132] Marianna Giassi and Malin Götema. “Parameter Optimization in Wave Energy Design by a Genetic Algorithm”. In: *Proceedings of 32nd International Workshop on Water Waves and Floating Bodies*. 2017 (cited on page 36).
- [133] Paula B. Garcia-Rosa and John V. Ringwood. “On the Sensitivity of Optimal Wave Energy Device Geometry to the Energy Maximizing Control System”. In: *IEEE Transactions on Sustainable Energy* 7.1 (2016). DOI: [10.1109/TSTE.2015.2423551](https://doi.org/10.1109/TSTE.2015.2423551) (cited on page 36).
- [134] Emma Edwards. “Optimization of the Geometry of Axisymmetric Point-absorber Wave Energy Converters”. PhD thesis. Massachusetts Institute of Technology, 2020 (cited on pages 36, 39).
- [135] Anna Garcia-Teruel. “Geometry Optimisation of Wave Energy Converters”. PhD thesis. University of Edinburgh, 2020 (cited on pages 37, 39, 45).

- [136] A. P. McCabe. “Constrained optimization of the shape of a wave energy collector by genetic algorithm”. In: *Renewable Energy* 51 (2013), pp. 274–284. DOI: [10.1016/j.renene.2012.09.054](https://doi.org/10.1016/j.renene.2012.09.054) (cited on page 37).
- [137] Saeed Azad et al. “Concurrent geometry, control and layout optimization of wave energy converter farms in probabilistic irregular waves using surrogate modeling”. In: *Ocean Engineering* 320.120183 (2025). DOI: [10.1016/j.oceaneng.2024.120183](https://doi.org/10.1016/j.oceaneng.2024.120183) (cited on pages 38, 45).
- [138] Andrei M. Ermakov et al. “Optimisation of heterogeneous wave energy converter arrays: A control co-design strategy”. In: *Renewable Energy* 244.122637 (2025). DOI: [10.1016/j.renene.2025.122637](https://doi.org/10.1016/j.renene.2025.122637) (cited on page 38).
- [139] Yeraí Peña-Sánchez, Demián García-Violini, and John V. Ringwood. “Control co-design of power take-off parameters for wave energy systems”. In: *IFAC-PapersOnLine* 55.27 (2022), pp. 311–316. DOI: [10.1016/j.ifacol.2022.10.531](https://doi.org/10.1016/j.ifacol.2022.10.531) (cited on page 38).
- [140] Carlos A. Michelén Ströfer et al. “Control co-design and sensitivity analysis of the LUPA’s PTO using WecOptTool”. In: *Proceedings of 15th European Wave and Tidal Energy Conference*. 2023. DOI: [10.36688/ewtec-2023-288](https://doi.org/10.36688/ewtec-2023-288) (cited on page 38).
- [141] Mehdi Neshat et al. “A hybrid evolutionary algorithm framework for optimising power take off and placements of wave energy converters”. In: *GECCO ’19: Proceedings of the Genetic and Evolutionary Computation Conference*. 2019. DOI: [10.1145/3321707.3321806](https://doi.org/10.1145/3321707.3321806) (cited on pages 38, 39).
- [142] B. Paduano et al. “Control co-design mooring optimisation for wave energy systems: a three-tethered point absorber case”. In: *IFAC-PapersOnLine* 56.2 (2023), pp. 11717–11722. DOI: [10.1016/j.ifacol.2023.10.537](https://doi.org/10.1016/j.ifacol.2023.10.537) (cited on pages 39, 45).
- [143] Stuart J. Russell and Peter Norvig. “Artificial Intelligence: A Modern Approach”. In: 4th. Hoboken: Pearson, 2021. Chap. 4. ISBN: 9780134610993 (cited on page 40).
- [144] Yuan Ma et al. “An innovative aerodynamic design methodology of wind turbine blade models for wind tunnel real-time hybrid tests based on genetic algorithm”. In: *Ocean Engineering* 257 (2022). DOI: [10.1016/j.oceaneng.2022.111724](https://doi.org/10.1016/j.oceaneng.2022.111724) (cited on page 42).
- [145] Bin Sang. “Application of genetic algorithm and BP neural network in supply chain finance under information sharing”. In: *Journal of Computational and Applied Mathematics* 384 (2021). DOI: [10.1016/j.cam.2020.113170](https://doi.org/10.1016/j.cam.2020.113170) (cited on page 42).

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- [146] Yangyu Zhang et al. “MultiCPRISPR-EGA: Optimizing Guide RNA Array Design for Multiplexed CRISPR Using the Elitist Genetic Algorithm”. In: *ACS Synthetic Biology* 14.3 (2025), pp. 919–930. DOI: [10.1021/acssynbio.4c00860](https://doi.org/10.1021/acssynbio.4c00860) (cited on page 42).
- [147] David E. Goldberg and Kalyanmoy Deb. “Foundations of Genetic Algorithms”. In: ed. by Gregory J. E. Rawlins. Vol. 1. Morgan Kaufmann Publishers, 1991. Chap. 5, pp. 69–93. DOI: [10.1016/B978-0-08-050684-5.50008-2](https://doi.org/10.1016/B978-0-08-050684-5.50008-2) (cited on page 43).
- [148] Hari Mohan Pandey, Ankit Chaudhary, and Deepti Mehrotra. “A comparative review of approaches to prevent premature convergence in GA”. In: *Applied Soft Computing* 24 (2014). DOI: [10.1016/j.asoc.2014.08.025](https://doi.org/10.1016/j.asoc.2014.08.025) (cited on page 43).
- [149] Heinz Mühlenbein and Dirk Schlierkamp-Voosen. “Predictive models for the breeder genetic algorithm i. continuous parameter optimization”. In: *Evolutionary Computation* 1 (1993). DOI: [10.1162/evco.1993.1.1.25](https://doi.org/10.1162/evco.1993.1.1.25) (cited on page 44).
- [150] Xinjie Yu and Mitsuo Gen. “Introduction to Evolutionary Algorithms”. In: Springer London, 2010. Chap. 3.2. DOI: [10.1007/978-1-84996-129-5](https://doi.org/10.1007/978-1-84996-129-5) (cited on page 44).
- [151] K. F. Man, K. S. Tang, and S. Kwong. “Genetic Algorithms”. In: Springer London, 1999. Chap. 2.4.2. ISBN: 1852330724 (cited on page 44).
- [152] Changwei Liang, Junxiao Ai, and Lei Zuo. “Design, fabrication, simulation and testing of an ocean wave energy converter with mechanical motion rectifier”. In: *Ocean Engineering* 136 (2017), pp. 190–200. DOI: [10.1016/j.oceaneng.2017.03.024](https://doi.org/10.1016/j.oceaneng.2017.03.024) (cited on page 46).

## CHAPTER II

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# *Modelling & Optimisation of Power Take-off Systems for use in Wave Energy Converters*

This chapter details the formulation of a novel method for optimising the design of power take-off systems (PTOs) specifically for use in wave energy converters. The chapter opens with a literature review on previous methods of modelling PTOs – both in wave energy and beyond – then describes how the permanent magnet synchronous generator is modelled and optimised as part of a control co-design methodology.

The theory is presented here in a pure mathematical format, agnostic of any computational framework. For the results in later chapters, this model is implemented in MATLAB.

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## *Review of Modelling & Optimisation of Power Take-off Systems*

As discussed in §I.3.2, there is not a great quantity of literature on the control co-design (CCD) of power take-offs (PTOs) in wave energy converters (WECs); §I.3.2 briefly covers the results of the PTO CCD literature that do exist, this chapter will cover the PTO models used in those works, and the methodologies for optimisation. Here more details of the methodologies of PTO CCD both within wave energy – in §II.1.1 – and in other fields – in §II.1.2 – are discussed.

### II.1.1 FOR WAVE ENERGY CONVERTERS

What does and does not class as a PTO for the sake of optimising varies in different works, but it can broadly be separated into two categories: constraints on the WEC’s operating zone; and modelling of the drive train’s dynamics and efficiency.

**MODELLING CONSTRAINTS:** Bacelli and Ringwood [1] created a geometric tool for analysing force and position constraints of PTOs. Taking FIGURE II.1.1 as an example, the optimal PTO force would be at position  $P_{\text{opt}}$ , but limits on the force and stroke (maximum displacement) of the PTO mean the controller can only operate in the shaded blue region; this means that the best place the controller can operate in is at point  $P_{\text{con}}$ . Intuitively, one can see that expanding the force constraint would lead to a greater improvement in performance than expanding the position constraint. Bacelli and Ringwood did not optimise the system within [1], but designed the tool for use in later optimisation.

Peña-Sanchez *et al.* [2] sought to account for the economic performance of the WEC by placing a monetary cost on expanding the constraints. The aim was to minimise the levelised cost of energy (LCOE). Calculating the capital and operational expenditures for an early stage technology such as WECs is incredibly difficult and requires lots of estimations and approximations, for example: in [2] the operational costs were assumed to be a fixed proportion of the capital costs; which were then assumed to be five times the cost of the PTO,  $C_{\text{PTO}}$ ; which were then dependant on the material costs of the different components of the PTO; which were then scaled to account for the varying

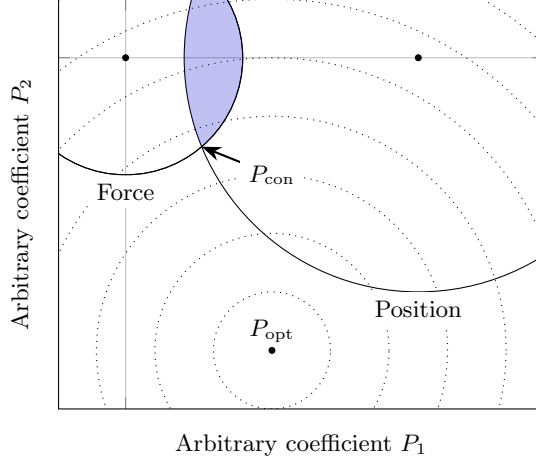


FIGURE II.1.1: Schematic pseudo-spectral representation of power take-off force constraints.  $(P_1, P_2)$  are arbitrary coefficients of the PTO force. The blue shaded region shows the area of allowable control. (Adapted from [1])

stroke and force limits. Putting all these together gets

$$\text{LCoE} \propto \frac{C_{\text{pto}}(S_{\text{max}}, F_{\text{max}})}{E(S_{\text{max}}, F_{\text{max}})} = \overline{\text{LCoE}}, \quad (\text{II.1.1})$$

where  $S_{\text{max}}$  and  $F_{\text{max}}$  were the limits of the stroke and force respectively.  $\overline{\text{LCoE}}$  was then used as the cost to minimise. For the optimisation, Peña-Sanchez *et al.* used a simple brute forcing methodology to find the best performing combination of  $(S_{\text{max}}, F_{\text{max}})$ , using Bacelli and Ringwood's methodology for optimising the control strategy.

**MODELLING DRIVE TRAIN EFFICIENCY:** A series of work by Grasberger [3], Michelén Ströfer *et al.* [4], Michelén Ströfer *et al.* [5] and Grasberger *et al.* [6] detailed the CCD optimisation of PTOS by considering the dynamics of the drive train – each work was for a different type of WEC: ranging from selecting the optimal sprocket [4] to abstractly selecting optimal values for the drive trains inertia, and stiffness [3, 5, 6].

The methodology involved considering the impedances between torque and speed, and current and voltage, to estimate the electrical losses in the generator. It is used as the foundation of this work's drive-train model, so is discussed in detail as part of the methodology, in § II.2.8. On the topic of their optimisation algorithms, the method works with different algorithms: they used both brute forcing [4, 5], and pseudo-spectral optimisation [3, 6].

## II.1.2 IN OTHER FIELDS

Beyond wave energy, there has been research in the CCD of PTOs and PTO-like systems. This subsection covers them in turn.

**WIND TURBINE GENERATORS:** Yazdanpanah *et al.* [7] used an analytical model of a permanent magnet synchronous generator (PMSG): based upon the magnetic equivalent circuit, and flux throughout the PMSG. The model designs a PMSG according to a given specification (power target, rated speed, *etc.*) and works to optimise according to one of five cost functions: total weight; total cost; temperature rise; efficiency; total weight times total cost times temperature rise over efficiency. All of these are minimised, except efficiency which was maximised. This optimisation was done using a genetic algorithm (GA).

The model was validated against a finite element method model and had a 2 kW (14%) disagreement in core losses, 0.3 mH (6.7%) in synchronous inductance, 0.40% in efficiency, and 3 °C (3.7%) in temperature rise. Yazdanpanah *et al.* investigated the difference between inner and outer rotor PMSGs, and found that the optimal inner rotor PMSGs tended to have a smaller temperature rise but a larger cost than the outer rotor, across all cost functions.

The optimisation of generators for wind turbines has been covered multiple times beyond the example by Yazdanpanah *et al.*, *eg.* [8, 9, 10, 11]. They all follow broadly the same methodology – using an analytical model of the magnetic equivalent circuit, and flux throughout the generator.

**MOTORS:** Cassimere [12] developed a model of a permanent magnet synchronous machine, designed to be used in a GA – this work was interested in developing motors but fundamentally the physics is the same for motors and generators, the inputs and outputs just swap between motion and electrical current. Cassimere’s model builds up the machine from the fundamental geometry of the device (rotor radius, number of magnets, *etc.*) and creates a lumped parameter model of the machine. The main difference between this and the wind turbine models above was the lack of a necessity for a design specification.

The methodology will not be covered in detail here as it forms the foundation of the generator model for this work, so is discussed as part of the methodology – in §II.2.

## II.1.3 IN SUMMARY

Whilst there has been some examination of CCD of PTOs for WECs in the literature, there has yet to be anything to the fidelity of that seen in the other fields. So this model will combine the PMSG model from Cassimere [12] and the WEC drive train modelling from [3, 4, 5, 6] to investigate if the application of a higher fidelity model shows significant changes between different WEC floaters and locations.

Cassimere’s motor model is chosen over the generator models, as it is found to be more adaptable: the wind turbine generator models tend to require a specification for the generator. The motor model on the other hand does not have those requirements, as is shown in page II, this gives it the potential of finding that a slightly smaller, less powerful generator is better economically.

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## *Modelling Power Take-off Systems for Wave Energy Converters*

This chapter details the theory of a model for a power take-off (PTO) system designed for a wave energy converter (WEC). As part of the wider PTO model, there is a lumped parameter model of a permanent magnet synchronous generator (PMSG): the model can be seen as a function that takes in inputs of the generator's geometry; and outputs the impedance, resistance, moment of inertia, and torque constant of the generator. This work mostly builds upon work by Cassimere [12], and Krause *et al.* [13].

The PMSG model is then included as part of a wider PTO drive train model, based upon works by Grasberger [3], Michelén Ströfer *et al.* [4], Michelén Ströfer *et al.* [5] and Grasberger *et al.* [6] for connecting the mechanical and electrical components, but in this work it is applied to a different drive train: a rack-and-pinion connecting the floater and PMSG.

At the outset it is important to note the PMSG model is a simplified model, but this is required for operation within a genetic algorithm where the PMSG model is run hundreds of thousands of times for a single algorithm run. So, in the interest of computational speed: structural, and thermal issues are not accounted for; and several loss mechanisms are neglected [14]. However, the model has been validated against hardware measurements by Cassimere in [12].

This section is laid out as follows: § II.2.1, and § II.2.2 define the physical design of the PMSG, namely the geometry and stator windings, respectively. Then § II.2.3 analyses the magnetic fields within the device. Using the physical, and magnetic definitions, § II.2.4, § II.2.5, § II.2.7, and § II.2.6 finds the inductance, winding resistance, moment of inertia, and generator constants, respectively. Finally, these properties are used in the drive train model in § II.2.8 to create an electrical power map of the PTO: taking inputs of force and velocity, and giving an output of the electrical power.

### II.2.1 GEOMETRY OF A PERMANENT MAGNET SYNCHRONOUS GENERATOR

The geometry of the PMSG is separated into three parts based of the dimensions: Lengths and angles in § II.2.1.1; Areas and volumes in § II.2.1.2; and a rectangular approximation of a slot in § II.2.1.3.

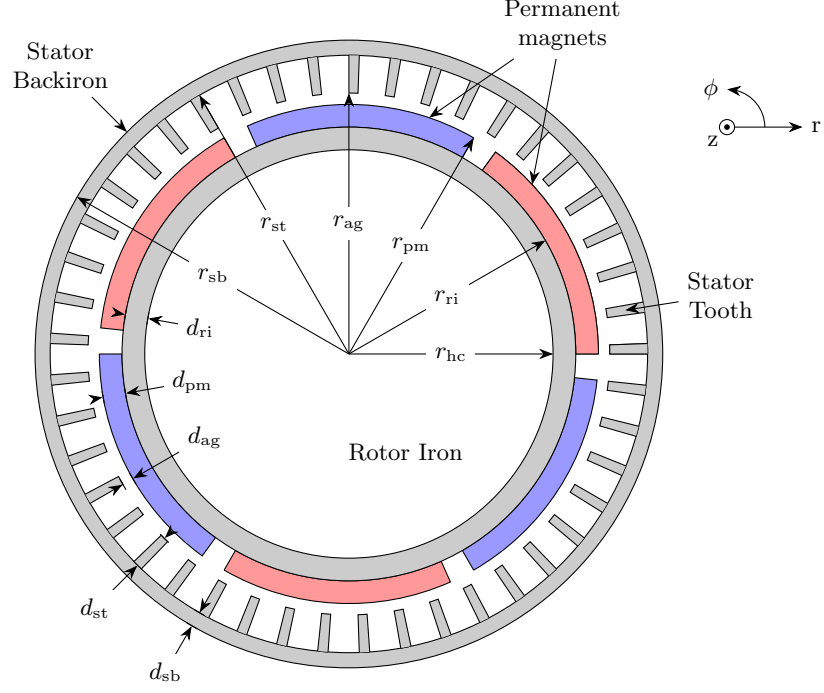


FIGURE II.2.1: Cross-sectional view of a permanent magnet synchronous generator showing key dimensions of the device. (Adapted from [12])

#### II.2.1.1 LENGTHS AND ANGLES

Shown in FIGURE II.2.1 is a collection of radii,  $r_{xx}$ , and depths,  $d_{xx}$  of the different components of a PMSG. The radii are as follows:  $r_{hc}$  is the radius of the hollow core;  $r_{ri}$  is the radius to the outer edge of the rotor iron;  $r_{pm}$  is the radius to the outer edge of the permanent magnets;  $r_{ag}$  is the radius the outer edge of the air gap;  $r_{st}$  is the radius to the outer edge of the stator teeth; and  $r_{sb}$  is the radius to the outer edge of the stator backiron. The depths can be found from the radii, where

$$d_{ri} = r_{ri} - r_{hc}, \quad (\text{II.2.1})$$

is the depth of the rotor iron;

$$d_{pm} = r_{pm} - r_{ri}, \quad (\text{II.2.2})$$

is the depth of the permanent magnets;

$$d_{ag} = r_{ag} - r_{pm}, \quad (\text{II.2.3})$$

is the depth of the air gap;

$$d_{st} = r_{st} - r_{ag}, \quad (\text{II.2.4})$$

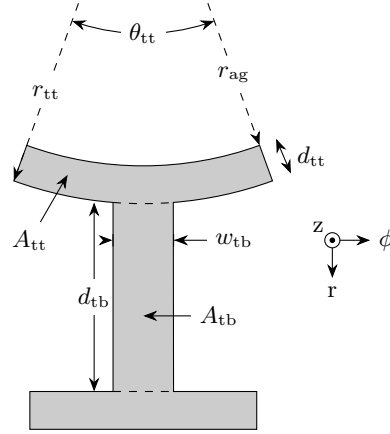


FIGURE II.2.2: Detailed view of a stator tooth showing the important widths and depths. Curvatures are exaggerated. (Adapted from [12])

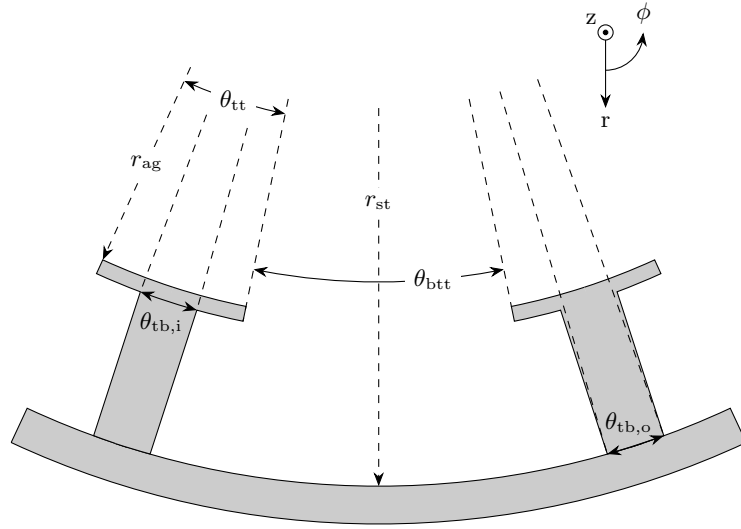


FIGURE II.2.3: Detailed view of the stator teeth showing the important radii and angles. Curvatures are exaggerated. (Adapted from [12])

is the depth of the stator teeth; and finally,

$$d_{sb} = r_{sb} - r_{st}, \quad (\text{II.2.5})$$

is the depth of the stator backiron.

FIGURE II.2.2 gives a detailed view of tooth dimensions, these are as follows:  $d_{tt}$  is the depth of the tooth tip;

$$r_{tt} = r_{ag} + d_{tt}, \quad (\text{II.2.6})$$

is the outer radius of the tooth tip;  $\theta_{tt}$  is the angle subtended by the tooth tip;  $w_{tb}$  is the width and the tooth base; and

$$d_{tb} = d_{st} - d_{tt}, \quad (\text{II.2.7})$$

is the depth of the tooth base.

FIGURE II.2.3 gives a more detailed view of the slot dimensions.

$$\theta_{\text{tb},i} = 2 \arctan\left(\frac{w_{\text{tb}}}{2r_{\text{tt}}}\right), \quad (\text{II.2.8})$$

is the angle subtended by the tooth at innermost point of the tooth base, and

$$\theta_{\text{tb},o} = 2 \arctan\left(\frac{w_{\text{tb}}}{2r_{\text{st}}}\right), \quad (\text{II.2.9})$$

is the angle subtended by the tooth at the outermost point; these are different due to the tooth base having a constant width, therefore having a different angular span at different radii.

$$\theta_{\text{btt}} = \frac{2\pi}{N_{\text{st}}} - \theta_{\text{tt}}, \quad (\text{II.2.10})$$

the angle subtended between adjacent tooth tips, where  $N_{\text{st}}$  is the number of stator teeth (or slots).

There are some other important quantities for describing a PMSG that have not been described so far. These are: the proportion of the permanent magnet region using magnetic material,  $\alpha_{\text{pm}} \in [0, 1]$ ; and the axial length of the generator (the length into the page),  $l_g$ .

### II.2.1.2 AREAS AND VOLUMES

The cross-sectional area of the rotor iron, and permanent magnets is given as:

$$A_{\text{ri}} = \pi (r_{\text{ri}}^2 - r_{\text{hc}}^2), \quad (\text{II.2.11})$$

$$A_{\text{pm}} = \pi (r_{\text{pm}}^2 - r_{\text{ri}}^2) \alpha_{\text{pm}}; \quad (\text{II.2.12})$$

their respective volumes are then

$$V_{\text{ri}} = A_{\text{ri}} l_g, \quad (\text{II.2.13})$$

$$V_{\text{pm}} = A_{\text{pm}} l_g. \quad (\text{II.2.14})$$

Similarly, the cross-sectional area of the stator backiron and the stator slot-teeth region are:

$$A_{\text{sb}} = \pi (r_{\text{sb}}^2 - r_{\text{st}}^2), \quad (\text{II.2.15})$$

$$A_{\text{slst}} = \pi (r_{\text{st}}^2 - r_{\text{ag}}^2); \quad (\text{II.2.16})$$

and their respective volumes are

$$V_{\text{sb}} = A_{\text{sb}} l_g, \quad (\text{II.2.17})$$

$$V_{\text{slst}} = A_{\text{slst}} l_g. \quad (\text{II.2.18})$$

The area of a single stator tooth can be found by separating the total area into two separate regions:  $A_{tt}$ , and  $A_{tb}$  – shown in FIGURE II.2.2 – the value of these are<sup>i</sup>

$$A_{tt} = \frac{\theta_{tt}}{2}(r_{tt}^2 - r_{ag}^2), \quad (\text{II.2.19})$$

$$A_{tb} = w_{tb}d_{tb}; \quad (\text{II.2.20})$$

the total area of a single stator tooth is then

$$A_{st, \text{single}} = A_{tt} + A_{tb}. \quad (\text{II.2.21})$$

The volume of a single stator tooth can then be found by

$$V_{st, \text{single}} = A_{st, \text{single}}l_g. \quad (\text{II.2.22})$$

This can then give the area and volume of all the stator teeth through multiplication by  $N_{st}$ :

$$A_{st} = A_{st, \text{single}}N_{st}; \quad (\text{II.2.23})$$

$$V_{st} = V_{st, \text{single}}N_{st}. \quad (\text{II.2.24})$$

The areas and volumes of all slots and a single slot are then simply

$$A_{sl} = A_{slst} - A_{st}, \quad (\text{II.2.25})$$

$$V_{sl} = V_{slst} - V_{st}, \quad (\text{II.2.26})$$

$$A_{sl, \text{single}} = \frac{A_{sl}}{N_{st}}, \quad (\text{II.2.27})$$

$$V_{sl, \text{single}} = \frac{V_{sl}}{N_{st}}, \quad (\text{II.2.28})$$

### II.2.1.3 RECTANGULAR APPROXIMATION OF A SLOT

In the calculations of the inductance, it is useful to have a rectangular approximation of the stator slots; this subsection derives that approximation. For clarity, all variables that utilise the rectangular approximation will have a ‘r’ appended to their subscript.

FIGURE II.2.4 shows the rectangular approximation of the slot dimensions. In this approximation, the depth of the tooth tip and width of the tooth base remain unchanged. The width of the tooth tip and the gap between tooth tips is chosen to be

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<sup>i</sup>Describing the tooth base as a rectangle is not entirely correct. At the interfaces with the tooth tip, this introduces gaps between the base and tip at the edges of the base. Conversely, at the interface with the backiron, there is an overlap at the edges of the base where the base would intersect with the backiron. However, the errors mostly counteract each other, leaving minimal error.

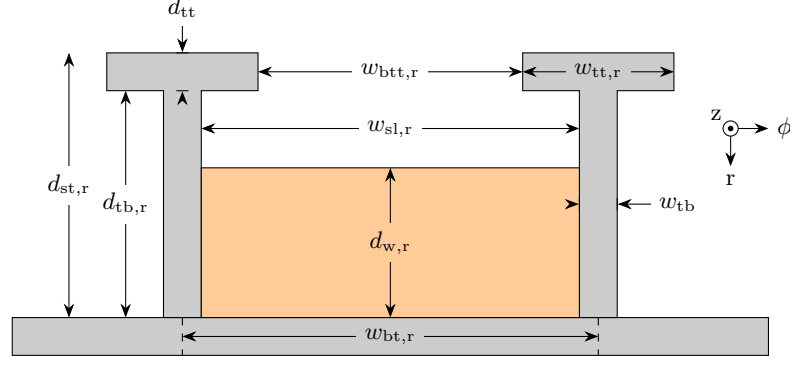


FIGURE II.2.4: Rectangular approximation of the stator slot dimensions. (Adapted from [12])

the length of the arc at a distance halfway through the depth of the tooth tip:

$$w_{tt,r} = \left( r_{ag} + \frac{d_{tt}}{2} \right) \theta_{tt}, \quad (\text{II.2.29})$$

$$w_{btt,r} = \left( r_{ag} + \frac{d_{tt}}{2} \right) \theta_{btt}. \quad (\text{II.2.30})$$

The width of the slot is taken to be the mean of the widths and the inner- and outer-most points:

$$w_{sl,r} = \frac{w_{sl,i} + w_{sl,o}}{2}, \quad (\text{II.2.31})$$

where the chords at the inner- and outer-most point of the slot are found by

$$w_{sl,i} = r_{tt} \left( \frac{2\pi}{N_{st}} - \theta_{tb,i} \right), \quad (\text{II.2.32})$$

$$w_{sl,o} = r_{st} \left( \frac{2\pi}{N_{st}} - \theta_{tb,o} \right). \quad (\text{II.2.33})$$

The distance between the centres of adjacent teeth is

$$w_{bt,r} = w_{sl,r} + w_{tb}. \quad (\text{II.2.34})$$

The depth of the tooth base is chosen to preserve the total area of a slot:

$$d_{tb,r} = \frac{A_{sl,single}}{w_{sl,r}}. \quad (\text{II.2.35})$$

Finally, the total depth of a tooth is:

$$d_{st,r} = d_{tb,r} + d_{tt}. \quad (\text{II.2.36})$$

The final length in FIGURE II.2.4 is  $d_{w,r}$ , this is the depth of the windings within the slot; this requires knowledge of the stator windings, so it is defined in § II.2.2.4.

## II.2.2 STATOR WINDINGS

This subsection discusses how the windings for the machine are formulated. Firstly, equations are found for a discretised number of conductors, both within the slots – in § II.2.2.1 – and between the slots – in § II.2.2.2. Then a continuous function for the number of turns is found in § II.2.2.3. Finally, the conductor area is found in § II.2.2.4.

## II.2.2.1 SLOT TURNS

Before finding the discrete number of windings located in each slot, it is first useful to define a continuous stator winding function for the conductor density: the number of conductors per unit angle as a function of position. The continuous density function is defined as

$$\vec{c}_{abc}(\phi) = \begin{bmatrix} c_a(\phi) \\ c_b(\phi) \\ c_c(\phi) \end{bmatrix} = \begin{bmatrix} N_{cd}^\dagger \sin(N_{pp}\phi) \\ N_{cd}^\dagger \sin\left(N_{pp}\phi - \frac{2\pi}{3}\right) \\ N_{cd}^\dagger \sin\left(N_{pp}\phi - \frac{4\pi}{3}\right) \end{bmatrix}, \quad (\text{II.2.37})$$

where  $c_a(\phi)$ ,  $c_b(\phi)$ , and  $c_c(\phi)$  are the conductor densities for each of the a-, b-, and c-phases respectively;  $N_{pp}$  is the number of pole pairs;  $\phi$  is the angular position in the stator; and  $N_{cd}^\dagger$  is the desired amplitude of the conductor density.

The discrete number of conductors in a slot can then be found via integrating the function between the centres of adjacent stator teeth:

$$\vec{C}_{abc}(j) = \begin{bmatrix} C_a(j) \\ C_b(j) \\ C_c(j) \end{bmatrix} = \text{round} \left[ \int_{\phi_{sl}(j) - \pi/N_{st}}^{\phi_{sl}(j) + \pi/N_{st}} \vec{c}_{abc}(\phi) d\phi \right], \quad (\text{II.2.38})$$

where  $\text{round}[\cdot]$  is a function which rounds the argument to the nearest integer;  $\phi_{sl}(j)$  is the angular position of the centre of slot  $j$  given by

$$\phi_{sl}(j) = \frac{2\pi(j-1)}{N_{st}}. \quad (\text{II.2.39})$$

Evaluating EQUATION (II.2.38) for each of the phases gives

$$C_a(j) = \text{round} \left[ \frac{2N_{cd}^\dagger}{N_{pp}} \sin(N_{pp}\phi_{sl}(j)) \sin\left(\frac{N_{pp}}{N_{st}}\pi\right) \right], \quad (\text{II.2.40})$$

$$C_b(j) = \text{round} \left[ \frac{2N_{cd}^\dagger}{N_{pp}} \sin\left(N_{pp}\phi_{sl}(j) - \frac{2\pi}{3}\right) \sin\left(\frac{N_{pp}}{N_{st}}\pi\right) \right], \quad (\text{II.2.41})$$

$$C_c(j) = \text{round} \left[ \frac{2N_{cd}^\dagger}{N_{pp}} \sin\left(N_{pp}\phi_{sl}(j) - \frac{4\pi}{3}\right) \sin\left(\frac{N_{pp}}{N_{st}}\pi\right) \right]. \quad (\text{II.2.42})$$

These equations can provide positive and negative results: a positive value represents turns coming out of the page, and a negative value represents a turn going into the page. Therefore, the total number of turns in slot  $j$  can be found by summing the magnitude of each phases contribution in that slot:

$$C_{\text{tot}}(j) = |C_a(j)| + |C_b(j)| + |C_c(j)|. \quad (\text{II.2.43})$$

Due to the rounding in the evaluation of  $\vec{C}_{\text{abc}}(j)$ , the true value of  $N_{\text{cd}}$  is different from the desired value; the true value can be estimated via Fourier analysis. Taking a generic equation of the form

$$y = a_1 \sin(\omega t), \quad (\text{II.2.44})$$

where the coefficient  $a_1$  can be found via [15]

$$a_1 = \frac{2}{T} \int_0^T y(t) \sin(\omega t) dt, \quad (\text{II.2.45})$$

where  $T$  is an integer multiple of the period of the fundamental. From comparison with EQUATION (II.2.40), it is found that<sup>ii</sup>

$$N_{\text{cd}} \approx \frac{N_{\text{pp}}}{2 \sin\left(\frac{N_{\text{pp}}}{N_{\text{st}}} \pi\right)} \frac{2\pi}{N_{\text{st}}} \sum_{i=1}^{N_{\text{st}}} C_a(i) \sin(N_{\text{pp}} \phi_{\text{sl}}(i)); \quad (\text{II.2.46})$$

noting that [16]

$$\int_0^T y(t) dt \approx \frac{T}{N} \sum_{i=1}^N Y(i), \quad (\text{II.2.47})$$

where  $Y(i)$  is the discretised formulation of  $y(t)$ , and assuming that there are  $N$  points in the discretisation, and that they are equally spaced.

### II.2.2.2 END TURNS

The number of end turns between slots  $j - 1$  and  $j$  is defined as

$$\vec{K}_{\text{abc}}(j) = \begin{bmatrix} K_a(j) \\ K_b(j) \\ K_c(j) \end{bmatrix} = \vec{K}_{\text{abc}}(j - 1) + \vec{C}_{\text{abc}}(j - 1), \quad (\text{II.2.48})$$

where  $K_a$ ,  $K_b$ , and  $K_c$  are the number of end turns associated with the a-, b-, and c-phase respectively. A positive value of one of the components of  $\vec{K}_{\text{abc}}(j)$  represents turns in an anticlockwise direction, as viewed from the front of the machine; this is shown diagrammatically in FIGURE II.2.5.

<sup>ii</sup>Similar equations can be found using  $C_b$  and  $C_c$ ; they should all yield the same value.

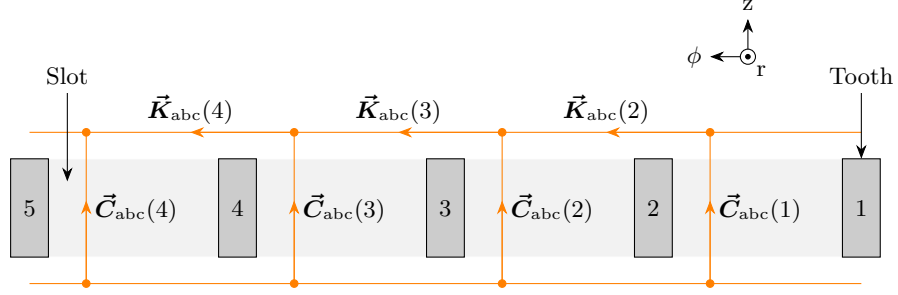


FIGURE II.2.5: Radial view of the uncurred stator slot-tooth region of a permanent magnet synchronous machine to show the end turns. Arrows show the direction of a positive value. (Adapted from [17])

EQUATION II.2.48 can be rewritten as the sum of contributions of many teeth from a reference point, say tooth  $N_{st}$ :

$$\vec{K}_{abc}(j) = \vec{K}_{abc}(N_{st}) + \sum_{k=1}^j \vec{C}_{abc}(k); \quad (\text{II.2.49})$$

to get a general form, a value for  $\vec{K}_{abc}(N_{st})$  needs to be found. By restricting the number of stator teeth to

$$N_{st} = 6kN_{pp}, \quad (\text{II.2.50})$$

where  $k$  is an integer value – the factor of six comes from a factor of two and a factor of three, to have at least one tooth per pole per phase to ensure the phases are balanced [8].<sup>iii</sup>  $\vec{K}_{abc}(N_{st})$  is now findable by considering the symmetry of the PMSG. The number of slot turns and end turns are odd-half wave symmetric over the number of slots corresponding to a pole:

$$\vec{C}_{abc}\left(j + \frac{N_{st}}{2N_{pp}}\right) = -\vec{C}_{abc}(j), \quad (\text{II.2.51})$$

$$\vec{K}_{abc}\left(j + \frac{N_{st}}{2N_{pp}}\right) = -\vec{K}_{abc}(j). \quad (\text{II.2.52})$$

By inputting EQUATION (II.2.52) into (II.2.49) with  $j = N_{st} + N_{st}/2N_{pp}$ , one can find

$$\vec{K}_{abc}(N_{st}) = -\frac{1}{2} \sum_{j=1}^{N_{st}/2N_{pp}} \vec{C}_{abc}(j), \quad (\text{II.2.53})$$

giving

$$\vec{K}_{abc}(i) = \sum_{j=1}^i \vec{C}_{abc}(j) - \frac{1}{2} \sum_{k=1}^{N_{st}/2N_{pp}} \vec{C}_{abc}(k), \quad (\text{II.2.54})$$

<sup>iii</sup>It should be noted that it is possible to create a PMSG that does not adhere to this condition, but the vast majority of PMSGs do [17].

which is the desired relationship: an equation for the number of end turns independent of the number of end turns between any other pair of slots.

Finally, the total number of end turns between slots  $j - 1$  and  $j$  is simply

$$K_{\text{tot}}(j) = |K_a(j)| + |K_b(j)| + |K_c(j)|. \quad (\text{II.2.55})$$

### II.2.2.3 CONTINUOUS WINDING FUNCTION

So far, the conductors have been described in relation to the physical slots of the generator; however, to calculate the inductances it is beneficial to have a continuous description of the conductors as a function of angular position. The continuous winding function defines the effective number of turns linking the magnetic flux at an angular position.

Let  $\vec{W}_{\text{abc}}$  be the vector of the winding functions for each phase such that

$$\vec{W}_{\text{abc}}(\phi) = \begin{bmatrix} W_a(\phi) \\ W_b(\phi) \\ W_c(\phi) \end{bmatrix}, \quad (\text{II.2.56})$$

where  $W_a(\phi)$ ,  $W_b(\phi)$ , and  $W_c(\phi)$  are the winding functions associated with the a-, b-, and c-phase respectively. Taking a small change in the angle,  $\Delta\phi$ , one finds

$$\vec{W}_{\text{abc}}(\phi + \Delta\phi) = \vec{W}_{\text{abc}}(\phi) + \vec{c}_{\text{abc}}(\phi)\Delta\phi; \quad (\text{II.2.57})$$

and in the limit  $\Delta\phi \rightarrow 0$ ,

$$\frac{d\vec{W}_{\text{abc}}(\phi)}{d\phi} = -\vec{c}_{\text{abc}}(\phi); \quad (\text{II.2.58})$$

thus

$$\vec{W}_{\text{abc}}(\phi) = - \int_0^\phi \vec{c}_{\text{abc}}(\phi) d\phi + \vec{W}_{\text{abc}}(0). \quad (\text{II.2.59})$$

To find  $\vec{W}_{\text{abc}}(0)$ , the same symmetry constraint on the discretised slot turns is used, EQUATION (II.2.51), such that

$$\vec{W}_{\text{abc}}\left(\phi + \frac{\pi}{N_{\text{pp}}}\right) = -\vec{W}_{\text{abc}}(\phi), \quad (\text{II.2.60})$$

and by setting  $\phi = \pi/N_{\text{pp}}$  in EQUATION (II.2.59) one finds

$$\vec{W}_{\text{abc}}(0) = \frac{1}{2} \int_0^{\pi/N_{\text{pp}}} \vec{c}(\phi) d\phi; \quad (\text{II.2.61})$$

finally, substituting this into EQUATION (II.2.59) gives

$$\vec{W}_{\text{abc}}(\phi) = \frac{1}{2} \int_0^{\pi/N_{\text{pp}}} \vec{c}(\phi) d\phi - \int_0^\phi \vec{c}(\phi) d\phi. \quad (\text{II.2.62})$$

Evaluating for each of the phases results in

$$W_a(\phi) = \frac{N_{cd}}{N_{pp}} \cos(N_{pp}\phi), \quad (\text{II.2.63})$$

$$W_b(\phi) = \frac{N_{cd}}{N_{pp}} \cos\left(N_{pp}\phi - \frac{2\pi}{3}\right), \quad (\text{II.2.64})$$

$$W_c(\phi) = \frac{N_{cd}}{N_{pp}} \cos\left(N_{pp}\phi - \frac{4\pi}{3}\right). \quad (\text{II.2.65})$$

#### II.2.2.4 WINDING AREA

The final objective of this section is to calculate the area taken up by the windings. Taking the area of a single turn of the windings to be  $A_{w,\text{single}}$ , the total area taken by the windings is

$$A_w = \frac{A_{w,\text{single}} \max[C_{\text{tot}}(i)]}{\alpha_{\text{pf}}} \quad (\text{II.2.66})$$

where  $\alpha_{\text{pf}} \in [0, 1]$  is the packing factor, and  $\max[\cdot]$  is a function that returns the maximum element of the argument. The maximum value of  $C_{\text{tot}}$  is used because there can be slight variations in the number of turns on each slot; however, it should be noted that variations are minor. Using the winding area, the depth of the windings – in the rectangular approximation of the slot – can be found via:

$$d_{w,r} = \frac{A_w}{w_{sl,r}}, \quad (\text{II.2.67})$$

### II.2.3 MAGNETIC FIELD ANALYSIS

This subsection describes how the magnetic field moves throughout the generator, for later use in § II.2.4 to find the inductance of the generator. FIGURE II.2.6 shows the relationship between the abc reference frame for the phases, and the qd0 reference frame. In here,  $\phi_{fa,m}$  and  $\phi_{fq,m}$  are the mechanical angles from the a- and q-axis respectively; they are related to their electrical counterparts via

$$\phi_{fa,e} = N_{pp}\phi_{fa,m}, \quad (\text{II.2.68})$$

$$\phi_{fq,e} = N_{pp}\phi_{fq,m}. \quad (\text{II.2.69})$$

Similarly  $\phi_{r,m}$  is the mechanical rotor position, which is related to the electrical rotor position via

$$\phi_{r,e} = N_{pp}\phi_{r,m}. \quad (\text{II.2.70})$$

For the magnetic field analysis, it is assumed that the magnetomotive force (MMF) drops across the steel portions of the machine are negligible; due to the relative permeability of the permanent magnets being very small compared to the steel, the MMF drop over the permanent magnet will dominate the drop over the steel [14].

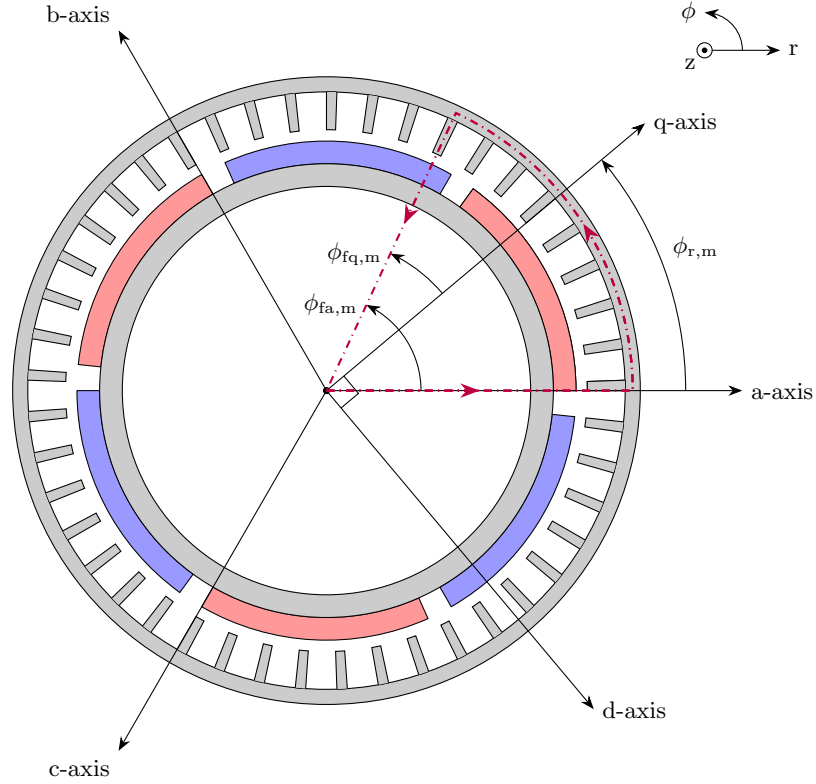


FIGURE II.2.6: Cross-sectional view of a permanent magnet synchronous generator, showing the axis and angles used in the calculation of the generator's inductance. The red dot-dashed line shows the path of the integral for EQUATION (II.2.84). (Adapted from [12])

The subsection is separated into two parts: § II.2.3.1 which derives an equation for the radial component of the flux density in the region containing the permanent magnets and airgap; and § II.2.3.2 which derives equations for the flux density in different parts of the steel.

#### II.2.3.1 THE RADIAL COMPONENT OF FLUX DENSITY

Consider a small angular section of the machine of size  $\Delta\phi$ ; at radii  $r = r_1$  and  $r = r_2$ , with  $r_1, r_2 \in (r_{ri}, r_{ag})$ . The flux going through these surfaces is

$$\Phi_1 = \mathcal{B}(r_1, \phi) l_g r_1 \Delta\phi, \quad (\text{II.2.71})$$

$$\Phi_2 = \mathcal{B}(r_2, \phi) l_g r_2 \Delta\phi; \quad (\text{II.2.72})$$

where  $\mathcal{B}(r, \phi)$  is the equation of the magnetic flux density at that point.

It is assumed that (near the permanent magnet interface) the magnetic field is purely radial; this means that, from Gauss' law, the flux going through each surface

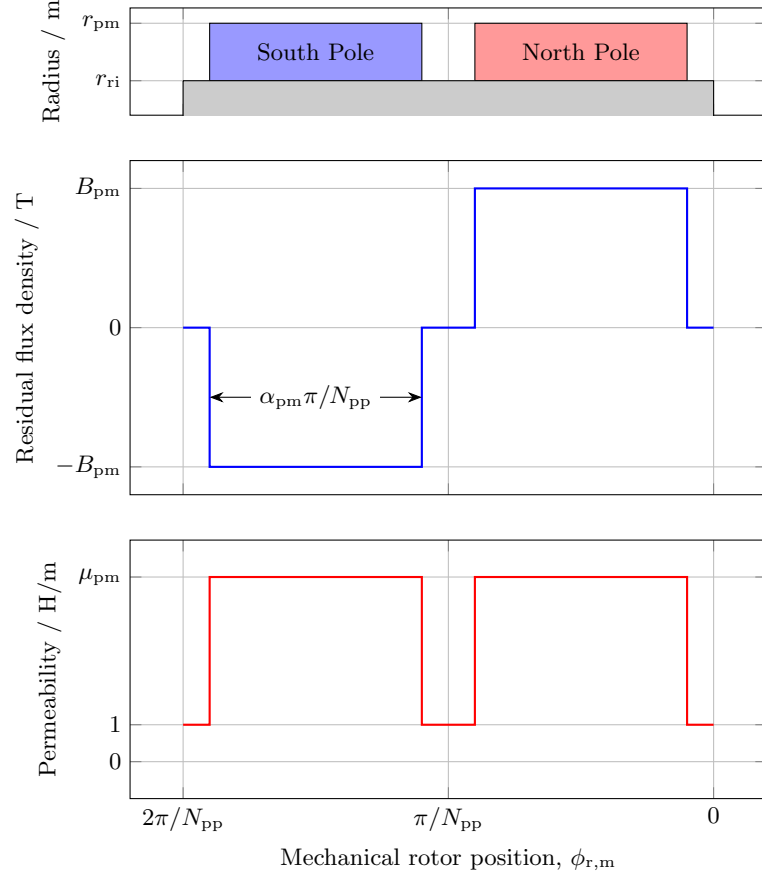


FIGURE II.2.7: Magnetisation in the permanent magnet region. Top plot shows the position on the pole pair within the radial interval. Middle plot shows the flux density. Bottom plot shows the permeability in this region.

must be equal,  $\Phi_1 = \Phi_2$ . Using this fact, and setting  $r_1 = r$  and  $r_2 = r_{ri}$ , one can find

$$\mathcal{B}(r, \phi) = \frac{r_{ri}}{r} \mathcal{B}(r_{ri}, \phi), \quad (\text{II.2.73})$$

it is important to note that this loses accuracy when  $r < r_{ri}$ , as the field takes on a significant tangential component; and when  $r > r_{ag}$  due to effects from the slots [12].

**MAGNETISM IN THE PERMANENT MAGNET REGION:** The PMSG includes a series of magnets, alternating in polarity, placed around the edge of the rotor, with spacers in between (in this case, air gaps); one pair of magnets is shown in FIGURE II.2.7 (top). Due to the alternating polarity of the magnets, the magnetic flux density in this region is given by

$$\mathcal{B}_{pmr}(\phi) = \mu_{pmr}(\phi) \mathcal{H}(\phi) + \mathcal{B}_{res}(\phi), \quad (\text{II.2.74})$$

which is shown in FIGURE II.2.7 (middle).  $\mathcal{H}(\phi)$  is the magnetic field strength; and  $\mu_{\text{pmr}}(\phi)$  is the permeability of the permanent magnet region, this is shown in FIGURE II.2.7 (bottom).

The MMF drop over the permanent magnet region (the annular region of the PMSG where  $r_{\text{ri}} < r < r_{\text{pm}}$ ) is found via

$$\mathcal{F}_{\text{pmr}}(\phi) = \int_{r_{\text{ri}}}^{r_{\text{pm}}} \mathcal{H}(r, \phi) dr, \quad (\text{II.2.75})$$

using this with EQUATIONS (II.2.73) and (II.2.74), one finds

$$\mathcal{F}_{\text{pmr}}(\phi) = \mathcal{R}_{\text{pmr}}(\phi) \mathcal{B}(r_{\text{ri}}, \phi) - \mathcal{F}_{\text{pm}}(\phi), \quad (\text{II.2.76})$$

where  $\mathcal{R}_{\text{pmr}}(\phi)$  is a *reluctance area* (with units of  $\text{m}^2/\text{H}$ ) of the permanent magnet region; this can take on two values depending on whether the region contains a magnet,

$$\mathcal{R}_{\text{pm}} = \frac{r_{\text{ri}}}{\mu_{\text{pm}}} \ln \left( 1 + \frac{d_{\text{pm}}}{r_{\text{ri}}} \right); \quad (\text{II.2.77})$$

or a spacer,

$$\mathcal{R}_{\text{sp}} = \frac{r_{\text{ri}}}{\mu_{\text{sp}}} \ln \left( 1 + \frac{d_{\text{pm}}}{r_{\text{ri}}} \right). \quad (\text{II.2.78})$$

$\mu_{\text{pm}}$  and  $\mu_{\text{sp}}$  are the permeabilities of the permanent magnet and spacer respectively.  $\mathcal{F}_{\text{pm}}(\phi)$  is the MMF from the permanent magnet:

$$\mathcal{F}_{\text{pm}}(\phi) = \frac{d_{\text{pm}}}{\mu_{\text{pm}}} \mathcal{B}_{\text{res}}(\phi). \quad (\text{II.2.79})$$

**MAGNETISM IN THE AIR GAP:** Finding the flux density in the air gap follows a similar process to that of the permanent magnet region, however it is a simpler case given there is no MMF source. The magnetic flux density of this region is

$$\mathcal{B}_{\text{ag}}(r, \phi) = \mu_0 \mathcal{H}(r, \phi), \quad (\text{II.2.80})$$

and the MMF drop over this region is found via

$$\mathcal{F}_{\text{ag}}(\phi) = \int_{r_{\text{pm}}}^{r_{\text{ag}}} \mathcal{H}(r, \phi) dr, \quad (\text{II.2.81})$$

substituting in EQUATIONS (II.2.73) and (II.2.80), and evaluating yields

$$\mathcal{F}_{\text{ag}}(\phi) = \mathcal{R}_{\text{ag}} \mathcal{B}(r_{\text{ri}}, \phi); \quad (\text{II.2.82})$$

where

$$\mathcal{R}_{\text{ag}} = \frac{r_{\text{ri}}}{\mu_0} \ln \left( 1 + \frac{d_{\text{ag}}}{r_{\text{pm}}} \right) \quad (\text{II.2.83})$$

is the reluctance area of the air gap.

FINDING THE RADIAL FLUX DENSITY: In FIGURE II.2.6, the red dash-dotted line shows a path through the generator. Applying Ampere's law to this path – in a counter clockwise direction – gives

$$\mathcal{F}_{\text{pmr}}(0) + \mathcal{F}_{\text{ag}}(0) - \mathcal{F}_{\text{ag}}(\phi_{\text{fa,m}}) - \mathcal{F}_{\text{pmr}}(\phi_{\text{fa,m}}) = i_{\text{enc}}(\phi_{\text{fa,m}}), \quad (\text{II.2.84})$$

where  $i_{\text{enc}}(\phi_{\text{fa,m}})$  is the current enclosed by the path, and  $\phi_{\text{fa,m}}$  is the mechanical angle from the a-axis. The enclosed current is found via

$$i_{\text{enc}}(\phi_{\text{fa,m}}) = \int_0^{\phi_{\text{fa,m}}} \vec{c}(\phi) \cdot \vec{i}_{\text{abc}}(\phi) d\phi, \quad (\text{II.2.85})$$

where  $\vec{i}_{\text{abc}}$  is the vector of the currents of the a-, b-, and c-phase respectively; it will be assumed to be of form

$$\vec{i}_{\text{abc}}(\phi_r) = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} i_{\text{peak}} \cos(\phi_{r,e} + \phi_a) \\ i_{\text{peak}} \cos(\phi_{r,e} + \phi_a - \frac{2\pi}{3}) \\ i_{\text{peak}} \cos(\phi_{r,e} + \phi_a - \frac{4\pi}{3}) \end{bmatrix} \quad (\text{II.2.86})$$

where  $i_{\text{peak}}$  is the peak magnitude of the phase current; and  $\phi_a$  is the phase of the a-phase current. Evaluating EQUATION (II.2.85) gives

$$i_{\text{enc}} = \mathcal{F}_i(0) - \mathcal{F}_i(\phi_{\text{fa,m}}), \quad (\text{II.2.87})$$

where

$$\mathcal{F}_i(\phi) = \frac{3N_{\text{cd}}i_{\text{peak}}}{2N_{\text{pp}}} \cos(N_{\text{pp}}\phi - \phi_{r,e} - \phi_a). \quad (\text{II.2.88})$$

Substituting EQUATIONS (II.2.76), (II.2.82), and (II.2.87) into EQUATION (II.2.84) and solving for  $B(r_{\text{ri}}, \phi_{\text{fa,m}})$  gives

$$B(r_{\text{ri}}, \phi_{\text{fa,m}}) = \frac{\mathcal{F}_{\text{pm}}(\phi_{\text{fa,m}}) - \mathcal{F}_{\text{pm}}(0) + \mathcal{F}_i(\phi_{\text{fa,m}}) - \mathcal{F}_i(0) + (\mathcal{R}_{\text{pmr}}(0) + \mathcal{R}_{\text{ag}})\mathcal{B}(r_{\text{ri}}, 0)}{\mathcal{R}_{\text{pmr}}(\phi_{\text{fa,m}}) + \mathcal{R}_{\text{g}}}. \quad (\text{II.2.89})$$

All that remains is a solution for  $\mathcal{B}(r_{\text{ri}}, 0)$ : it is found by considering the symmetry of the field. From observation of FIGURE II.2.7 (middle and bottom), one can see that

$$\mathcal{B}_{\text{res}}\left(\phi + \frac{\pi}{N_{\text{pp}}}\right) = -\mathcal{B}_{\text{res}}(\phi), \quad (\text{II.2.90})$$

meaning the flux density is odd-half wave symmetric; and

$$\mu_{\text{pmr}}\left(\phi + \frac{\pi}{N_{\text{pp}}}\right) = \mu_{\text{pmr}}(\phi), \quad (\text{II.2.91})$$

meaning the permeability is half wave symmetric. Therefore,

$$\mathcal{R}_{\text{pmr}}\left(\phi + \frac{\pi}{N_{\text{pp}}}\right) = \mathcal{R}_{\text{pmr}}(\phi). \quad (\text{II.2.92})$$

Combining with EQUATION (II.2.76) gives

$$\mathcal{F}_{\text{pmr}}\left(\phi + \frac{\pi}{N_{\text{pp}}}\right) = -\mathcal{F}_{\text{pmr}}(\phi). \quad (\text{II.2.93})$$

From observation of EQUATION (II.2.88), since the cosine function is odd half wave symmetric,  $\mathcal{F}_{\text{i}}$  must be as well:

$$\mathcal{F}_{\text{i}}\left(\phi + \frac{\pi}{N_{\text{pp}}}\right) = -\mathcal{F}_{\text{i}}(\phi). \quad (\text{II.2.94})$$

Lastly, it is assumed that the flux density is opposite in sign at diametrically opposed electrical positions, such that

$$\mathcal{B}\left(r_{\text{ri}}, \phi + \frac{\pi}{N_{\text{pp}}}\right) = -\mathcal{B}(r_{\text{ri}}, \phi). \quad (\text{II.2.95})$$

Substituting all of these, and setting  $\phi_{\text{fa,m}} = \pi/N_{\text{pp}}$  gives

$$\mathcal{B}(r_{\text{ri}}, 0) = \frac{\mathcal{F}_{\text{pm}}(0) + \mathcal{F}_{\text{i}}(0)}{\mathcal{R}_{\text{pmr}}(0) + \mathcal{R}_{\text{ag}}}; \quad (\text{II.2.96})$$

and substituting this into EQUATION (II.2.89) gives

$$\mathcal{B}(r_{\text{ri}}, \phi_{\text{fa,m}}) = \frac{\mathcal{F}_{\text{pm}}(\phi_{\text{fa,m}}) + \mathcal{F}_{\text{i}}(\phi_{\text{fa,m}})}{\mathcal{R}_{\text{pmr}}(\phi_{\text{fa,m}}) + \mathcal{R}_{\text{ag}}}. \quad (\text{II.2.97})$$

Finally, with substitution into EQUATION (II.2.73), one finds

$$\mathcal{B}(r, \phi) = \frac{r_{\text{ri}}}{r} \frac{\mathcal{F}_{\text{pm}}(\phi) + \mathcal{F}_{\text{i}}(\phi)}{\mathcal{R}_{\text{pmr}}(\phi) + \mathcal{R}_{\text{ag}}}, \quad (\text{II.2.98})$$

which is the general equation of the magnetic flux density in the radial direction, in the domain  $r \in (r_{\text{ri}}, r_{\text{ag}})$ .

## II.2.3.2 FERROMAGNETIC FIELD ANALYSIS

This subsection derives equations for the flux density within the stator tooth base and in the rotor iron.

STATOR TOOTH BASE: Because stator slots do not conduct magnetic flux well, the flux converges into the stator teeth within the slot-tooth region; this causes a high flux density within the teeth. Consequently, the total amount of flux within tooth  $i$  is

$$\Phi_{\text{st}}(i) = w_{\text{tb}} l_g \mathcal{B}_{\text{tb}}(i), \quad (\text{II.2.99})$$

where  $\mathcal{B}_{\text{tb}}(i)$  is the flux density in the base of tooth  $i$ .

The flux in a span from the centre of one slot to the centre of the next is given as

$$\Phi_{\text{sl}}(i) = r_{\text{ri}} l_g \int_{\phi_{\text{st}}(i) - \pi/N_{\text{st}}}^{\phi_{\text{st}}(i) + \pi/N_{\text{st}}} \mathcal{B}(r_{\text{ri}}, \phi) d\phi, \quad (\text{II.2.100})$$

where

$$\phi_{\text{st}}(i) = \frac{2\pi(i-1)}{N_{\text{st}}} \quad (\text{II.2.101})$$

is the angular position of the centre of tooth  $i$ .

Assuming that all of the flux from the span enters the tooth, EQUATIONS (II.2.99) and (II.2.100) can be equated, and rearranged for the flux density in the tooth base:

$$\mathcal{B}_{\text{tb}}(i) = \frac{r_{\text{ri}}}{w_{\text{tb}}} \int_{\phi_{\text{st}}(i) - \pi/N_{\text{st}}}^{\phi_{\text{st}}(i) + \pi/N_{\text{st}}} \mathcal{B}(r_{\text{ri}}, \phi) d\phi. \quad (\text{II.2.102})$$

ROTOR IRON: For the rotor iron, depending on the current situation and design, the maximum flux density can be at any of a number of points [12]. These points are, for a rotor position  $\phi_{\text{fq,e}} = 0$ : the point just beyond the boundary of a magnet,

$$\mathcal{B}_{\text{ri}}(1) = \frac{1}{\mathcal{R}_{\text{sp}} + \mathcal{R}_{\text{ag}}} \left( \frac{3N_{\text{cd}} i_{\text{peak}}}{2N_{\text{pp}}} \cos \left( \frac{(1 - \alpha_{\text{pm}})\pi}{N_{\text{pp}}} \right) \right); \quad (\text{II.2.103})$$

the point just within the permanent magnet,

$$\mathcal{B}_{\text{ri}}(2) = \frac{1}{\mathcal{R}_{\text{pm}} + \mathcal{R}_{\text{ag}}} \left( \frac{d_{\text{pm}} B_{\text{res}}}{\mu_{\text{pm}}} + \frac{3N_{\text{cd}} i_{\text{peak}}}{2N_{\text{pp}}} \cos \left( \frac{(1 - \alpha_{\text{pm}})\pi}{N_{\text{pp}}} \right) \right); \quad (\text{II.2.104})$$

the point just within the permanent magnet on the other side,

$$\mathcal{B}_{\text{ri}}(3) = \frac{1}{\mathcal{R}_{\text{pm}} + \mathcal{R}_{\text{ag}}} \left( \frac{d_{\text{pm}} B_{\text{res}}}{\mu_{\text{pm}}} + \frac{3N_{\text{cd}} i_{\text{peak}}}{2N_{\text{pp}}} \cos \left( \frac{\alpha_{\text{pm}}\pi}{N_{\text{pp}}} \right) \right); \quad (\text{II.2.105})$$

the point just beyond the permanent magnet on the other side,

$$\mathcal{B}_{\text{ri}}(4) = \frac{1}{\mathcal{R}_{\text{sp}} + \mathcal{R}_{\text{ag}}} \left( \frac{3N_{\text{cd}} i_{\text{peak}}}{2N_{\text{pp}}} \cos \left( \frac{\alpha_{\text{pm}}\pi}{N_{\text{pp}}} \right) \right); \quad (\text{II.2.106})$$



FIGURE II.2.8: The points at which the rotor flux density is evaluated. The positions of the points are not to scale; in reality the points are infinitesimally inside or outside of the magnet.

and the point of peak stator MMF,

$$\mathcal{B}_{ri}(5) = \frac{1}{\mathcal{R}_{pmr}(\phi_a/N_{pp}) + \mathcal{R}_{ag}} \left( B_{res}(\phi_a/N_{pp}) + \frac{3N_{cd}i_{peak}}{2N_{pp}} \right). \quad (\text{II.2.107})$$

The first four points are shown in FIGURE II.2.8; the fifth is not shown because it can appear anywhere depending on the present phase shift. Due to only the magnitude of the flux density being important, only one pole of magnet needs to be considered.

#### II.2.4 INDUCTANCE

The inductance of the PMSG is found from consideration of the paths of permeance through the generator. These paths can be split into two categories: magnetising inductance and leakage inductance. The distinction between magnetising and leakage inductance can differ depending on the source; this work follows the definition from Krause *et al.* [17] in that magnetising inductance is that associated with the flux crossing the air gap, and leakage is associated with the flux that does not.

The equations of the inductance in qd0 variables are

$$L_q = L_{qm} + L_{ls}, \quad (\text{II.2.108})$$

$$L_d = L_{dm} + L_{ls}, \quad (\text{II.2.109})$$

$$L_0 = L_{ls} + 3L_{lm}, \quad (\text{II.2.110})$$

where  $L_{qm}$  and  $L_{dm}$  are the magnetising inductances in the q- and d- axis respectively. They are derived in § II.2.4.1.

$$L_{ls} = L_{lp} - L_{lm} \quad (\text{II.2.111})$$

where  $L_{lp}$  and  $L_{lm}$  are the self- and mutual-leakage inductances. They are derived in § II.2.4.2.

## II.2.4.1 MAGNETISING INDUCTANCE

The magnetising inductances in the q- and d-axis are found via

$$L_{qm} = \frac{\lambda_{qm}}{i_q}, \quad (\text{II.2.112})$$

$$L_{dm} = \frac{\lambda_{dm}}{i_d}, \quad (\text{II.2.113})$$

where  $\lambda_{qm}$  and  $\lambda_{dm}$  are the magnetising flux linkages in the q- and d-axis respectively; and  $i_q$ ,  $i_d$  are the q- and d-axis currents respectively.

The flux linkage in abc variables is given as

$$\vec{\lambda}_{abc} = \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = \vec{\lambda}_{m,abc} + \vec{\lambda}_{l,abc}, \quad (\text{II.2.114})$$

where  $\vec{\lambda}$  is a vector of flux linkages for the a-, b-, and c-phases; similarly,  $\vec{\lambda}_{m,abc}$ , and  $\vec{\lambda}_{l,abc}$  are vectors for the magnetising and leakage flux linkages, respectively.

The magnetising flux linkages are given by

$$\vec{\lambda}_{m,abc} = l_g r_{ag} \int_0^{2\pi} \mathcal{B}(r_{ag}, \phi_{fa,m}) \vec{W}_{abc}(\phi_{fa,m}) d\phi_{fa,m}. \quad (\text{II.2.115})$$

where  $\phi_{fa,m}$  is the angle measured from the a-axis.

However, given that both  $B(r_{ag}, \phi)$ , and  $\vec{W}_{abc}(\phi)$  are periodic over  $2\pi/N_{pp}$  and odd half wave symmetric (see § II.2.3.1 and § II.2.2.3, respectively) the magnetising flux linkages can be rewritten as (with a change of variable to  $\phi_{fq,e}$ )

$$\vec{\lambda}_{m,abc} = 2l_g r_{ag} \int_0^\pi \mathcal{B}\left(r_{ag}, \frac{\phi_{fq,e}}{N_{pp}} + \phi_{r,m}\right) \vec{W}_{abc}\left(\frac{\phi_{fq,e}}{N_{pp}} + \phi_{r,m}\right) d\phi_{fq,e}, \quad (\text{II.2.116})$$

where

$$\phi_{fa,m} = \frac{\phi_{fq,e}}{N_{pp}} + \phi_{r,m}. \quad (\text{II.2.117})$$

Due to the symmetries of the function, the bounds of the integral can be shifted without affecting the value – as long as they span an angle of  $\pi$ . By shifting the limits by  $-(1 - \alpha_{pm})\pi/2$  the integral can be split into a region consisting of just spacer, and a region of just magnet:

$$\begin{aligned} \vec{\lambda}_{m,abc} = & 2l_g r_{ag} \int_{-(1-\alpha_{pm})\pi/2}^{(1-\alpha_{pm})\pi/2} \mathcal{B}_{sp}\left(r_{ag}, \frac{\phi_{fq,e}}{N_{pp}} + \phi_{r,m}\right) \vec{W}_{abc}\left(\frac{\phi_{fq,e}}{N_{pp}} + \phi_{r,m}\right) d\phi_{fq,e} \\ & + 2l_g r_{ag} \int_{(1-\alpha_{pm})\pi/2}^{\pi-(1-\alpha_{pm})\pi/2} \mathcal{B}_{pm}\left(r_{ag}, \frac{\phi_{fq,e}}{N_{pp}} + \phi_{r,m}\right) \vec{W}_{abc}\left(\frac{\phi_{fq,e}}{N_{pp}} + \phi_{r,m}\right) d\phi_{fq,e}, \end{aligned} \quad (\text{II.2.118})$$

where  $\mathcal{B}_{\text{sp}}(r_{\text{ag}}, \phi)$  and  $\mathcal{B}_{\text{pm}}(r_{\text{ag}}, \phi)$  are the flux densities in the spacer and permanent magnet respectively; they are given by

$$\mathcal{B}_{\text{sp}}(r_{\text{ag}}, \phi) = \frac{r_{\text{ri}}}{r_{\text{ag}}} \frac{\mathcal{F}_{\text{i}}(\phi)}{\mathcal{R}_{\text{sp}} + \mathcal{R}_{\text{ag}}}, \quad (\text{II.2.119})$$

$$\mathcal{B}_{\text{pm}}(r_{\text{ag}}, \phi) = \frac{r_{\text{ri}}}{r_{\text{ag}}} \frac{\mathcal{F}_{\text{pm}}(\phi) + \mathcal{F}_{\text{i}}(\phi)}{\mathcal{R}_{\text{pm}} + \mathcal{R}_{\text{ag}}}. \quad (\text{II.2.120})$$

From rewriting EQUATION (II.2.88), one can express the stator MMF as

$$\mathcal{F}_{\text{i}} \left( \frac{\phi_{\text{fq,e}}}{N_{\text{pp}}} + \phi_{\text{r,m}} \right) = \sqrt{\frac{3}{2}} \frac{N_{\text{cd}}}{N_{\text{pp}}} (i_{\text{q}} \sin(\phi_{\text{fq,e}}) + i_{\text{d}} \cos(\phi_{\text{fq,e}})), \quad (\text{II.2.121})$$

where  $i_{\text{q}}$ , and  $i_{\text{d}}$  are the currents in the q- and d-axis respectively. They are found via the length invariant a-phase to q-axis alignment Park transformation:

$$\mathbf{K}_{\text{p}}(\theta) = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{4\pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{4\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}; \quad (\text{II.2.122})$$

The current in qd0 variables can therefore be found to be

$$\vec{\mathbf{i}}_{\text{qd0}} = \begin{bmatrix} i_{\text{q}} \\ i_{\text{d}} \\ i_0 \end{bmatrix} = \mathbf{K}_{\text{p}}(\phi_{\text{r}}) \vec{\mathbf{i}}_{\text{abc}}(\phi_{\text{r}}) = \sqrt{\frac{3}{2}} i_{\text{peak}} \begin{bmatrix} \cos(\phi_{\text{a}}) \\ \sin(\phi_{\text{a}}) \\ 0 \end{bmatrix}. \quad (\text{II.2.123})$$

Similarly, the winding function transformed into qd0 variables is

$$\vec{W}_{\text{qd0}} \left( \frac{\phi_{\text{fq,e}}}{N_{\text{pp}}} + \phi_{\text{r,m}} \right) = \begin{bmatrix} W_{\text{q}} \\ W_{\text{d}} \\ W_0 \end{bmatrix} = \mathbf{K}_{\text{p}}(\phi_{\text{r}}) \vec{W}_{\text{abc}} \left( \frac{\phi_{\text{fq,e}}}{N_{\text{pp}}} + \phi_{\text{r,m}} \right) = \frac{N_{\text{c1}}^*}{N_{\text{pp}}} \begin{bmatrix} \cos(\phi_{\text{fq,e}}) \\ -\sin(\phi_{\text{fq,e}}) \\ 0 \end{bmatrix}. \quad (\text{II.2.124})$$

Solving the integral of the magnetic flux linkages, EQUATION (II.2.118), in qd0 variables, gives

$$\vec{\lambda}_{\text{m,qd0}} = \begin{bmatrix} \lambda_{\text{m,q}} \\ \lambda_{\text{m,d}} \\ \lambda_{\text{m,0}} \end{bmatrix} = \begin{bmatrix} L_{\text{m,q}} i_{\text{q}} \\ L_{\text{m,d}} i_{\text{d}} + \lambda_{\text{m}} \\ 0 \end{bmatrix}, \quad (\text{II.2.125})$$

where  $L_{\text{m,q}}$  and  $L_{\text{m,d}}$  are the magnetising inductances in the q- and d-axis respectively, given by

$$L_{\text{m,q}} = \frac{3}{2} l_{\text{g}} r_{\text{ri}} \frac{N_{\text{cd}}^2}{N_{\text{pp}}^2} \left[ \frac{(1 - \alpha_{\text{pm}})\pi + \sin(\alpha_{\text{pm}})}{\mathcal{R}_{\text{sp}} + \mathcal{R}_{\text{ag}}} + \frac{\alpha_{\text{pm}}\pi - \sin(\alpha_{\text{pm}})}{\mathcal{R}_{\text{pm}} + \mathcal{R}_{\text{ag}}} \right], \quad (\text{II.2.126})$$

$$L_{\text{m,d}} = \frac{3}{2} l_{\text{g}} r_{\text{ri}} \frac{N_{\text{cd}}^2}{N_{\text{pp}}^2} \left[ \frac{(1 - \alpha_{\text{pm}})\pi - \sin(\alpha_{\text{pm}})}{\mathcal{R}_{\text{sp}} + \mathcal{R}_{\text{ag}}} + \frac{\alpha_{\text{pm}}\pi + \sin(\alpha_{\text{pm}})}{\mathcal{R}_{\text{pm}} + \mathcal{R}_{\text{ag}}} \right], \quad (\text{II.2.127})$$

and  $\lambda_m$  is the magnetic flux linkage from the permanent magnets:

$$\lambda_m = \frac{4r_{ri}l_g N_{c1}}{N_{pp}(\mathcal{R}_{pm} + \mathcal{R}_{ag})} \frac{d_{pm}}{\mu_{pm}} \mathcal{B}_{res} \sin\left(\frac{\alpha_{pm}\pi}{2}\right). \quad (\text{II.2.128})$$

Some of the magnetic flux leaks beyond the ends of the PMSG. Therefore from a magnetic viewpoint, the generator is longer than it is physically. The remainder of the subsection derives the axially-compensated length – given in EQUATION (II.2.152) – by considering the permeance paths beyond the end of the PMSG. The axially-compensated length can be simply inputted into EQUATIONS (II.2.126) and (II.2.127) in place of  $l_g$ . Note that it is not required in the equation of the magnetic flux linkage [14].

PERMEANCE BEYOND THE END OF THE GENERATOR: The majority of the flux passes directly between the rotor and stator. However there is a component that travels around the ends of the generator, effectively extending its length [12]; these paths are shown on a two-dimensional cross section in FIGURE II.2.9 and in three-dimensions in FIGURE II.2.10. FIGURE II.2.9 shows the four regions that the flux paths can be separated into; end flux paths will cross through one of the regions –  $\mathcal{P}_1$ ,  $\mathcal{P}_2$ ,  $\mathcal{P}_3$ , or  $\mathcal{P}_4$  – in the rotor then through the same region in the stator. FIGURE II.2.10 shows a three dimensional view of flux line through the centre of each of the paths. The total permeance for the end leakage is

$$\mathcal{P}_{end} = 2(\mathcal{P}_1 + \mathcal{P}_2 + \mathcal{P}_3 + \mathcal{P}_4), \quad (\text{II.2.129})$$

where the factor of two accounts for both ends of the PMSG.

The paths can be combined into two similar pairs due to the shape of their magnetic field path:  $\mathcal{P}_1$  and  $\mathcal{P}_2$ ;  $\mathcal{P}_3$  and  $\mathcal{P}_4$ . They are referred to as  $\mathcal{P}_a$  and  $\mathcal{P}_b$  respectively.

The  $\mathcal{P}_a$  paths are characterised by a quarter turn coming out of the page; followed by a straight section; then a quarter turn back into the page. In FIGURE II.2.11 (top), the bounds for path  $\mathcal{P}_1$  are shown, where the all the magnetic paths associated with  $\mathcal{P}_1$  must pass through. The general equation for the permeance of a path is [18]

$$\mathcal{P} = \frac{\mu A_p}{l_p}, \quad (\text{II.2.130})$$

where  $\mu$  is the permeability of the path;  $A_p$  is the area of the path; and  $l_p$  is the mean magnetic path length. Given this path goes through the air gap,  $\mu = \mu_0$ , where  $\mu_0 = 1.257 \times 10^{-6}$  H/m is the permeability of free space. The area and mean magnetic path length are

$$A_p = wd, \quad (\text{II.2.131})$$

$$l_p = \frac{1}{d} \int_0^d \frac{\pi r_{rz}}{2} + \frac{\pi r_{rz}}{2} + g \, dr_{rz} = \frac{\pi d}{2} + g. \quad (\text{II.2.132})$$

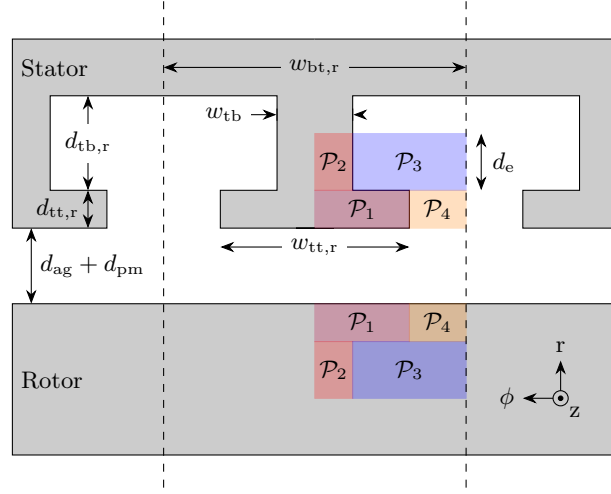


FIGURE II.2.9: Paths of permeance for the end leakage magnetic flux lines. Flux paths will pass through one of the  $\mathcal{P}$  regions in the rotor, and through the same region in the stator. (Adapted from [12])

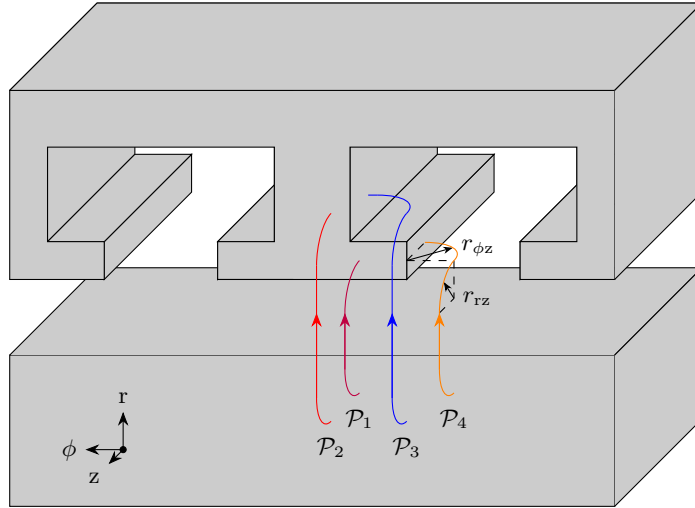


FIGURE II.2.10: Oblique projection of the four paths of permeance for the end leakage magnetic flux lines. Radii of curvature in the  $rz$ -plane are denoted  $r_{rz}$ , and radii of curvature in the  $\phi z$ -plane are denoted  $r_{\phi z}$ . (Adapted from [12])

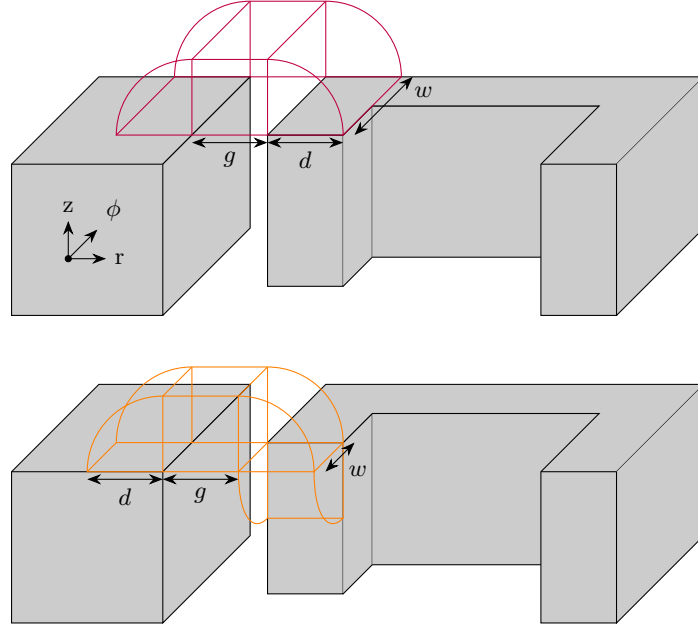


FIGURE II.2.11: Oblique projection of the bounds for end leakage permeance paths. Paths shown are  $\mathcal{P}_1$  (top, purple) and  $\mathcal{P}_4$  (bottom, orange); all magnetic field lines associated with these paths will pass through these bounds. The generator is cut along the right dashed line in FIGURE II.2.9.

Inputting these into EQUATION (II.2.130) results in

$$\mathcal{P}_a = \frac{2\mu_0 w d}{\pi d + 2g}, \quad (\text{II.2.133})$$

The  $\mathcal{P}_b$  paths are characterised by a quarter turn coming out of the page; followed by a straight radial section; then a quarter turn back into the page; finally, a quarter turn in the  $\phi z$  plane. The bounding box of the  $\mathcal{P}_4$  permeance path is shown in FIGURE II.2.11 (bottom). This has the same area as the  $\mathcal{P}_a$  path, and a mean magnetic path length of

$$l_p = \frac{1}{wd} \int_0^w \int_0^d \frac{\pi r_{rz}}{2} + \frac{\pi r_{rz}}{2} + g + \frac{\pi r_{\phi z}}{2} dr_{rz} dr_{\phi z} = \frac{\pi d}{2} + g + \frac{\pi w}{4}, \quad (\text{II.2.134})$$

giving the permeance as<sup>iv</sup>

$$\mathcal{P}_b = \frac{4\mu_0 w d}{2\pi d + 4g + \pi w}. \quad (\text{II.2.135})$$

<sup>iv</sup>A small error is found in Cassimere's work [12]. Instead of computing the integral for the length then finding the reciprocal; he found the reciprocal then integrated. It has a very minor effect on the resulting permeance but has been corrected in this work.

Each of the four paths have their own lengths:  $w$ ,  $d$ , and  $g$ ; these are as follows, where the subscript denotes which path the length is associated with:

$$w_1 = \frac{w_{\text{tt},r}}{2}, \quad (\text{II.2.136})$$

$$d_1 = d_{\text{tt},r}, \quad (\text{II.2.137})$$

$$g_1 = d_{\text{ag}} + d_{\text{pm}}, \quad (\text{II.2.138})$$

$$w_2 = \frac{w_{\text{tb}}}{2}, \quad (\text{II.2.139})$$

$$d_2 = d_{\text{e}}, \quad (\text{II.2.140})$$

$$g_2 = d_{\text{ag}} + d_{\text{pm}} + 2d_{\text{tt},r}, \quad (\text{II.2.141})$$

$$w_3 = \frac{(w_{\text{bt},r} - w_{\text{tb}})}{2}, \quad (\text{II.2.142})$$

$$d_3 = d_{\text{e}}, \quad (\text{II.2.143})$$

$$g_3 = d_{\text{ag}} + d_{\text{pm}} + 2d_{\text{tt},r}, \quad (\text{II.2.144})$$

$$w_4 = \frac{(w_{\text{bt},r} - w_{\text{tt},r})}{2}, \quad (\text{II.2.145})$$

$$d_4 = d_{\text{tt},r}, \quad (\text{II.2.146})$$

$$g_4 = d_{\text{ag}} + d_{\text{pm}}, \quad (\text{II.2.147})$$

where

$$d_{\text{e}} = \sqrt{\left(l_{\text{eo}} + \frac{l_{\text{ew}}}{2}\right)^2 + \left(d_{\text{st},r} - d_{\text{tt},r} - \frac{d_{\text{ew}}}{2}\right)^2}, \quad (\text{II.2.148})$$

is the maximum distance the flux can travel without passing through the centre of the end winding bundle; this is shown in FIGURE II.2.12.

THE AXIALLY-COMPENSATED LENGTH: Considering a segment of the generator from the centre of one tooth to the centre of the next, the permeance is

$$\mathcal{P}_{\text{seg}} = \frac{\mu_0 w_{\text{seg}} l_{\text{g}}}{d_{\text{ag}} + d_{\text{pm}}} + 2\mathcal{P}_{\text{end}}, \quad (\text{II.2.149})$$

where

$$w_{\text{seg}} = \frac{\pi(r_{\text{ri}} + r_{\text{ag}})}{N_{\text{st}}}. \quad (\text{II.2.150})$$

However, the permeance can also be written in the more general form of

$$\mathcal{P}_{\text{seg}} = \frac{\mu_0 w_{\text{seg}} l_{\text{g,ac}}}{d_{\text{ag}} + d_{\text{pm}}}, \quad (\text{II.2.151})$$

where  $l_{\text{g,ac}}$  is the axially-compensated length. By equating EQUATIONS (II.2.149) and (II.2.151) and solving for  $l_{\text{g,ac}}$ , one finds

$$l_{\text{g,ac}} = l_{\text{g}} + \frac{2(d_{\text{ag}} + d_{\text{pm}})}{\mu_0 w_{\text{seg}}} \mathcal{P}_{\text{end}}, \quad (\text{II.2.152})$$

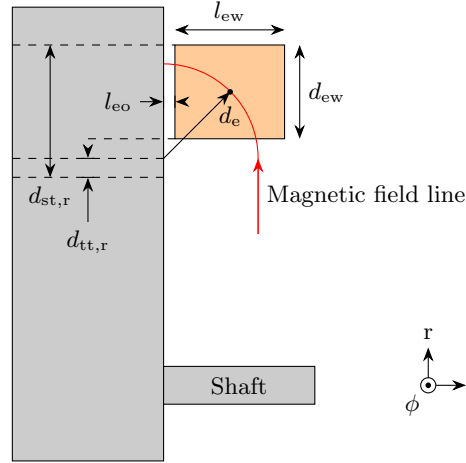


FIGURE II.2.12: Radial view of a permanent magnet synchronous machine. Showing the required dimensions to calculate the distance to the centre of the end winding bundle,  $d_e$ . (Adapted from [12])

which replaces  $l_g$  in the equations of magnetising inductance – EQUATIONS (II.2.127) and (II.2.126).

#### II.2.4.2 LEAKAGE INDUCTANCE

As mentioned at the start of § II.2.4, the leakage inductance is associated with the flux that does not cross the air gap; it can be further broken down into two components [19]: the self-inductance,

$$L_{lp} = \mathcal{P}_{sl} \sum_{i=1}^{N_{st}} C_a(i)^2 + \mathcal{P}_{el} \sum_{i=1}^{N_{st}} K_a(i)^2, \quad (\text{II.2.153})$$

which is the inductance caused by a phase, interacting with itself; and the mutual-inductance,

$$L_{lm} = \mathcal{P}_{sl} \sum_{i=1}^{N_{st}} C_a(i)C_b(i) + \mathcal{P}_{el} \sum_{i=1}^{N_{st}} K_a(i)K_b(i), \quad (\text{II.2.154})$$

which is the inductance caused by a phase, interacting with another phase.  $\mathcal{P}_{sl}$  and  $\mathcal{P}_{el}$  are the leakage permeances in the slots, and in the end turn region; they are derived in turn

**SLOT LEAKAGE PERMEANCE:** Rather than pathing between the rotor and stator as desired, some of the flux will path between adjacent stator teeth; these paths are categorised into two groups:  $\mathcal{P}_5$ ,  $\mathcal{P}_6$ ,  $\mathcal{P}_7$ , and  $\mathcal{P}_8$ , which are contained within the generator, and shown in FIGURE II.2.13;  $\mathcal{P}_9$ ,  $\mathcal{P}_{10}$ , and  $\mathcal{P}_{11}$ , which pass beyond the ends of the generator, shown in FIGURE II.2.14.

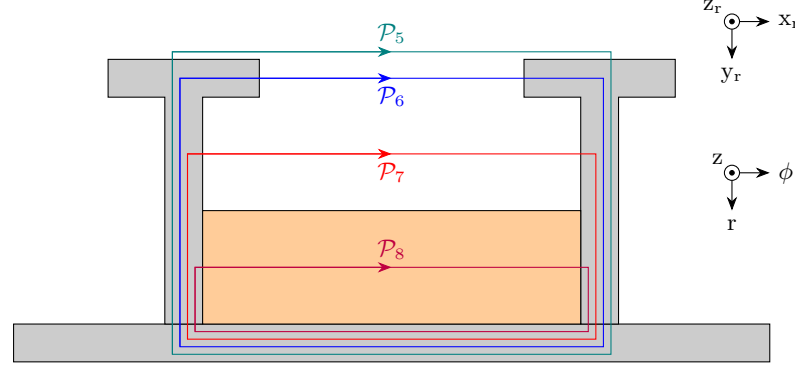


FIGURE II.2.13: Slot leakage permeance. (Adapted from [12])

Starting with the first group: path  $\mathcal{P}_5$  contains all the flux that travels into the air gap but returns to a neighbouring tooth, rather than into the rotor; path  $\mathcal{P}_6$  contains the flux that travels between adjacent tooth tips; path  $\mathcal{P}_7$  contains the flux that travels within the slot, but above the conductors; finally, path  $\mathcal{P}_8$  contains the flux that travels within the slot and through the conductors.

Paths  $\mathcal{P}_5$ ,  $\mathcal{P}_6$ , and  $\mathcal{P}_7$  are derived using the same methodology for the end permeance in § ??; using this, it can be found that<sup>v</sup>

$$\mathcal{P}_5 = \frac{\mu_0 l_g (d_{ag} + d_{pm})}{w_{btt,r} + w_{tt,r}}; \quad (\text{II.2.155})$$

$$\mathcal{P}_6 = \frac{\mu_0 l_g d_{tt,r}}{w_{btt,r}}; \quad (\text{II.2.156})$$

$$\mathcal{P}_7 = \frac{\mu_0 l_g (d_{tb,r} - d_{w,r})}{w_{sl,r}}. \quad (\text{II.2.157})$$

Path  $\mathcal{P}_8$  is more complicated, due to it travelling through the conductors, and not all the flux will link the entire conductor; for this, an energy analysis approach is used. Assume there was a single conductor running through the slot, with a current  $i$  flowing through it, such that this singular conductor can produce the same MMF as true number of conductors, from all of the phases. The energy stored by the inductance of the slot is

$$E = \frac{1}{2} Li^2, \quad (\text{II.2.158})$$

and since the effective turns is one, this is equivalent to

$$E = \frac{1}{2} \mathcal{P} i^2, \quad (\text{II.2.159})$$

<sup>v</sup>Note that for path  $\mathcal{P}_5$ , this ignores the radial component of the path, however the component is small should not have a significant effect of the result [12].

since  $\mathcal{P} = L/N^2$ , where  $N$  is the number of turns [20]; rearranging gives

$$\mathcal{P} = \frac{2E}{i^2}. \quad (\text{II.2.160})$$

Now consider the energy stored within a magnetic field:

$$E = \frac{1}{2} \int \vec{\mathcal{B}} \cdot \vec{\mathcal{H}} \, dV, \quad (\text{II.2.161})$$

in the slot  $\mathcal{B} = \mu_0 \mathcal{H}$ , therefore the energy in the field can be expressed as

$$E = \frac{1}{2} \int \mu_0 \mathcal{H}^2 \, dV, \quad (\text{II.2.162})$$

where  $\mathcal{H}$  is purely transversal (in the rectangular approximation):

$$\mathcal{H}_8 w_{\text{sl},r} = \frac{y_r i}{d_{w,r}}. \quad (\text{II.2.163})$$

Inputting into EQUATION (II.2.162), with  $dV = l_g w_{\text{sl},r} \, dy_r$ . The energy in the slot is found to be

$$E_8 = \frac{\mu_0}{2} \int_0^{d_{w,r}} \frac{i^2 y_r^2}{d_{w,r}^2 w_{\text{sl},r}} l_g \, dy_r = \frac{\mu_0 i^2 l_g d_{w,r}}{6 w_{\text{sl},r}}, \quad (\text{II.2.164})$$

which when inputted into EQUATION (II.2.160) gives

$$\mathcal{P}_8 = \frac{\mu_0 l_g d_{w,r}}{3 w_{\text{sl},r}}. \quad (\text{II.2.165})$$

The second group –  $\mathcal{P}_9$ ,  $\mathcal{P}_{10}$ , and  $\mathcal{P}_{11}$ , shown in FIGURE II.2.14 – move as follows: path  $\mathcal{P}_9$  passes between the tooth tips; path  $\mathcal{P}_{10}$  passes over the slot, above the level of the windings (akin to  $\mathcal{P}_7$ ); path  $\mathcal{P}_{11}$  passes over the slot, in the section occupied by the windings (akin to  $\mathcal{P}_8$ ).

Path  $\mathcal{P}_9$  can be found using the same energy analysis; in this case, the magnetic field strength in the direction of the path is

$$\mathcal{H}_9 \left( \frac{\pi r}{2} + w_{\text{btt},r} + \frac{\pi r}{2} \right) = i, \quad (\text{II.2.166})$$

substitution into EQUATION (II.2.162), using  $dV = (\pi r + w_{\text{btt},r}) \, dr \, dy_r$

$$E_9 = \frac{\mu_0}{2} \int_0^{d_{\text{tt},r}} \int_0^{w_{\text{tt},r}} \frac{i^2}{(\pi r + w_{\text{btt},r})} \, dr \, dy_r = \frac{\mu_0 i^2}{2\pi} \ln \left( 1 + \frac{\pi w_{\text{tt},r}}{w_{\text{btt},r}} \right) d_{\text{tt},r}. \quad (\text{II.2.167})$$

Solving then inputting into EQUATION (II.2.160), gives the permeance as

$$\mathcal{P}_9 = \frac{\mu_0}{\pi} \ln \left( 1 + \frac{\pi w_{\text{tt},r}}{w_{\text{btt},r}} \right) d_{\text{tt},r}. \quad (\text{II.2.168})$$

For path  $\mathcal{P}_{10}$ , the process is very similar as to  $\mathcal{P}_9$ , the magnetic field strength is

$$\mathcal{H}_{10} \left( \frac{\pi r}{2} + w_{\text{sl},r} + \frac{\pi r}{2} \right) = i, \quad (\text{II.2.169})$$

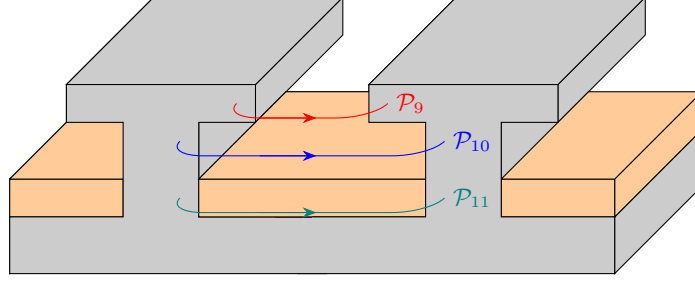


FIGURE II.2.14: End leakage permeance. (Adapted from [12])

which when substituted into EQUATION (II.2.162) with a differential volume element of  $dV = (\pi r + w_{sl,r}) dr dy_r$ , one finds

$$E_{10} = \frac{\mu_0}{2} \int_{d_{w,r}}^{d_{sl,r}} \int_0^{w_{tb,r}} \frac{i^2}{(\pi r + w_{sl,r})} dr dy_r = \frac{\mu_0 i^2}{2\pi} \ln \left( 1 + \frac{\pi w_{tb,r}}{w_{sl,r}} \right) (d_{sl,r} - d_{w,r}), \quad (\text{II.2.170})$$

and then substituted into EQUATION (II.2.160), gives a path permeance of

$$\mathcal{P}_{10} = \frac{\mu_0}{\pi} \ln \left( 1 + \frac{\pi w_{tb,r}}{w_{sl,r}} \right) (d_{sl,r} - d_{w,r}). \quad (\text{II.2.171})$$

Path  $\mathcal{P}_{11}$  requires a slight modification to the methodology for the previous two paths, due to it passing through the windings. This gives a magnetic field strength of

$$\mathcal{H}_{11} \left( \frac{\pi r}{2} + w_{sl,r} + \frac{\pi r}{2} \right) = \frac{y_r}{d_{w,r}} i. \quad (\text{II.2.172})$$

Substitution into EQUATION (II.2.162) with the same differential volume element as path  $\mathcal{P}_{10}$ , the energy in the slot is

$$E_{11} = \frac{\mu_0}{2} \int_0^{d_{w,r}} \int_0^{w_{tb,r}} \frac{i^2 y_r^2}{d_{w,r}^2 (\pi r + w_{sl,r})} dr dy_r = \frac{\mu_0 i^2}{6\pi} \ln \left( 1 + \frac{\pi w_{tb,r}}{w_{sl,r}} \right) d_{w,r}, \quad (\text{II.2.173})$$

which gives a permeance of

$$\mathcal{P}_{11} = \frac{\mu_0}{3\pi} \ln \left( 1 + \frac{\pi w_{tb,r}}{w_{sl,r}} \right) d_{w,r}. \quad (\text{II.2.174})$$

Finally, paths 5 through 11, can be combined to give the total slot leakage permeance

$$\mathcal{P}_{sl} = \mathcal{P}_5 + \mathcal{P}_6 + \mathcal{P}_7 + \mathcal{P}_8 + 2(\mathcal{P}_9 + \mathcal{P}_{10} + \mathcal{P}_{11}); \quad (\text{II.2.175})$$

the final three paths are multiplied by two to account for each end of the PMSG.

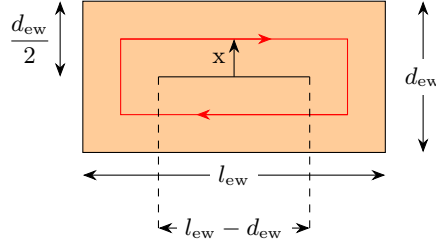


FIGURE II.2.15: End leakage permeance interior path. (Adapted from [12])

END LEAKAGE PERMEANCE: The permeance in the end turn region is separated into two paths:  $\mathcal{P}_{12}$ , and  $\mathcal{P}_{13}$ . Path  $\mathcal{P}_{12}$  describes the flux flowing within the end turn; path  $\mathcal{P}_{13}$  describes the flux flowing outside the turn.

Starting with  $\mathcal{P}_{12}$ , this path accounts for the flux flowing within the end windings. Similarly to the slot leakage permeance, assume there is a single end turn with a current  $i$ , that can recreate the total MMF from all the end turns; this singular winding is shown in FIGURE II.2.15. Within the winding is a rectangular permeance path at a distance  $x$  from the centre of the end winding. Ampere's law states that<sup>vi</sup>

$$\mathcal{H}_{12} (2(l_{ew} - d_{ew}) + 8x) = i \left( \frac{4x^2 + 2x(l_{ew} - d_{ew})}{l_{ew}d_{ew}} \right). \quad (\text{II.2.176})$$

The energy stored within this path is found using EQUATION (II.2.162), using  $dV = w_{\text{seg}}(2(l_{ew} - d_{ew}) + 8x) dx$

$$E_{12} = \frac{\mu_0}{2} \int_0^{d_{ew}/2} \frac{i^2 (4x^2 + 2x(l_{ew} - d_{ew}))^2}{l_{ew}^2 d_{ew}^2 (2(l_{ew} - d_{ew}) + 8x)} w_{\text{seg}} dx, \quad (\text{II.2.177})$$

evaluating and substituting into EQUATION (II.2.160) gives the permeance as

$$\mathcal{P}_{12} = \frac{\mu_0 w_{\text{seg}}}{128 l_{ew}^2 d_{ew}^2} \left( 2(l_{ew} - d_{ew})^4 \tanh^{-1} \left( \frac{d_{ew}}{l_{ew}} \right) - 2d_{ew}l_{ew}(l_{ew}^2 - 4l_{ew}d_{ew} + d_{ew}^2) \right). \quad (\text{II.2.178})$$

Path  $\mathcal{P}_{13}$  accounts for the permeance exterior to the end windings; FIGURE II.2.16 shows the route of this path. To calculate the permeance of the path, it is best to calculate the reluctance (inverse of the permeance) of the path, which is broken into three sections:

$$\frac{1}{\mathcal{P}_{13}} = \frac{l_{eo} + l_{ew} + x/2}{d_{sb}w_{\text{seg}}\mu_0} + \frac{d_{ew} + d_{sb}/2 + y/2}{2xw_{\text{seg}}\mu_0} + \frac{l_{eo} + l_{ew} + x/2}{2yw_{\text{seg}}\mu_0}. \quad (\text{II.2.179})$$

All that remains is to find the values of  $x$  and  $y$ ; they can not be found analytically, but since the flux flows in a path to minimise reluctance, a numerical method can be

<sup>vi</sup>The derivation assumes  $l_{ew} > d_{ew}$ . Should this be false, the values of the two variables can simply be swapped.

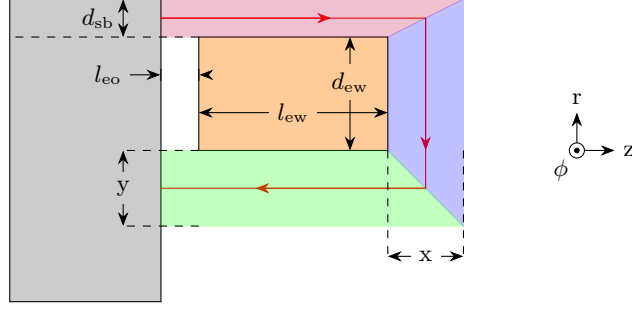


FIGURE II.2.16: End leakage permeance exterior path. (Adapted from [12])

applied to find the pair of values for  $x$  and  $y$  that minimises  $1/\mathcal{P}_{13}$ . There is only a singular minima in the map of  $1/\mathcal{P}_{13}(x, y)$ ; therefore a simpler numerical method than a GA can be used to find it. For this work, the gradient descent method is used, as described in § I.4.3.

The total end turn leakage permeance, is found via

$$\mathcal{P}_{el} = 2(\mathcal{P}_{12} + \mathcal{P}_{13}); \quad (\text{II.2.180})$$

again, the factor of two is to account for both ends of the PMSG.

## II.2.5 WINDING RESISTANCE

The resistance of each phase can be calculated by

$$\vec{R}_{abc} = \begin{bmatrix} R_a \\ R_b \\ R_c \end{bmatrix} = \frac{\rho_{\text{cond}}}{A_{\text{cond}}} \vec{l}_{\text{cond}}, \quad (\text{II.2.181})$$

where  $\rho_{\text{cond}}$  and  $A_{\text{cond}}$  are the resistivity of the conductor material, and the area of a single conductor respectively; and

$$\vec{l}_{\text{cond}} = \begin{bmatrix} l_{\text{cond},a} \\ l_{\text{cond},b} \\ l_{\text{cond},c} \end{bmatrix} = \vec{l}_{\text{cond},sl} + \vec{w}_{\text{cond},end}, \quad (\text{II.2.182})$$

is the length of conductor associated with the  $x$ -phase; it is split into two sections.

$$\vec{l}_{\text{cond},sl} = (l_g + 2l_{eo} + l_{ew}) \sum_{i=1}^{N_{st}} |\vec{C}_{abc}(i)|, \quad (\text{II.2.183})$$

is the length of conductor within the slots, where  $l_{eo}$  and  $l_{ew}$  are the lengths of the end winding offset and end winding bundles respectively. The end winding offset is

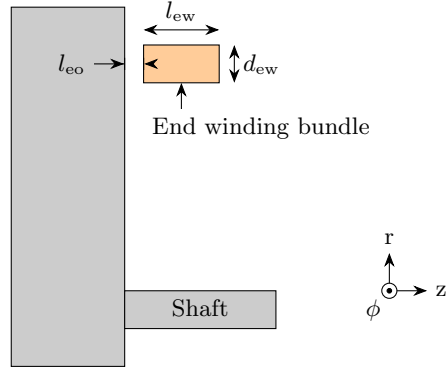


FIGURE II.2.17: Radial view of a permanent magnet synchronous machine, to show the dimensions of the end winding bundle. (Adapted from [12])

the distance from the end of the generator to the end winding. It is dependent on the assembly process [12]. Both of these are shown in FIGURE II.2.17.

$$\vec{w}_{\text{cond,end}} = w_{\text{seg}} \sum_{i=1}^{N_{\text{st}}} |\vec{K}_{\text{abc}}(i)|, \quad (\text{II.2.184})$$

is the length in the end turn region, where  $w_{\text{seg}}$  is the mean length of a segment of the end winding in the tangential direction; it is approximated as

$$w_{\text{seg}} = \frac{2\pi}{N_{\text{st}}} \left( r_{\text{ag}} + d_{\text{st}} - \frac{d_{\text{w,r}}}{2} \right). \quad (\text{II.2.185})$$

## II.2.6 GENERATOR CONSTANTS

In the drive train model, two generator constants will be required: the torque constant,  $\kappa_T$ , and the back electromotive force (EMF) constant,  $\kappa_V$ ; these relate electromagnetic torque to generator current, and the EMF to generator speed respectively. However, in PMSGs, these two constants have an identical value [21], so only one needs to be derived.

The electromagnetic torque of a generator is given, in qd0 variables, as [21]

$$T_e = N_{\text{pp}}((L_q - L_d)i_d i_q - \lambda_m i_q); \quad (\text{II.2.186})$$

in permanent magnetic machines  $L_d \approx L_q$ , which means that the magnitude of the torque is approximately

$$|T_e| \approx |N_{\text{pp}} \lambda_m i_q|. \quad (\text{II.2.187})$$

Defining the torque constant as the ratio of electromagnetic torque to the current, it follows that

$$\kappa_T \approx N_{\text{pp}} \lambda_m; \quad (\text{II.2.188})$$

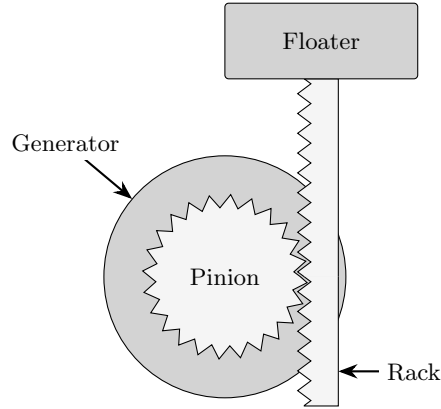


FIGURE II.2.18:  
Schematic of the chosen  
wave energy converter's  
drive train.

and since the two constants are equal,

$$\kappa_V \approx N_{pp} \lambda_m. \quad (\text{II.2.189})$$

## II.2.7 MOMENT OF INTERIA

The final required property of the generator is mechanical, the moment of inertia. For a hollow cylinder, it is given by

$$J = \frac{1}{2} \rho l \pi (r_{\text{out}}^4 - r_{\text{in}}^4), \quad (\text{II.2.190})$$

where  $\rho$  is the density of the cylinder;  $l$  is the length of the cylinder;  $r_{\text{out}}$  is the outer radius of the cylinder; and  $r_{\text{in}}$  the inner radius.

The moment of inertia of the rotor is therefore found via

$$J_{\text{rotor}} = J_{\text{ri}} + J_{\text{pm}}, \quad (\text{II.2.191})$$

where  $J_{\text{ri}}$ , and  $J_{\text{pm}}$  are the moments components of inertia coming from the rotor iron, and permanent magnets respectively. They can be found using EQUATION (II.2.190) to be

$$J_{\text{ri}} = \frac{1}{2} \rho_{\text{ri}} l \pi (r_{\text{ri}}^4 - r_{\text{hc}}^4), \quad (\text{II.2.192})$$

$$J_{\text{pm}} = \frac{1}{2} \rho_{\text{pm}} l \pi (r_{\text{pm}}^4 - r_{\text{ri}}^4) \alpha_{\text{pm}}; \quad (\text{II.2.193})$$

in the case of  $J_{\text{pm}}$ , there is the extra factor  $\alpha_{\text{pm}}$  to account for the fact that the permanent magnets do not fill the entire ring.

## II.2.8 MODELLING DRIVE TRAINS FOR WAVE ENERGY CONVERTERS

The design of the drive train used in this work is mentioned in §1.5, and consists of a WEC assumed to be perfectly moored such that it is free to move in heave, but fully

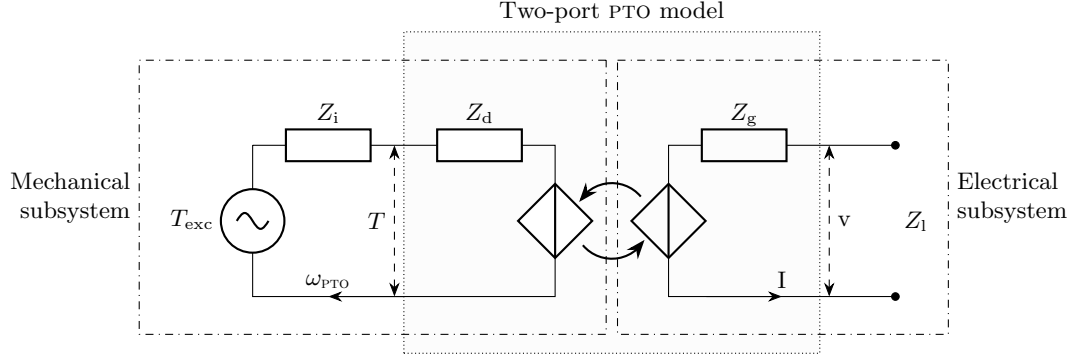


FIGURE II.2.19: System diagram of the drive-train system connecting the mechanical (left hand circuit) and electrical (right hand circuit) subsystems via the power take-off, represented by a two-port network. (Adapted from [6])

constrained in every other degree of freedom. A schematic is shown in FIGURE II.2.18. This subsection builds up a model of the drive train that accounts for losses to create a map from inputs of mechanical force and speed to an output of electrical power.

#### II.2.8.1 A TWO-PORT SYSTEM

The power take-off (PTO) can be modelled as a two port system, as shown in FIGURE II.2.19, connecting the mechanical and electrical subsystems. In equation form, the system is described as [3]

$$\begin{bmatrix} F_{\text{PTO}}(\omega_{\text{PTO}}) \\ v_q(\omega_{\text{PTO}}) \end{bmatrix} = \mathbf{Z}_{\text{PTO}} \begin{bmatrix} u(\omega_{\text{PTO}}) \\ i_q(\omega_{\text{PTO}}) \end{bmatrix} = \begin{bmatrix} Z_{Fu}(\omega_{\text{PTO}}) & Z_{Fi}(\omega_{\text{PTO}}) \\ Z_{vu}(\omega_{\text{PTO}}) & Z_{vi}(\omega_{\text{PTO}}) \end{bmatrix} \begin{bmatrix} u(\omega_{\text{PTO}}) \\ i_q(\omega_{\text{PTO}}) \end{bmatrix} \quad (\text{II.2.194})$$

where  $\mathbf{Z}_{\text{PTO}}$  is the impedance matrix of the PTO, and  $\omega_{\text{PTO}}$  is the angular velocity of the PTO. To find the components, first consider the torque over the PTO,

$$T_{\text{PTO}} = Z_d \omega_{\text{PTO}} - \sqrt{\frac{3}{2}} \kappa_T i_q, \quad (\text{II.2.195})$$

where

$$Z_d = i \omega_{\text{PTO}} J, \quad (\text{II.2.196})$$

is the impedance of the drive train. Rewriting this to get the PTO force as a function of generator speed and current, one gets

$$F_{\text{PTO}} = \frac{i \omega_{\text{PTO}} J}{r_p^2} u - \sqrt{\frac{3}{2}} \kappa_T i_q, \quad (\text{II.2.197})$$

using  $F_{\text{PTO}} = T_{\text{PTO}}/r_p$  and  $u = \omega_{\text{PTO}} r_p$ .

Similarly, the voltage of the PTO can be found to be

$$v_q = Z_g i_q - \sqrt{\frac{3}{2}} \kappa_v \omega_{\text{PTO}}, \quad (\text{II.2.198})$$

where

$$Z_g = R_g + i\omega_{\text{PTO}} L_g, \quad (\text{II.2.199})$$

is the impedance of the generator. Again, rewriting to get voltage as a function of generator speed and current yields

$$v_q = -\sqrt{\frac{3}{2}} \frac{\kappa_v}{r_p} u + (R_g + i\omega_{\text{PTO}} L_g) i_q. \quad (\text{II.2.200})$$

From comparing EQUATION (II.2.194) with EQNS. (II.2.197) and (II.2.200), the impedance of the PTO can be found from open circuit conditions (setting either  $u = 0$  or  $i_q = 0$ ) to be

$$Z_{Fu} := \left. \frac{F_{\text{PTO}}}{u} \right|_{i_q=0} = \frac{i\omega_{\text{PTO}} J}{r_p^2}; \quad (\text{II.2.201})$$

$$Z_{Fi} := \left. \frac{F_{\text{PTO}}}{i_q} \right|_{u=0} = -\sqrt{\frac{3}{2}} \frac{\kappa_T}{r_p}; \quad (\text{II.2.202})$$

$$Z_{vu} := \left. \frac{v_q}{u} \right|_{i_q=0} = -\sqrt{\frac{3}{2}} \frac{\kappa_v}{r_p}; \quad (\text{II.2.203})$$

$$Z_{vi} := \left. \frac{v_q}{i_q} \right|_{u=0} = R + i\omega_{\text{PTO}} L_q. \quad (\text{II.2.204})$$

It is useful to re-arrange the matrix equation of the PTO – EQUATION (II.2.194) – to bring together the electrical and mechanical variables:

$$\begin{bmatrix} v_q \\ i_q \end{bmatrix} = \begin{bmatrix} Z_{vi}/Z_{Fi} & Z_{vu} - Z_{Fu}Z_{vi}/Z_{Fi} \\ 1/Z_{Fi} & -Z_{Fu}/Z_{Fi} \end{bmatrix} \begin{bmatrix} F_{\text{PTO}} \\ u \end{bmatrix}. \quad (\text{II.2.205})$$

### II.2.8.2 ELECTRICAL POWER LOOKUP TABLE

The final step is to create a lookup table for the outputted electrical power as a function of PTO force and speed,  $P_{\text{elec}}(F_{\text{PTO}}, u)$ . This can simply be done using the fact that the instantaneous real power is

$$P_{\text{elec}} = \text{real}[v_q] \text{real}[i_q]; \quad (\text{II.2.206})$$

in the equation:  $\text{real}[\cdot]$  is a function that returns the real component of the argument;  $v_q$  and  $i_q$  are found from EQUATION (II.2.205).

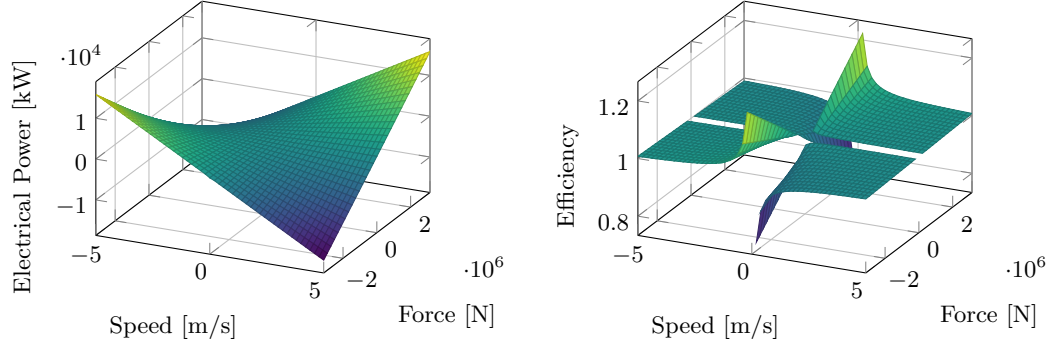


FIGURE II.2.20: Example efficiency and power maps. (Figure from [24])

It is commonplace to use efficiency maps when discussing losses in a drive train or generator, *eg.* [22, 23]; and one could easily be found using this method via

$$\eta(F_{\text{PTO}}, u) = \frac{P_{\text{elec}}(F_{\text{PTO}}, u)}{F_{\text{PTO}} u}. \quad (\text{II.2.207})$$

This does however introduce numerical issues and discontinuities at very low forces or speeds where the mechanical power,  $F_{\text{PTO}} u$ , is practically zero, as seen in FIGURE II.2.20. It is therefore found that using a power map in the optimisation, is much more stable; and it saves on calculation as the efficiency would have to be multiplied by mechanical force again at a later point anyway.

An example power map and efficiency map are shown in FIGURE II.2.20. Here the electrical power is negative and positive in different quadrants: the negative quadrants correspond to conditions in which the WEC would be generating, and the positive quadrants correspond to motoring conditions. In the efficiency map, the efficiency is always less than one when generating, as the losses in the motor mean that less electrical power is output than mechanical was inputted. Conversely, in the motoring regions it is always greater than one because to achieve a demanded mechanical power, a higher value of electrical power needs to be inputted.

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## *Optimising & Evaluating Power Take-off Systems for Wave Energy Converters*

In § II.2, the theory behind the models of the permanent magnet synchronous generator (PMSG) and the drive train is described, to create an overall model of the power take-off (PTO). This section discusses how that model is integrated into a genetic algorithm (GA): starting with the definition of the genes to produce the PTO in § II.3.1; the inputs to the model are described in § II.3.2; then the cost function is defined in § II.3.3; ending with an overview of the power simulation and control strategy optimisation in § II.3.4.

### II.3.1 IMPLEMENTATION IN A GENETIC ALGORITHM

The model uses a GA to design the PTO: the PTO is defined using a genome of various geometric properties of the PMSG and ranks the relative performance of different PTO designs to select the best performing solutions to *breed* to create the next generation of solutions; this repeats until stagnation in the design. A more in-depth overview of the theory of GAs is presented in § I.4.4. The model builds up the PTO design from the genome, following the theory presented in § II.2; using the genes and some fixed relationships between different variables as a data source. In this section both are explained, with the rationale for why they were picked.

#### II.3.1.1 GENE SELECTION AND BOUNDS

There are many combinations of genes that can define the PTO, for example building the generator from the centre outwards; or from the outer radius inwards. However, from testing, it is found that defining the PMSG around the air gap radius: defining the rotor inwards from there, and the stator outwards produced the most consistent results. The other methods have issues with stagnation, as when the GA resized one component a knock on effect is caused that would resize another – and another, and another – leading to an invalid design.

Another consequence of the interactivity between different parts of the PMSG, is that the bounds of some of the genes are defined as functions of the current values of other genes. This makes the ordering of the genes pivotal. The final selection of genes – in the order of evaluation – are as follows: the number of pole pairs,  $N_{pp}$ ; the

TABLE II.3.1: Upper and lower bounds of the genes to define the power take-off.

| GENE                          | MINIMUM VALUE                       | MAXIMUM VALUE | UNIT |
|-------------------------------|-------------------------------------|---------------|------|
| $N_{pp}$ <sup>a</sup>         | 1                                   | 100           | -    |
| $N_{st}/6N_{pp}$ <sup>a</sup> | 1                                   | 5             | -    |
| $\alpha_{pm}$                 | 20                                  | 100           | %    |
| $l_g$                         | 0.2                                 | 5             | m    |
| $r_{ag}$                      | – VARIES –                          | 6             | m    |
| $d_{tb}$                      | $2\sqrt{A_{wind}}/\pi$ <sup>b</sup> | 1             | m    |
| $d_{pm}$                      | 0.001                               | – VARIES –    | m    |
| $\theta_{tt}/\theta_{sl}$     | – VARIES –                          | 1             | -    |

Gene bounds marked ‘VARIES’ are a function of the genome and are described in the text.

<sup>a</sup> Fixed to an integer value.

<sup>b</sup> *i.e.* The tooth base is at least as deep as a single turn of winding.

number of stator teeth per pole per phase,  $N_{st}/6N_{pp}$ ; the proportion of the permanent magnet region with magnets,  $\alpha_{pm}$ ; the length of the generator,  $l_g$ ; the outer radius of the air gap,  $r_{ag}$ ; the depth of the tooth base,  $d_{tb}$ ; the depth of the permanent magnets,  $d_{pm}$ ; the ratio of angular spans of the tooth tip and slot,  $\theta_{tt}/\theta_{sl}$ .

The bounds of the genes are the maximum and minimum values the genes can take. Most of the bounds are fixed – with the values in TABLE II.3.1 – but for some of the bounds they change depending on the current values of other genes. How these are found are described throughout the remainder of the section. Note that  $N_{pp}$  and  $N_{st}/6N_{pp}$  are fixed to an integer value; the rest of the genes are non-integers.

On the creation of a new PMSG design – be that in the initialisation of the population, or the after the mutation of a new design – firstly, the static bounds for all genes are enforced. Then, the variable bounds are enforced in genome order (the order of TABLE II.3.1).

**LOWER BOUND OF THE OUTER AIR GAP RADIUS:** The minimum air gap radius is set such that a winding can fit at the narrowest point of the slot.

On initialisation, the width of the tooth base is not known, so instead it is assumed that the width of the tooth base takes up  $1/10^{\text{th}}$  the span of the slot.

On later iterations – when the width of the tooth base is known – the minimum air gap radius can be found precisely. FIGURE II.3.1 shows the geometry of the problem.

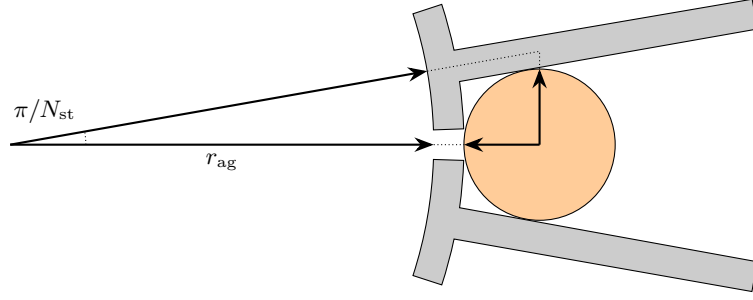


FIGURE II.3.1: Geometry of the minimum outer air gap radius problem. Relative dimensions are exaggerated.

Taking the triangle shown to be a right angle triangle, the adjacent length is equal to

$$r_{\text{adj}} = r_{\text{ag,min}} + w_{\text{tb}} + w_{\text{wind}}, \quad (\text{II.3.1})$$

and the opposite length is

$$w_{\text{opp}} = w_{\text{wind}} + \frac{w_{\text{tb}}}{2} \sin \left( \pi - \frac{\pi}{N_{\text{st}}} \right). \quad (\text{II.3.2})$$

This gives the minimum air gap radius as

$$r_{\text{ag,min}} = \frac{w_{\text{opp}}}{\tan(\pi/N_{\text{st}})} - w_{\text{tb}} - w_{\text{wind}}. \quad (\text{II.3.3})$$

**LOWER BOUND OF THE TOOTH TIP ANGULAR SPAN:** The lower bound of the tooth tip angular span is set such that the tooth tip is not narrower than the tooth base. From simple trigonometry:

$$\theta_{\text{tt,min}} = \arctan \left( \frac{w_{\text{tb}}}{2r_{\text{tt}}} \right); \quad (\text{II.3.4})$$

note that this information is stored in the genome as a ratio of this to slot span,  $\theta_{\text{sl}}$ .

**UPPER BOUND OF THE DEPTH OF PERMANENT MAGNETS:** The upper bound on the depth of the permanent magnets is set to avoid over saturation of flux in the rotor iron: the point of maximum flux can be at any of five points as defined by EQUATIONS (II.2.103)-(II.2.107) in §II.2.3.2. The GA ensures that all values of flux in the rotor iron are less than  $\mathcal{B}_{\text{steel}}$ , the maximum flux density in the steel – a user-defined material property. On initialisation, it is impossible to know if this limit is broken – due to other genes not being initialised yet – so the upper bound is set to 20 cm<sup>i</sup>, but a true limit is enforced from the second iteration onwards.

<sup>i</sup>This value is chosen to be so large that the chance of breaking the flux density limit is minute. The algorithm quickly brings the tooth width to a more reasonable value.

On all subsequent iterations, the upper bound is found by testing the current value of the gene for the depth of the permanent magnet. If it has broken any of the flux density limits, the gene is set to 95% of its current value; and the test is undertaken again, repeating until a valid value is found.

### II.3.1.2 RELATIONSHIPS BETWEEN DIFFERENT DIMENSIONS

Beyond the genes, and their bounds described above, there are design parameters where it is objectively best to maximise or minimise its size. Take for example the air gap depth: there is a drop in magnetomotive force over the air gap so the best performing generators have the smallest air gap possible. The lower limit comes from accounting for manufacturing defects, mechanical problems, or assembly errors, in which there is an eccentricity in the rotor's motion (the rotor's centre is misaligned from the stator's centre); the air gap is therefore sized to counteract eccentricity causing the rotor and stator to collide [25].<sup>ii</sup> To make the air gap a suitable size, the outer radius of the permanent magnets is therefore assumed to be [26]

$$r_{\text{pm}} = 0.999r_{\text{ag}}. \quad (\text{II.3.5})$$

This is a common assumption in the design of PMSGs for wind turbines; it has been used for a range of power ratings, for example: 500 kW in [26], 3 MW in [27], 5 MW in [28], 10 MW in [29], and 15 MW in [30].

The fundamental amplitude of conductor density is found by the model to maximise the number of conductors within the slot. The maximum number of windings in a slot is

$$N_{\text{wind}} = \left\lfloor \frac{A_{\text{sl}}\alpha_{\text{pm}}}{A_{\text{wind}}} \right\rfloor, \quad (\text{II.3.6})$$

where  $\lfloor \cdot \rfloor$  returns the floor of the argument. The desired fundamental conductor density is given as

$$N_{\text{cd}}^{\dagger} = \frac{N_{\text{wind}}}{2\pi N_{\text{st}}}; \quad (\text{II.3.7})$$

which is an overestimate, so the model removes one conductor from each phase (to ensure the phases stay balanced) until  $C_{\text{tot}}(j) \leq N_{\text{wind}}$  for all  $j$ . An argument could be made for allowing the GA to reduce the number of turns – lessening the material cost of the PMSG – but this is better achieved through reducing the depth of the stator teeth – an action the GA can already take – as doing so also reduces the amount of steel used.

There is a similar argument in allowing the GA to alter the width of the tooth base, increasing it could be used as a method for reducing the number of turns – but again

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<sup>ii</sup>Rotor eccentricity has an effect on the performance of a PMSG but a higher fidelity model than the one used in this work would be required to account for it.

reducing the depth of the stator teeth is more effective. The width is therefore set to be the minimum value that does not cause over saturation of flux: the minimum width of each tooth is

$$w_{\text{tb},\min}(j) = \frac{r_{\text{ri}}}{\mathcal{B}_{\text{steel}}} = \int_{\phi_{\text{st}}(j)-\pi/N_{\text{st}}}^{\phi_{\text{st}}(j)+\pi/N_{\text{st}}} \mathcal{B}(r_{\text{ri}}, \phi) \, d\phi. \quad (\text{II.3.8})$$

Each tooth needs to be assessed individually because there is a possibility of different minima due to the way the windings are distributed in the machine, although all the values are very similar. EQUATION (II.3.8) is a rearrangement of the equation of flux density in the tooth base – EQUATION (II.2.102). The chosen value of the width of the tooth base is then

$$w_{\text{tb}} = \max[w_{\text{tb},\min}(j)]. \quad (\text{II.3.9})$$

The depth of the tooth tips is set to be equal to the width of the tooth base:

$$d_{\text{tt}} = w_{\text{tb}} \quad (\text{II.3.10})$$

The depth of the stator backiron and rotor iron are set to be equal, and set to be equal to the combined width of all the tooth bases over the span of a pole, because they need to be able to handle all of the flux that builds up over a pole. Hence,

$$d_{\text{sb}} = d_{\text{ri}} = \frac{N_{\text{st}}}{2N_{\text{pp}}} w_{\text{tb}}. \quad (\text{II.3.11})$$

Finally, the radius of the pinion is set to be equal to the air gap radius:

$$r_{\text{p}} = r_{\text{ag}}. \quad (\text{II.3.12})$$

Testing with a fully independent pinion radius showed that, the GA would always drive  $r_{\text{p}}$  to its minimum value; this is to be expected as a smaller pinion leads to faster speeds where the PMSG is more efficient. The pinion radius is therefore tied to another parameter.<sup>iii</sup>

## II.3.2 INPUTS TO THE MODEL

To apply control co-design (CCD) to a PTO for a specific use case, there needs to be inputs to the model; in this model they are: the control strategy; the floater design; the sea conditions; and the material properties for the different components of the PMSG. The material properties are given in TABLE II.3.2. It is the focus of § III to investigate differences in PMSG design when varying all of the inputs, except the material properties.

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<sup>iii</sup>In reality, a limit on the speed – and therefore size – of the pinion would be dependent on structural factors, such as the wear on the pinion’s teeth, but this is not modelled in the model as it exists.

TABLE II.3.2: Material properties used in the optimiser

| MATERIAL             | PROPERTY              | VALUE | UNIT              | REF. |
|----------------------|-----------------------|-------|-------------------|------|
| COPPER<br>WINDINGS   | Density               | 8930  | kg/m <sup>3</sup> | [31] |
|                      | Resistivity           | 16.8  | nΩm               | [31] |
|                      | Cost                  | 5.65  | £/kg              | [32] |
| STEEL                | Density               | 7650  | kg/m <sup>3</sup> | [33] |
|                      | Maximum flux density  | 1.8   | T                 | [33] |
|                      | Cost                  | 1.21  | £/kg              | [32] |
| PERMANENT<br>MAGNETS | Density               | 7600  | kg/m <sup>3</sup> | [34] |
|                      | Relative permeability | 1.05  | -                 | [34] |
|                      | Residual flux density | 1.4   | T                 | [34] |
|                      | Cost                  | 51.63 | £/kg              | [32] |

## II.3.2.1 FLOATER DESIGN

The floater designs in this thesis come from the IMPACT toolbox [35]: taking the floaters from WEC 1, and WEC 2, and renaming them to TYPE-N and TYPE-B respectively due to their narrow- and broad-banded designs. The effect of changing the floater is the topic of § III.2; so in avoidance of repetition – and for the ease of comparison in later sections – the reader is directed to § III.2 to find exact details of the reference floater designs.

## II.3.2.2 SEA CLIMATE

When designing a WEC, ideally every sea state it could experience would be tested; this however requires large amounts of computational effort. Instead, by using a  $k$ -means clustering algorithm, the total number of sea states is reduced to a manageable number without altering the statistical properties of the sea climate. This method has previously been used in the CCD of WECs literature (*e.g.* in [36]). The  $k$ -means clustering algorithm works to minimise the intercluster variance [37]:

$$\arg \min_{\underline{\mu}_j} \sum_{j=1}^k \sum_{\underline{x}_n \in K_j} \|\underline{x}_n - \underline{\mu}_j\|^2, \quad (\text{II.3.13})$$

where  $\underline{\mu}_j$  is the mean of the  $j^{\text{th}}$  cluster;  $\underline{x}_n$  is the  $n^{\text{th}}$  data point; and  $K_j$  represents the data points associated with the  $j^{\text{th}}$  cluster. The steps of the algorithm are shown in FIGURE II.3.2, and an example of the algorithm in action can be found in [38]. The

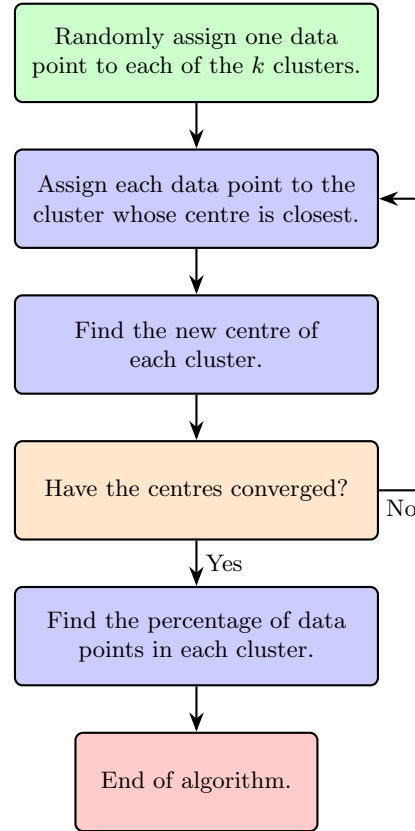


FIGURE II.3.2: Flow chart of the operation of a  $k$ -means clustering algorithm.

effect on PTO design by varying the number of sea states is investigated across § III – in conjunction with other changes. The effect of moving locations is the focus of § III.3.

### II.3.3 COST FUNCTION

The GA works by optimising to a cost function. Working to maximise power alone however causes the PMSG to balloon in size – since larger generators are more efficient. Therefore, there also needs to be a factor to provide downwards pressure on the PMSG’s size. More established technologies use the levelised cost of energy (LCoE) to account for the cost to produce a unit of power (see § I.2.5.3 for a full description); but its use wave energy introduces a lot of uncertainty due the number of unknowns.<sup>iv</sup>

There are risks in using estimates and assumptions in optimisation work, in that the model ends up optimising to the assumption – and if that assumption turns out to be false, then the outputs are useless. Therefore, for this work, a simple measure was chosen to avoid reliance on estimates as much as possible:

$$C_{\text{WEC}} = C_{\text{elec}} + C_{\text{gen}}. \quad (\text{II.3.14})$$

<sup>iv</sup>There are ways to estimate these parameters, as done by Peña-Sanchez *et al.* [2]; see § II.1.1 for a summary.

$C_{\text{elec}}$  is the estimated amount of money generated over the lifetime of the WEC:

$$C_{\text{elec}} = C_e T_{\text{life}} A_e P_{\text{avg}}, \quad (\text{II.3.15})$$

where  $C_e = 90$  GBP/MWhr is the cost of energy [39];  $T_{\text{life}} = 30$  yr is the lifetime of the WEC; and  $A_e = 0.9$  is the energy based availability of the WEC [40].  $P_{\text{avg}}$  is the average electrical power output from the WEC, derived in § II.3.4. Note that due to the sign conventions used, money earned is negative.

$$C_{\text{gen}} = C_{\text{steel}} M_{\text{steel}} + C_{\text{pm}} M_{\text{pm}} + C_{\text{winding}} M_{\text{winding}} \quad (\text{II.3.16})$$

is the material cost of the generator.  $C_{\text{steel}}$ ,  $C_{\text{pm}}$ , and  $C_{\text{winding}}$  are the costs per kg for the steel, permanent magnet, and windings respectively; whose values are given in TABLE II.3.2.  $M_{\text{steel}}$ ,  $M_{\text{pm}}$ , and  $M_{\text{winding}}$  are the total mass in the generator of each of the three materials. The inclusion of the material cost of the PMSG is required, because larger generators tend to be more efficient, so without any downwards pressure on PMSG size, the model returns incredibly large PMSGs for sites with minimal power.

#### II.3.4 POWER SIMULATION AND CONTROL OPTIMISATION

Power production is critical for the cost function; which is itself heavily dependent on the control strategy. This section describes both how the power simulation works, and how a unique control strategy is optimised for every sea state. Due to the generator being nonlinear, there is no analytical method to find the optimal control design – as is possible for optimising for mechanical power output (see § I.2.4). Therefore a numerical method is used.

##### II.3.4.1 POWER SIMULATION

Previous work by Stock [41] and Coe *et al.* [42] have each found that – for the purposes of control design – the excitation force spectrum acting on a point absorber is sufficiently narrow banded that it can be approximated as a regular wave. In the regular wave approximation, the velocity of the WEC is described as

$$A_v e^{i(\omega t + \phi_v)} = \left( A_{\text{exc}} e^{i\omega t} + A_{\text{PTO}} e^{i(\omega t + \phi_{\text{PTO}})} \right) A_Y e^{i\phi_Y}, \quad (\text{II.3.17})$$

where  $\omega$  is the wave frequency;  $A_v$  and  $\phi_v$  are the amplitude and phase of the velocity;  $A_{\text{ex}}$  is the amplitude of the excitation force<sup>v</sup>;  $A_{\text{PTO}}$  and  $\phi_{\text{PTO}}$  are the amplitude and phase of the PTO control force; and  $A_Y$  and  $\phi_Y$  are the amplitude and phase of the floater's

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<sup>v</sup>The excitation force also introduces a phase shift from the true position of the wave,  $\phi_{\text{exc}}$ , but for this work absolute phase is irrelevant – only the relative phase between the excitation and PTO forces – so  $\phi_{\text{exc}}$  is ignored.

mechanical admittance (inverse of impedance) from force to velocity. The excitation force variables are from converting the sinusoidal definition of wave excitation force in EQUATION (I.2.17) to complex exponential form. The PTO control variables are optimised for each WEC and sea state – as described in §II.3.4.3. The admittance of the floater is found from the impedance from position to force:

$$Z_{\text{float}} = -\omega^2(M + A(\omega) + G) + i\omega R(\omega) + C, \quad (\text{II.3.18})$$

where  $M$ ,  $A$ ,  $R$ , and  $C$  are the mass, added mass, damping, and hydrostatic restoring coefficients, as defined in §I.2.3.1;

$$G = \frac{J}{r_p^2}, \quad (\text{II.3.19})$$

is the effective mass from the generator's inertia. Using tools from the IMPACT toolbox [35] a transfer function is fitted for the admittance from force to velocity; which when evaluated at the wave frequency gives  $A_Y$  and  $\phi_Y$ .

The average electrical power over a wave period can then be found from

$$P_{\text{avg}} = \frac{\omega}{2\pi} \sum_{t=0}^{2\pi/\omega} P_{\text{elec}} \left( \text{real} \left[ A_{\text{PTO}} e^{i(\omega t + \phi_{\text{PTO}})} \right], \text{real} \left[ A_v e^{i(\omega t + \phi_v)} \right] \right) \Delta t; \quad (\text{II.3.20})$$

using the definition of  $P_{\text{elec}}$  in EQUATION (II.2.206),

$$\Delta t = \frac{1}{N_t} \frac{2\pi}{\omega}, \quad (\text{II.3.21})$$

and  $N_t$  is the number of time points.

#### II.3.4.2 BOUNDS OF THE POWER MAP

The power map is bounded by the maximum allowable force and linear speed on the PTO: these are set to avoid overheating in the generator by limiting the root mean squared (RMS) current density to 4 A/mm<sup>2</sup> – the value chosen is on the lower end of current densities but ensures no complex cooling mechanisms would be required [43]. The RMS current is then

$$i_{\text{RMS}} = 4 \times 10^6 \cdot \frac{A_{\text{sl}} \alpha_{\text{pf}}}{\max[C_{\text{tot}}(j)]}; \quad (\text{II.3.22})$$

and the peak current is

$$i_{\text{peak}} = i_{\text{RMS}} \sqrt{2}. \quad (\text{II.3.23})$$

The peak electromagnetic force the PMSG can provide is given by

$$F_{\text{PTO,max}} = \frac{\kappa_T}{r_p} i_{\text{peak}} \sqrt{\frac{3}{2}}, \quad (\text{II.3.24})$$

using the equation of the electromagnetic torque, EQUATION (II.2.187).

The peak allowable speed is found from Faraday's law of induction: where the electromotive force is given by

$$\varepsilon = N \frac{d\Phi}{dt}, \quad (\text{II.3.25})$$

assuming the flux is of the form

$$\Phi = H_\Phi \sin(\omega_{\text{PTO}} t), \quad (\text{II.3.26})$$

where  $\omega_{\text{PTO}}$  is the angular speed of the PTO; and  $H_\Phi$  is the amplitude of the flux. The peak current is found when  $\sin(\omega_{\text{PTO}} t) = 1$ :

$$i_{\text{peak}} = \frac{\varepsilon}{R} = \frac{NH_\Phi \omega_{\text{PTO}}}{R}; \quad (\text{II.3.27})$$

which gives a speed limit of

$$v_{\text{PTO}} = \left| \frac{R i_{\text{peak}}}{r_p N H_\Phi} \right|. \quad (\text{II.3.28})$$

In practice however, WECs operate at high forces and low speeds, this means that to build the generator to handle the large forces, the speed limit is often well beyond the operating region of the WEC. Hence the speed points in the map are spread out over a region where the WEC does not operate, reducing the fidelity of the map in the region of interest. The speed limit is therefore capped at a speed of 5 m/s:

$$v_{\text{PTO,max}} = \begin{cases} v_{\text{PTO}} & \text{for } v_{\text{PTO}} < 5, \\ 5 & \text{otherwise;} \end{cases} \quad (\text{II.3.29})$$

the value of 5 m/s is chosen from testing to be well beyond the operating regions of the WEC – which peak at less than 2 m/s in the harshest of seas – so it does not affect the results whilst improving the fidelity of the map in the operating region.

#### II.3.4.3 CONTROL OPTIMISATION

As part of the power simulation, the user picks between two control strategies and the algorithm finds the optimal controller of that type: the options are linear damping and impedance matching. The remainder of this section describes how both types of controller are designed; and in § III.1 the differences in PTO design when using different control strategies is investigated.

**LINEAR DAMPING:** In the case of linear damping  $\phi_{\text{PTO}} = \phi_v$ , making this substitution into EQUATION (II.3.17) and solving for  $\phi_{\text{PTO}}$  gives

$$e^{i\phi_{\text{PTO}}} = \frac{A_{\text{ex}} A_y e^{i\phi_y}}{A_v - A_{\text{PTO}} A_y e^{i\phi_y}}, \quad (\text{II.3.30})$$

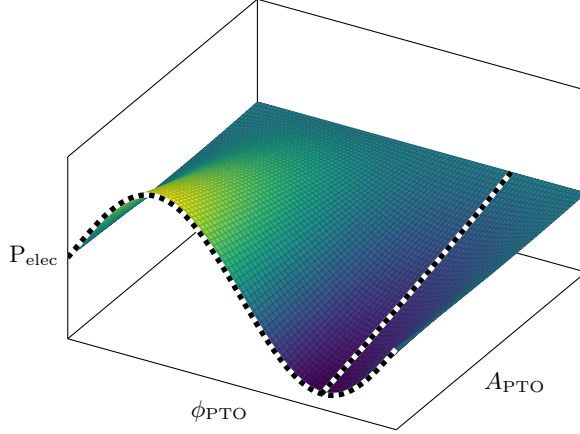


FIGURE II.3.3: Generic example of the tuning of the impedance matching control strategy. The optimal tuning is found by examining the dashed lines. (Figure from [44])

which leaves the only unknown as  $A_v$ . By taking the magnitude of the above equation and solving for  $A_v$ , one finds

$$A_v = A_{\text{PTO}} A_y \cos(\phi_y) \pm A_y \sqrt{A_{\text{ex}}^2 - A_{\text{PTO}}^2 \sin^2(\phi_y)}. \quad (\text{II.3.31})$$

One solution of the equation causes the PTO force and WEC velocity to be exactly in phase, the other causes the two to be in anti-phase.

The optimal value of  $A_{\text{PTO}}$  is then found by stepping through values up to the maximum allowable value (see § II.3.4.2) to find which returns the best electrical power.

**IMPEDANCE MATCHING:** The optimal impedance matching controller – for electrical power output – is more complicated to calculate than the linear damping as there are two free variables:  $(A_{\text{PTO}}, \phi_{\text{PTO}})$ . In FIGURE II.3.3, a generic map of this design space is shown; to calculate this entire map for each sea state, for each PTO, for each generation would be incredibly computationally expensive (at a resolution high enough to be accurate). Therefore, a more refined method is required.

The tuning is completed as part of a two-step process: firstly, the optimal phase is found by stepping through  $\phi_{\text{PTO}} \in [0, 2\pi)$  at the maximum allowable value of  $A_{\text{PTO}}$ , limited by the overheating limit (see § II.3.4.2). Then the optimal force is found by stepping  $A_{\text{PTO}}$  back to find the value that produces the maximum electrical power. This method is shown by the black and white dashed lines in FIGURE II.3.3.

In the case of a controller breaking the speed limit (see § II.3.4.2) during the phase test, a sub-optimal control strategy is used that produces less power but keeps the WEC within its limits. There is no analytical way to know what combination of sub-optimal force or phase will avoid the speed limit, so the full power map is tested in this case.

## An Example Output

To end the chapter, an example output of the model is presented. This section shows what an output power take-off (PTO) from the model looks like and how it performs, but does not investigate how the design varies with varying input conditions (control type, floater, site conditions); that investigation is the focus of § III.

### II.4.1 COST CONVERGENCE

For this example, a PTO is developed for a wave energy converter (WEC) using linear damping control; a TYPE-N floater (a narrow-banded surface-piercing cylinder – see the beginning of § III.2 for further details of its design); and designed to be located at the Tyne-Tees WaveNet site [45] (see the beginning of § III.3 for details of the location). To show the optimiser consistently arrives at the same solution it is run three times. FIGURE II.4.1 shows the best cost of each run’s population as it iterates through the design space. Each run makes different progress throughout its operation but all arrive at a very similar cost – with a maximum difference of less than 0.15% between any two runs. Each run arrives at the solution in a different number of iterations, because of the stochastic nature of a genetic algorithm: *i.e.* a run may be *luckier* in its random initialisation and be in a better position to find the optimal solution quicker. The quickest was RUN 2 which reached its solution in 1967 iterations, whereas RUN 3 took 3264 iterations to do the same.

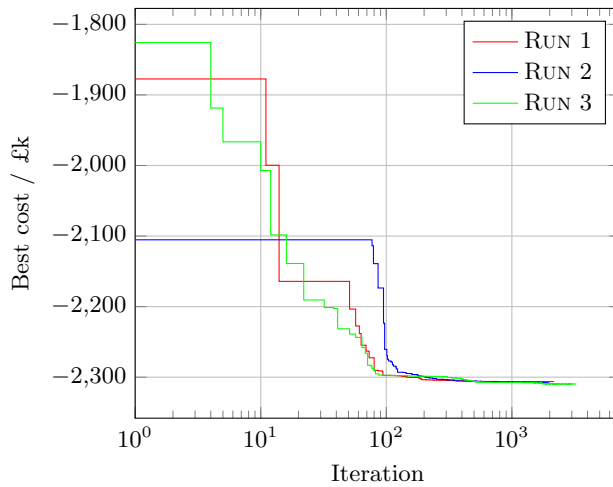


FIGURE II.4.1: Cost of the best performing solution throughout the operation of the optimiser.

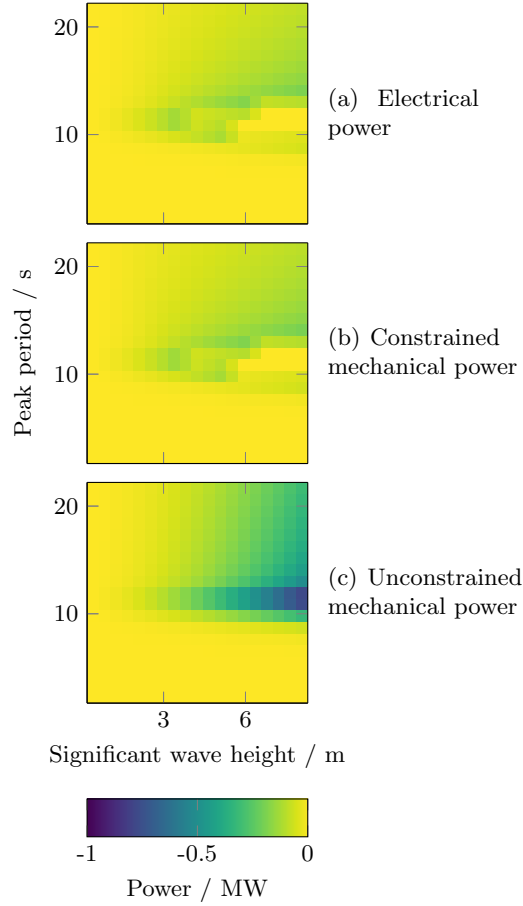


FIGURE II.4.2: Power performance of an example power take-off.

## II.4.2 ELECTRICAL POWER PERFORMANCE

FIGURE II.4.3 shows a number of power maps: for these, the power simulation is run using the control optimiser to find the best performing controller for the desired conditions. The (a) subplot uses the power map directly from the optimisation. The (b) subplot optimises for mechanical power, but is only allowed to operate within the limits of the PTO. And, the (c) subplot ignores all limits and just works to maximise mechanical power capture.

The constrained scenarios (the (a) and (b) subplots), show that limits of the PTO constrain the performance of the WEC, as the power drops with increasing significant wave height at a constant period of around 11 s<sup>i</sup>. In the unconstrained scenario (c), the power continually increases with increasing wave height. At the highest wave heights, the power absorbed in the constrained cases (a) drops to zero, due the control optimiser

<sup>i</sup>As is described in § III.2, the TYPE-N floater's resonant point is at a period of roughly 10.9 s, so the response at this frequency is very large which causes the limits to be hit at around this period. And as is seen in § III.2 there are no test sea points at this period greater than 3 m, so the PTO has not been sized by the optimiser to cope in those seas.

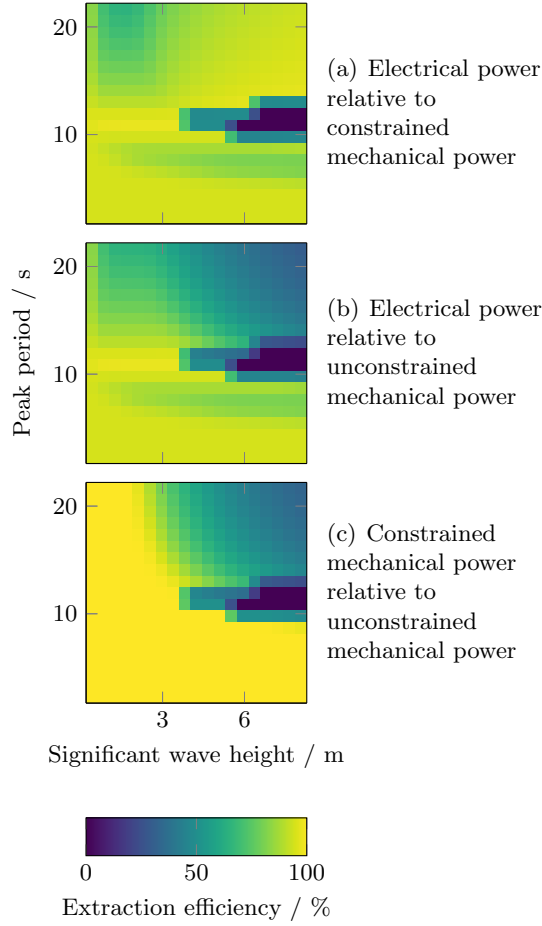


FIGURE II.4.3: Power extraction efficiency of an example power take-off.

not being able to find a safe operating regime, and the WEC needing to be switched off.

For ease of comparison between the scenarios, FIGURE II.4.3 shows the relative power extraction efficiencies of each pair of scenarios. Here, another place where the operating limits are affecting performing is seen: in the (c) subplot, the power is significantly less in the top right corner of the plot – where the seas have the most energy. In this region, the operating limits cause the control optimiser to use a different control regime which harnesses less power but keeps the WEC running within the operating limits.

Finally, in the (a) plot of FIGURE II.4.3, the electrical efficiency of the PTO is shown. The PTO is most efficient in the region around 11 s period, and less than 3 m significant wave height. The region in which the WEC is moving the fastest is the region in which the PTO is most efficient. Moving away from this point, efficiency drops off as the waves move away from the resonant point<sup>ii</sup>.

<sup>ii</sup>The WEC would also be moving very fast and with large forces – a good operating point for permanent magnet synchronous generators – in the top right of the plot, but, as is seen in § III.2, there are no test seas in this region so the PTO has not been sized to cope in those seas.

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## Bibliography

- [1] Giorgio Bacelli and John V. Ringwood. “A geometric tool for the analysis of position and force constraints in wave energy converters”. In: *Ocean Engineering* 65.1 (2013), pp. 10–18. DOI: [10.1016/j.oceaneng.2013.03.011](https://doi.org/10.1016/j.oceaneng.2013.03.011) (cited on pages [63](#), [64](#)).
- [2] Yeraí Peña-Sánchez, Demián García-Violini, and John V. Ringwood. “Control co-design of power take-off parameters for wave energy systems”. In: *IFAC-PapersOnLine* 55.27 (2022), pp. 311–316. DOI: [10.1016/j.ifacol.2022.10.531](https://doi.org/10.1016/j.ifacol.2022.10.531) (cited on pages [63](#), [108](#)).
- [3] Jeff Grasberger. “Co-design Investigation and Optimization of an Oscillating-Surge Wave Energy Converter”. MA thesis. Virginia Polytechnic Institute and State University, 2022 (cited on pages [64](#), [66](#), [67](#), [99](#)).
- [4] Carlos A. Michelén Ströfer et al. “Control co-design and sensitivity analysis of the LUPA’s PTO using WecOptTool”. In: *Proceedings of 15th European Wave and Tidal Energy Conference*. 2023. DOI: [10.36688/ewtec-2023-288](https://doi.org/10.36688/ewtec-2023-288) (cited on pages [64](#), [66](#), [67](#)).
- [5] Carlos A. Michelén Ströfer et al. “Control Co-Design of Power Take-Off Systems for Wave Energy Converters Using WecOptTool”. In: *IEEE Transactions on Sustainable Energy* 14.4 (2023). DOI: [10.1109/TSTE.2023.3272868](https://doi.org/10.1109/TSTE.2023.3272868) (cited on pages [64](#), [66](#), [67](#)).
- [6] Jeff Grasberger et al. “Control co-design and optimization of oscillating-surge wave energy converter”. In: *Renewable Energy* 225.120234 (Feb. 2024). DOI: [10.1016/j.renene.2024.120234](https://doi.org/10.1016/j.renene.2024.120234) (cited on pages [64](#), [66](#), [67](#), [99](#)).
- [7] Reza Yazdanpanah et al. “Resign optimization of inner and outer-rotor PMSGs for X-ROTOR wind turbines”. In: *IET Renewable Power Generation* 18.14 (2023). DOI: [10.1049/rpg2.13111](https://doi.org/10.1049/rpg2.13111) (cited on page [65](#)).
- [8] Henda Zorgani Agrebi et al. “Integrated Optimal Design of Permanent Magnet Synchronous Generator for Smart Wind Turbine Using Genetic Algorithm”. In: *Energies* 14.4642 (2021). DOI: [10.3390/en14154642](https://doi.org/10.3390/en14154642) (cited on pages [65](#), [75](#)).

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- [9] Warda Gul, Qiang Gao, and Wanchak Lenwari. “Optimal Design of a 5-MW Double-Stator Single-Rotor PMSG for Offshore Direct Drive Wind Turbines”. In: *IEEE Transactions on Industry Applications* 56.1 (2020). DOI: [10.1109/TIA.2019.2949545](#) (cited on page 65).
  - [10] A. Abdelli, B. Sareni, and X. Roboam. “Optimization of a small passive wind turbine generator with multiobjective genetic algorithms”. In: *International Journal of Applied Electromagnetics and Mechanics* 26.3-4 (2007). DOI: [10.3233/JAE-2007-906](#) (cited on page 65).
  - [11] A. Bensalah, G. Barakat, and Y. Amara. “Comparative Design Optimization of 15 MW Rare- Earth Permanent Magnet Synchronous Generators for Offshore Semi-Direct Wind Turbines”. In: *2022 International Conference on Electrical Machines (ICEM)*. 2022. DOI: [10.1109/ICEM51905.2022.9910675](#) (cited on page 65).
  - [12] Brandon Cassimere. “Modeling and Evolutionary Design of Permanent Magnet Synchronous Machines”. PhD thesis. Purdue University, May 2008 (cited on pages [65–69](#), [72](#), [78](#), [79](#), [83](#), [87–89](#), [91](#), [92](#), [94–97](#)).
  - [13] Paul Krause et al. *Analysis of Electric Machinery and Drive Systems*. 3rd ed. Hoboken, New Jersey, U.S.A.: John Wiley & Sons, Inc., 2013. ISBN: 978-1-118-02429-4. DOI: [10.1002/9781118524336](#) (cited on page 67).
  - [14] Paul Krause et al. “Analysis of Electric Machinery and Drive Systems”. In: 3rd ed. Hoboken, New Jersey, U.S.A.: John Wiley & Sons, Inc., 2013. Chap. 15. ISBN: 978-1-118-02429-4. DOI: [10.1002/9781118524336.ch15](#) (cited on pages [67](#), [77](#), [87](#)).
  - [15] W. Bolton. “Fourier Series”. In: 1st ed. Harlow, Essex, England: Longman Scientific & Technical, 1995. Chap. 2. ISBN: 978-0-582-23934-0 (cited on page [74](#)).
  - [16] Wilfred Kaplan. “Advanced Calculus”. In: ed. by Eric Reissner. 1st ed. Reading, Massachusetts, U.S.A.: Addison-Wesley Publishing Company, Inc., 1952. Chap. 4. ISBN: 978-0-201-03611-4 (cited on page [74](#)).
  - [17] Paul Krause et al. “Analysis of Electric Machinery and Drive Systems”. In: 3rd ed. Hoboken, New Jersey, U.S.A.: John Wiley & Sons, Inc., 2013. Chap. 2. ISBN: 978-1-118-02429-4. DOI: [10.1002/9781118524336.ch2](#) (cited on pages [75](#), [84](#)).
  - [18] Edward Hughes et al. “Hughes Electrical and Electronic Technology”. In: 12th ed. Harlow, Essex, England: Pearson Education Limited, 2016. Chap. 7. ISBN: 978-1-292-09304-8 (cited on page [87](#)).

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- [19] Paul Krause et al. “Analysis of Electric Machinery and Drive Systems”. In: 3rd ed. Hoboken, New Jersey, U.S.A.: John Wiley & Sons, Inc., 2013. ISBN: 978-1-118-02429-4. DOI: [10.1002/9781118524336.app3](https://doi.org/10.1002/9781118524336.app3) (cited on page 91).
- [20] Steven E. Schwarz. “Electromagnetics for Engineers”. In: Oxford Series in Electrical and Computer Engineering. 198 Madison Avenue, New York, New York, 10016: Oxford University Press, 1996. Chap. 6. ISBN: 978-0-19-511597-0 (cited on page 93).
- [21] D. Lin, P. Zhou, and Z. J. Cendes. “In-Depth Study of the Torque Constant for Permanent-Magnet Machines”. In: *IEEE Transactions on Magnetics* 45.12 (2009). DOI: [10.1109/TMAG.2009.2026043](https://doi.org/10.1109/TMAG.2009.2026043) (cited on page 97).
- [22] *PowerPod Datasheet*. Tech. rep. Trident Energy Ltd. (cited on page 101).
- [23] Jonas Sjolte et al. “All-electric wave energy Power Take Off system with improved power quality at the grid connection point”. In: *2012 IEEE PES Transmission and Distribution Conference and Exposition*. 2012. DOI: [10.1109/TDC.2012.6281645](https://doi.org/10.1109/TDC.2012.6281645) (cited on page 101).
- [24] Matthew Onslow and Adam Stock. “Genetic Algorithms for the Control Co-Design of Permanent Magnet Synchronous Generators for Wave Energy Converters”. In: *Applied Ocean Research* (Under Review) (cited on page 101).
- [25] Zijian Liu, Jin Hunag, and Bingnan Li. “Diagnosing and distinguishing rotor eccentricity from partial demagnetisation of interior PMSM based on fluctuation of high-frequency d-axis inductance and rotor flux”. In: *IET Electric Power Applications* 11.7 (2017), pp. 1151–1355. DOI: [10.1049/iet-epa.2016.0814](https://doi.org/10.1049/iet-epa.2016.0814) (cited on page 105).
- [26] Anders Grauers. “Design of Direct-driven Permanent-magnet Generators for Wind Turbines”. PhD thesis. Chalmers University of Technology, 1996 (cited on page 105).
- [27] Pablo Jaen-Sola, Alasdair S. McDonald, and Erkan Oterkus. “Dynamic structural design of offshore direct-drive wind turbine electrical generators”. In: *Ocean Engineering* 161 (2018), pp. 1–19. DOI: [10.1016/j.oceaneng.2018.04.074](https://doi.org/10.1016/j.oceaneng.2018.04.074) (cited on page 105).
- [28] Latha Sethuraman et al. “A 5MW direct-drive generator for floating spar-buoy wind turbine: Drive-train dynamics”. In: *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* 231.4 (2016). DOI: [10.1177/0954406215623306](https://doi.org/10.1177/0954406215623306) (cited on page 105).

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- [29] H. Polinder et al. “10 MW Wind Turbine Direct-Drive Generator Design with Pitch or Active Speed Stall Control”. In: *2007 IEEE International Electric Machines & Drives Conference*. 2007. DOI: [10.1109/IEMDC.2007.383632](https://doi.org/10.1109/IEMDC.2007.383632) (cited on page 105).
  - [30] Evan Gaertner et al. *Definition of the IEA Wind 15-Megawatt Offshore Reference Wind Turbine*. Tech. rep. National Renewable Energy Laboratory, 2020 (cited on page 105).
  - [31] R. A. Matula. “Electrical resistivity of copper, gold, palladium, and silver”. In: *Journal of Physical and Chemical Reference Data* 8.4 (1979), pp. 1147–1298. DOI: [10.1063/1.555614](https://doi.org/10.1063/1.555614) (cited on page 107).
  - [32] Garrett E. Barter et al. “Beyond 15MW: A cost of energy perspective on the next generation of drivetrain technologies for offshore wind turbines”. In: *Applied Energy* 344.121272 (2023). DOI: [10.1016/j.apenergy.2023.121272](https://doi.org/10.1016/j.apenergy.2023.121272) (cited on page 107).
  - [33] *Selection of Electrical Steels for Magnetic Cores*. Tech. rep. AK Steel (cited on page 107).
  - [34] *N48SH*. Tech. rep. Arnold Magnetic Technologies Corp. (cited on page 107).
  - [35] Adam Stock, Carlos Gonzalez, and David Robb. *Integrated marine point absorber control tool final stage 2 project report*. Tech. rep. SgurrControl Ltd., 2018 (cited on pages 107, 110).
  - [36] Carlos A. Michelén Ströfer et al. “Control Co-Design of Power Take-Off Systems for Wave Energy Converters Using WecOptTool”. In: *IEEE Transactions on Sustainable Energy* 14.4 (Oct. 2023), pp. 2157–2167. DOI: [10.1109/TSTE.2023.3272868](https://doi.org/10.1109/TSTE.2023.3272868) (cited on page 107).
  - [37] Steven L. Brunton and J. Nathan Kutz. “Data-Driven Science and Engineering - Machine Learning, Dynamical Systems, and Control”. In: 2nd ed. Cambridge: Cambridge University Press, 2022. Chap. 5.3. ISBN: 978-1-009-09848-9 (cited on page 107).
  - [38] Daniel T. Larose and Chantal D. Larose. “Discovering Knowledge in Data”. In: Hoboken: John Wiley & Sons, Inc., 2005. Chap. 8. ISBN: 0-471-66657-2 (cited on page 107).
  - [39] Siya Jin and Deborah Greaves. “Wave energy in the UK: Status review and future perspectives”. In: *Renewable and Sustainable Energy Reviews* (2021). DOI: [10.1016/j.rser.2021.110932](https://doi.org/10.1016/j.rser.2021.110932) (cited on page 109).

- [40] Khalid Abdulla et al. “Statistical Availability Analysis of Wave Energy Converters”. In: *Proceedings of the Twenty-first (2011) International Offshore and Polar Engineering Conference*. 2011 (cited on page 109).
- [41] Adam Stock. “Accounting for power take-off efficiency in optimal velocity tracking control of wave energy converters”. In: *Ocean Engineering* 312 (2024). DOI: [10.1016/j.oceaneng.2024.119191](https://doi.org/10.1016/j.oceaneng.2024.119191) (cited on page 109).
- [42] Ryan G. Coe, Giorgio Bacelli, and Dominic Forbush. “A practical approach to wave energy modeling and control”. In: *Renewable and Sustainable Energy Reviews* 142 (2021). DOI: [10.1016/j.rser.2021.110791](https://doi.org/10.1016/j.rser.2021.110791) (cited on page 109).
- [43] Mohammadali Abbasian and Arash Hassanpour Isfahani. “Optimal Design of a Direct-Drive Permanent Magnet Synchronous Generator for Small-Scale Wind Energy Conversion Systems”. In: *Journal of Magnetism* 16.4 (2011), pp. 379–385. DOI: [10.4283/JMAG.2011.16.4.379](https://doi.org/10.4283/JMAG.2011.16.4.379) (cited on page 110).
- [44] Matthew Onslow and Adam Stock. “Genetic algorithm based co-design of wave energy converter power take-off, floater geometry, and control”. In: *Proceedings of the 16th European Wave and Tidal Energy Conference*. 2025. DOI: [10.36688/ewtec-2025-1013](https://doi.org/10.36688/ewtec-2025-1013) (cited on page 112).
- [45] Centre for Environment Fisheries and Aquaculture Science. *Tyne/Tees WaveNet Site*. Last Accessed: 19/7/26. URL: <https://wavenet.cefas.co.uk/details/TYNETEESWN/INT> (cited on page 113).

## CHAPTER III

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# *Control Co-design of Power Take-off Systems for use in Wave Energy Converters*

This chapter takes the model designed in §II, and investigates how the optimal power take-off design changes when varying: the control strategy; the floater design; and the site location.

## *Varying the control strategy*

In this section, the differences in the optimal power take-off (PTO) design are investigated when varying the control strategy between a linear damping (LD) and an impedance matching (IM) approach.

Preliminary versions of these results were presented at the XI International Conference on Computational Methods in Marine Engineering and are published in [1]. In that work, PTOs were only designed according to a 10-sea-state representation; in this work that is expanded to a range of representations to investigate the PTO designs dependence on the size of the representation.

### III.1.1 DIFFERENCES IN COST FUNCTION VALUES

The cost function is dependent upon the power capture and the material costs. The cost, power capture, and material costs are referred to collectively as properties.

FIGURE III.1.1 shows the cost of the best solution when running the algorithm with either LD or IM control. The test uses a TYPE-B floater<sup>i</sup> – a broad-banded, submerged, pill-shaped floater – and site data from the Tyne-Tees WaveNet site [3] (see FIGURE III.1.2 for an incidence plot of the location, and APPENDIX A for tables of the test sea states used). Each example is run three times to ensure the algorithm converges in a range of scenarios; as can be seen the solutions are very similar in all scenarios – the largest percentage difference between any two runs is 0.53%, and 0.50% for the LD- and IM-control results respectively.<sup>ii</sup> In all situations, the IM-controlled wave energy converter (WEC) outperforms the LD-controlled WEC, in both electrical power output<sup>iii</sup> and cost function value: this is not unexpected, as it is well known that more advanced control methods improve WEC performance, for example in [4].

Both control types show the same trend: a well performing solution for the single-sea-state scenario, then a reduction in performance for the five-sea-state case before improving with increasing numbers of sea states, with diminishing improvements, as the solution tends to the *true* optimal solution. Looking at the constituent parts of

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<sup>i</sup>This floater comes from the IMPACT toolbox [2], where it is part of the ‘WEC 2’ WEC but is re-named to TYPE-B for this work for easier readability (see the beginning of § III.2, or APPENDIX B for further details of its design).

<sup>ii</sup>A breakdown of maximum percentage difference by number of sea states is found in TABLE III.2.1.

<sup>iii</sup>The sign convention used is for electrical power outputted from the WEC to be negative.

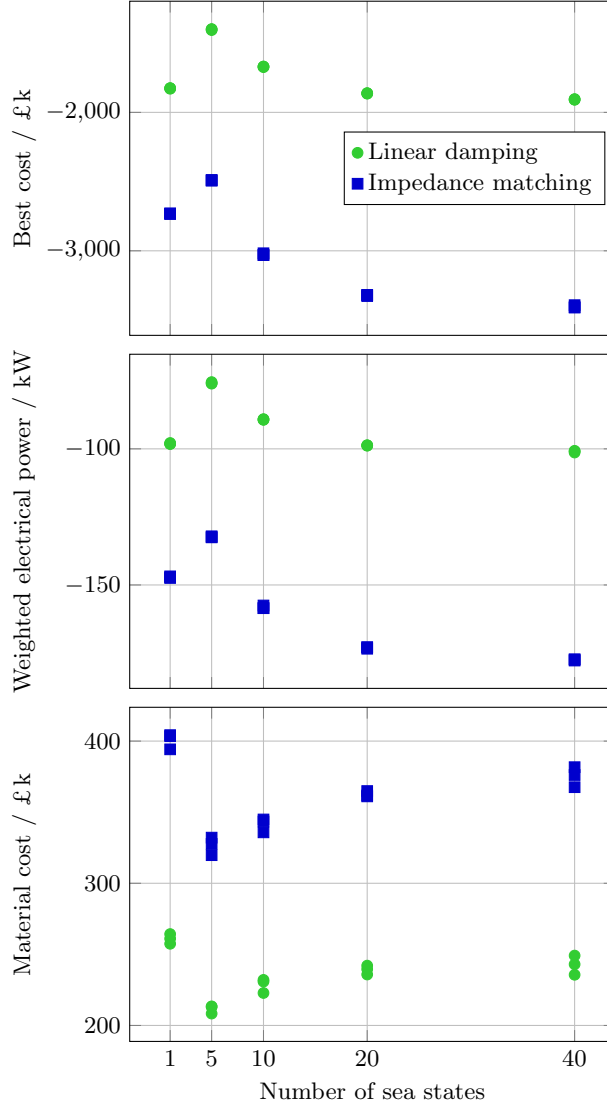


FIGURE III.1.1: Best cost solutions from the optimiser, when varying the control strategy optimised for. WECs are designed with a TYPE-B floater, at the Tyne-Tees WaveNet site.

the cost function: weighted electrical power, and material cost in FIGURE III.1.1. The weighted electrical power follows the same trend as seen in the overall cost function value, and with similar maximum percentage differences: 0.61% and 0.29% for the LD- and IM-controlled results respectively. The shape of these results is explained by looking at the sea state representations in relation to the incidence and power matrices in FIGURE III.1.2. The single-sea-state solution has coincidentally been placed at nearly the resonant frequency of the WEC<sup>iv</sup> – where it produces the most power: shown by it being almost in line with the period of the darkest blue region of the the power matrices. The introduction of more sea states in the five-state representation adds in

<sup>iv</sup>As is discussed more in § III.2, this floater is broad-banded in its frequency response so although being preferential, this is not as necessary for performance as it would be with a narrow-banded floater.

seas further away from the resonant conditions – which produce less power. Then, in the 10-sea-state sample, states are generally added in between those of the five state sample – and therefore closer to the resonant point; as well as the first introduction of a sea state with a larger wave height which provides more available energy. Finally the 20- and 40-sea-state representations introduce even more sea states with higher wave heights – therefore providing even more available energy.

In the material costs, things are slightly different. The one-sea-state generator is most expensive, then there's a large drop to the five sea state generator (the cheapest of all the representations), and finally there's a gradual increase of the costs of the generators with increasing numbers of sea states. As with the trends in power, the changes can be explained by the sea conditions and resonance with the floater. Where the sea state representation is heavily focussed about a resonant point (*e.g.* one sea state) or with a higher proportion of larger wave heights (*e.g.* 20 and 40 sea states), it is rational that a more expensive generator is required to cope with the higher forces and speeds. The variance in material costs is also larger than in the other properties: maximum percentage differences between runs are 5.7% and 3.9%, for LD- and IM-controlled respectively.

In all three of the properties – cost function value, weighted electrical power, and material cost – the differences with number of sea states are hitting diminishing returns at the 20 sea state point. For example, the change in average cost function value for the IM-optimised devices is 21% from five to 10 sea states, but the change from 20 to 40 is only 2.5%. For LD-optimised it is 19% and 2.4% respectively.

### III.1.2 DIFFERENCES IN POWER TAKE-OFF DESIGN

FIGURE III.1.3 shows the optimised genomes of all of the runs of the optimiser for this comparison: there is high variance in the gene values within any one set of results, such that no discernible trends can be made from looking at the genes alone. However, the electrical power performance of the generators are within around 0.5% of each other; and, although larger, the maximum percentage difference in force demand between any two runs is 18% (the average percentage difference is 7%), showing they are still fairly similar. Hence, there must be multiple designs of generators that return a very similar cost function value; suggesting that the cost function is leading to the optimisation of some *meta variable* that is not explicitly shown here in the genes. For example it could be the overall volume of the generator,

$$V_{\text{gen}} = l_g r_{\text{sb}}^2 \pi; \quad (\text{III.1.1})$$

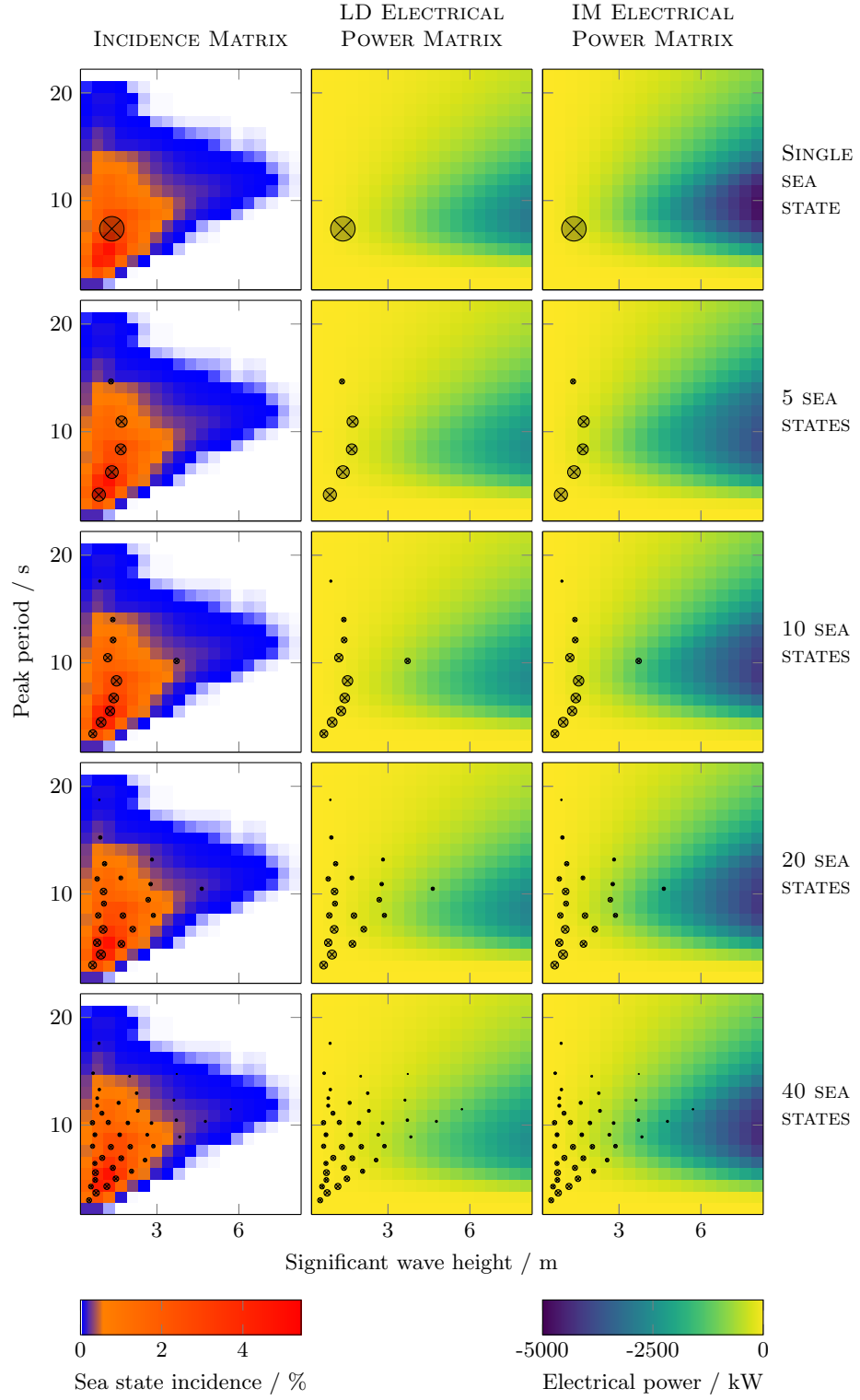


FIGURE III.1.2: Incidence, and power matrices for WECs optimised for varying control strategies. WECs are designed with a TYPE-B floater, at the Tyne-Tees WaveNet site: test points are shown by the circles, with relative areas representing the weighting of each sea state.

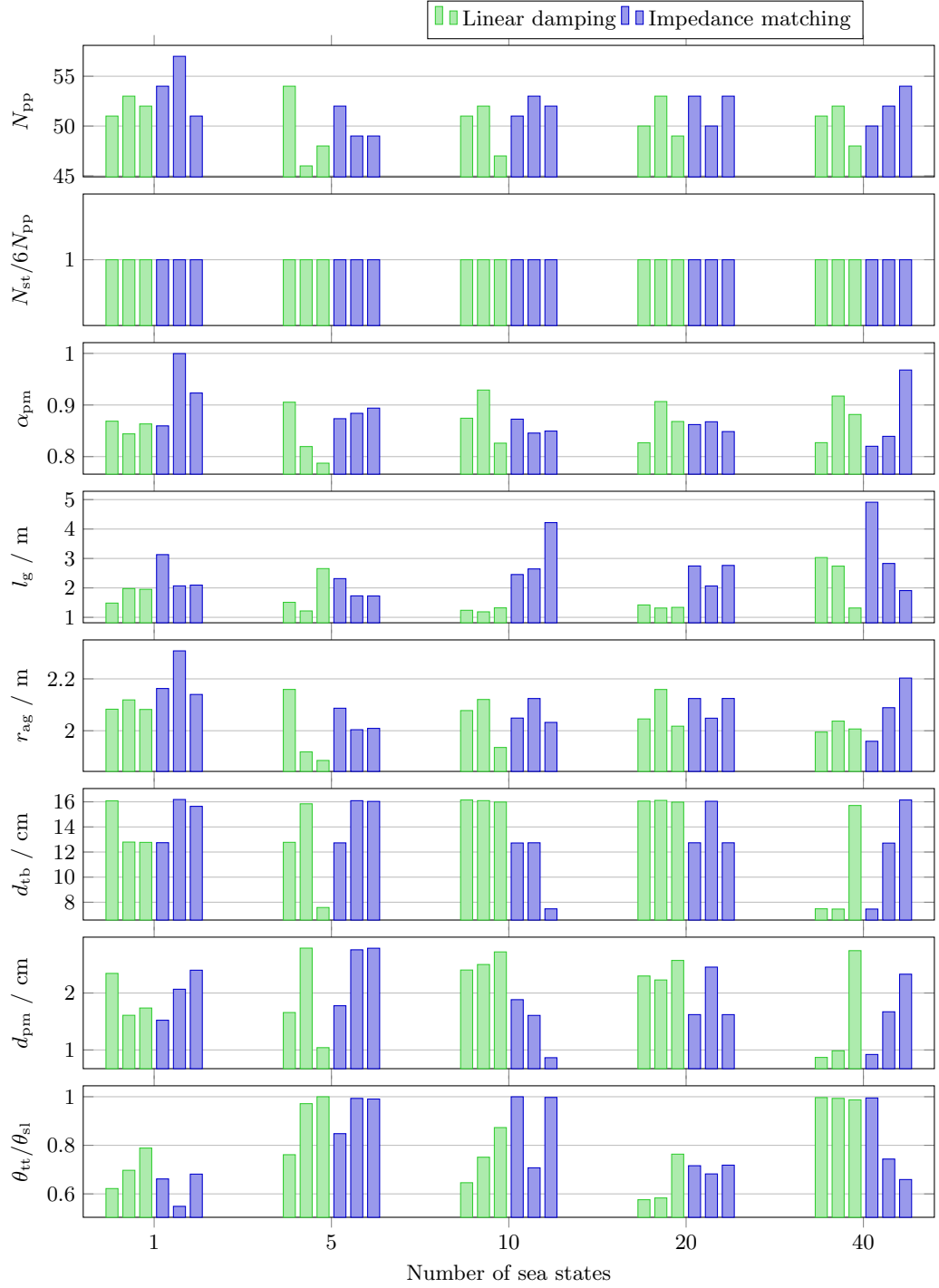


FIGURE III.1.3: Optimised genes for multiple runs of the optimiser, when varying the control strategy used. Note that the first two genes are fixed to integer values. WECs are designed with a TYPE-B floater, at the Tyne-Tees WaveNet site.

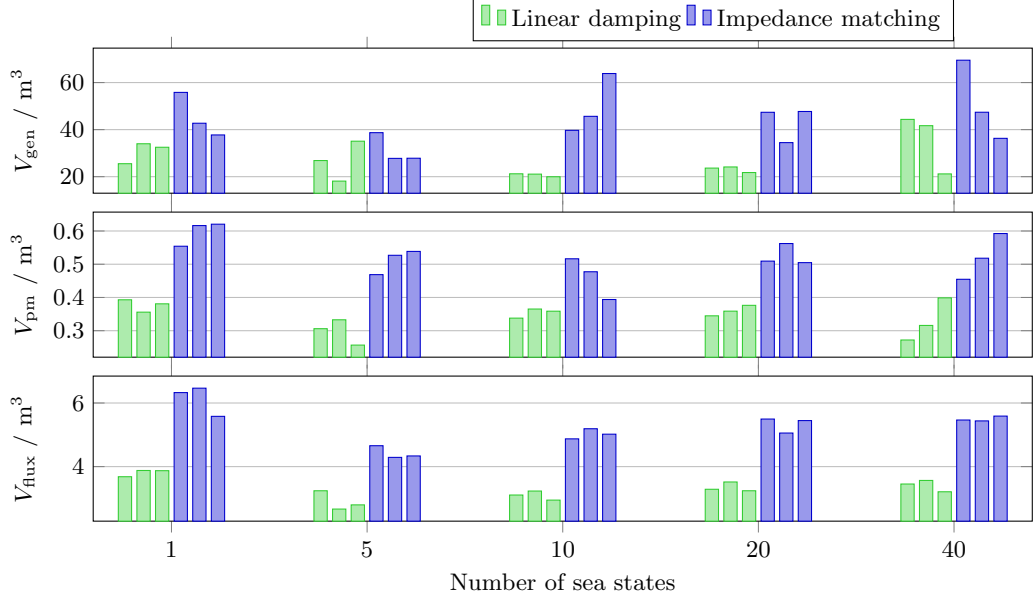


FIGURE III.1.4: Meta variables for multiple runs of the optimiser, when varying the control strategy used. The ordering of PTOs corresponds to the order of the genes in FIGURE III.1.3. WECS are designed with a TYPE-B floater, at the Tyne-Tees WaveNet site.

or volume of permanent magnet material in the generator,

$$V_{\text{pm}} = \alpha_{\text{pm}} l_g (r_{\text{pm}}^2 - (r_{\text{pm}} - d_{\text{pm}})^2) \pi; \quad (\text{III.1.2})$$

or the volume through which the flux flows,

$$V_{\text{flux}} = \alpha_{\text{pm}} l_g (r_{\text{st}}^2 - (r_{\text{ri}} - d_{\text{pm}})^2) \pi. \quad (\text{III.1.3})$$

The values of each of these candidate meta variables are shown in FIGURE III.1.4; they are all more stable than the genes, showing that some meta variable is indeed being optimised throughout the model. The  $V_{\text{flux}}$  variable is the most stable of the set – as shown in TABLE III.1.1 – suggesting it is closest to this unknown variable. But none of them are stable enough to state definitively that this is the meta variable being optimised.

However, what all of these variables do show is that generators optimised for LD control are generally smaller than those optimised for IM control. And it is likely that a more advanced cost function that can account for more properties, such as manufacturability, transportation, and installation, could be able to push the optimiser into favouring a more certain design type.

TABLE III.1.1: Maximum percentage difference in meta variables between like runs of the optimiser.

| META VARIABLE     | LD-CONTROL | IM-CONTROL |
|-------------------|------------|------------|
| $V_{\text{gen}}$  | 109%       | 91%        |
| $V_{\text{pm}}$   | 32%        | 23%        |
| $V_{\text{flux}}$ | 11%        | 2.7%       |

These are for the 40-sea-state representation only.

### III.1.3 CROSS-PERFORMANCE OF THE POWER TAKE-OFF SYSTEMS

In the absence of a baseline generator, FIGURE III.1.5 shows that the PTOs have been optimised for their specific control types by cross-evaluating their performance – showing generators optimised for LD-control operating with IM-control, and *vice versa*. The generators with the best overall cost for LD- and IM-control in the 40-sea-state representation are selected and run through the power simulation and control optimisation steps of the optimiser – to find the best control strategy within the generator’s limits. The LD-optimised generator returns a better cost when used with LD-control than the IM-optimised generator; and *vice versa* when IM-control is applied. Of note however, is that when the IM-optimised generator is used with LD control, it produces slightly more electrical power (roughly 5 kW) than the LD-optimised generator with LD-control, but this uplift in power is not enough to counteract the 60% increase cost of the larger, more expensive IM-optimised generator. This therefore shows that the generators have been fine-tuned, and sized to their respective control strategy.

To summarise, even though the LD-optimised generator does perform better when used with IM-control the results show that if the decision to use LD-control is made early on in the design process, a smaller generator can be used to create a WEC that is more economically designed.

### III.1.4 DIFFERENCES IN COMPUTATIONAL EFFORT

The final point of investigation for this section is how the computation time is affected by changes in both the number of sea states being computed, and the control strategy used. FIGURE III.1.6 shows the number of iterations, and total computation time for all of the runs of the simulation analysed in this section. Owing to the stochastic nature of genetic algorithms, there is a lot of variation in each of the parameters within any set of runs – therefore it is of more interest to focus on trends in varying number of sea states or control strategy rather than absolute runtimes. The runtimes are also dependent on

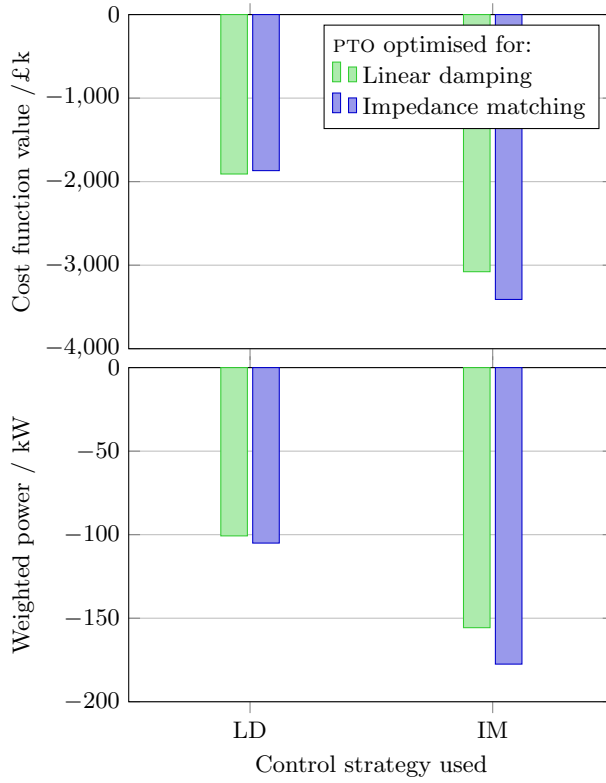


FIGURE III.1.5: Cross performance of generators optimised for different control strategies. WECs are designed with a TYPE-B floater, at the Tyne-Tees WaveNet site.

the computer hardware<sup>v</sup> used and any background processes running throughout the optimisation; so trends are more relevant than absolute values.

For the LD-optimised designs, there is a steady upwards trend in both number of iterations, and total runtime, leading to a fairly consistent average runtime per iteration. In the IM-optimised designs, this trend shows again in the one-sea-state through to the 20-sea-state simulations – with very little difference to the LD values – but there is a divergence in the 40-sea-state scenarios. For these, the time to compute the IM-optimised designs are significantly longer than the LD-optimised, but without an increase in the number of iterations. This is due to the 40-sea-state representation introducing more sea states with a higher wave height, causing the optimiser to use the more-computationally-intensive brute forcing backstop to avoid breaking the WEC’s force and speed constraints (see § II.3.4.3 for the definition of this process).

<sup>v</sup>Simulations are run on a computer with a 10-core Apple M4 CPU and 16 GB of RAM.

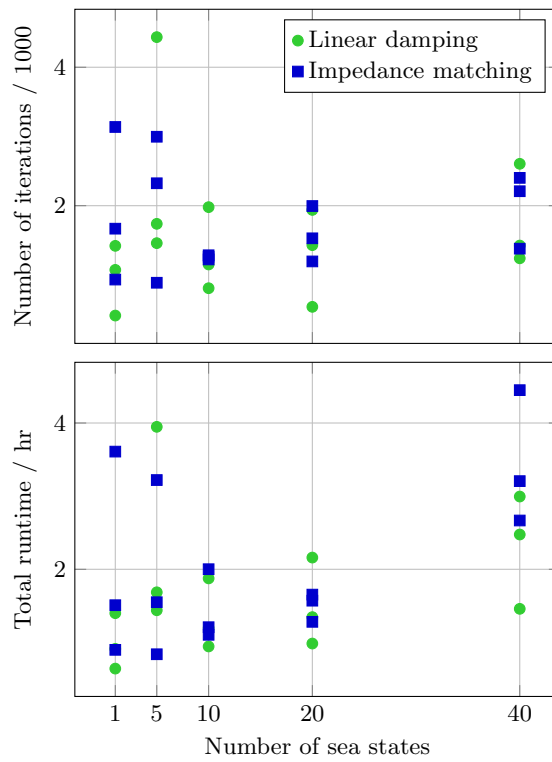


FIGURE III.1.6: The number of iterations, and total runtime of the optimiser, when varying the control strategy used. WECs are designed with a TYPE-B floater, at the Tyne-Tees WaveNet site.

## *Varying the floater design*

Here, differences in the optimal power take-off (PTO) design are investigated when varying the wave energy converter (WEC) floater design. The floaters from this work come from the IMPACT toolbox [2]; for this thesis they are labelled TYPE-N, and TYPE-B, in the IMPACT toolbox they are associated with WEC 1, and WEC 2 respectively. The wetted surface areas of the floaters are shown in FIGURE III.2.1.<sup>i</sup> The TYPE-N floater is a surface-piercing cylinder with a 20 m diameter and a 24 m draft. And the TYPE-B floater is a fully-submerged pill-shaped design with a 20 m diameter and a 5 m height; the floater is designed such that at rest it is 5 m below the surface of the water. Their frequency responses are in FIGURE III.2.2: showing that TYPE-N is a narrow-banded device, and TYPE-B is broad-banded.

The remainder of this section investigates how the optimal PTO design changes when varying the floater design<sup>ii</sup> as well as when changing the control strategy – between linear damping (LD) and impedance matching (IM) – and when changing the number of sea states in the representation of the location. As in the previous section, the site used for all of these tests is the Tyne-Tees WaveNet Site [3] (see FIGURE III.2.4 for an

<sup>i</sup>The panels shown in the figure are not those used in the evaluation of the hydrodynamic parameters. For that the TYPE-N floater had circa 1700 panels, and the TYPE-B floater had circa 1600 panels. The evaluation was done as a part of the IMPACT project [2], using Nemoh as the BEM model.

<sup>ii</sup>The TYPE-B results are the same results from the §III.1 on varying control strategy brought forward for ease of comparison.

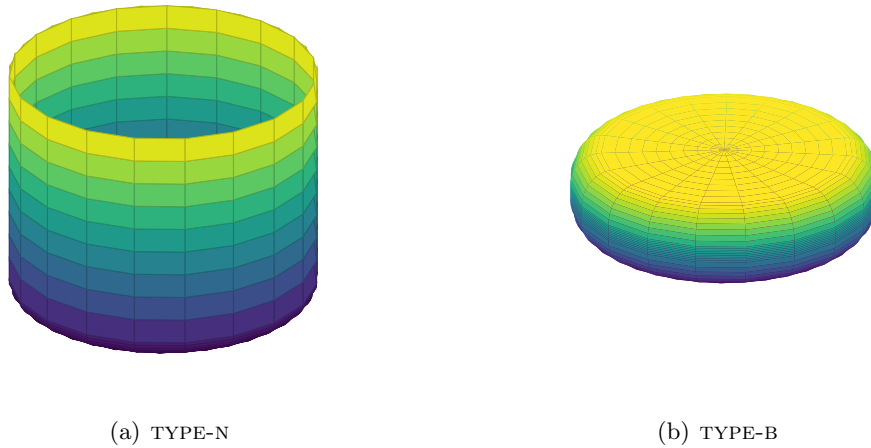


FIGURE III.2.1: Wetted surface area of the reference floaters.

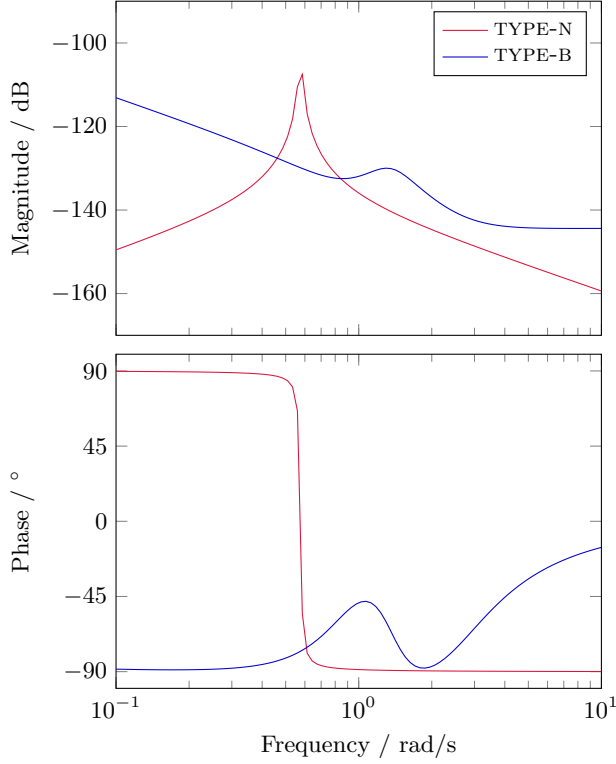


FIGURE III.2.2: Bode plots for the response of the reference floaters from velocity to force.

incidence plot of the location, and APPENDIX A for the test sea states used).

### III.2.1 DIFFERENCES IN COST FUNCTION VALUES

FIGURE III.2.3 shows how cost function, and its constituent parts, vary when using the different floaters. The effect on the TYPE-B costs of varying the number of sea states is discussed in § III.1.1, so is not repeated here. The most notable feature of the TYPE-N costs is at the single-sea-state representation: two of the LD-control runs and one of the IM-control runs have a positive value for their cost function. This means that the money earned from electricity production did not overcome the material cost of the generator – never mind the cost of manufacturing and the rest of the components. The runs that did return a negative value had a cost function value of -£2k for the LD-control result, and -£6k and -£11k for the IM-control results; this being the profit over a 30 year lifespan of the WEC. Due to a severe mismatch between the resonance of the TYPE-N floater and the singular sea state – the resonant point is roughly 10.9 s period, and the sea state is at 7.38 s – the algorithm finds more improvement from shrinking the size of the generator (reducing material cost whilst harming power production) than from tuning the generator to fit the seas (increasing power production). It is conceivable that the LD control strategy is more likely to experience this failure to produce a viable

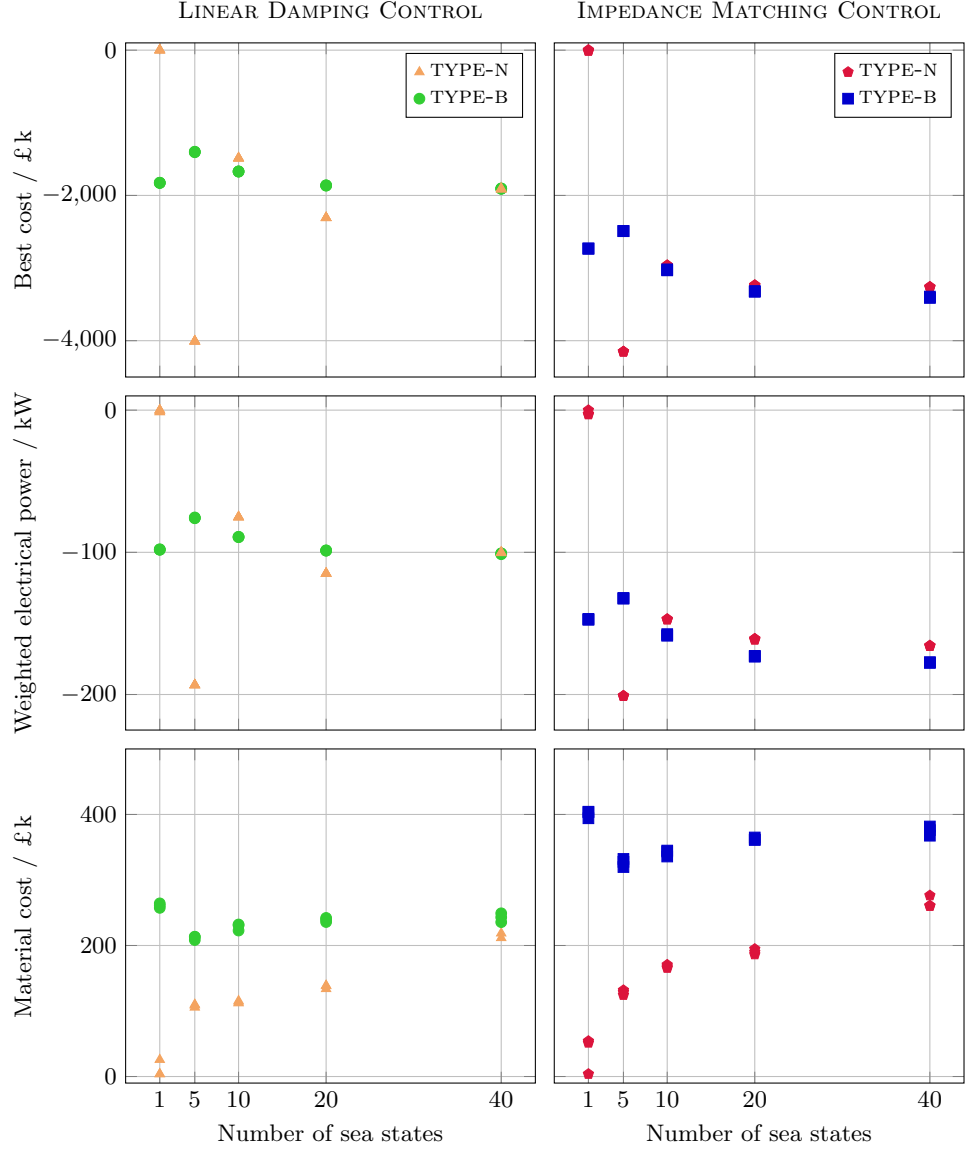


FIGURE III.2.3: Best cost solutions from the optimiser, when varying the floater used. WECS are designed for the Tyne-Tees WaveNet site.

generator, as IM control can shift the resonance of a system: as observed by the 2/3 failures for LD-control compared to 1/3 for IM-control. After the one-sea-state case, all runs produced a viable generator design. Neglecting the one-sea-state TYPE-N case, the maximum percentage difference between any two like runs of the optimiser is 0.89% showing good convergence regardless of floater or control strategy; a full breakdown by number of sea states is found in TABLE III.2.1.

The TYPE-N floater experiences a greater variability with increasing numbers of sea states, oscillating around the optimal value whereas TYPE-B asymptotes towards

TABLE III.2.1: Maximum percentage differences between like runs of the optimiser at the Tyne-Tees site.

| N° of<br>sea states | TYPE-N | TYPE-N | TYPE-B | TYPE-B |
|---------------------|--------|--------|--------|--------|
|                     | LD     | IM     | LD     | IM     |
| 1                   | -      | -      | 0.23%  | 0.15%  |
| 5                   | 0.091% | 0.037% | 0.53%  | 0.16%  |
| 10                  | 0.36%  | 0.25%  | 0.25%  | 0.44%  |
| 20                  | 0.15%  | 0.32%  | 0.19%  | 0.24%  |
| 40                  | 0.89%  | 0.25%  | 0.25%  | 0.50%  |

The one-sea-state TYPE-N results are blanked out due to the failures to find viable PTOs.

its optimal. Again, this can be explained by the narrow-bandedness of the floater: making it more sensitive to the selection of sea states, and whether or not they are close to the resonant frequency of the floater. The test sea states overlaid on the power matrices of the LD-optimised and IM-optimised WECs can be seen in FIGURE III.2.4 and FIGURE III.2.5 respectively. Discontinuities can be seen in the power matrices of many of the TYPE-N-optimised WECs, which are due to the WEC hitting the overheating constraints and the power therefore being limited, and even requiring the WEC be turned off in those seas.<sup>iii</sup> The effect is not as extreme in the TYPE-B-optimised WECs, due to the lower peak magnitude in the transfer function (see FIGURE III.2.2), so the response of the WEC is not as large, reducing the speeds and forces in the more extreme seas – the power does get limited but it does not get to the point where the WEC must to be switched off.

The prior result of IM control always outperforming LD control (see § III.1) still holds when optimising for the TYPE-N floater. Similarly to when varying the control strategy, the differences going from a 20- to a 40-sea-state representation is fairly small for both control types with TYPE-B and IM-control with TYPE-N; suggesting this is the point of negligible improvement. TYPE-N with LD-control settles at this point but the change in cost function value is much larger than the others at this point. A full breakdown of the percentage differences in average cost is shown in TABLE III.2.2.

<sup>iii</sup>This is primarily in regions of significant wave heights greater than 5 m; in reality, in those seas, survivability would be the top priority, not optimal power production.

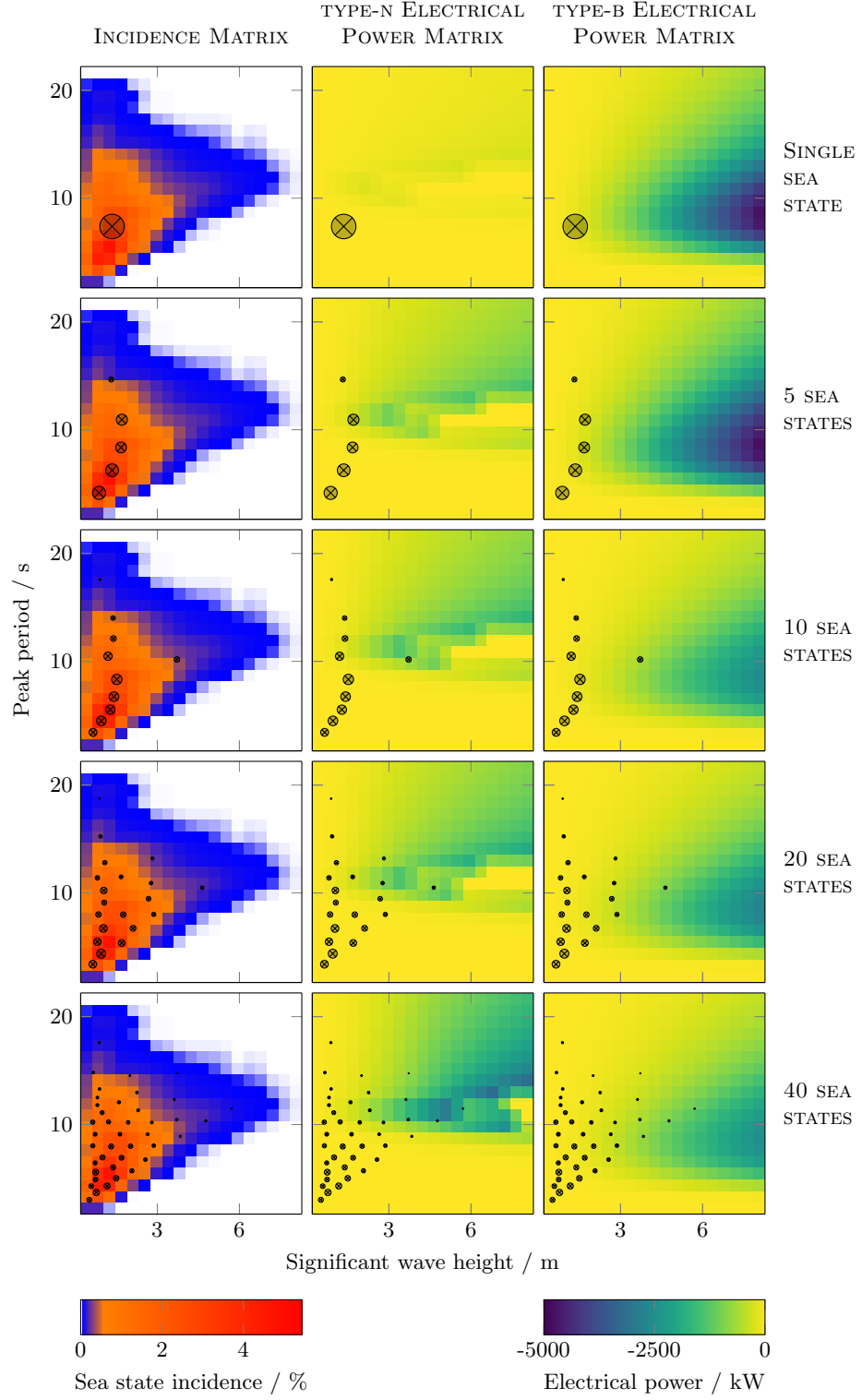


FIGURE III.2.4: Incidence, and power matrices for WECS optimised for linear damping control and varying floater designs. WECS are designed for use at the Tyne-Tees WaveNet site: test points are shown by the circles, with relative areas representing the weighting of each sea state.

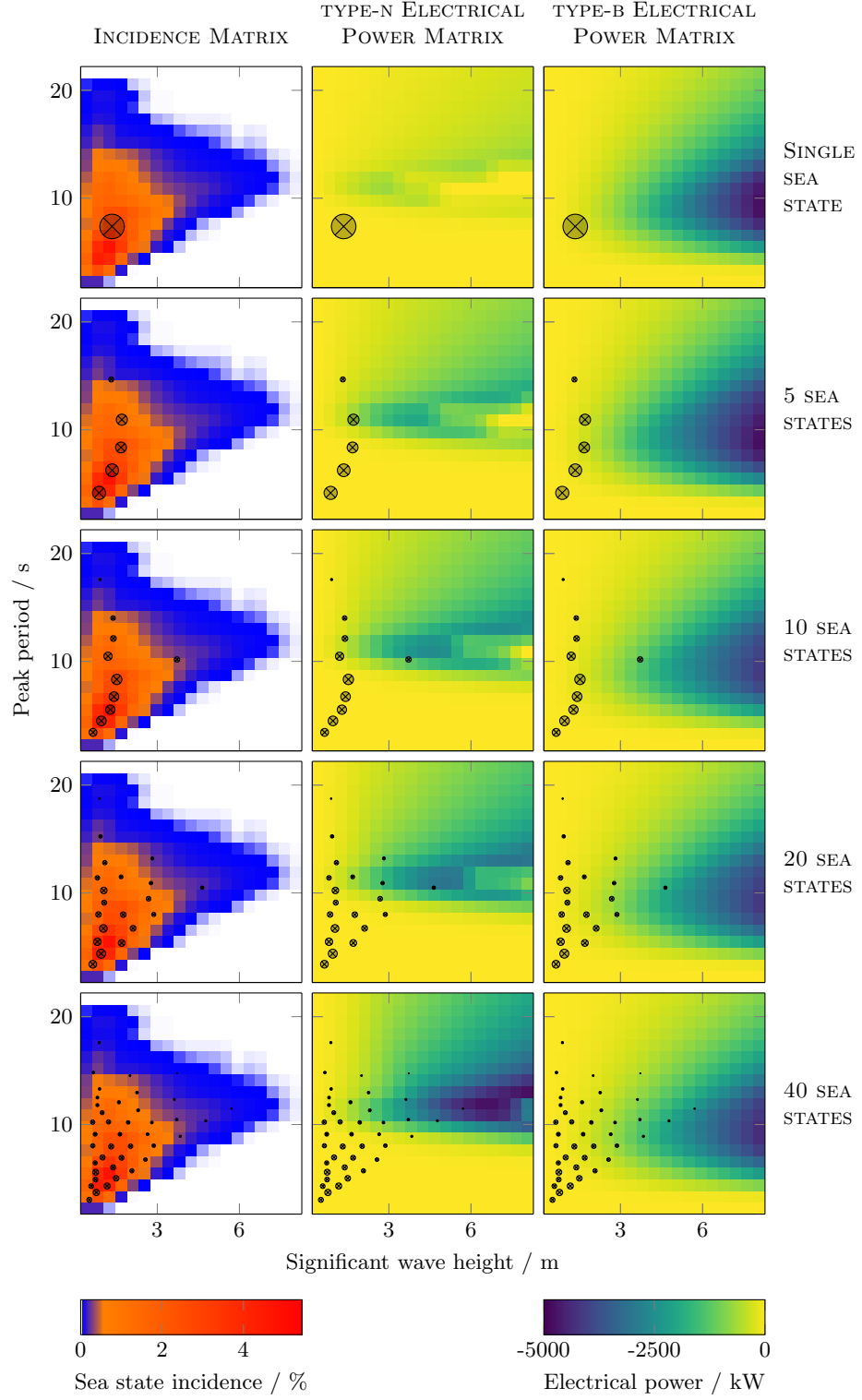


FIGURE III.2.5: Incidence, and power matrices for WECs optimised for impedance matching control and varying floater designs. WECs are designed for use at the Tyne-Tees WaveNet site: test points are shown by the circles, with relative areas representing the weighting of each sea state.

TABLE III.2.2: Percentage difference in average cost when increasing numbers of sea states

| CHANGE IN<br>N <sup>o</sup> OF SEA STATES | TYPE-N | TYPE-N | TYPE-B | TYPE-B |
|-------------------------------------------|--------|--------|--------|--------|
|                                           | LD     | IM     | LD     | IM     |
| 1 to 5                                    | -      | -      | 23%    | 8.8%   |
| 5 to 10                                   | 63%    | 29%    | 19%    | 21.5%  |
| 10 to 20                                  | 55%    | 9.3%   | 12%    | 9.76%  |
| 20 to 40                                  | 17%    | 0.73%  | 2.4%   | 2.5%   |

The 1 to 5 TYPE-N results are blanked out due to the failures to find viable PTOS.

TABLE III.2.3: Maximum percentage difference in meta variables between like runs of the optimiser.

| META VARIABLE     | TYPE-N | TYPE-N | TYPE-B | TYPE-B |
|-------------------|--------|--------|--------|--------|
|                   | LD     | IM     | LD     | IM     |
| $V_{\text{gen}}$  | 38%    | 35%    | 109%   | 91%    |
| $V_{\text{pm}}$   | 26%    | 33%    | 32%    | 23%    |
| $V_{\text{flux}}$ | 26%    | 5.0%   | 11%    | 2.7%   |

These are for the 40-sea-state representation only.

### III.2.2 DIFFERENCES IN POWER TAKE-OFF DESIGN

The genes of the optimised generator designs are shown in FIGURE III.2.6 and FIGURE III.2.7 for LD- and IM-control respectively. As with the varying control results, there is little to glean from looking at the genes directly; therefore the *meta variables* (introduced in § III.1.2) are shown in FIGURE III.2.8 and FIGURE III.2.9, which show clearer trends. As with the varying control results, the  $V_{\text{flux}}$  meta variable appears to be the most constant meta variable between like runs – although there is still some variability, as seen in TABLE III.2.3.

The biggest difference between WECs optimised for different floaters is seen in the one-sea-state case, but this is due to the TYPE-N-floater’s very low power capture at the frequency of the sea state, and so the two floaters are effectively working towards different goals: TYPE-B optimisation works to maximise power and minimise generator material cost; TYPE-N optimisation works only to minimise generator material cost. The difference is shown in the results: in the LD case, the TYPE-B-optimised devices have overall volumes ( $V_{\text{gen}}$ ) around 30 m<sup>3</sup>; and for the TYPE-N-optimised devices, the

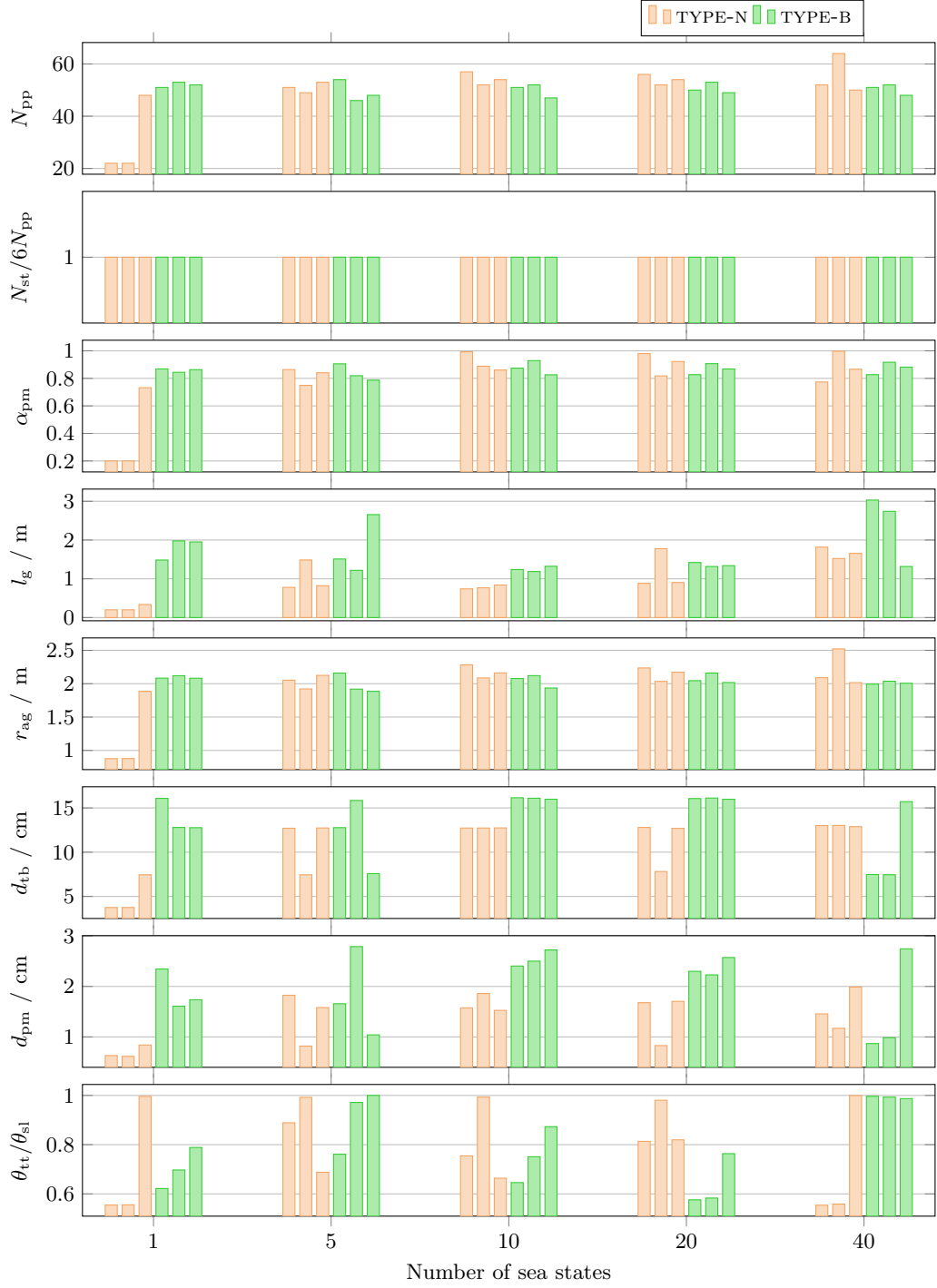


FIGURE III.2.6: Optimised genes for multiple runs of the optimiser, when using linear damping control and varying the floater used. Note that the first two genes are fixed to integer values. WECs are designed for use at the Tyne-Tees WaveNet site.

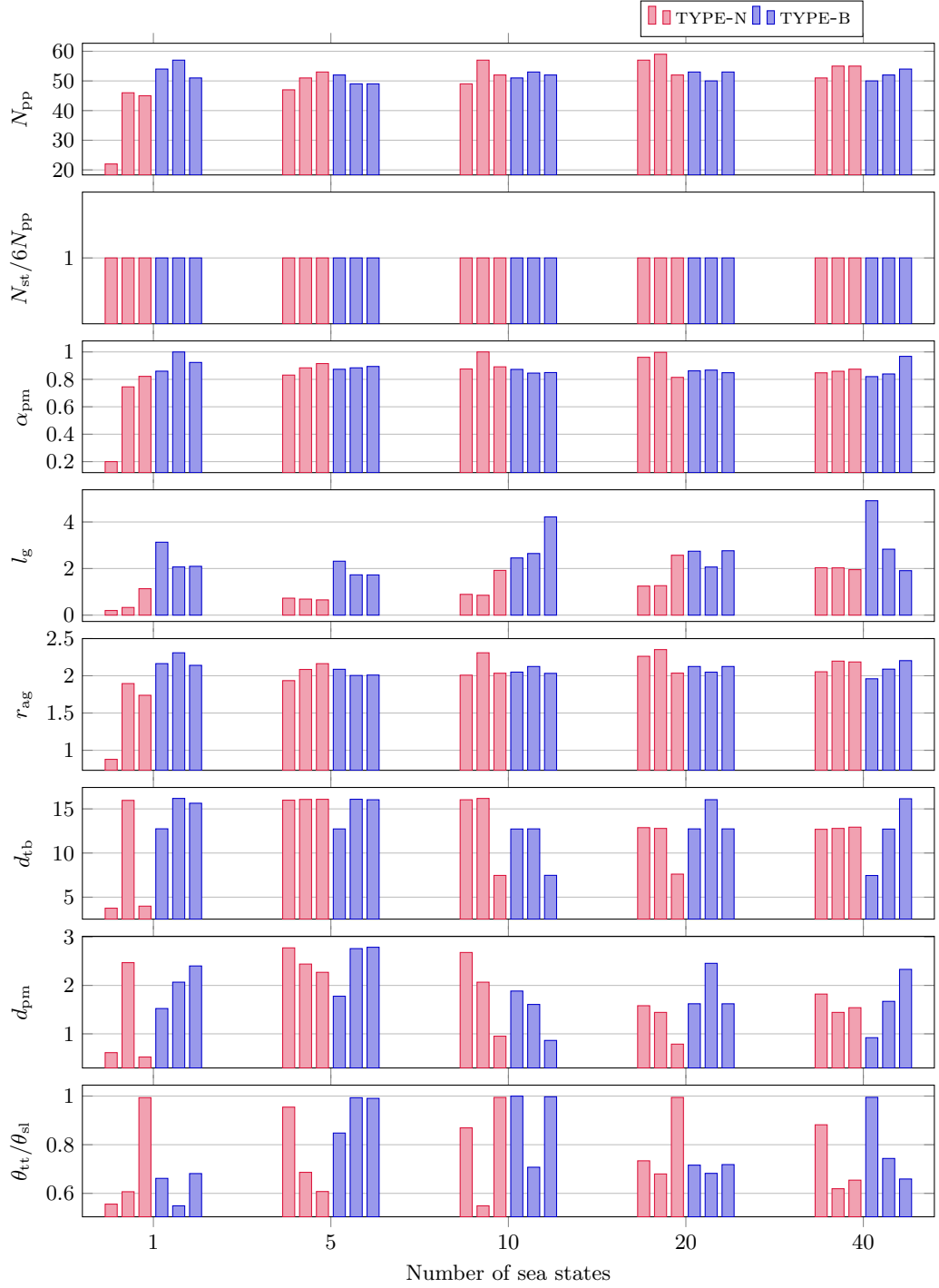


FIGURE III.2.7: Optimised genes for multiple runs of the optimiser, when using impedance matching control and varying the floater used. Note that the first two genes are fixed to integer values. WECs are designed for use at the Tyne-Tees WaveNet site.

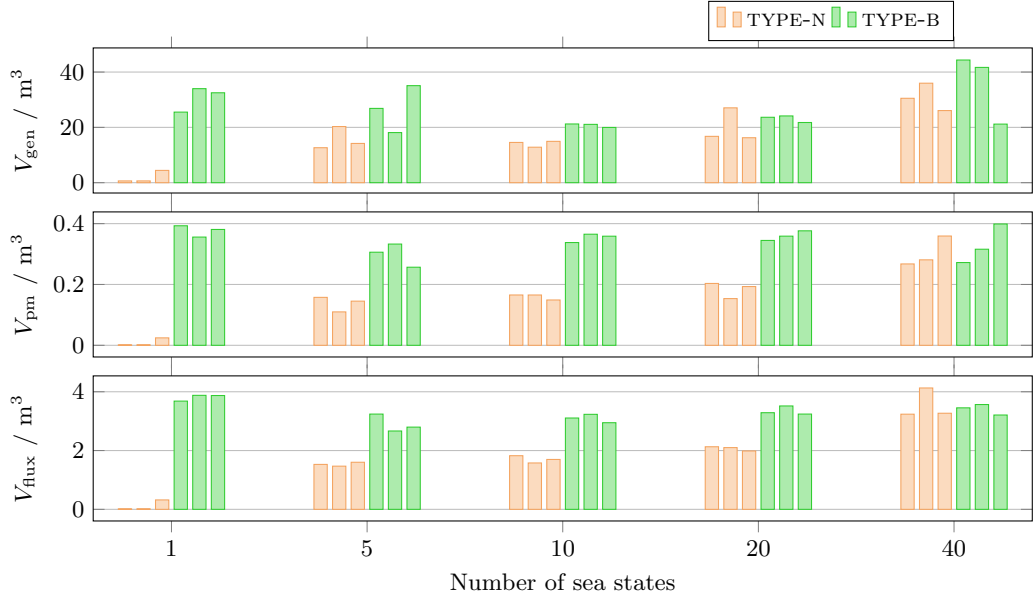


FIGURE III.2.8: Meta variables for multiple runs of the optimiser, when using linear damping control and varying the floater used. The ordering of PTOs corresponds to the order of the genes in FIGURE III.2.6. WECs are designed for use at the Tyne-Tees WaveNet site.

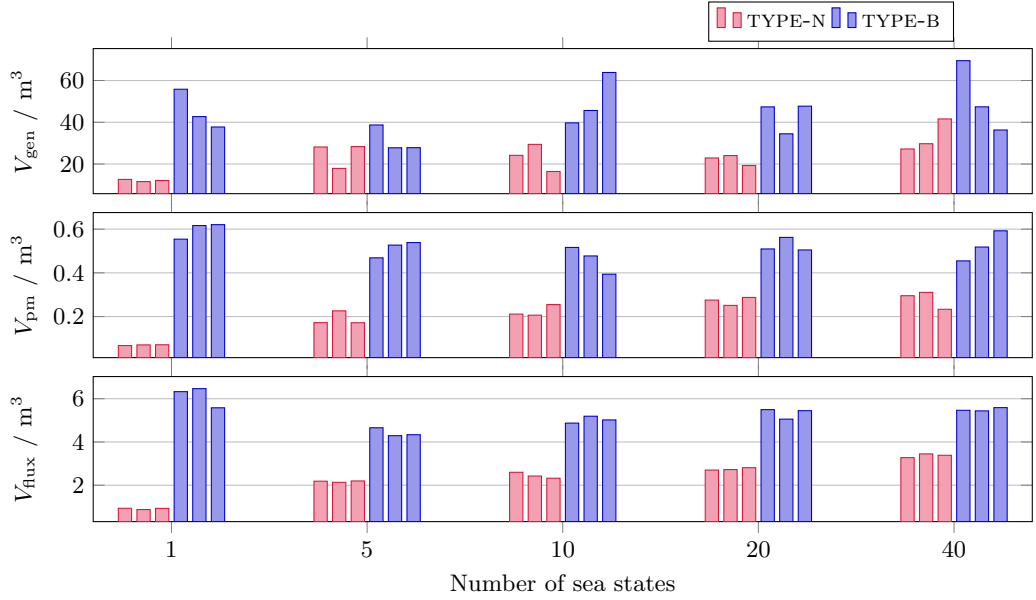


FIGURE III.2.9: Meta variables for multiple runs of the optimiser, when using impedance matching control and varying the floater used. The ordering of PTOs corresponds to the order of the genes in FIGURE III.2.7. WECs are designed for use at the Tyne-Tees WaveNet site.

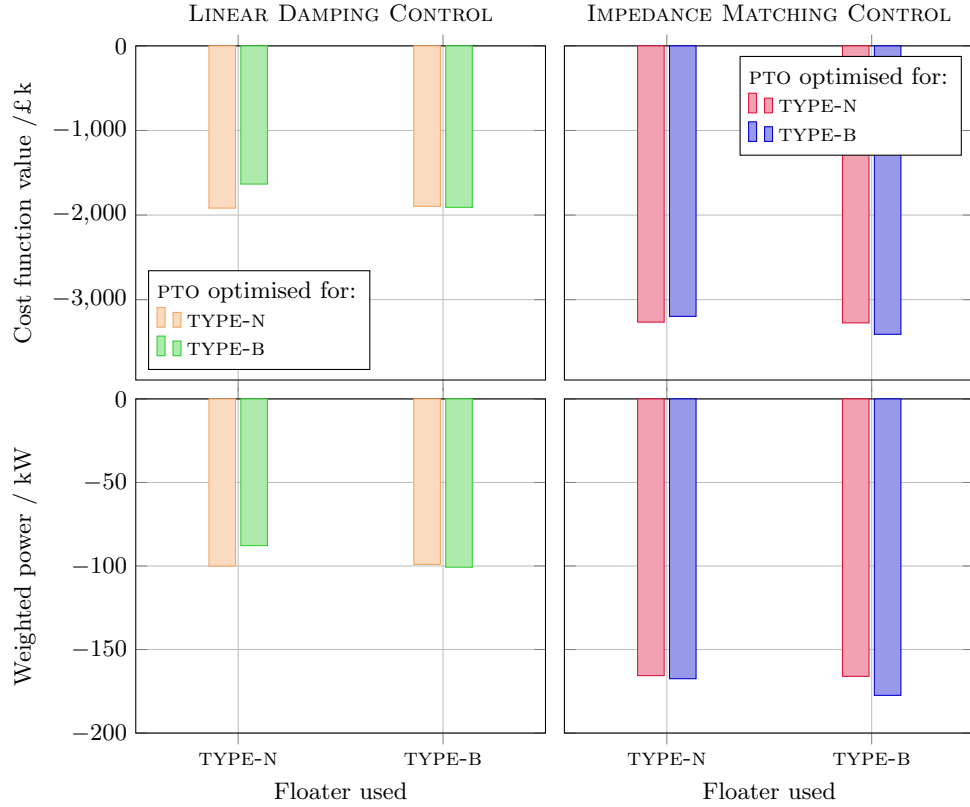


FIGURE III.2.10: Cross performance of generators optimised for different floater designs. WECS are designed for use at the Tyne-Tees WaveNet site.

only run that returned a negative cost function value had a volume of  $4.4 \text{ m}^3$ , whilst those that had a positive cost function value have volumes of around  $0.64 \text{ m}^3$ .

### III.2.3 CROSS-PERFORMANCE OF THE POWER TAKE-OFF SYSTEMS

When taking a PTO optimised for the TYPE-N floater and using it with the TYPE-B floater and *vice versa*, the WECS perform worse than in optimised pairings. This holds regardless of the control strategy, as shown in FIGURE III.2.10, showing that the PTOs have been tuned to their respective floater. Similarly to results in § III.1.3, in the IM-control case, using the TYPE-B-optimised PTO with the TYPE-N floater increases power output compared to its own optimised PTO, but the increase in power output is not able to counteract the increased cost of the more expensive generator. However, using the TYPE-N-optimised PTO with TYPE-B causes a reduction in power output as the WEC is limited by a smaller generator.

In the LD-control case however, the broad-banded TYPE-B floater is also better able to cope with an ill-tuned PTO, and therefore has less of a reduction in power when

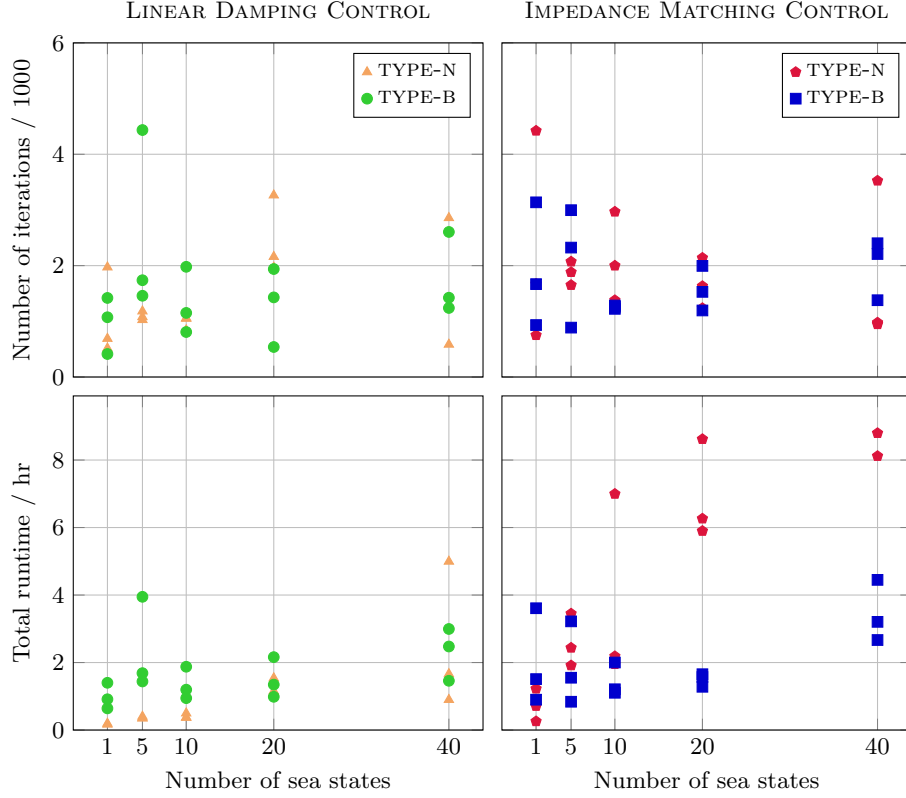


FIGURE III.2.11: The number of iterations, and total runtime of the optimiser, when using different floater designs. WECs are designed for use at the Tyne-Tees WaveNet site.

used with a non-optimised PTO than the TYPE-N floater does. The TYPE-B floater loses only 1 kW in performance compared to the 13 kW in the TYPE-N floater, when using LD-control.

### III.2.4 DIFFERENCES IN COMPUTATIONAL EFFORT

Finally, FIGURE III.2.11 shows the number of iterations and runtime to reach the final solutions. Similarly to § III.1.4, a caveat is made that this is a highly random process so there should be the expectation of lots of variability between like runs. The absolute runtime is highly dependent on the computer hardware<sup>iv</sup> used, so trends in the runtime are the focus of this subsection.

When using LD control, there is not any major difference in runtimes. TYPE-N runs tend to be marginally – but not significantly – quicker. The same is seen for the one and five sea state IM control runs, whilst running the code with TYPE-N takes significantly

<sup>iv</sup>Simulations are run on a computer with a 10-core Apple M4 CPU and 16 GB of RAM.

longer than with TYPE-B. This is due to the 10-sea-state representations and beyond adding seas within the resonant zone of the WEC (shown by the introduction of sea states within the darker blue regions in FIGURE III.2.5): these seas are more likely to break the overheating constraint of the PTO, leading to the full set of the possible controllers being tested (see §II.3.4.3 for the definition of the process), leading to much longer runtimes.

### *Varying the location*

In this section, power take-off (PTO) systems are optimised at various locations alongside changing the control strategy and floater design, with the results indicative of whether the previous two sections can be generalised to different locations.

To accompany the Tyne-Tees site [3] used in the previous sections, two others are selected: Liverpool Bay [5], and the Isles of Scilly [6]; these are selected as a lower and higher energy location than Tyne-Tees respectively. The locations of the sites are shown on a map in FIGURE III.3.1, and wave incidence plots are shown in FIGURE III.3.2. Data for all of these sites comes from WaveNet [7]. Given that the cost functions in both varying control strategy, and varying floater design had mostly stabilised by a 20-sea-state representation, only that representation is run in simulations here – but each test is still run three times – the breakdown of the 20 sea states for each site is found in APPENDIX A.

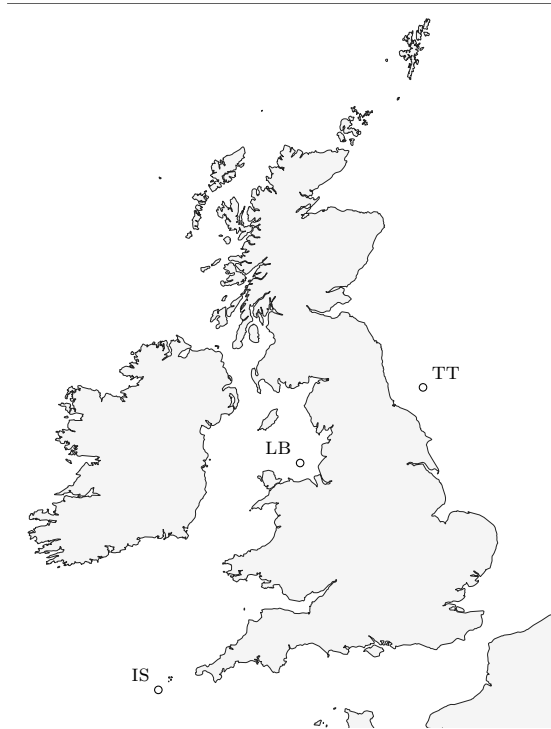


FIGURE III.3.1: Map of site locations. Locations are approximate. Initialisms are as follows: TT, Tyne-Tees; LB, Liverpool Bay; and IS, Isles of Scilly.

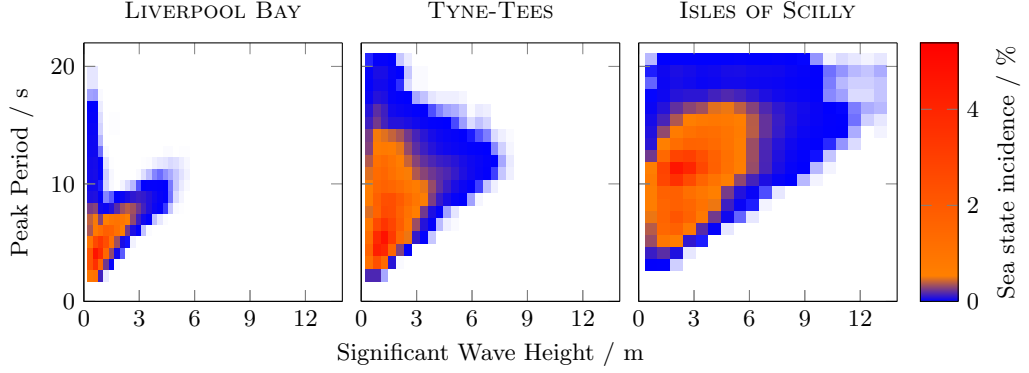


FIGURE III.3.2: Incidence plots of the three test sites.

### III.3.1 DIFFERENCES IN COST FUNCTION VALUES

FIGURE III.3.3 shows the cost function of all the WEC combinations – using either linear damping (LD) or impedance matching (IM) control, and either a TYPE-N or TYPE-B floater – when optimised at each of the three sites. The first point to make is that there is not a single successful run of the TYPE-N floater at the Liverpool Bay site, regardless of control strategy. As with the one-sea-state representation at Tyne-Tees (seen in § III.2) where some – but not all – TYPE-N runs failed to produce viable generators, here there is a miss-match in the site’s most common wave peak periods, and TYPE-N’s resonant period. In the 20 sea states used, there was only one sea state at the Liverpool Bay site within  $\pm 1$  s of the resonant point of the TYPE-N floater, and it is weighted at 0.397% of the total time. And there is only one other sea state with a peak period above the resonant point (*i.e.* with higher energy), so the vast majority of the sea states are low in energy, and in a region the floater is bad at extracting energy from. Apart from TYPE-N-Liverpool Bay runs, all other runs produced a workable PTO. Amongst these other runs, results are as expected: when using the same floater and location, IM control always performs better than LD control; and when using the same floater and control strategy, a higher energy site returns a better performing WEC – both in terms of cost function value and weighted electrical power.

At the Tyne-Tees site, the best performing floater is the TYPE-B, but at the Isles of Scilly it is the TYPE-N; this is because the resonance of a TYPE-N is coincidentally incredibly similar to the most common sea state at the Isles of Scilly site. The sea state with the highest incidence at the site has a peak period of 10.85 s and a significant wave height of 1.7 m (the resonance of the TYPE-N is 10.9 s); in fact, the TYPE-N floater is so well suited to the site that the LD TYPE-N WEC outperformed the IM TYPE-B WEC. And the IM TYPE-N produces 106% more electrical power at the IM-TYPE-B at the Isles

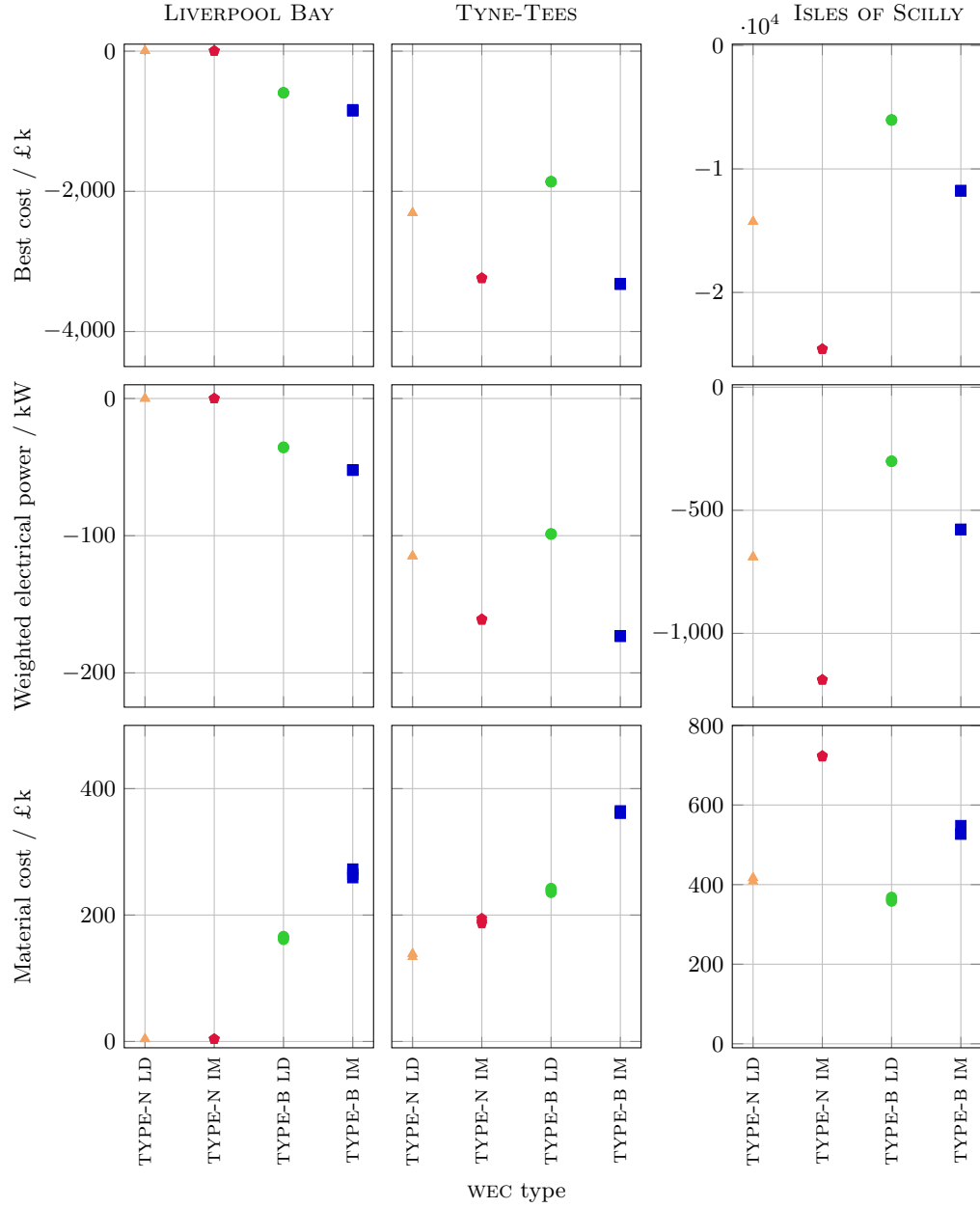


FIGURE III.3.3: Best cost solutions from the optimiser, when varying the site location. Note that due to the Isles of Scilly site being so much more energetic than the others, the y-axis on its plots works to a different scale – but they continue to have the same units.

TABLE III.3.1: Maximum percentage differences between like-runs of the optimiser at varying sites.

| SITE            | TYPE-N | TYPE-N  | TYPE-B | TYPE-B |
|-----------------|--------|---------|--------|--------|
|                 | LD     | IM      | LD     | IM     |
| Liverpool Bay   | 1.6%   | 0.21%   | 0.33%  | 1.6%   |
| Tyne-Tees       | 0.15%  | 0.32%   | 0.19%  | 0.24%  |
| Isles of Scilly | 0.11%  | 0.0028% | 0.11%  | 0.28%  |

 TABLE III.3.2: Percentage increase in average  $V_{\text{flux}}$  from a linear damping to impedance matching controlled WEC.

| SITE            | TYPE-N | TYPE-B |
|-----------------|--------|--------|
| Liverpool Bay   | -      | 70%    |
| Tyne-Tees       | 47%    | 59%    |
| Isles of Scilly | 89%    | 42%    |

of Scilly site, whilst the IM TYPE-B only produces 7.44% more electrical power than the IM TYPE-N at the Tyne-Tees site. Showing that a well-tuned floater design can greatly improve power production to such a level that it can counteract increased material costs, and a poorly-tuned floater can counteract the benefits of a well-designed PTO.

Convergence in the optimiser remains across sites, with the largest percentage difference between like-runs being 1.6%, 0.32%, and 0.28% at Liverpool Bay, Tyne-Tees, and the Isles of Scilly respectively. The complete breakdown by WEC type is found in TABLE III.3.1.

### III.3.2 DIFFERENCES IN POWER TAKE-OFF DESIGN

The optimised genes for the various WECs at each location are found in FIGURE III.3.4, as with each of the previous sections, there is no discernible trend from the genes alone – bar the TYPE-N Liverpool Bay PTOs being much smaller than all the other combinations of components. As in previous sections, the meta variables are shown in FIGURE III.3.5. The intra-location trends in terms of control strategy remain here: IM-controlled WECs can make use of a larger PTO than LD-controlled; although the amount of uplift in PTO size is heavily dependent on location and floater, ranging from 40% to 90% as shown in TABLE III.3.2.

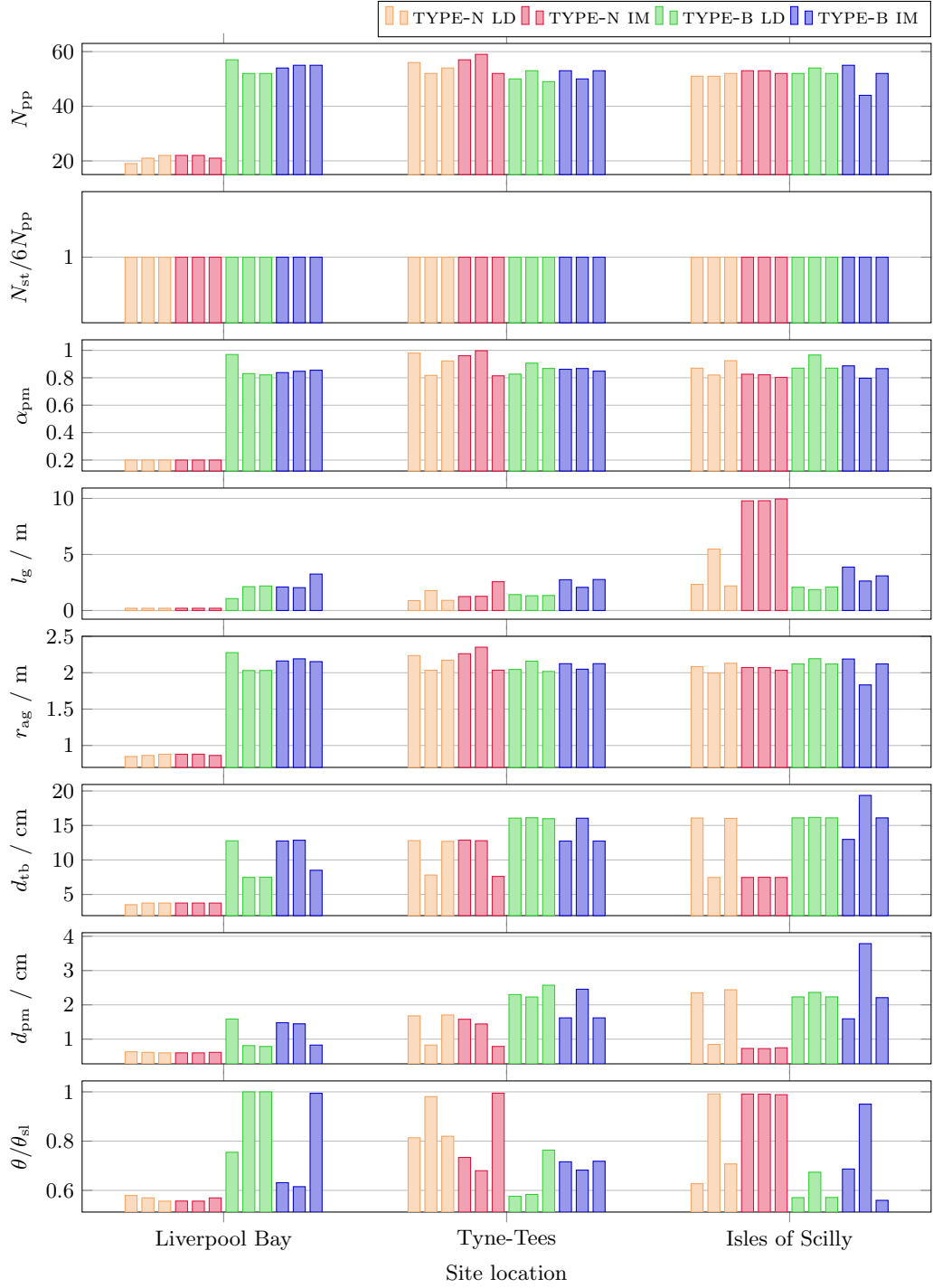


FIGURE III.3.4: Optimised genes for multiple runs of the optimiser, when varying the site location. Note that the first two genes are fixed to integer values.

### III.3.3. CROSS-PERFORMANCE OF THE POWER TAKE-OFF DESIGNS

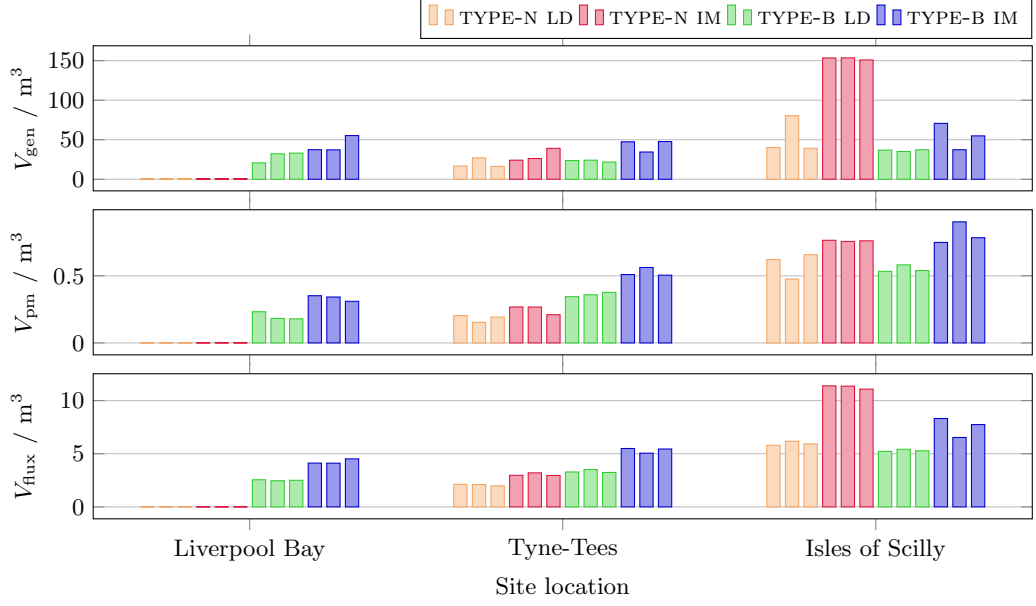


FIGURE III.3.5: Meta variables for multiple runs of the optimiser, when varying the site location. The ordering of PTOs corresponds to the order of the genes in FIGURE III.3.4.

### III.3.3 CROSS-PERFORMANCE OF THE POWER TAKE-OFF DESIGNS

FIGURES III.3.6, III.3.7, and III.3.8 show WECs optimised for each of the three sites being used at Liverpool Bay, Tyne-Tees, and the Isles of Scilly respectively. PTOs optimised for the Isles of Scilly always produce more electrical power than their counterparts (same control strategy and floater design) optimised for the other locations. This is because the Isles of Scilly-optimised WECs are larger so are able to eke out more power from the seas: either from having larger thermal limits, or from being more efficient. This does however not make these PTOs better suited for the other sites, as shown by the cost function values; the PTOs optimised for each site always have a better cost function, regardless of the control strategy or floater design, showing that the PTOs have been optimised and sized to their respective sites.

### III.3.4 DIFFERENCES IN COMPUTATIONAL EFFORT

The number of iterations and runtime of each of the optimiser runs for this section are found in FIGURE III.3.9. Results here follow the same trends as before: IM-controlled runs are generally either take roughly the same amount of time as LD-controlled, or longer; with IM TYPE-N taking significantly longer. The reason for IM TYPE-N WECs in particular taking significantly longer is covered in § III.2.4, but to summarise is due to the resonant point of the floater having a much higher chance of hitting the

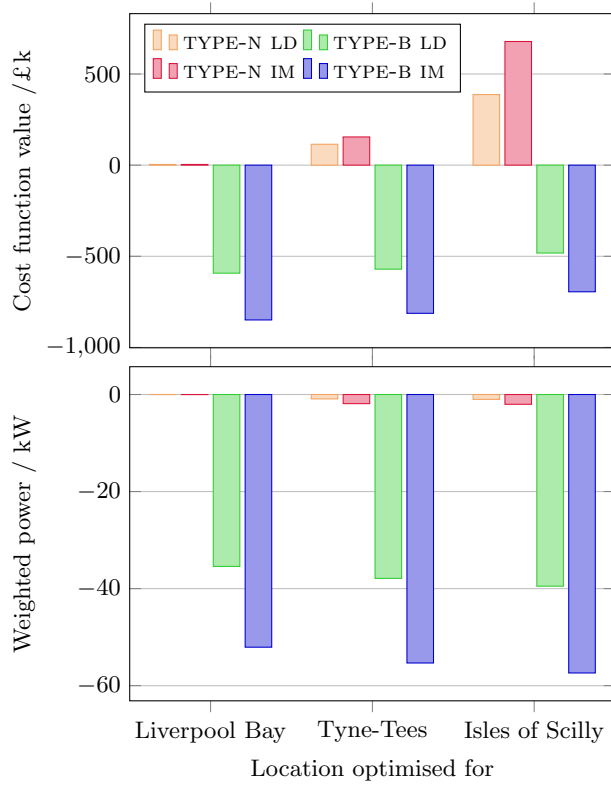


FIGURE III.3.6: Cross performance of generators optimised for different sites, used at the Liverpool Bay site.

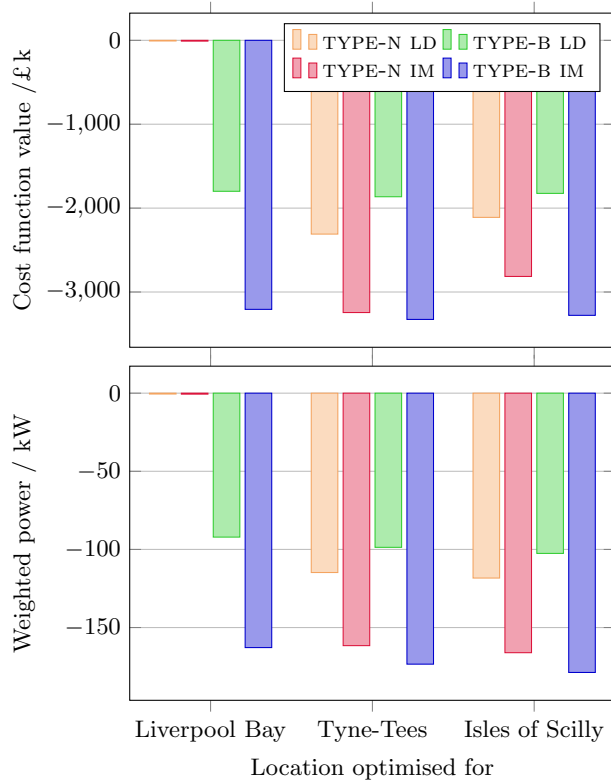


FIGURE III.3.7: Cross performance of generators optimised for different sites, used at the Tyne-Tees site.

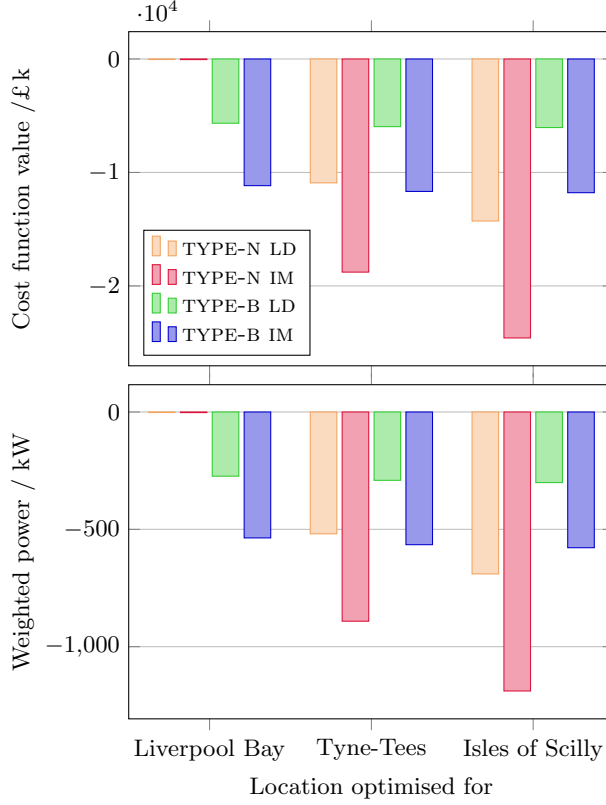


FIGURE III.3.8: Cross performance of generators optimised for different sites, used at the Isles of Scilly site.

overheating limits and causing the optimiser to use a slower brute-forcing approach to control optimisation. This is exacerbated for the Isles of Scilly site because a higher proportion of the sea states are around the resonant point of the TYPE-N floater, and with a very high significant wave height (*i.e.* states very likely to cause the WEC to move very fast). For example, the number of sea states with a peak period within  $\pm 2$  s of the resonant point, and with a significant wave height over 2 m is: 6/20 for the Isles of Scilly site; 0/20 for the Liverpool Bay site; and 3/20 for the Tyne-Tees. Halving the runtime for the IM TYPE-N runs at the Isles of Scilly site brings the runtime only marginally longer than that of the same runs at the Tyne-Tees site, suggesting that it is the number of sea states within this period band that drive the total runtime.

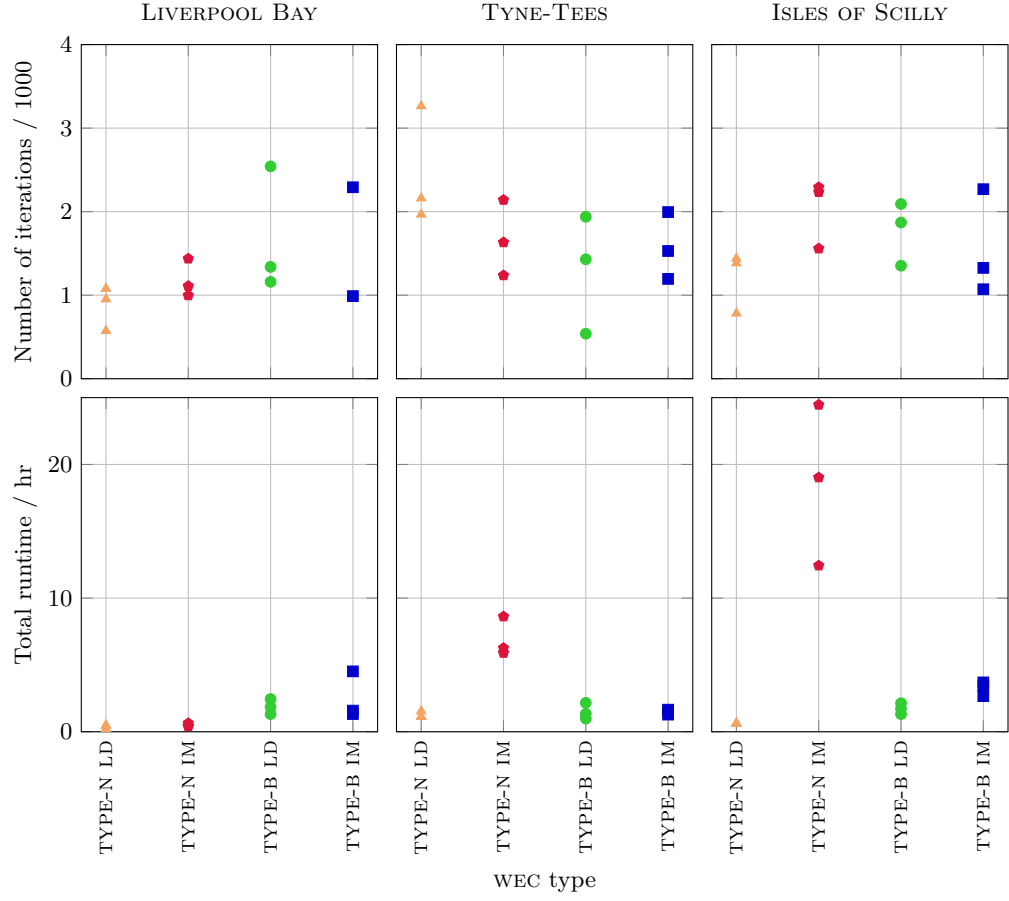


FIGURE III.3.9: The number of iterations, and total runtime of the optimiser, when designing for different site locations.

## *Discussion of Results*

Investigations into varying each of the components of a wave energy converter (WEC) – control strategy, floater design, and site location – have found that the optimal power take-off (PTO) design depends heavily upon each component. Changing any of the three components whilst keeping the PTO design the same results in a worse WEC overall (in terms of the cost function).

There has not been the discovery of a definitive *best* PTO design; none of the PTOS in this thesis are presented as such. However, the optimiser is instead finding a *family* of designs that all perform to the same level. The optimiser is finding this family due to the looseness of the cost function, which is deliberately chosen to avoid relying on assumptions wherever possible (see §II.3.3). In designing a WEC for physical production, there are a multitude of factors that need to be accounted for: the loadings on the WEC; the costs, and sizings of the other components<sup>i</sup>; costs of transportation; and many more. It stands to reason that adding any of these to the cost function will reduce the size of this family of solutions, until a single solution remains – or the combination of requirements is found to be unworkable.

Conversely, loosening the requirements can find solutions previously thought impossible. Take, for example, the case of an impedance-matching-controlled TYPE-N WEC placed at the Liverpool Bay site; using the cost function definition defined in the methodology – accounting for electrical power output and generator material cost – the optimiser was unable to produce a viable design over three attempted runs. But, in testing the cross-performance of PTOS, it was found that WECs designed for the other sites were able to extract non-insignificant amounts of electrical power, so it is possible to design a WEC that would work at Liverpool Bay (although its advisability is another matter). So in running the optimiser again for a simpler goal: maximise electrical power (see EQUATION (II.3.20)); a PTO can be produced to create a WEC that can extract even more electrical power than the one designed for the Isles of Scilly site; as shown in FIGURE III.4.1.<sup>ii</sup> The simpler cost function has produced a WEC that produces 5.9% more electrical power (53 W) but at an increase in material costs of 67% (£90k); this

<sup>i</sup>For example: a larger generator pulling large amounts of reactive power in an impedance matching regime would likely require a larger and more expensive power electronics system to a small generator only resisting under linear damping.

<sup>ii</sup>This example is run three times and the maximum percentage difference between any two runs in weighted electrical power is less than 0.06%.

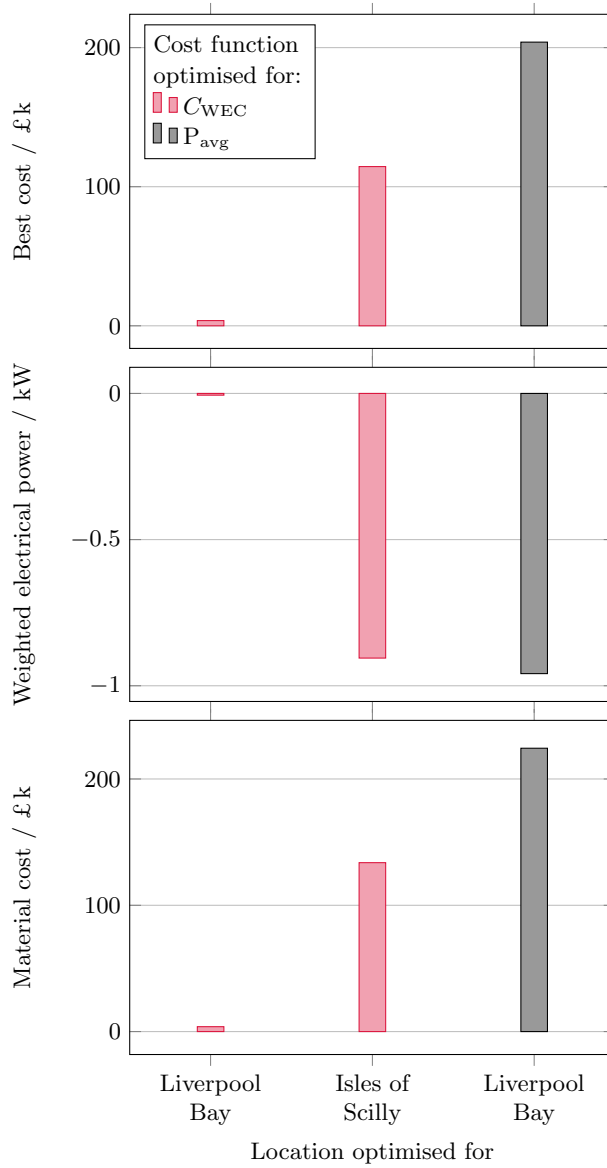


FIGURE III.4.1: Best cost solutions from the optimiser, when optimising for different cost function. WECs are designed for: impedance matching control; for a TYPE-N floater; and at the site given in the x-axis label.

would not be recommended, but it demonstrates that loosening constraints could help design WECs in more challenging situations.

To conclude, this chapter has shown a large degree of interactivity in the dynamics of the various parts of a WEC through the changing of the optimal PTO design. The model as it exists does not account for all the complexities of WEC design and operation; but through the inclusion of these neglected factors, the improvements from control co-design would likely become more important as the window of where a *good* WEC can exist only narrows.

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## Bibliography

- [1] Matthew Onslow and Adam Stock. “Genetic Algorithms for the Control Co-Design of Permanent Magnet Synchronous Generators for Wave Energy Converters”. In: *Applied Ocean Research* (Under Review) (cited on page 122).
- [2] Adam Stock, Carlos Gonzalez, and David Robb. *Integrated marine point absorber control tool final stage 2 project report*. Tech. rep. SgurrControl Ltd., 2018 (cited on pages 122, 131).
- [3] Centre for Environment Fisheries and Aquaculture Science. *Tyne/Tees WaveNet Site*. Last Accessed: 19/7/26. URL: <https://wavenet.cefasc.co.uk/details/TYNETEESWN/INT> (cited on pages 122, 131, 144).
- [4] Yue Hong et al. “Review on electrical control strategies for wave energy converting systems”. In: *Renewable and Sustainable Energy Reviews* (Dec. 2014). DOI: <https://doi.org/10.1016/j.rser.2013.11.053> (cited on page 122).
- [5] Centre for Environment Fisheries and Aquaculture Science. *Liverpool Bay WaveNet Site*. Last Accessed: 19/7/26. URL: <https://wavenet.cefasc.co.uk/details/LIVBAYWN/INT> (cited on page 144).
- [6] Centre for Environment Fisheries and Aquaculture Science. *SW Isles of Scilly WaveNet Site*. Last Accessed: 19/7/26. URL: <https://wavenet.cefasc.co.uk/details/SWSCILLYWN/INT> (cited on page 144).
- [7] Centre for Environment Fisheries and Aquaculture Science. *WaveNet*. Last Accessed: 19/7/26. URL: <https://wavenet.cefasc.co.uk> (cited on page 144).

## CHAPTER IV

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# *Control Co-design of Floaters and Power Take-off Systems for use in Wave Energy Converters*

Control co-design of floaters has had a lot more interest in the published literature than the CCD of power take-off systems (PTOs); though much of the work on floater optimisation neglected any effect of PTO dynamics and efficiency. This chapter expands upon the power take-off (PTO) optimiser developed in §II, to perform the novel co-optimisation of the floater and PTO. The chapter starts with a literature review of previous methods of floater control co-design and then investigates the differences in optimal design when: serially optimising the PTO and floater, and co-optimising the PTO and floater to see how the level of control co-design integration improves overall wave energy converter performance.

## *Review of Descriptions & Optimisation of Floaters*

The discussion here builds upon §I.3.2, in which the control co-design (CCD) of wave energy converters (WECS) is briefly covered. In that section floater optimisation is mentioned at a high level; this section goes into more detail into the literature on floater geometric definitions, and the methodologies for their optimisation; ending with a rationale for the choice of floater optimisation method used in this work. Due to how the model has been designed so far, this section is limited to talking about point absorbers; for a broader review the reader is directed to [1].

### IV.1.1 SIMPLE SHAPES

There is a plethora of literature on the optimisation of simple floater shapes (see [1] for a full review); shapes such as spheres, cylinders, cones, and cuboids (and compounds of these). These shapes are typically modelled in a boundary element method (BEM) program to give information on the hydrodynamic parameters of the floater, which is then used to evaluate device performance.

Given their simplicity in shape, fully describing the floaters requires few parameters (*e.g.* radius and draught); therefore the optimisation problem is also fairly easy and the literature tends to favour using exhaustive searches (*eg.* [2]) due to their ease of implementation. Exhaustive searching also comes with the benefit of creating a map of all the possible geometries, giving a visual representation of the relationship between the floater dimensions and power production.

These works used a variety of cost functions to optimise the system, which can be broadly separated into two categories. A pure measure of the device's effectiveness at extracting energy from the seas: average power, response amplitude operator, and capture width ratio. Or as a function of energy extraction and (a proxy for) device cost: average power over volume; average power over wetted area; and levelised cost of energy.

More advanced evolutionary algorithms are used in some investigations, *e.g.* in [3] or [4]; but these are generally have greater complexity in another part of the system. In these examples the complexities are: multiple degrees of freedom in [3]; or modelling two-body point absorbers in [4].

## IV.1.2 COMPLEX SHAPES

Beyond the simple shapes described in the previous subsection, more complex shaped floaters have been considered.

**AXISYMMETRIC FLOATERS:** Edwards [5] designed axisymmetric floaters about the heave axis using Chebyshev polynomials of the first kind: these are defined by the recurrence relation

$$T_0(x) = 1, \quad (\text{IV.1.1})$$

$$T_1(x) = x, \quad (\text{IV.1.2})$$

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x); \quad (\text{IV.1.3})$$

using these polynomials, any function can be described in the domain  $x \in [-1, 1]$  by the expansion

$$f(x) = \sum_{n=0}^{\infty} a_n T_n(x). \quad (\text{IV.1.4})$$

This is applied to parametrically define the profile of the floater as a function of the radius,  $r$ , and draught,  $z$ :

$$r_j(s) = \sum_{i=0}^{N_j} a_{ji} T_i(s), \quad (\text{IV.1.5})$$

$$z_j(s) = \sum_{i=0}^{M_j} b_{ji} T_i(s). \quad (\text{IV.1.6})$$

This method can create piecewise continuous profiles, where  $j$  is the piece number, and  $N_j$  and  $M_j$  are the number of polynomials for piece  $j$  in the radius and draught respectively. This profile is then revolved about the heave axis to create the surface of the floater. Then each profile was ran through the BEM programme WAMIT to find the hydrodynamic parameters, which were then used to find the mechanical power.

The optimisation uses a multi-objective genetic algorithm to vary the coefficients  $a_{ji}$  and  $b_{ji}$ . The cost functions used in this work were to maximise the mechanical power whilst minimising either volume or surface area (used as a proxy of material cost).

**NON-AXISYMMETRIC FLOATERS:** Garcia-Teruel [7] and McCabe [6] used floaters described by a complex surface; which is itself defined by a bi-cubic B-spline surface drawn through a number of control points. FIGURE IV.1.1 gives an example of these control points: both Garcia-Teruel and McCabe use a genetic algorithm to position the control points in space to find the surface.

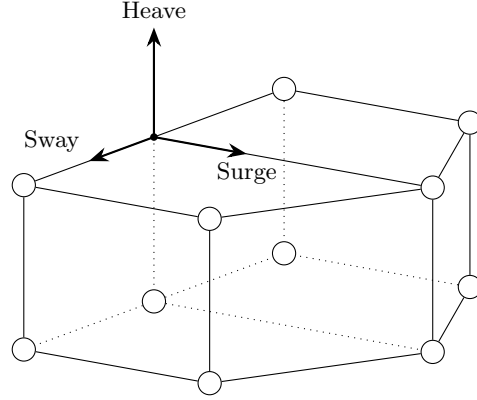


FIGURE IV.1.1: Example of control points to define a floaters surface. (Recreated from [6])

The bi-cubic B-spline surface is defined parametrically by

$$\mathbf{S}(u, v) = \sum_{i=0}^V \sum_{j=0}^U \mathbf{C}_{ij} N_i^k(u) N_j^l(v), \quad (\text{IV.1.7})$$

where  $\mathbf{C}_{ij}$  is the control point matrix,  $N_i^k(u)$  and  $N_j^l(v)$  are the  $i^{\text{th}}$  and  $j^{\text{th}}$  basis functions of orders  $k$  and  $l$  respectively, see [7] for a definition.

McCabe used three different cost functions:

$$C_1 = P_{\text{avg}}, \quad (\text{IV.1.8})$$

$$C_2 = \frac{P_{\text{avg}}}{V}, \quad (\text{IV.1.9})$$

$$C_3 = \frac{P_{\text{avg}}}{\sqrt[3]{V}}, \quad (\text{IV.1.10})$$

where  $P_{\text{avg}}$  is the average annual mechanical power, and  $V$  is the volume. The cube root of the volume in the third cost function is used to get the characteristic length of the floater. Garcia-Teruel expanded upon these to focus on the wetted surface area:

$$C_4 = \frac{P_{\text{avg}}}{V^{2/3}}, \quad (\text{IV.1.11})$$

$$C_5 = \frac{P_{\text{avg}}}{A}, \quad (\text{IV.1.12})$$

$$C_6 = \frac{P_{\text{avg}}}{\sqrt{A}}, \quad (\text{IV.1.13})$$

where  $V^{2/3}$  is used as a proxy for the wetted surface area;  $A$  is the true wetted surface area; and  $\sqrt{A}$  is a proxy for the characteristic length.

### IV.1.3 FROUDE SCALING

The final method discussed here is Froude scaling. As the name suggests, when Froude scaling the relative shape of the floater does not change, the whole design gets scaled

up or down; the theory as to how this is achieved is discussed in §IV.2.1. As a result, the optimiser can not decide whether the optimal design is broad- or narrow-banded.

However, Froude scaling has benefits over the other methods: primarily in the runtime. When using the Froude scaling approach, the BEM code does not have to be run for each candidate design, as pre-existing BEM data is simply scaled. Running a BEM model can take up to 10 minutes [8]<sup>i</sup>; whereas the scaling takes a couple of hundredths of a second. As such the runtime is a key advantage of the Froude scaling approach.

The author is not aware of any work that focussed solely on the Froude scaling of floater designs, but there has been work that used the method in conjunction with another optimisation problem. For example in [9], where Ermakov *et al.* co-optimised the individual floater scales of an array of WECS as well as their individual positions.

#### IV.1.4 IN SUMMARY

This section has shown there is a variety of methods for optimising floater geometries to varying degrees of fidelity. The Froude scaling approach is selected for this work as, due to the amount of computation that is already required to optimise the PTO design, the Froude scaling's reduced computation cost is a key benefit over the other methods.

Whilst Froude scaling can not directly decide if the optimal floater design is narrow- or broad-banded; the optimal device could be found indirectly by running the model multiple times with different reference floater designs.

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<sup>i</sup>This is highly dependant on the number of panels and the computational power, so this value is only presented as a rough figure.

## *Control Co-design of Floaters for use in Wave Energy Converters*

A question is posed: do the benefits of control co-design (CCD) compound when co-optimising multiple components of a wave energy converter (WEC). This section describes the theory of Froude scaling; and then applies it to a range of floater scales in WECs with a pre-optimised power take off (PTO) design – as a form of serial CCD – to investigate if further improvements can be found over the nominal-scale floaters. In § IV.3, the optimisation of floater and PTO is combined to co-optimize the two components. Then, in § IV.4, the performance of WECs in varying levels of CCD is compared.

### IV.2.1 THEORY OF FROUDE SCALING

Varying the size of an object always introduces scaling effects [10] due to different phenomena scaling with different properties of the device. For example, friction scales with surface area and gravitational forces scale with volume (assuming density is fixed). A variety of force ratios exist to measure the impacts of different scaling effects. These include the Froude number (inertial force over gravitational force); the Reynolds number (inertial force over viscous force); and Weber force (inertial force over surface-tension force). The inertial force is normally most relevant in fluid dynamics, hence its inclusion in all of the ratios [10].

For the vast majority of WECs – including the WECs used in this work – the Froude number is considered the most important [10]<sup>i</sup> and is given by

$$\text{Fr} = \frac{v}{\sqrt{gL}}, \quad (\text{IV.2.1})$$

where  $\text{Fr}$  is the Froude number;  $v$  is the velocity of the floater;  $g$  is the acceleration due to gravity; and  $L$  is the characteristic length of the floater. Defining the length scale of the floater to scale via

$$L_a := \lambda L_b, \quad (\text{IV.2.2})$$

where  $\lambda$  is the scale factor, and subscripts ‘a’ and ‘b’ denote the parameters of two scales of the same WEC. The scaling factors of the other properties of the WEC can be

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<sup>i</sup>In other types of WEC the dynamics of the device do not Froude scale, so another scaling technique would be required. An example of this is oscillating water columns, in which the air stiffness does not Froude scale [11].

found with two assumptions: the Froude number of the two scales is identical; and the density of the floater is constant.

From the assumption of constant Froude number,

$$\frac{v_a}{\sqrt{gL_a}} = \frac{v_b}{\sqrt{gL_b}}, \quad (\text{IV.2.3})$$

from substituting in EQUATION (IV.2.2), one finds that the velocity scales with  $\lambda^{1/2}$ :

$$v_a = \lambda^{1/2} v_b. \quad (\text{IV.2.4})$$

Then from dimensional analysis, it follows that time also scales with  $\lambda^{1/2}$ .

Under the assumption of constant density of the floater,<sup>ii</sup>

$$\frac{M_a}{V_a} = \rho = \frac{M_b}{V_b}; \quad (\text{IV.2.5})$$

where  $M_a$  and  $M_b$  are the masses of the two scales of floater;  $V_a$  and  $V_b$  are the volumes; and  $\rho$  is the density. The volume and mass scales with  $\lambda^3$ :

$$V_a = \lambda^3 V_b; \quad (\text{IV.2.6})$$

$$M_a = \lambda^3 M_b. \quad (\text{IV.2.7})$$

From these three properties – length scaling with  $\lambda$ , time scaling with  $\lambda^{1/2}$ , and mass scaling with  $\lambda^3$  – all other relevant properties can be found using dimensional analysis. All the factors for the properties scaled in this thesis are given in TABLE IV.2.1: a point to note is that the mass has been altered to scale with  $\lambda^2$  because the floater is better modelled as a thin shell rather than a solid object.

## IV.2.2 VARIABLE FROUDE SCALING OF FLOATERS

The optimal buoy scale is found by brute forcing<sup>iii</sup> a range of buoy scales; then for each scale of buoy, the control optimisation routine is run to find the best control for that WEC in each sea state.

The results for each WEC type are shown in FIGURES IV.2.1, IV.2.2, and IV.2.3 for the Tyne-Tees, Isles of Scilly, and Liverpool Bay sites respectively. The biggest determiner of WEC performance is the reference floater design, so the analysis of results is separated accordingly.

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<sup>ii</sup>The density of a floater is required to at least stay somewhat similar to its original value because the water density does not change, so any changes to the density would effect the floaters buoyancy. In some wave tanks there is a slight discrepancy in water density due the tanks tending to use fresh water ( $\sim 1000 \text{ kg/m}^3$ ) instead of salt water ( $\sim 1025 \text{ kg/m}^3$ ) which can introduce some *density effects* – see [12] for more details.

<sup>iii</sup>Another form of optimisation technique would have likely been more computationally efficient method for calculating the best buoy scale, but the brute forcing method is chosen to be able to produce figures to better understand the relationship between buoy scale and electrical power. Although the current method is not time intensive either: taking 0.05 to 0.25 s per test point (the time for floater scaling, control optimisation, and power simulation).

TABLE IV.2.1: Froude scaling factors for the floater variables [13].

| VARIABLE                     | SCALE FACTOR     |
|------------------------------|------------------|
| Mass                         | $\lambda^2$ *    |
| Frequency                    | $\lambda^{-0.5}$ |
| Added mass                   | $\lambda^3$      |
| Radiation damping            | $\lambda^{2.5}$  |
| Stiffness                    | $\lambda^2$      |
| Excitation force coefficient | $\lambda^2$      |
| Force                        | $\lambda^3$      |
| Speed                        | $\lambda^{0.5}$  |
| Power                        | $\lambda^{3.5}$  |

\* This is changed from  $\lambda^3$  to model the floater as a thin shell, see the end of §IV.2.1 for more details.

#### IV.2.2.1 SCALING TYPE-B FLOATERS

Across all the sites, the broad-banded TYPE-B floater optimised WECs follow the same shape: a parabolic curve with differing gradients on each side of the minima. To the left (smaller floaters) the gradient is much steeper than to the right (larger floaters), suggesting that it is better to have a slightly oversized floater than for it to be slightly undersized.

The optimal size varies according to location, and control type. Generally, the more energetic the site, the larger the floater. Impedance matching (IM) control generally prefers a smaller floater to linear damping (LD), likely due to the LD-controlled WECs needing the floater to match the resonant frequency of the site, whilst the IM-controlled WECs can operate effectively further from resonance, so favour a slightly smaller, lighter, more-nimble floater.

#### IV.2.2.2 SCALING TYPE-N FLOATERS

Results for the narrow-banded TYPE-N floater are very different to those of the TYPE-B floater. The relationship between floater scale and electrical power is not smooth: there are many local minima and maxima along the curve. This can, at least partly, be explained by the overheating limits in the PTO as the different sized buoys react differently to the waves. Take, as an example, the LD-controlled TYPE-N WEC at the Tyne-Tees site, in FIGURE IV.2.1: there is a sudden reduction in power between 0.8

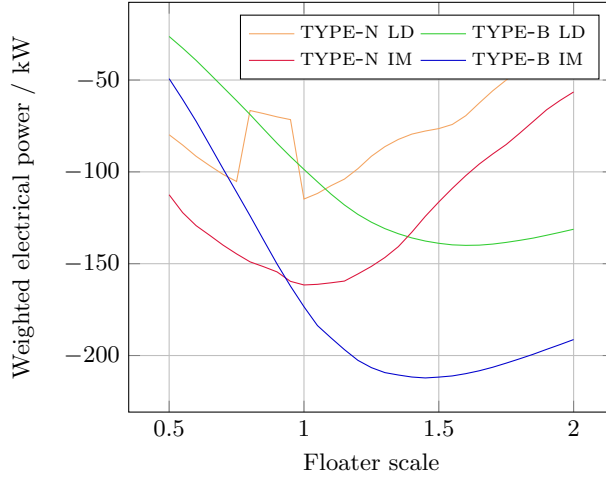


FIGURE IV.2.1: Weighted electrical power for WECS at the Tyne-Tees WaveNet Site, as a function of floater scale.

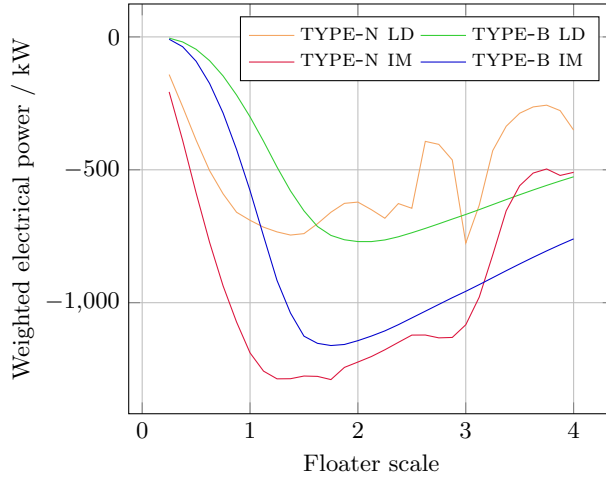


FIGURE IV.2.2: Weighted electrical power for WECS at the Isles of Scilly WaveNet Site, as a function of floater scale.

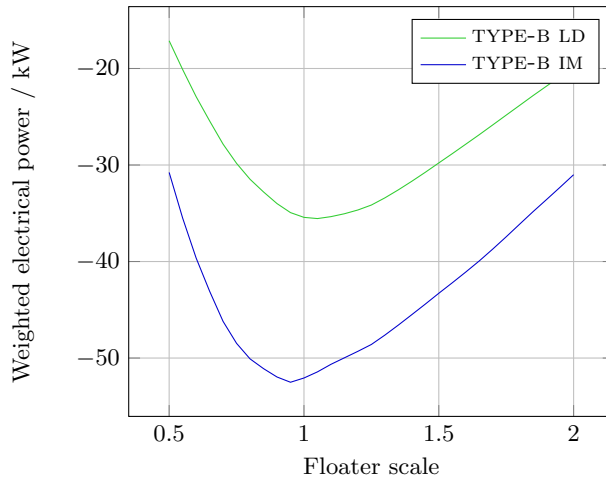


FIGURE IV.2.3: Weighted electrical power for WECS at the Liverpool Bay WaveNet Site, as a function of floater scale. There are no TYPE-N-WEC results because there are no successfully optimised PTOs for that floater type at Liverpool Bay, see § III.3 for more details.

and 0.95 times floater scale. In this region one of the sea states pushes the WEC into a zone in which the WEC can not operate safely and so the power for this sea state is 0 W. This is the largest of the sea states, that would otherwise contribute significantly to the overall power production of the WEC. Other discontinuities in power production are caused by the WEC approaching the limits so the control optimisation selects a controller that allows the WEC to operate within thermal limits.

#### IV.2.2.3 DISCUSSION OF RESULTS

For all but one of the 10 WECs scaled in this section, the best performing floater scale was not the nominal size. This shows that the best performing WEC can not be made by optimising the PTO alone, as other components (*e.g.* the floater, moorings, *etc.*) must be accounted for in the CCD procedure as well.

The one WEC that is best performing at nominal scale, is the TYPE-N LD-controlled WEC at Tyne-Tees. This WEC has the thermal limit problems slightly below nominal scale, so it is entirely possible that, had the WEC been optimised for this slightly smaller floater, the PTO would have been made larger to cope with the larger forces, and the WEC would perform better.

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## *Control Co-design of Power Take-off Systems & Floaters for use in Wave Energy Converters*

This section combines the power take-off (PTO) and floater optimisation stages – from § II and § IV.2.2 respectively – into a single optimisation process. Here, the methodology is described, and the results of the simultaneous optimisation presented; comparison between serial and simultaneous optimisation results is the focus of the next section.

Preliminary versions of results presented here were presented at the 16<sup>th</sup> European Wave and Tidal Energy Conference and are published in [14]<sup>i</sup>. In that work, the differences in PTO design and floater scale were investigated when varying control strategy, and location, when using the TYPE-B floater, using a 10-sea-state representation. However, following on from the findings of § II, the results for this section use the 20-sea-state representation of each of the sites (see APPENDIX A for tables of the sea states).

### IV.3.1 ALTERATIONS TO THE MODEL

To facilitate scaling within the genetic algorithm (GA), an extra gene  $\lambda_s$ , representing the floater scale, is added and a processing stage is added to scale the floater. The GA is free to scale the floater between 0.2 and 5 times nominal size. The updated model flow chart is shown in FIGURE IV.3.1. Hydrodynamic properties of the floater are scaled according to the appropriate Froude scaling factors: given in TABLE IV.2.1.

Finally, it is noted that there are no alterations to the cost function to account for the differences in cost to building varying sizes of floaters. This follows on from the ambition in formulating of the original cost function: to minimise assumptions (see § II.3.3). When unconstrained, the generator tends towards an incredibly large device (as those are more efficient). As such the material cost is used as a constraint; the floater scaling however, does not exhibit the behaviour and the optimal scale can be larger or smaller depending on the site conditions (as is seen in § IV.2.2). It is therefore decided to not account for a floater material cost in the cost function.

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<sup>i</sup>Note that in the paper the site labelled ‘North Sea’ is the same site as ‘Tyne-Tees’ in this thesis.

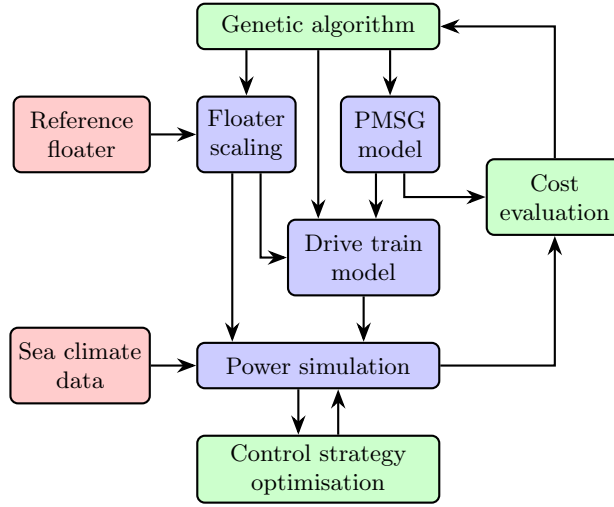


FIGURE IV.3.1: Flow chart of the optimisation model including floater scaling. Red blocks show data inputs; blue blocks show physical models; and green blocks show optimisation tools.

#### IV.3.2 CO-OPTIMISATION OF FLOATER SCALE AND POWER TAKE-OFF DESIGN

The adapted model is run for a similar set of runs to the varying location investigation in § III.3: three times for each floater and control type pairing at each location. The final costs for these runs are shown in FIGURE IV.3.2. When comparing to the similar results for optimising just the PTO (FIGURE III.3.3 in § III.3.1), there are differences in the relative performance of the different WEC types. Now at every location, the ordering of WEC performance from worst to best is: LD-controlled TYPE-N, IM-controlled TYPE-N, LD-controlled TYPE-B, and IM-controlled TYPE-B. When optimising just the PTO, performance between floater type was similar at Tyne-Tees; and TYPE-N outperforms TYPE-B at the Isles of Scilly.<sup>ii</sup> When only optimising the PTO at the Liverpool Bay site, there is not a single successful run of the optimiser out of the three attempts<sup>iii</sup>, but when co-optimising with the floater two-of-the-three runs for each control type created a successful PTO-floater design. This shows that by adding the floater design into the control co-design process there is a larger design space in which to find suitable WEC designs. However, the fact that one-of-the-three runs still failed shows some limitations in the genetic algorithm’s ability to explore the design space – this could possibly be solved by increasing the number of individuals in the population, but would have negative effects on the total runtime of the optimiser. Ignoring those failed runs, the results – as when just optimising the PTO – converge very well: having a maximum percentage difference between like runs of 3.88%, 2.51%, and 0.490% for the Liverpool Bay, Tyne-Tees, and Isles of Scilly sites respectively.

<sup>ii</sup>The reasons for this are discussed in § III.3; put briefly it is due to the resonance of the TYPE-N floater aligning well with the sites.

<sup>iii</sup>Mainly due to resonance reasons: due to very poor alignment between the floater and the site conditions. Compounded with the fact that the site does not have much energy to harness, designing a WEC under these conditions is incredibly hard. See § III.3.

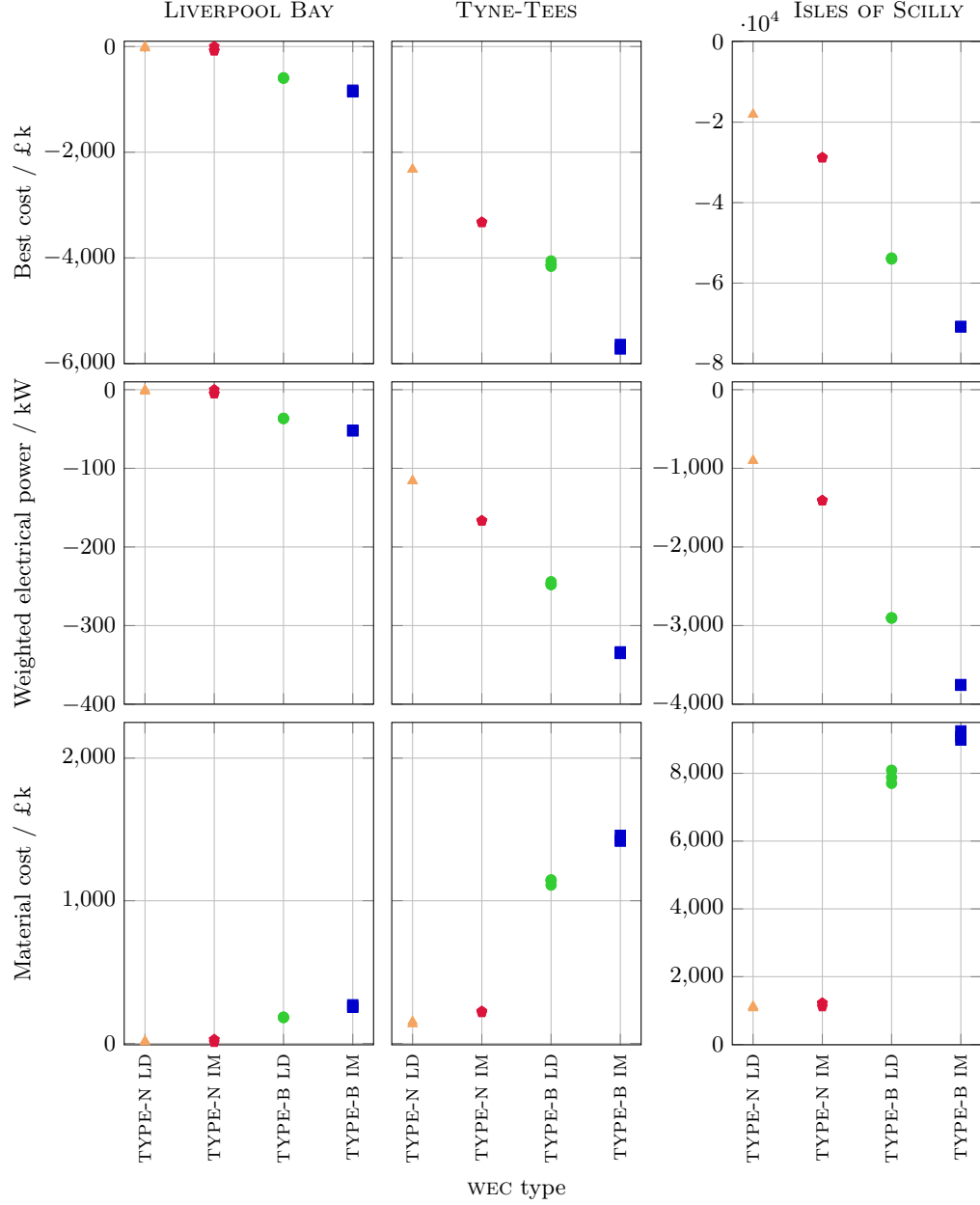


FIGURE IV.3.2: Best cost solutions from the PTO and floater co-optimiser, when varying the site location. Note that due to the Isles of Scilly site being so much more energetic than the others, the y-axis on its plots works to a different scale – but they continue to have the same units.

#### IV.3.2.1 DIFFERENCES IN FLOATER AND POWER TAKE-OFF DESIGN

The final genes for each of the runs is shown in FIGURE IV.3.3; there are two significant points of note. Firstly, the genes for the floater scale ( $\lambda_{fs}$ ) are much more stable than the genes determining the PTO – excluding the two Liverpool Bay runs that returned a positive cost function value. Secondly, the immense length of the generators ( $l_g$ ) for the TYPE-B WECs at the Isles of Scilly. These points are investigated in more detail in turn.

The concluding discussion of § III introduced the idea of *families* of solutions: the idea that the cost function (as it currently exists) does not limit the design space enough such that there is a single PTO design that gives that best cost function value, but instead a family of solutions that all perform to practically the same level. This explains the variability in the gene values. For the floater scale however – ignoring the two failed runs – the gene values are very similar: having maximum percentage differences between like runs of 5.16%, 1.05%, and 0.455% for the Liverpool Bay<sup>iv</sup>, Tyne-Tees, and Isles of Scilly sites respectively. This means that the cost function has reduced the family of solutions to a set with very little floater scale variance. The inclusion of floater scaling does not appear to have any significant effect on the variance of the PTO genes.

The length of the generators for the TYPE-B WECs at the Isles of Scilly site are around 40 and 50 m for LD- and IM-controlled respectively – the upper limit for the gene is 50 m so the *optimal* may be an even larger value. This is obviously absurd, and shows the limits of the model as it exists: without accounting for factors such as manufacturability and installation of the WEC, the floater<sup>v</sup> and PTO are free to balloon in size (assuming there are the waves large enough to require such a behemoth). To properly account for manufacturability and installation would require a more detailed cost analysis of the various stages of a WEC development – introducing several assumptions. But as a simple first pass, encouraging smaller generators helps towards both goals. Hence, an alteration to the cost function is proposed:

$$C_{vol} = \frac{C_{WEC}}{\pi r_{sb}^2 l_g}, \quad (IV.3.1)$$

where  $C_{WEC}$  is the standard cost function (defined in EQUATION (II.3.14));  $r_{sb}$  is the outer radius of the stator backiron (the outer radius of the generator); and  $l_g$  is the

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<sup>iv</sup>The Liverpool Bay has repeatedly been a site found to be ill-suited to the installation of WECs which could explain why its variance is larger: *i.e.* it is just very hard to find a good design at this site.

<sup>v</sup>A related concern with the increased floater size is that the floater has been scaled such that its diameter is roughly 80 m – well above the guidelines for size relative to the waves. Count suggests that the floater be roughly  $1/20^{\text{th}}$  the wavelength for a point absorber [15]. Assuming deep water, the wavelength of the waves is approximately 156 m (for a 10 s period); this puts the floater well above the suggested size, at approximately  $1/2^{\text{th}}$ : likely causing the assumptions of point absorber motion to break down.

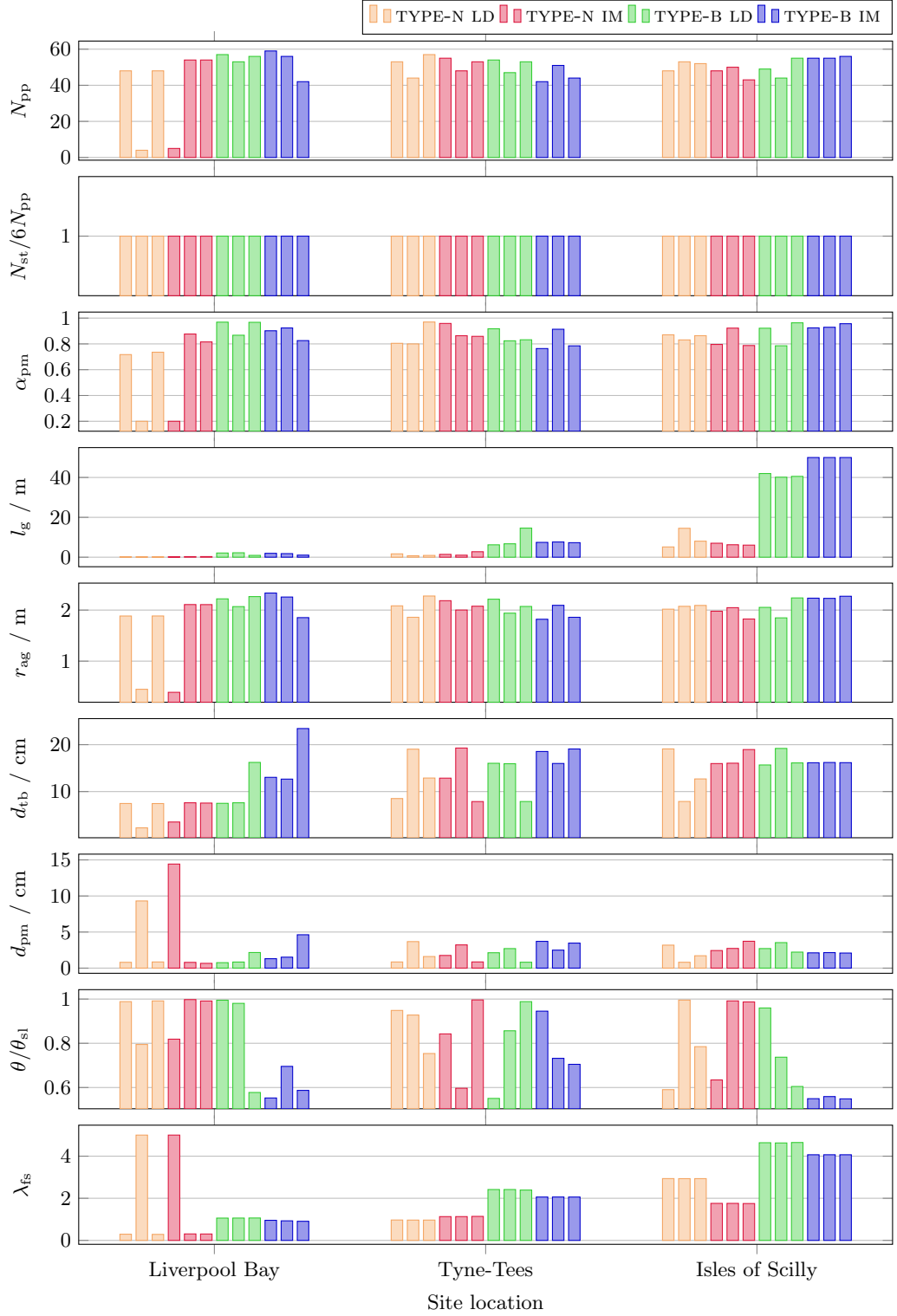


FIGURE IV.3.3: Optimised genes for multiple runs of the PTO and floater co-optimiser, when varying the site location. Note that the first two genes are fixed to integer values.

TABLE IV.3.1: A selection of properties for WECs optimised to two different cost functions and control strategies. WECs are optimised for a TYPE-B floater at the Isles of Scilly site.

| PROPERTY                      | OPTIMISED TO $C_{\text{WEC}}$ WITH |            | OPTIMISED TO $C_{\text{vol}}$ WITH |            |
|-------------------------------|------------------------------------|------------|------------------------------------|------------|
|                               | LD-control                         | IM-control | LD-control                         | IM-control |
| $C_{\text{WEC}} / \text{£ m}$ | -54.0                              | -70.8      | -3.96                              | -4.29      |
| $C_{\text{vol}} / \text{£ k}$ | -68.2                              | -72.8      | -1460                              | -1590      |
| $P_{\text{avg}} / \text{kW}$  | -2900                              | -3750      | -194                               | -209       |
| $C_{\text{gen}} / \text{£ k}$ | 7700                               | 8980       | 162                                | 162        |
| Volume / $\text{m}^3$         | 792                                | 973        | 2.71                               | 2.70       |
| Axial length / m              | 40.5                               | 50.0       | 0.213                              | 0.232      |
| Floater scale factor          | 4.65                               | 4.07       | 1.30                               | 0.979      |

$C_{\text{WEC}}$  and  $C_{\text{vol}}$  are the two cost functions;  $P_{\text{avg}}$  is the weighted electrical power output; and  $C_{\text{gen}}$  is the material cost of the generators.

axial length of the generator. A selection of properties for WECs optimised to both cost functions are found in TABLE IV.3.1. Using this alternative cost function has brought down the axial lengths to much more reasonable values around 0.2 m; and due to the generator being much smaller, the floaters have been scaled down as well to match the generators. This shows that through the inclusion of more properties to the cost function, a designer can better define the WEC they wish to produce.

#### IV.3.2.2 DIFFERENCES IN COMPUTATIONAL EFFORT

Finally, for completeness, the number of iterations and runtime for the co-optimised runs are shown in FIGURE IV.3.4. The trends are broadly similar to those when just optimising the PTO so the reader is directed to §III.3.4 for that analysis. The only difference of note is that the very long runtimes seen for the IM-control runs seen when just optimising the PTO are not as extreme here: they still exist but not are not as large. This suggests that by scaling the PTO, the IM-optimisation overheating fallback is not needed as often.

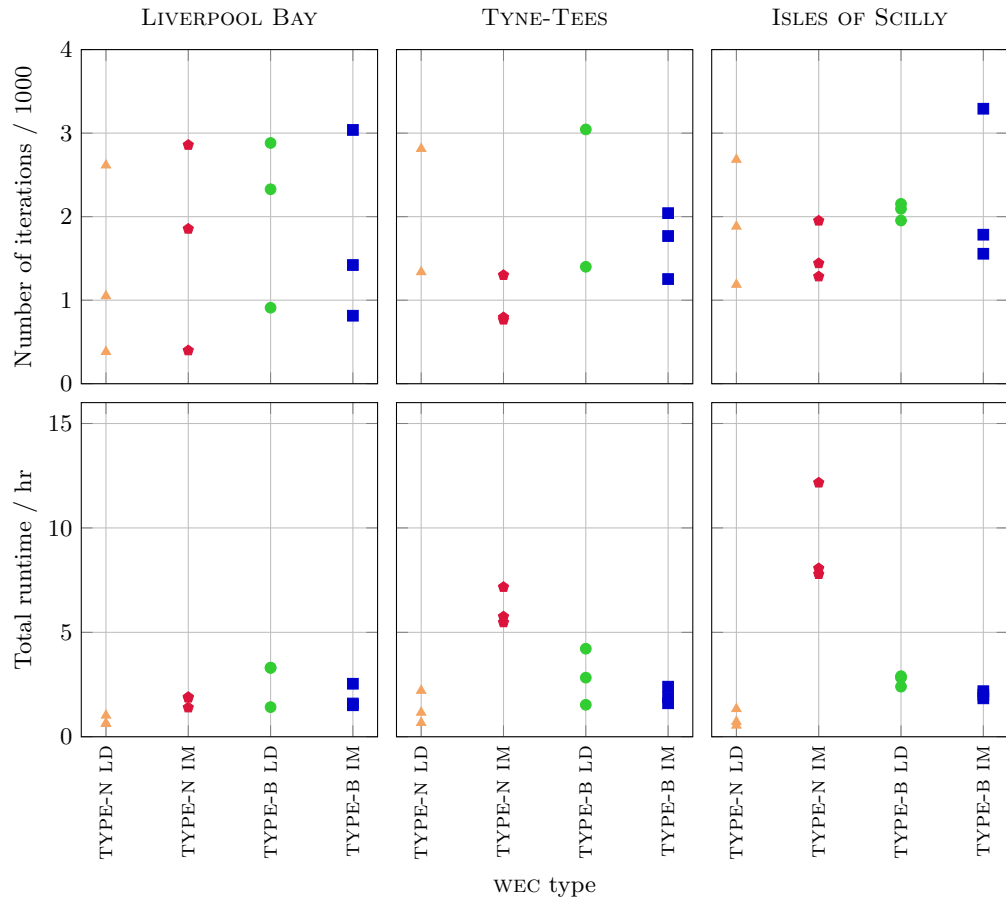


FIGURE IV.3.4: The number of iterations, and total runtime of the PTO and floater co-optimiser, when designing for different site locations.

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### *Comparing Levels of Control Co-design Integration*

Throughout the thesis results are presented of increasing levels of control co-design (CCD) integration. These are: optimising just the power take-off (PTO), in § III; serially optimising the PTO and then the floater scale, in § IV.2; and simultaneously optimising the PTO and floater scale in § IV.3. Here, results from these three sections are compared to investigate the improvements in power production from increasing the levels of integration in the CCD process.

Due to the Liverpool Bay site not producing any PTO-only and serial optimised results for a TYPE-N floater, and the Isles of Scilly site having problems with its TYPE-B floater results, only Tyne-Tees results are compared in this section. The comparison is found in FIGURE IV.4.1. For all wave energy converter (WEC) types, the trends are the same: the PTO-only optimisation performs the worst – in terms of cost function and electrical power output, the simultaneous optimisation performs the best, and the serial optimisation in between – but closer to the PTO-only optimisation: showing that the benefits of CCD compound when more of the WEC design process is included.

The floater scale is also different between the serial and simultaneous optimisations meaning that there is a multidirectional dependancy between the WEC components: *i.e.* the PTO design does not just depend upon the floater design, the floater design depends upon the PTO design as well. Whilst only scaling the floater design does not give the optimiser complete freedom to find the optimal design, the work presented here shows that CCD is important, and further improvements in the fidelity of the WEC design should only lead to further improvements in performance.

When combined with the results of § III.3 – which found that the poor matching of floater to site conditions can counteract the benefits of PTO CCD entirely – the case for the co-optimisation of floater and PTO is clear.

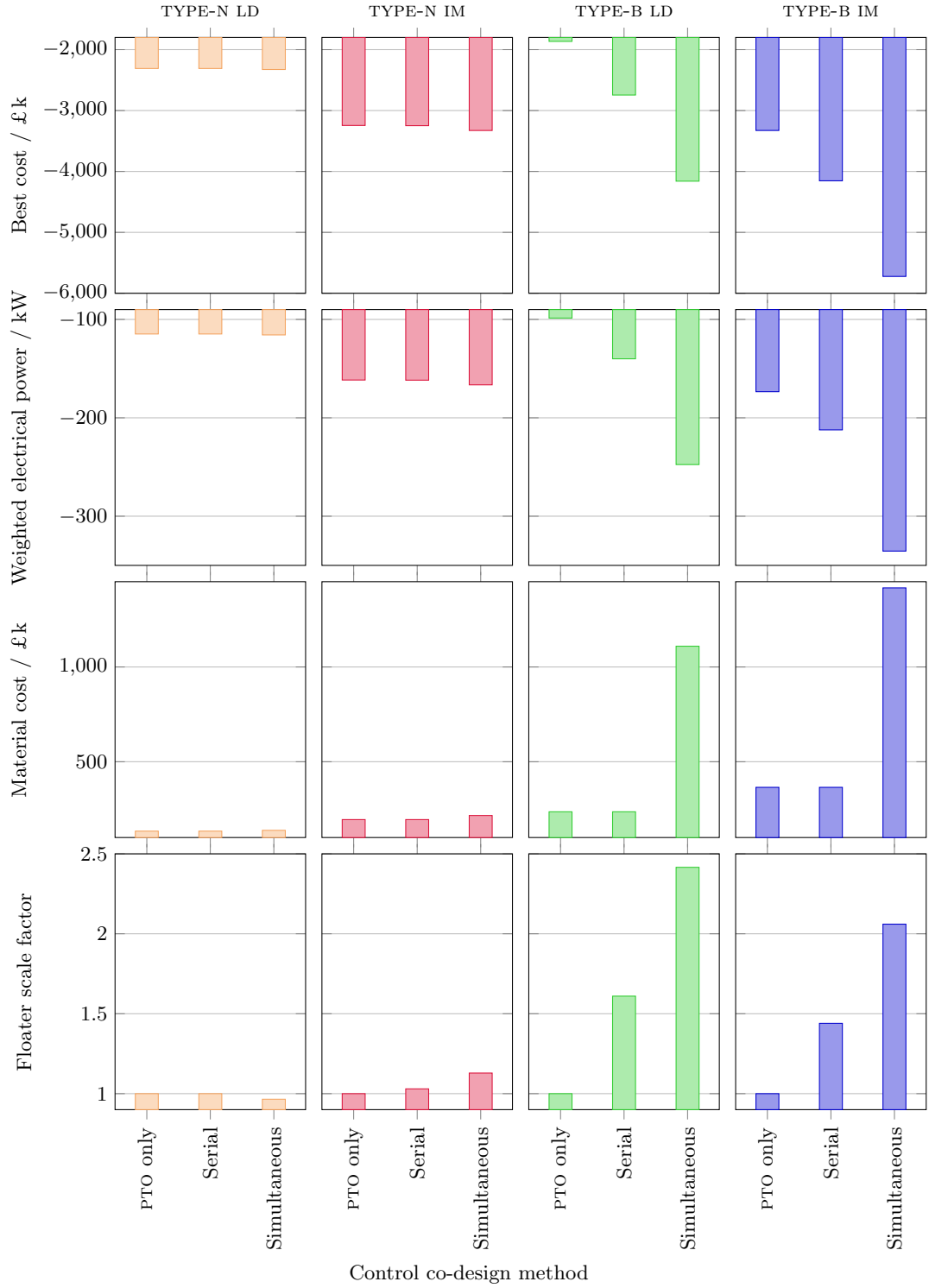


FIGURE IV.4.1: Best cost solutions from the optimiser with varying levels of control co-design integration. WECs are optimised for the Tyne-Tees site.

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## Bibliography

- [1] Bingyong Guo and John V. Ringwood. “Geometric optimisation of wave energy conversion devices: A survey”. In: *Applied Energy* 297 (2021). DOI: [10.1016/j.apenergy.2021.117100](https://doi.org/10.1016/j.apenergy.2021.117100) (cited on page 157).
- [2] Paula B. Garcia-Rosa and John V. Ringwood. “On the Sensitivity of Optimal Wave Energy Device Geometry to the Energy Maximizing Control System”. In: *IEEE Transactions on Sustainable Energy* 7.1 (2016). DOI: [10.1109/TSTE.2015.2423551](https://doi.org/10.1109/TSTE.2015.2423551) (cited on page 157).
- [3] Mehdi Neshat et al. “Multi-Mode Wave Energy Converter Design Optimisation Using an Improved Moth Flame Optimisation Algorithm”. In: *Energies* 14.3737 (2021). DOI: [10.3390/en14133737](https://doi.org/10.3390/en14133737) (cited on page 157).
- [4] Elie Al Shami et al. “A parameter study and optimization of two body wave energy converters”. In: *Renewable Energy* 131 (2019), pp. 1–13. DOI: [10.1016/j.renene.2018.06.117](https://doi.org/10.1016/j.renene.2018.06.117) (cited on page 157).
- [5] Emma Edwards. “Optimization of the Geometry of Axisymmetric Point-absorber Wave Energy Converters”. PhD thesis. Massachusetts Institute of Technology, 2020 (cited on page 158).
- [6] A. P. McCabe. “Constrained optimization of the shape of a wave energy collector by genetic algorithm”. In: *Renewable Energy* 51 (2013), pp. 274–284. DOI: [10.1016/j.renene.2012.09.054](https://doi.org/10.1016/j.renene.2012.09.054) (cited on pages 158, 159).
- [7] Anna Garcia-Teruel. “Geometry Optimisation of Wave Energy Converters”. PhD thesis. University of Edinburgh, 2020 (cited on pages 158, 159).
- [8] V. Raghavan et al. “A comparative study on BEM solvers for Wave Energy Converters”. In: *Trends in Renewable Energies Offshore* (2022). DOI: [10.1201/9781003360773-50](https://doi.org/10.1201/9781003360773-50) (cited on page 160).
- [9] Andrei M. Ermakov et al. “Optimisation of heterogeneous wave energy converter arrays: A control co-design strategy”. In: *Renewable Energy* 244.122637 (2025). DOI: [10.1016/j.renene.2025.122637](https://doi.org/10.1016/j.renene.2025.122637) (cited on page 160).
- [10] V. Heller. “Comprehensive Renewable Energy”. In: Elsevier, 2012. Chap. 8.04.3.2. DOI: [10.1016/B978-0-08-087872-0.00804-0](https://doi.org/10.1016/B978-0-08-087872-0.00804-0) (cited on page 161).

- [11] Krisna Adi Pawitan. “Wave loadings and scaling effects on and within an Oscillating Water Column (OWC) caisson breakwater”. PhD thesis. University of Edinburgh, 2019 (cited on page 161).
- [12] Bernad Le Méhauté. “An Introduction to Hydrodynamics & Water Waves”. In: Springer-Verlag, 1976. Chap. B.3. ISBN: 9780387072326 (cited on page 162).
- [13] M. Islam, F. Jahra, and S. Hiscock. “Data Analysis Methodologies for Hydrodynamic Experiments in Waves”. In: *Journal of Naval Architecture and Marine Engineering* (2016). DOI: [10.3329/jname.v13i1.25347](https://doi.org/10.3329/jname.v13i1.25347) (cited on page 163).
- [14] Matthew Onslow and Adam Stock. “Genetic algorithm based co-design of wave energy converter power take-off, floater geometry, and control”. In: *Proceedings of the 16th European Wave and Tidal Energy Conference*. 2025. DOI: [10.36688/ewtec-2025-1013](https://doi.org/10.36688/ewtec-2025-1013) (cited on page 166).
- [15] B. M. Count. “Power from Sea Waves”. In: ed. by Brian Count. Academic Press, 1980. Chap. 2.3. ISBN: 0-12-193550-7 (cited on page 169).

## CHAPTER V

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# *The OSPREY-I Wave Energy Converter & Polychromatic Power Simulations*

This chapter re-applies the floater scaling methodology from §IV to the scaling of the power take-off (PTO) in a novel method to develop a *synthetic* PTO model for a wave-tank scale PTO. The synthetic PTO designs are then used in a polychromatic-wave power simulation to better understand the importance of PTO dynamics in the overall performance of a wave energy converter.

## *The HAPiWEC Power Simulation Model*

As a final point of investigation, this chapter utilises a polychromatic-wave power simulation to test the validity of using a monochromatic-wave simulation within the power take-off optimisation. The polychromatic simulation is introduced and described in this section, and the results of using it are in the next section.

The polychromatic-wave power simulation used was created as a part of the HAPiWEC (Holistic Advanced Prototyping and Interfacing for Wave Energy Control) project. The project sought to create remotely accessible hardware and tools for the rapid prototyping of novel control algorithms, see [1] for more details. The HAPiWEC project is solely at wave-tank scale. As such the polychromatic power simulation, and the WEC used are also at wave-tank scale and alterations are made to the optimiser model to account for the change in scale. This section first details the changes made to the optimiser to create a tank-scale power take-off (PTO) in § V.1.1; then the operation of the wider polychromatic power simulation are described in § V.1.2.

### V.1.1 POWER TAKE-OFF DESIGNS FOR THE OSPREY-I WAVE ENERGY CONVERTER

The OSPREY-I (Open System-wide Prototyping for Renewable Energy Yield Improvement) is a tank-scale WEC developed as part of the HAPiWEC project. The floater shape for OSPREY-I is a scaled down design from the COERbuoy1 WEC [3], a picture of the OSPREY-I in the tank is shown in FIGURE V.1.1 and a schematic of its design is in FIGURE V.1.2. The floater is connected to the generator via a taught cable; and a system of pulleys connects it to the generator, which puts a preload force on the floater of 850 N. The OSPREY-I is installed at the FloWave wave tank at the University of Edinburgh [4]: the nominal scale of FloWave is 1:20, the same scaling is therefore also chosen to scale the OSPREY-I floater. Due to limitations in the PTO-optimisation

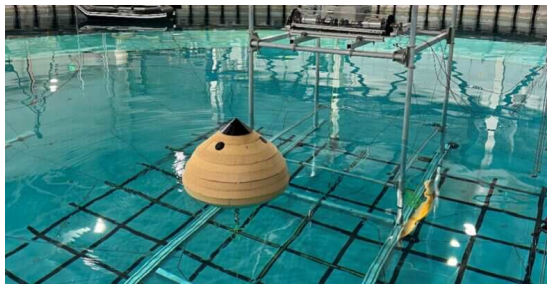


FIGURE V.1.1: Picture of the OSPREY-I WEC in the FloWave wave tank. (Picture provided by the HAPiWEC team; available at [2])

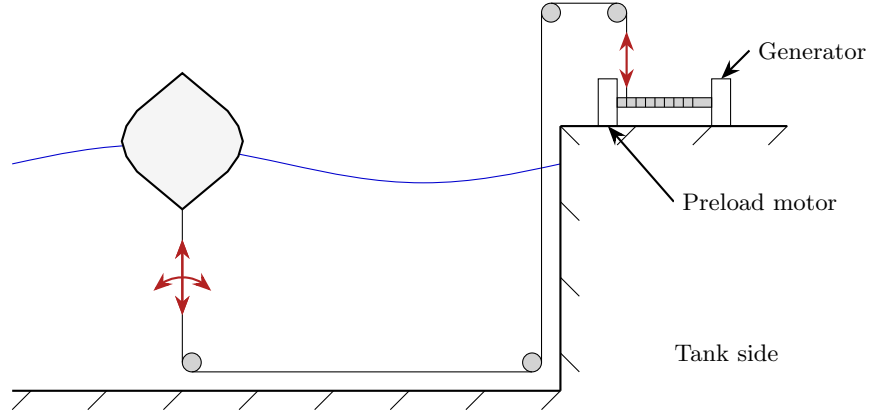


FIGURE V.1.2: Design of the OSPREY-I wave energy converter. Not to scale. (Adapted from [5])

model<sup>i</sup>, the OSPREY-I is modelled to move only in heave; but in the tank, it can move in six degrees of freedom.

As WEC at tank scale, the actual PTO built for OSPREY-I is much less efficient than its sea-scale counterpart would be: because electrical resistance scales according to  $1/\lambda$ , where  $\lambda$  is the scale factor, whilst forces scale with  $\lambda^3$ . So, as the size of a WEC decreases, the forces on it decrease rapidly, but the electrical resistance in the PTO increases reducing efficiency. Therefore, to get a more useful electrical efficiency conversion a *synthetic* PTO model would be useful: one that has the efficiency of a sea-scale PTO, but at the electrical power scale of a wave-tank-scale PTO. So far, Froude scaling has been applied to the floater only, but applying the same methods to down-scale the PTO, a synthetic PTO can be created.

To create a synthetic PTO the following procedure is taken: firstly, the OSPREY-I is scaled up to sea scale; then a PTO optimised for the upscaled device; and finally, the power map is scaled down to tank scale to get the synthetic PTO model. During a tank test, data of the mechanical speed and force of the OSPREY-I's motion can be recorded and a simulated electrical power output can be calculated as a post-processing step.

The methodology for creating the optimised PTO is broadly the same as previously discussed in § II.3. The only differences are the pre-optimisation scaling up of the floater; the post-optimisation scaling of the PTO; and a new test location.

When running wave tank tests, the range of sea states simulated must be within the range that the tank can accurately, and repeatedly produce. Draycott showed that it is possible to simulate the sea states of the European Marine Energy Centre (EMEC) site in the FloWave wave tank [6, 7]. Therefore, a recreation of the EMEC site that

<sup>i</sup>This is a limitation in the model as it exists currently; it could be hypothetically expanded to account for more degrees of freedom.

TABLE V.1.1: Sea states for the OSPREY-I optimisation.

| TANK SCALE |              | SEA SCALE |              | WEIGHT / % |
|------------|--------------|-----------|--------------|------------|
| $T_p$ / s  | $H_{m0}$ / m | $T_p$ / s | $H_{m0}$ / m |            |
| 0.225      | 2.66         | 4.50      | 11.9         | 7.55       |
| 0.225      | 2.35         | 4.50      | 10.5         | 3.77       |
| 0.175      | 2.35         | 3.50      | 10.5         | 22.6       |
| 0.125      | 2.21         | 2.50      | 9.88         | 9.43       |
| 0.125      | 2.03         | 2.50      | 9.08         | 13.2       |
| 0.125      | 1.72         | 2.50      | 7.69         | 20.8       |
| 0.075      | 2.35         | 1.50      | 10.5         | 3.77       |
| 0.075      | 1.72         | 1.50      | 7.69         | 18.9       |

is chosen for both the optimisation of the synthetic PTO and the tank tests for the HAPiWEC project more generally.

Using the set of sea states in TABLE 4 of [6] and an incidence plot of the conditions at the EMEC site in [8], a set of sea states with relative weightings is created, as shown in TABLE V.1.1<sup>ii</sup>.

### V.1.2 THE POLYCHROMATIC POWER SIMULATION

The polychromatic power simulation was created in Simulink by Dr. David Forehand as a part of the HAPiWEC project; FIGURE V.1.3 gives a block diagram representation of its operation. It is a linear model of a WEC found by solving the Cummins equation, see § I.2.3.2 for more details on the general theory behind the use of the Cummins equation. The remainder of the subsection describes the specifics of the implementation in this simulation.

The radiation convolution is approximated by a state-space model: directly computing a convolution is computationally expensive, so it is commonplace to approximate it. As well as state-space representations, transfer functions are also commonly used; see [9] for a review. The preload force is due to the OSPREY-I being attached to the power take-off (PTO) via a tether; the preload is used to pretension the tether. It is fixed at 850 N. The wave spectrum used is a modified Pierson-Moskowitz spectrum using a linearly-spaced set of 541 wave frequencies between 1.25 and 8.00 s. The spectrum is then converted to a time series of wave elevations using the methodology explained

<sup>ii</sup>This set is missing a few sea states seen in TABLE 4 of [6]: some sea states went beyond the bounds of the incidence plot; and there were some repeats from the exclusion of the energy period that were removed.

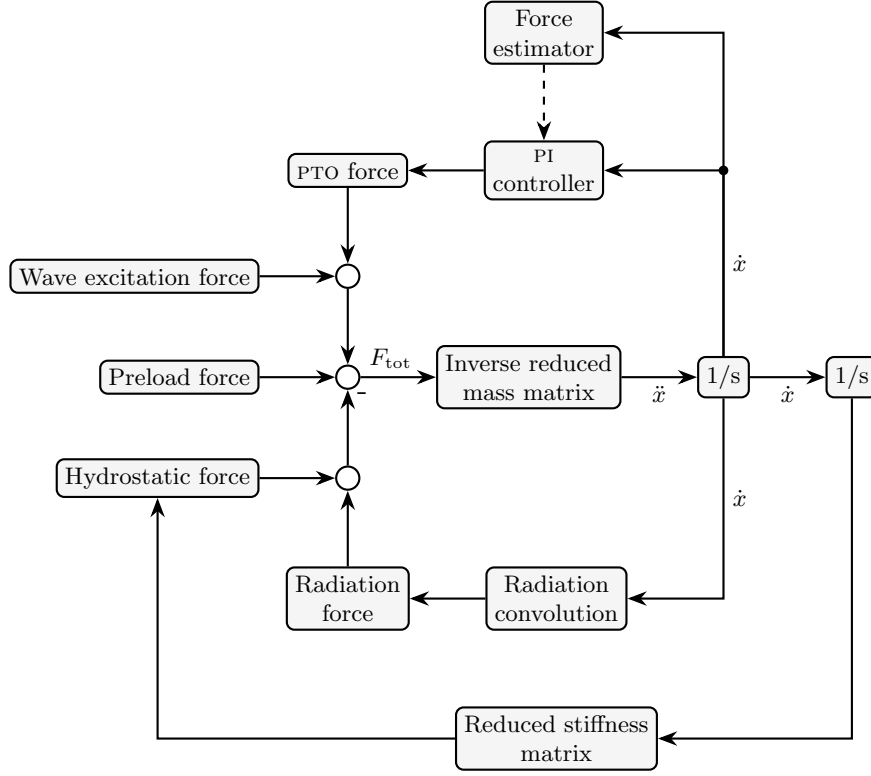


FIGURE V.1.3: Block diagram for the polychromatic power simulation for the OSPREY-I WEC.

in §I.1.3.2; and then to a time series of excitation forces using the excitation-force coefficients from WAMIT, and the relationship given in EQUATION (I.2.17).

Finally, the controller is based on the PI-design by Coe *et al.* [10]. This is not a regular PI controller – where the controller is before the plant and after the comparator. Instead the controller is on the feedback loop where it acts as a simplified model of the WEC’s impedance. Coe *et al.* [10] tuned their controller for mechanical power capture, but this has been changed to instead focus for electrical power capture. Using a similar methodology to that used in the PTO optimisation, a range of PTO amplitudes and phases are tested in a set of regular wave tests to find the best performing pair of values. The appropriate gains for the PI controller to achieve that response are found via

$$K_p - \frac{1}{i\omega} K_i = -\frac{A_{\text{PTO}} e^{i(\omega t + \phi_{\text{PTO}})}}{A_v e^{i(\omega t + \phi_v)}}, \quad (\text{V.1.1})$$

where  $K_p$  and  $K_i$  are the proportional and integral gains, respectively, and the rest of the parameters are as defined in §II.3.4. Evaluating for the set of regular wave tests creates a look-up table that is used within the wider HAPiGYM simulation to actively change the gains throughout the simulation by using an extended Kalman filter based

force estimator.

The controller is not activated from the beginning of the simulation; it turns on after 40 s: this is to give the force estimator the time to get a good estimate of the peak excitation force frequency before it is used. The average power is then measured from 60 s to the end of the simulation at 100 s. An example time series of various parameters is shown in [FIGURE V.1.4](#).

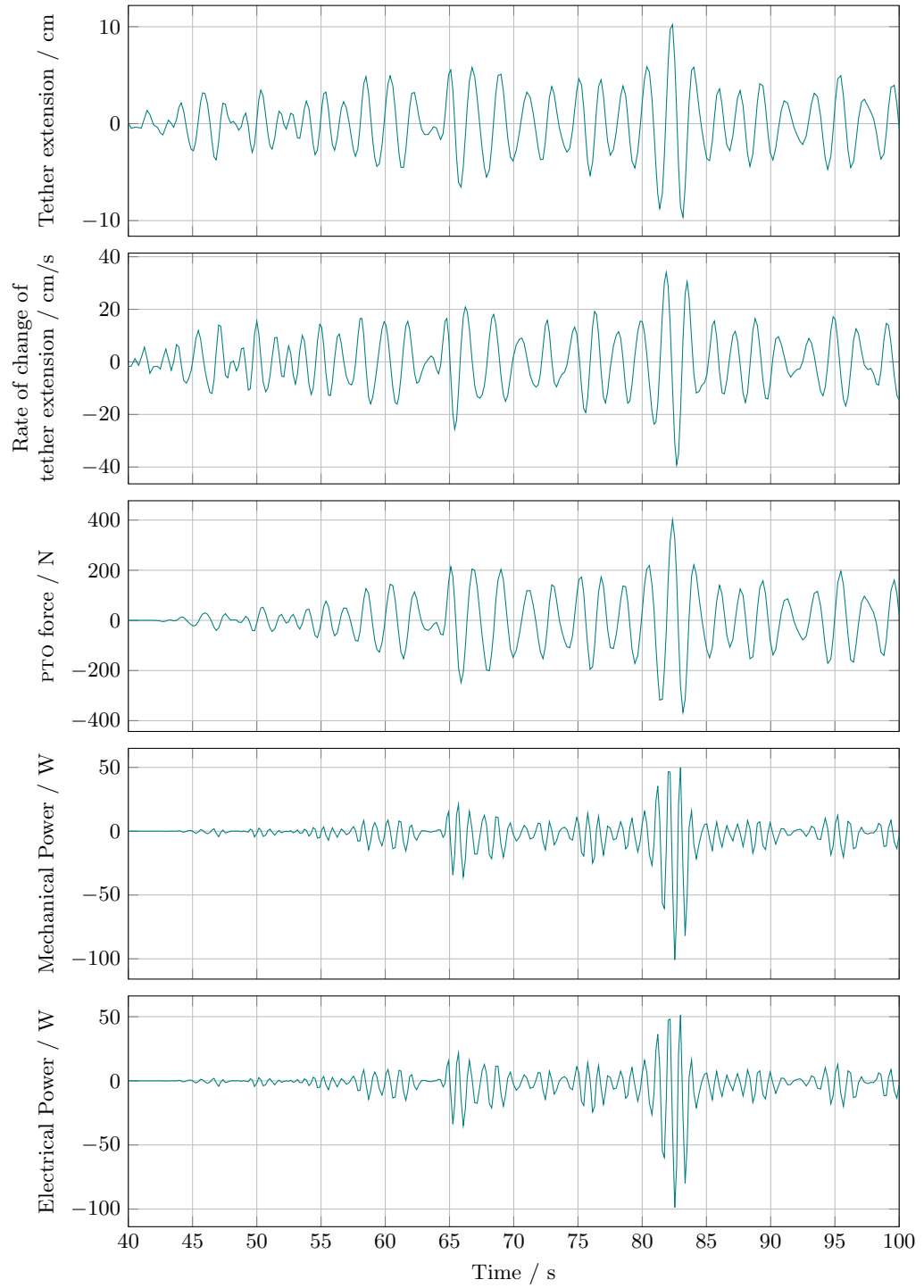


FIGURE V.1.4: Example time series of properties in a polychromatic power simulation.

## *Comparing Optimised and Non-optimised Power Take-off Designs*

Here, optimised and non-optimised power take-off designs for the OSPREY-I WEC are compared. The optimised design is created following the methodology in § V.1.1. The non-optimised design is the best solution found one quarter of the way through the same optimisation: this gives a generator for comparison that is partially designed to the OSPREY-I, but not optimised.

The breakdown of electrical power output per sea state is shown in FIGURE V.2.1: the trend that the non-optimised generator produced marginally more power – with an average power output increase of 1.0%, and a maximum increase of 2.4% – is consistent across all sea states. The final weighted average power for each design is 5.60 and 5.65 W for the optimised and non-optimised respectively.

Whilst it appears that the non-optimised generator performs better than the optimised, the cost function shows the benefits of the optimised design. FIGURE V.2.2 shows the cost function values when using the polychromatic and monochromatic power simulations: polychromatic power values at sea scale are found by Froude scaling the power

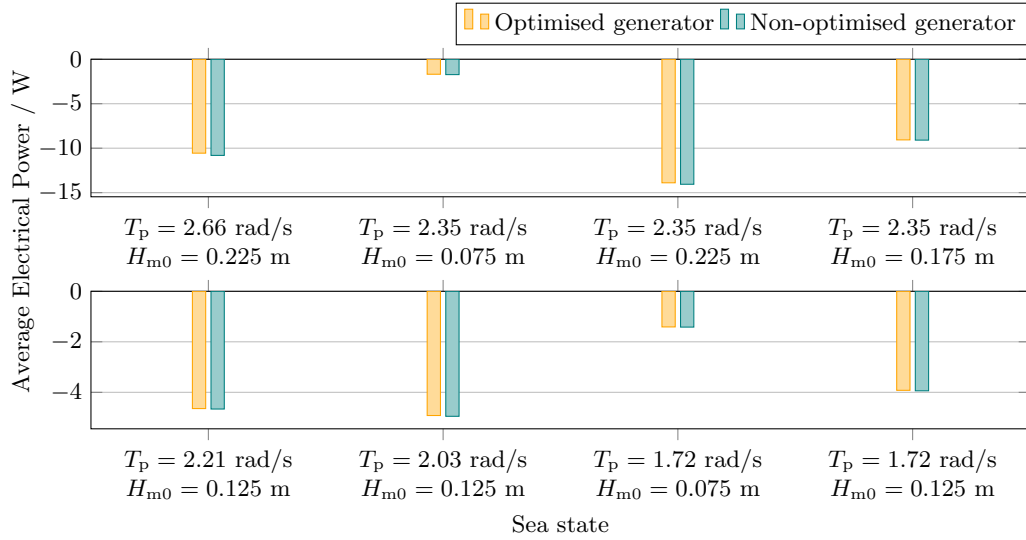


FIGURE V.2.1: Average electrical power output from the OSPREY-I WEC in a variety of polychromatic sea states using different generator designs.

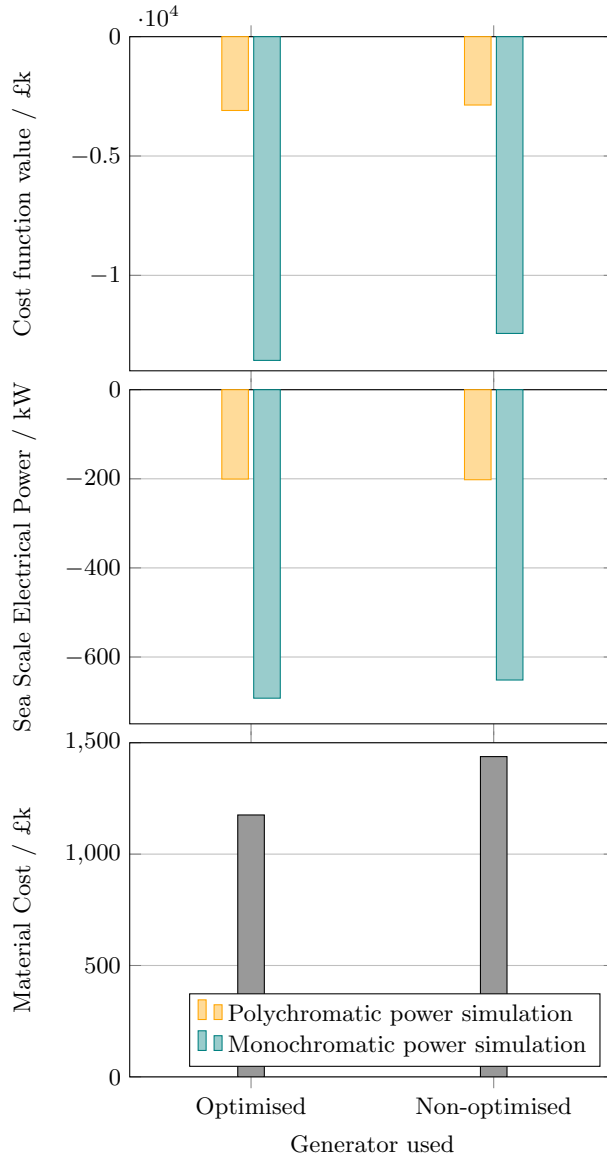


FIGURE V.2.2: Comparing the cost function for optimised and non-optimised generator designs for the OSPREY-I WEC in monochromatic and polychromatic power simulations. Both power simulations use the same generator, hence the material costs are the same for both.

up to the equivalent sea scale. The 1.0% increase in power output for the non-optimised design comes at a cost of a 22% increase in generator material cost. Obviously, that is not a desired trade-off and hence it was not the returned optimised design.

### V.2.1 POWER ESTIMATES IN MONOCHROMATIC AND POLYCHROMATIC SEAS

The biggest takeaway from this section is the differences in estimated power output between the monochromatic and polychromatic power simulations: the polychromatic power simulation gives a value less than one third (see FIGURE V.2.2) than that of the monochromatic power simulation in the example tested here. It is expected that the power output is lower; it can be found across the literature (*e.g.* in [11] and [12]) and

occurs because the WEC can no longer be controlled perfectly without future knowledge of the waves, as explained in §I.2.4.

The fact that both generators have basically the same electrical power output in the polychromatic simulation suggests that both are large enough to fully saturate the available power throughput in the given seas: looking into the PTO force demands throughout the power simulations shows the maximum force demand is 980 and 990 N for the optimal- and non-optimal-generator WECs when the force limits are 1070 and 1000 N respectively. Conversely, there is a 5.9% difference between power outputs in the monochromatic power simulation, and in them, force limits are being hit – for both generators. Therefore, it is plausible that an optimisation run using the polychromatic power simulation would return a smaller, cheaper generator design; the problem in doing so is the time to run each of the simulations.

In the monochromatic case, the power simulation and control strategy optimisation happens in tandem for each sea state, taking approximately 5.5 ms. In the polychromatic case, the controller can be tuned once for each design then used in all sea states (within the defined tuning window); to tune the controller takes approximately 55 s, and to run one sea state simulation takes 2 s of computation time. Therefore to compute a suite of 20 sea state simulations takes roughly 0.11 and 95 s in the mono- and poly-chromatic cases respectively<sup>1</sup>; this would need to be repeated hundreds of times per generation of the genetic algorithm, for hundreds of generations. There are options that could improve the runtime of the program: such as reducing the number of sea states, but §III found that reducing the number of sea states affects the design of the PTO.

Whether the increased computational cost of using polychromatic sea states is worthwhile remains an open question, but it has been found that designing a PTO using a control co-design methodology in monochromatic sea states does still improve the performance of the WEC in a more realistic scenario.

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<sup>1</sup>Simulations are run on a computer with a 10-core Apple M4 CPU and 16 GB of RAM.

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## Bibliography

- [1] *HAPiWEC - Holistic Advanced Prototyping & Interfacing for Wave Energy Control*. Last Accessed: 19/7/26. URL: <https://hapiwec.eee.strath.ac.uk> (cited on page 178).
- [2] *HAPiWEC Update: Remote Wave Energy Testing Just Got Even Smarter*. Last Accessed: 19/7/26. URL: <https://hapiwec.eee.strath.ac.uk/2025/05/21/%F0%9F%8C%8A-hapiwec-update-remote-wave-energy-testing-just-got-even-smarter/> (cited on page 178).
- [3] Simon H. Thomas, Jørgen Hals Todalshaug, and John V. Ringwood. “A realistic nonlinear benchmark problem for wave energy controllers - COERbuoy1”. In: *Proceedings of 14th European Wave and Tidal Energy Conference*. 2021 (cited on page 178).
- [4] University of Edinburgh. *FloWave Ocean Energy Research Facility*. Last Accessed: 19/7/26. URL: <https://www.flowavett.co.uk> (cited on page 178).
- [5] Alexandra Price et al. “HAPiGYM: Two Rapid Prototyping Environments for Wave Energy Control”. In: *Proceedings of 15th European Wave and Tidal Energy Conference*. Vol. 15. 2023. DOI: [10.36688/ewtec-2023-499](https://doi.org/10.36688/ewtec-2023-499) (cited on page 179).
- [6] Samuel Draycott. *Wave Energy Scotland NWE2: 20th Scale Standardised Sea States Analysis and Correction*. Tech. rep. FloWave TT Ltd., 2017 (cited on pages 179, 180).
- [7] Samuel Draycott. “On the Re-creation of Site-Specific Directional Wave Conditions”. PhD thesis. University of Edinburgh, 2016 (cited on page 179).
- [8] Pierre Benregui et al. “Wave-to-Wire Model Development and Validation for Two OWC Type Wave Energy Converters”. In: *Energies* 12(20).3977 (2019). DOI: [10.3390/en12203977](https://doi.org/10.3390/en12203977) (cited on page 180).
- [9] Salman Husain et al. “Radiation Force Modeling for a Wave Energy Converter Array”. In: *Energies* 17.6 (2024). DOI: [10.3390/en17010006](https://doi.org/10.3390/en17010006) (cited on page 180).
- [10] Ryan G. Coe, Giorgio Bacelli, and Dominic Forbush. “A practical approach to wave energy modeling and control”. In: *Renewable and Sustainable Energy Reviews* 142 (2021). DOI: [10.1016/j.rser.2021.110791](https://doi.org/10.1016/j.rser.2021.110791) (cited on page 181).

- [11] Adam Stock. “Accounting for power take-off efficiency in optimal velocity tracking control of wave energy converters”. In: *Ocean Engineering* 312 (2024). DOI: [10.1016/j.oceaneng.2024.119191](https://doi.org/10.1016/j.oceaneng.2024.119191) (cited on page 185).
- [12] Zhipeng Zang et al. “Hydrodynamic responses and efficiency analyses of a heaving-buoy wave energy converter with PTO damping in regular and irregular waves”. In: *Renewable Energy* 116 (2018), pp. 527–542. DOI: [10.1016/j.renene.2017.09.057](https://doi.org/10.1016/j.renene.2017.09.057) (cited on page 185).

## CHAPTER VI

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### *Conclusions & Future Work*

The final chapter of the thesis gives some final conclusions of the work: a summary of the findings and discussion of recommendations for wave energy converter design going forward. The chapter ends with example avenues for further work.

## *Conclusions*

To summarise the findings of the thesis in a few sentences: implementing a control co-design (CCD) methodology improves the performance of wave energy converters. Further, the more components included in co-design process, the greater the improvements in electrical power output. Finally, power take-off (PTO) dynamics and efficiencies have a significant effect on the electrical performance of a wave energy converter (WEC), and neglecting such effects disguises the actual performance of the WEC.

### SUMMARY OF KEY FINDINGS

The novel application of a lumped parameter model of a permanent magnet synchronous generator (PMSG), originally developed by Cassimere [1], and included in a wider drive train model, created by Gasberger [2], has created a PTO model at a much higher fidelity than has been seen in the CCD literature to date.

When used within a genetic algorithm, designs are consistently produced of a similar electrical performance, showing that the optimiser is able to effectively explore the design space: providing a strong foundation for implementing CCD methodologies.

There is variance in the physical design of final PMSG designs, showing there is a degree of freedom in the cost function allowing multiple PMSG designs to return the same cost. The greater degree of freedom in the cost function opens the opportunity for the WEC designer to have a greater influence on the desired PMSG design by altering the cost function to tighten the design constraints, as is found in § IV.3.2.1. Conversely, § III.4 has shown that loosening design constraints can create viable designs in conditions previously thought impossible.

In § III, significant differences in optimal PTO designs are found when varying control type, floater design, and location. Broadly speaking, WECs that are more able to perform well across a wide range of frequencies – *e.g.* when using impedance matching control, or a broad-banded floater design – are able to operate more effectively and can utilise the capacity of larger PMSG designs to counteract their increased cost. Conversely, if it is known that linear damping control will be used, an optimal generator can be produced at around 60% the material cost of one designed for impedance matching control with only a 5% drop in average electrical power output. It is also shown that WECs placed in more energetic sites are able to utilise larger PMSGs.

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In §IV, the optimiser model is expanded to co-optimize the novel combination of floater and PTO design, by introducing the Froude scaling of the floater. It is found that the design dependence between floater and PTO works both ways, and so the simultaneous co-design of the two components leads to greater performance over individual design processes – with an up to 90% improvement in electrical power output with IM control, and 50% improvement with LD control. This adds to the knowledge of the importance of CCD methodologies in WEC design. When combined with previous work by Garcia-Rosa and Ringwood [3] who found that the optimal floater design was dependant on the control type, and work by Garcia-Teruel [4] who found the optimal floater design depended on the wave climate; there is a strong interdependency between all of the components of a WEC.

Finally, in §V it is found that using the idealised monochromatic sea states in the optimisation leads to an over estimate of the power output when compared to polychromatic sea states. However, the optimal design still outperforms a non-optimal design in the polychromatic tests: showing that the model is capturing an intrinsic property of the waves, and allowing for the optimisation of the PTO.

To conclude, it is clear that the implementation of CCD methodologies is advisable for future WEC design processes, and PTO efficiency must be accounted for in all models of WECs to give a clearer view of the true performance of a WEC.

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## *Further Work*

This thesis clearly identifies avenues for future work that improve the levelised cost of energy of wave energy converter (WEC) designs. Any alteration to the optimiser to improve accuracy must be implemented carefully to avoid increasing the runtime of the model further. Hence, improvements to the computational efficiency of the optimiser model must accompany any additional complexity to prevent the model becoming overly cumbersome. The main computational sink is currently in the impedance matching optimisation technique, due to the fallback brute forcing of the design space required when overheating limits are exceeded. Investigations into constrained optimisation techniques are recommended to more quickly find the optimal solution in cases that currently require the brute-force method.

Potential additions to the model are discussed below and all focus on increasing the fidelity of particular components of the optimisation model.

**POLYCHROMATIC WAVES AND NON-LINEAR MODELLING:** In § V.2, an optimised power take-off (PTO) is shown to outperform a non-optimised PTO, demonstrating that the monochromatic approximation is harnessing an intrinsic property of the waves. However, the polychromatic simulation shows a significant overestimation in the electrical power when the monochromatic approximation is used in the optimisation. It is therefore likely that incorporating polychromatic power simulations into the optimisation would improve overall generator design. Computational limits have meant polychromatic simulations could not be included in this work, but future improvements in available computation resources, or improvements in the efficiency of WEC modelling could open the possibility of including them in the future.

Similarly, it is known that WECs have significant non-linear dynamics [5], but can be modelled as a linear process under certain conditions [6]; this is analogous to using monochromatic waves to estimate polychromatic waves. So, similarly, it is likely that incorporating non-linear WEC models into the optimiser would improve the overall PTO design.

**DRIVE-TRAIN DESIGN:** Future work could use many different types of drive train depending on the required usecase. The drive train model is based on a series of work by Grasberger [2], Michelén Ströfer *et al.* [7], Michelén Ströfer *et al.* [8] and Grasberger

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*et al.* [9]. In these works the drive trains were modelled as using a series of pulleys in [7], a belt transmission mechanism in [8]; and was applied to a terminator-type WEC in [2, 9]. As such, future work could explore the use of drive trains outwith the rack-and-pinion paradigm used here. Moving away from a rack-and-pinion design presents an opportunity to model power extraction from multiple degrees of freedom.

**FLOATER DESIGN:** One of the key discoveries of the thesis is the interdependency in optimal design of the PTO and floater, which is found in a simplified form of floater optimisation via scaling of reference designs. Implementing a more advanced form of floater co-design – for example those described in § IV.1 – would therefore likely lead to better performing WECS as the optimiser has more freedom to tune the floater design to the given sea conditions.

**COST FUNCTION:** In § III.4 and § IV.3.2.1, it is shown that altering the cost function allows better define of the design space. The formulation of the cost function for this work is limited by a lack of availability of information regarding both capital expenditure, and operation and maintenance costs. However, as the industry becomes more mature, and more is known about the day-to-day operations of a WEC the cost function could be expanded to better account for these factors.

Crucially, any alterations to the cost function should also include a re-investigation of the required number of sea states as different components of the cost function may be more sensitive to the sea-state representation.

**WEC TYPE AND ARRAYS:** More drastic changes could be made to allow the modelling of different types of WEC – for example attenuators or terminators. Arrays of WECS could also be considered. Such alterations would likely require a complete re-design of the power simulation and control optimisation stages, especially if a global controller that controls all WECS in an array is included (such as that developed in [10]).

**DESIGNING FOR MULTIPLE SITES:** In reality, designing a WEC from the ground up for each and every location would cost an immense amount of money when compared to placing the same WEC at many locations. This is due to economies of scale.

Setting up production lines to build many WECS, and learning from design iterations is a way to bring down the cost of a product. However, such an approach is contrary to the findings of the rest of the thesis: in that the most effective WECS are those finely tuned to the given location. A compromise could be the design of regional WECS – *i.e.* there is one WEC design for all across the North Sea, one for the Mediterranean, *etc.* – to combine the benefits of control co-design and economies of scale.

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## Bibliography

- [1] Brandon Cassimere. “Modeling and Evolutionary Design of Permanent Magnet Synchronous Machines”. PhD thesis. Purdue University, May 2008 (cited on page 190).
- [2] Jeff Grasberger. “Co-design Investigation and Optimization of an Oscillating-Surge Wave Energy Converter”. MA thesis. Virginia Polytechnic Institute and State University, 2022 (cited on pages 190, 192, 193).
- [3] Paula B. Garcia-Rosa and John V. Ringwood. “On the Sensitivity of Optimal Wave Energy Device Geometry to the Energy Maximizing Control System”. In: *IEEE Transactions on Sustainable Energy* 7.1 (2016). DOI: [10.1109/TSTE.2015.2423551](https://doi.org/10.1109/TSTE.2015.2423551) (cited on page 191).
- [4] Anna Garcia-Teruel. “Geometry Optimisation of Wave Energy Converters”. PhD thesis. University of Edinburgh, 2020 (cited on page 191).
- [5] C.J. Fitzgerald. “Numerical Modelling of Wave Energy Converters”. In: ed. by Matt Folley. Academic Press, 2016. Chap. 5.1. ISBN: 9780128032107 (cited on page 192).
- [6] M. Alves. “Numerical Modelling of Wave Energy Converters”. In: ed. by Matt Folley. Academic Press, 2016. Chap. 2.1. ISBN: 9780128032107 (cited on page 192).
- [7] Carlos A. Michelén Ströfer et al. “Control co-design and sensitivity analysis of the LUPA’s PTO using WecOptTool”. In: *Proceedings of 15th European Wave and Tidal Energy Conference*. 2023. DOI: [10.36688/ewtec-2023-288](https://doi.org/10.36688/ewtec-2023-288) (cited on pages 192, 193).
- [8] Carlos A. Michelén Ströfer et al. “Control Co-Design of Power Take-Off Systems for Wave Energy Converters Using WecOptTool”. In: *IEEE Transactions on Sustainable Energy* 14.4 (2023). DOI: [10.1109/TSTE.2023.3272868](https://doi.org/10.1109/TSTE.2023.3272868) (cited on pages 192, 193).
- [9] Jeff Grasberger et al. “Control co-design and optimization of oscillating-surge wave energy converter”. In: *Renewable Energy* 225.120234 (Feb. 2024). DOI: [10.1016/j.renene.2024.120234](https://doi.org/10.1016/j.renene.2024.120234) (cited on page 193).

- [10] Paula B. Garcia-Rosa, Giorgio Bacelli, and John V. Ringwood. “Control-Informed Optimal Array Layout for Wave Farms”. In: *IEEE Transactions on Sustainable Energy* 6.2 (2015), pp. 575–582. DOI: [10.1109/TSTE.2015.2394750](https://doi.org/10.1109/TSTE.2015.2394750) (cited on page 193).

# APPENDIX

## *Wave Data*

This appendix contains a record of the historical wave data used within the thesis. Three locations are used throughout the thesis: Tyne-Tees [1], Liverpool Bay [2], and the Isles of Scilly [3]; this gives an example of a medium-, low-, and high-energy location respectively.

All data comes from WaveNet [4], which gives a peak period, and significant wave height record every 30 minutes. TABLE A.1 gives a list of the locations, the time spans, and amount of data available. Time spans were chosen to start and end on the same date, to not have an overrepresentation of any season in the data; sites are also selected with the longest timespan of available data to reduce the effect a single abnormal season can have on the data.

For the power simulations, the locations have been reduced to a representative sample of sea states for each location; these are found using a  $k$ -means clustering algorithm, as described in §II.3.2.2. The Tyne-Tees site is ran in a range of samples: one, five, 10, 20, and 40; the values of the sea states used are found in TABLES A.2, A.3, A.4, A.5, and A.6 respectively. For the Liverpool Bay and Isles of Scilly sites, only a 20 sea state representation is used, these seas are found in TABLES A.7, and A.8 respectively.

TABLE A.1: List of locations and available data.

| LOCATION        | START DATE <sup>*</sup> | END DATE <sup>**</sup> | Available Data <sup>†</sup> / % | REF. |
|-----------------|-------------------------|------------------------|---------------------------------|------|
| Tyne-Tees       | 09/04/2009              | 08/04/2020             | 99.72                           | [1]  |
| Liverpool Bay   | 06/08/2009              | 05/08/2017             | 93.63 <sup>‡</sup>              | [2]  |
| Isles of Scilly | 12/10/2014              | 11/10/2019             | 99.97                           | [3]  |

<sup>\*</sup> The first data point is at 00:00:00.

<sup>\*\*</sup> The final data point is at 23:30:00.

<sup>†</sup> There are sometimes missing data points: this is the percentage of data available.

<sup>‡</sup> The longest span of intermittent data (missing data due to events such as repairs taking place) is less than one week.

TABLE A.2: One sea state representation of the Tyne-Tees site.

| $T_p$ / s | $H_{m0}$ / m | WEIGHT / % |
|-----------|--------------|------------|
| 7.38      | 1.35         | 100        |

TABLE A.3: Five sea state representation of the Tyne-Tees site.

| $T_p$ / s | $H_{m0}$ / m | WEIGHT / % |
|-----------|--------------|------------|
| 14.7      | 1.32         | 4.23       |
| 10.9      | 1.70         | 20.5       |
| 8.36      | 1.67         | 19.4       |
| 6.24      | 1.35         | 27.0       |
| 4.15      | 0.87         | 28.6       |

TABLE A.4: 10 sea state representation of the Tyne-Tees site.

| $T_p$ / s | $H_{m0}$ / m | WEIGHT / % |
|-----------|--------------|------------|
| 17.6      | 0.906        | 0.775      |
| 14.0      | 1.38         | 3.43       |
| 12.1      | 1.39         | 5.41       |
| 10.5      | 1.20         | 11.5       |
| 10.2      | 3.72         | 4.99       |
| 8.33      | 1.52         | 17.9       |
| 6.73      | 1.42         | 15.4       |
| 5.52      | 1.28         | 14.3       |
| 4.48      | 0.955        | 15.6       |
| 3.41      | 0.650        | 10.4       |

TABLE A.5: 20 sea state representation of the Tyne-Tees site.

| $T_p$ / s | $H_{m0}$ / m | WEIGHT / % |
|-----------|--------------|------------|
| 18.7      | 0.888        | 0.336      |
| 15.2      | 0.925        | 1.86       |
| 13.2      | 2.82         | 1.46       |
| 12.8      | 1.08         | 3.12       |
| 11.5      | 1.68         | 2.46       |
| 11.4      | 0.810        | 3.36       |
| 10.9      | 2.77         | 2.01       |
| 10.5      | 4.64         | 1.77       |
| 10.2      | 1.04         | 7.14       |
| 9.46      | 2.68         | 3.60       |
| 9.10      | 1.06         | 4.10       |
| 8.01      | 2.87         | 2.86       |
| 8.01      | 0.850        | 4.98       |
| 7.98      | 1.75         | 5.12       |
| 6.72      | 2.11         | 5.59       |
| 6.71      | 1.03         | 9.94       |
| 5.47      | 0.811        | 9.12       |
| 5.36      | 1.70         | 7.60       |
| 4.37      | 0.948        | 13.5       |
| 3.39      | 0.643        | 9.95       |

TABLE A.6: 40 sea state representation of the Tyne-Tees site.

| $T_p$ / s | $H_{m0}$ / m | WEIGHT / % | $T_p$ / s | $H_{m0}$ / m | WEIGHT / % |
|-----------|--------------|------------|-----------|--------------|------------|
| 17.6      | 0.879        | 0.768      | 9.10      | 1.63         | 2.10       |
| 14.8      | 0.660        | 1.09       | 9.10      | 0.730        | 2.43       |
| 14.7      | 3.72         | 0.141      | 8.90      | 3.84         | 0.787      |
| 14.5      | 2.00         | 0.500      | 8.04      | 2.87         | 2.08       |
| 13.3      | 0.888        | 1.03       | 8.03      | 0.642        | 2.91       |
| 13.0      | 2.26         | 1.06       | 7.99      | 2.02         | 3.37       |
| 12.5      | 0.812        | 1.21       | 7.97      | 1.31         | 4.28       |
| 12.3      | 3.62         | 0.620      | 6.95      | 0.996        | 4.28       |
| 12.1      | 1.60         | 1.62       | 6.92      | 1.69         | 3.96       |
| 11.8      | 0.810        | 1.49       | 6.75      | 2.56         | 2.14       |
| 11.5      | 5.71         | 0.302      | 6.44      | 0.727        | 2.86       |
| 11.3      | 2.30         | 1.35       | 6.03      | 1.39         | 4.32       |
| 11.1      | 0.984        | 2.59       | 5.71      | 2.07         | 2.91       |
| 10.5      | 3.71         | 1.00       | 5.58      | 0.746        | 5.78       |
| 10.3      | 4.77         | 0.728      | 5.02      | 1.50         | 5.10       |
| 10.2      | 1.22         | 2.98       | 4.90      | 0.742        | 3.70       |
| 10.2      | 0.638        | 3.11       | 4.30      | 1.17         | 6.18       |
| 10.2      | 1.93         | 2.01       | 4.29      | 0.582        | 3.65       |
| 10.2      | 2.80         | 1.19       | 3.70      | 0.772        | 6.71       |
| 9.10      | 2.64         | 1.29       | 3.02      | 0.515        | 4.18       |

TABLE A.7: 20 sea state representation of the Liverpool Bay site.

| $T_p$ / s | $H_{m0}$ / m | WEIGHT / % |
|-----------|--------------|------------|
| 14.3      | 0.307        | 0.246      |
| 11.8      | 0.345        | 0.397      |
| 9.79      | 0.341        | 0.461      |
| 8.19      | 3.17         | 1.69       |
| 7.34      | 0.369        | 1.22       |
| 7.04      | 2.38         | 3.84       |
| 6.51      | 1.63         | 3.49       |
| 6.38      | 0.370        | 2.44       |
| 6.11      | 2.04         | 3.02       |
| 5.62      | 1.47         | 7.24       |
| 5.47      | 0.939        | 5.24       |
| 4.93      | 0.635        | 4.87       |
| 4.90      | 1.18         | 5.84       |
| 4.36      | 0.946        | 7.84       |
| 4.23      | 0.493        | 7.86       |
| 3.81      | 0.839        | 6.31       |
| 3.70      | 0.464        | 6.46       |
| 3.25      | 0.542        | 10.3       |
| 2.80      | 0.441        | 9.52       |
| 2.21      | 0.335        | 5.38       |

TABLE A.8: 20 sea state representation of the Isles of Scilly site.

| $T_p$ / s | $H_{m0}$ / m | WEIGHT / % |
|-----------|--------------|------------|
| 18.7      | 2.45         | 0.500      |
| 17.4      | 5.77         | 1.64       |
| 15.9      | 2.63         | 2.32       |
| 14.7      | 4.50         | 5.17       |
| 14.7      | 7.47         | 1.56       |
| 13.7      | 2.20         | 5.56       |
| 12.9      | 3.57         | 6.66       |
| 12.6      | 5.32         | 4.02       |
| 12.1      | 1.93         | 9.51       |
| 11.3      | 3.55         | 6.59       |
| 10.6      | 2.32         | 9.48       |
| 10.5      | 5.19         | 2.56       |
| 10.4      | 1.33         | 10.9       |
| 9.51      | 3.35         | 3.92       |
| 9.10      | 1.42         | 5.42       |
| 8.01      | 2.19         | 3.59       |
| 7.97      | 1.04         | 5.15       |
| 7.86      | 3.34         | 3.26       |
| 6.66      | 1.83         | 7.36       |
| 5.23      | 1.20         | 4.80       |

## *Floater Data*

This appendix contains a record of the hydrodynamic data for the floater designs used within the thesis. The floaters come from the IMPACT toolbox [5], from here two floaters are used, from WEC 1, and WEC 2; for the ease of readability, in this thesis they have been named TYPE-N, and TYPE-B respectively, due to their narrow- and broad-banded frequency responses. The hydrostatic and hydrodynamic data was found using the boundary element method code NEMOH (this was completed as part of the IMPACT project [5]).

As a part of §IV, these floaters are Froude scaled, it is important to note that the data in this appendix is for the reference, nominal size. For both the floaters, the water density was assumed to be  $1025 \text{ kg/m}^3$ . The hydrostatic data can be found in TABLE B.1. The hydrodynamic data can be found in FIGURES B.1, B.2, and B.3, for the added mass, radiation damping, and excitation force coefficients respectively.

TABLE B.1: Hydrostatic data for the two reference floaters.

| FLOATER | MASS / kg | RESTORING FORCE / N |
|---------|-----------|---------------------|
| TYPE-N  | 7,705,900 | 3,157,352           |
| TYPE-B  | 730,000   | 0                   |

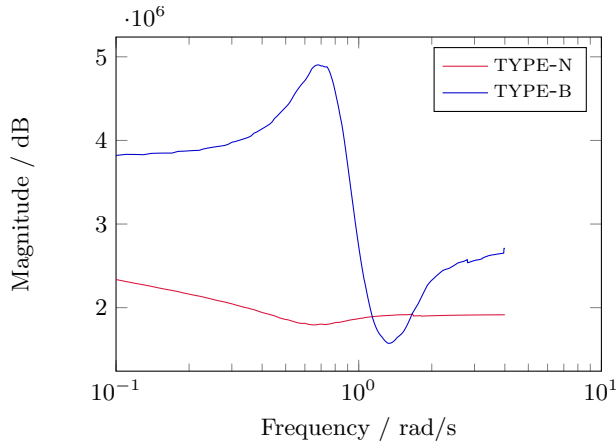


FIGURE B.1: Added mass in heave for the two reference floaters.

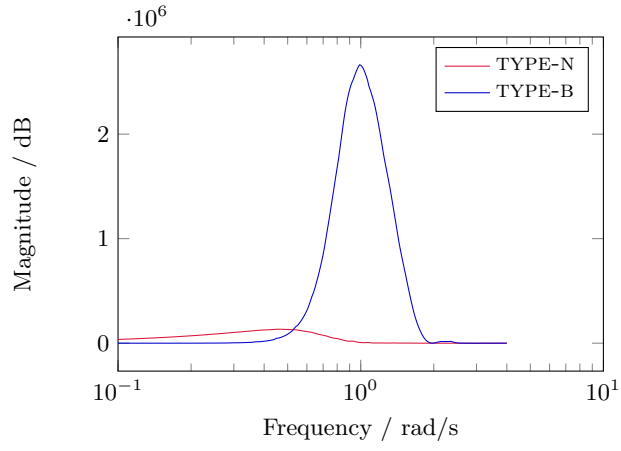


FIGURE B.2: Radiation damping in heave for the two reference floaters.

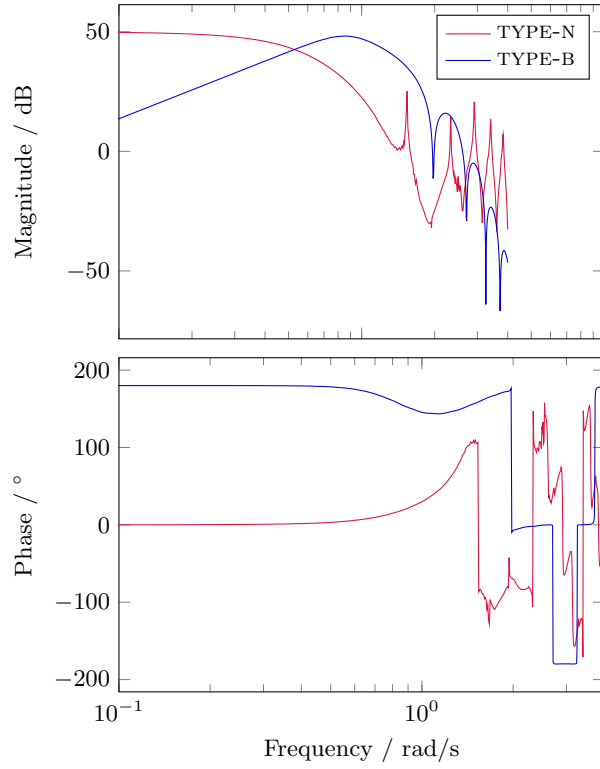


FIGURE B.3: Excitation force coefficients for the two reference floaters.

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## *Bibliography*

- [1] Centre for Environment Fisheries and Aquaculture Science. *Tyne/Tees WaveNet Site*. Last Accessed: 19/7/26. URL: <https://wavenet.cefasc.co.uk/details/TYNETEESWN/INT> (cited on page 197).
- [2] Centre for Environment Fisheries and Aquaculture Science. *Liverpool Bay WaveNet Site*. Last Accessed: 19/7/26. URL: <https://wavenet.cefasc.co.uk/details/LIVBAYWN/INT> (cited on page 197).
- [3] Centre for Environment Fisheries and Aquaculture Science. *SW Isles of Scilly WaveNet Site*. Last Accessed: 19/7/26. URL: <https://wavenet.cefasc.co.uk/details/SWSCILLYWN/INT> (cited on page 197).
- [4] Centre for Environment Fisheries and Aquaculture Science. *WaveNet*. Last Accessed: 19/7/26. URL: <https://wavenet.cefasc.co.uk> (cited on page 197).
- [5] Adam Stock, Carlos Gonzalez, and David Robb. *Integrated marine point absorber control tool final stage 2 project report*. Tech. rep. SgurrControl Ltd., 2018 (cited on page 201).